
Realistic Image Synthesis

- Progressive Photon Mapping &
Vertex Connection and Merging -

Philipp Slusallek

Karol Myszkowski

Gurprit Singh

Corentin Salaun

Overview

- **Today**

- Progressive Photon Mapping (PPM)
 - Basic Approach
 - Probabilistic Approach
- Combining Photon Mapping and Bidirectional Path Tracing
 - Vertex connection and merging

- **Next lecture**

- Radar / Spectral

Reading Materials:

- HACHISUKA, T., OGAKI, S., AND JENSEN, H. W. 2008. Progressive photon mapping. *ACM Trans. Graph.* 27, 5
- HACHISUKA, T. AND JENSEN, H. W. 2009. Stochastic progressive photon mapping. *ACM Trans. Graph.* 28, 5.
- HACHISUKA, T., OGAKI, S., AND JENSEN, H. W. 2008. Progressive photon mapping. *ACM Trans. Graph.* 27, 5
- Knaus, C., and Zwicker, M. 2011. Progressive Photon Mapping: A Probabilistic Approach. *ACM Trans. Graph.* 30.
- Iliyan Georgiev, Jaroslav Křivánek, Tomáš Davidovič, and Philipp Slusallek. 2012. Light transport simulation with vertex connection and merging. *ACM Trans. Graph.* 31, 6.

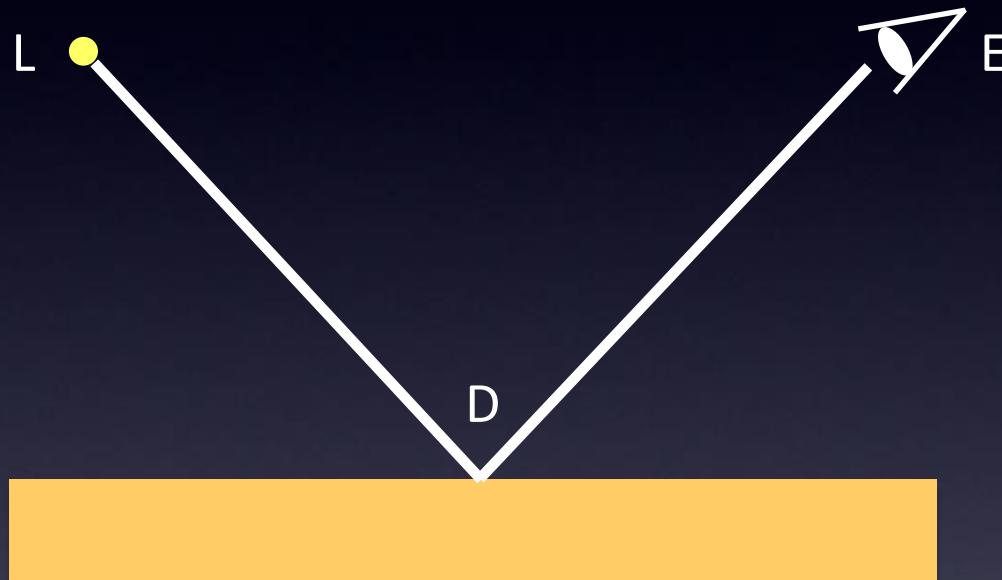
Progressive Photon Mapping

Toshiya Hachisuka* Shinji Ogaki† Henrik Wann Jensen*

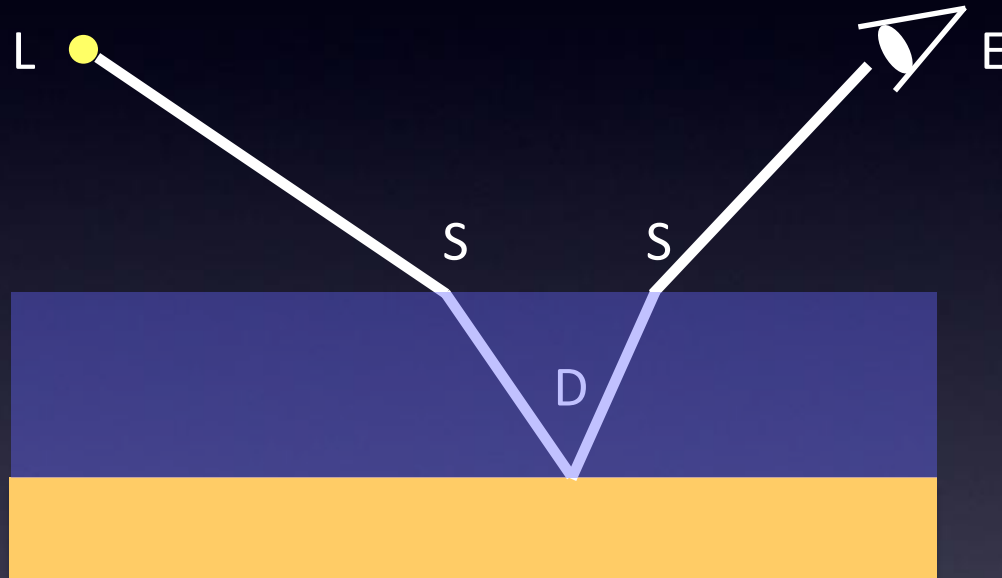
*University of California, San Diego

†University of Nottingham

LDE Path



LSDSE Path



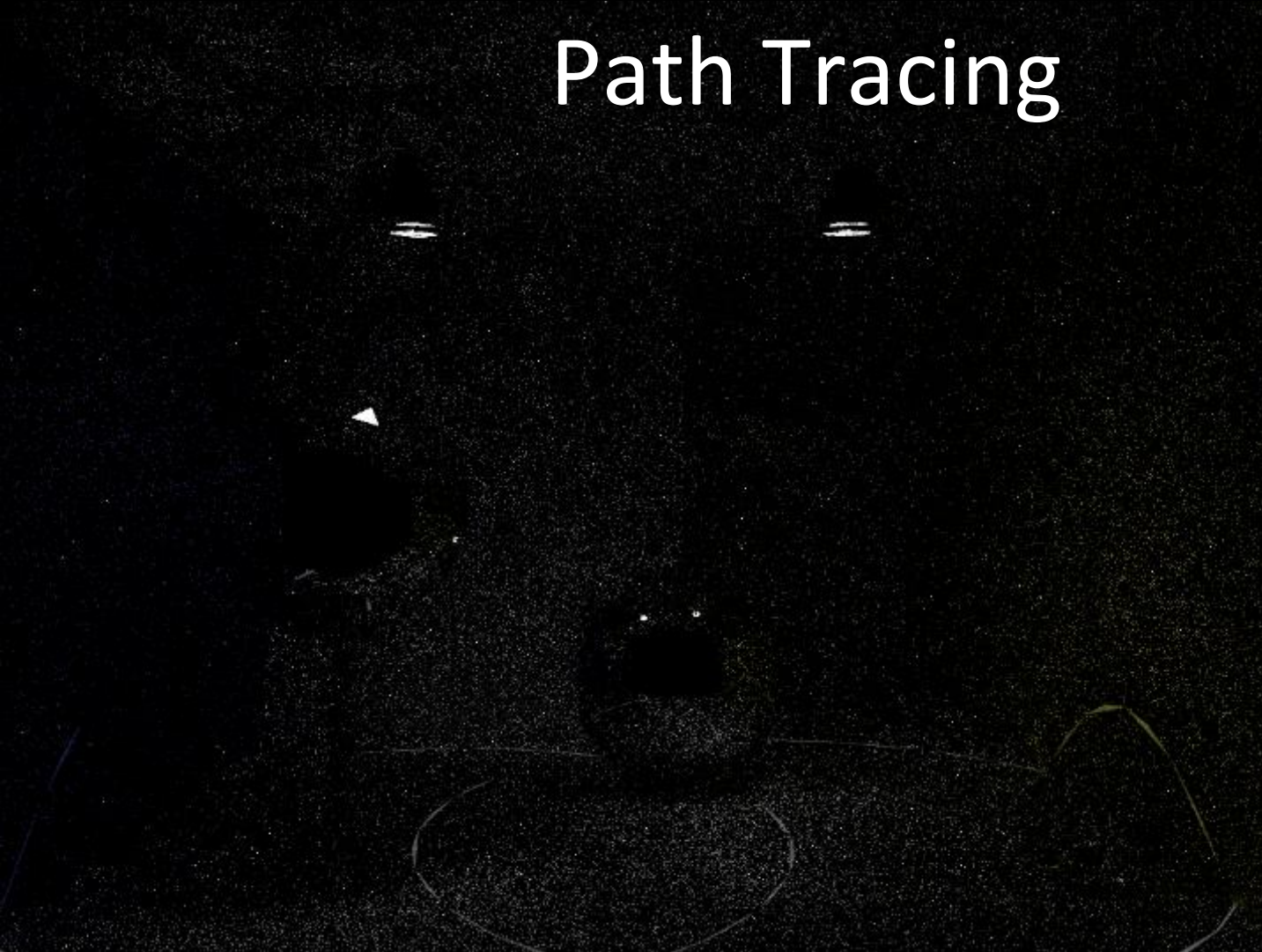




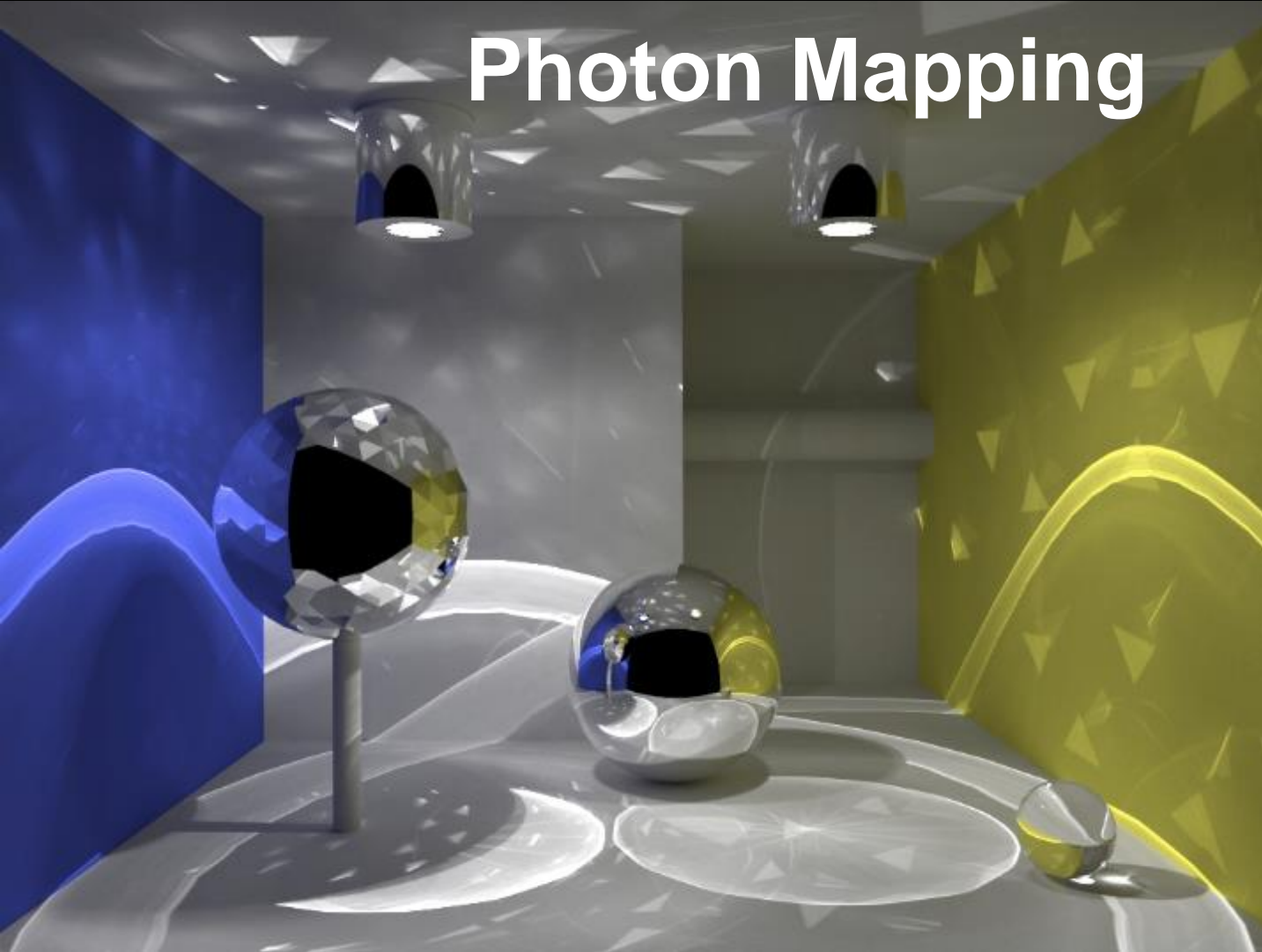




Path Tracing

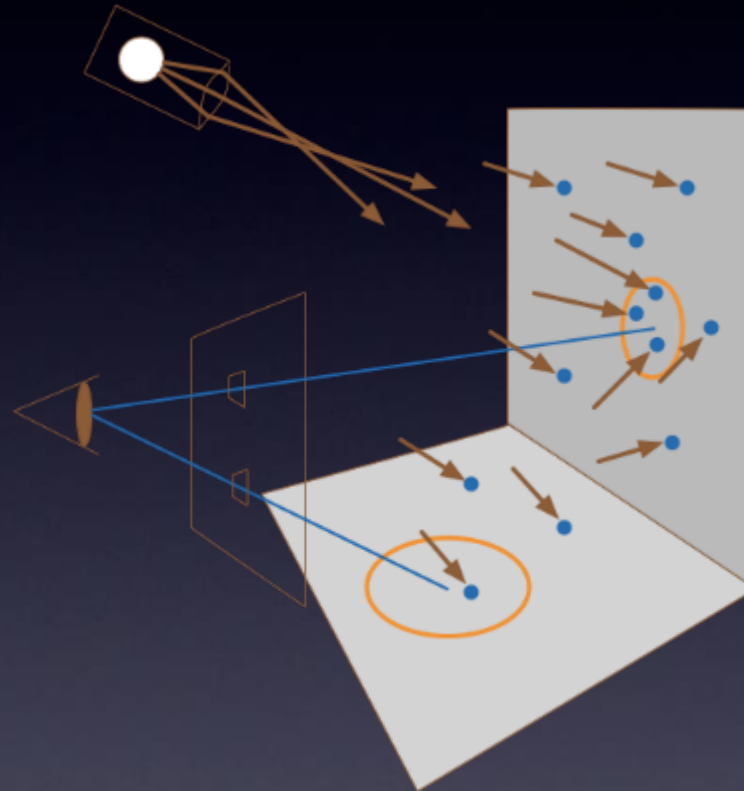
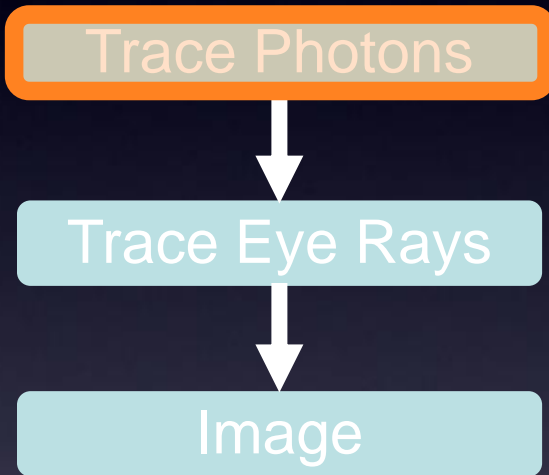


Photon Mapping

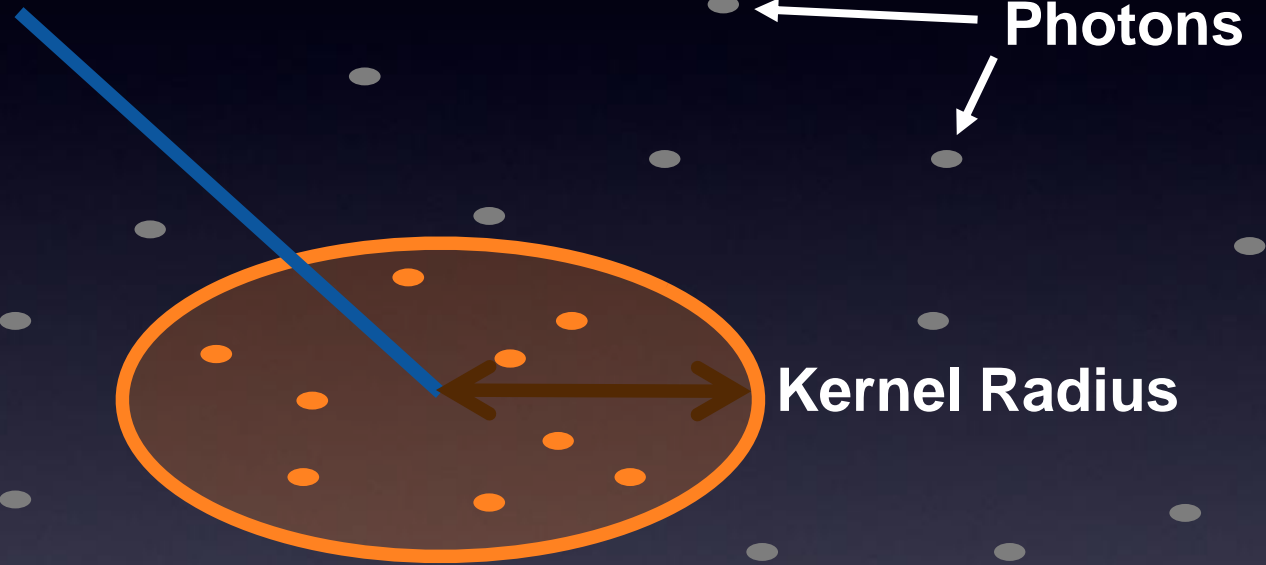


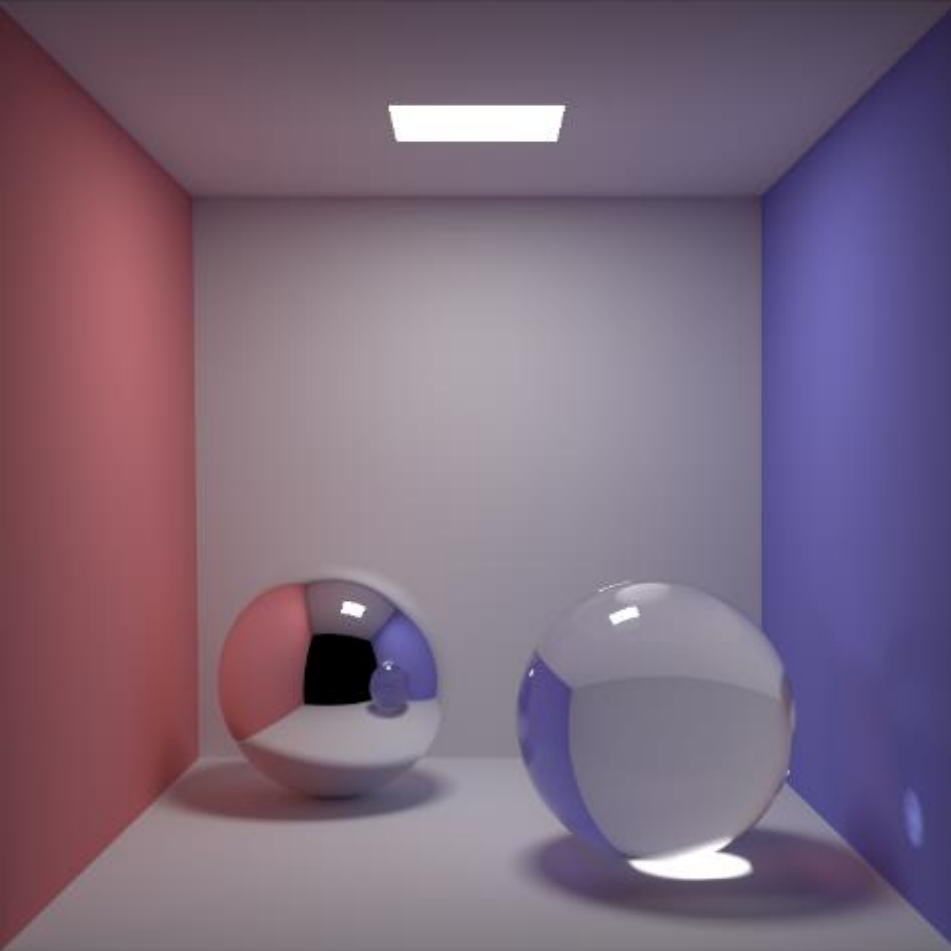
Scene courtesy of
Toshiya Hachisuka

Photon Mapping

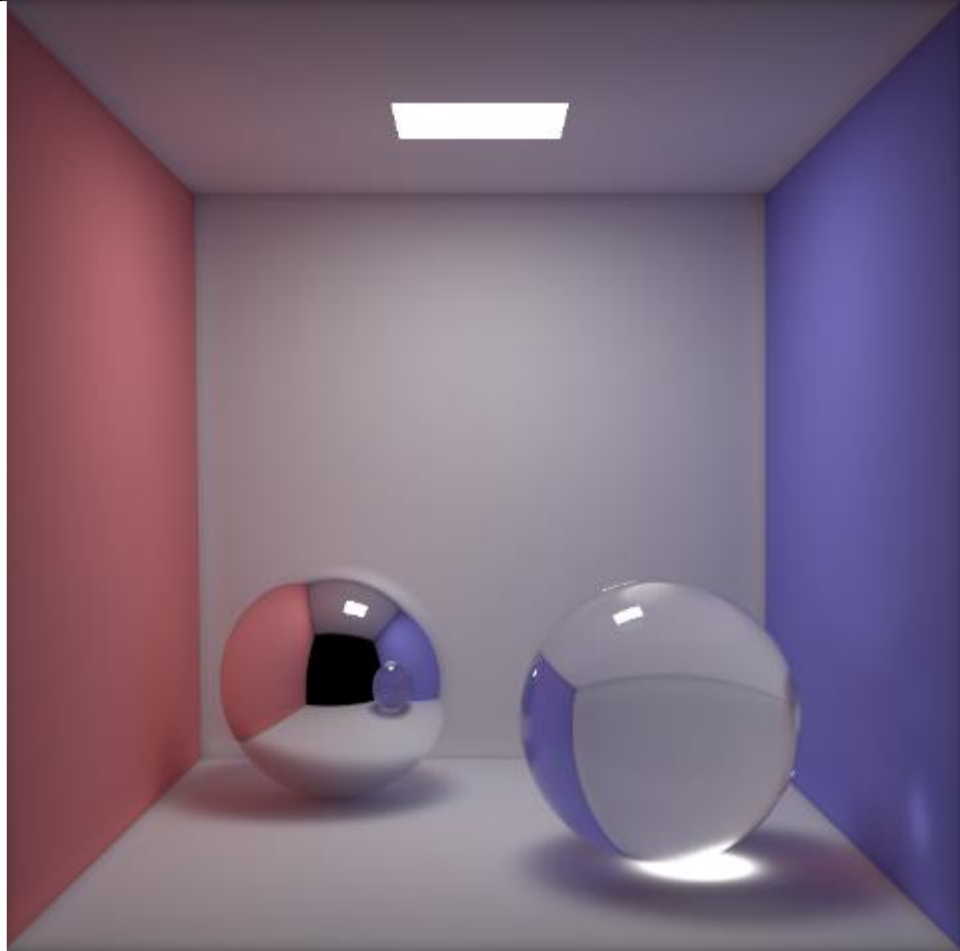


Radiance Estimation

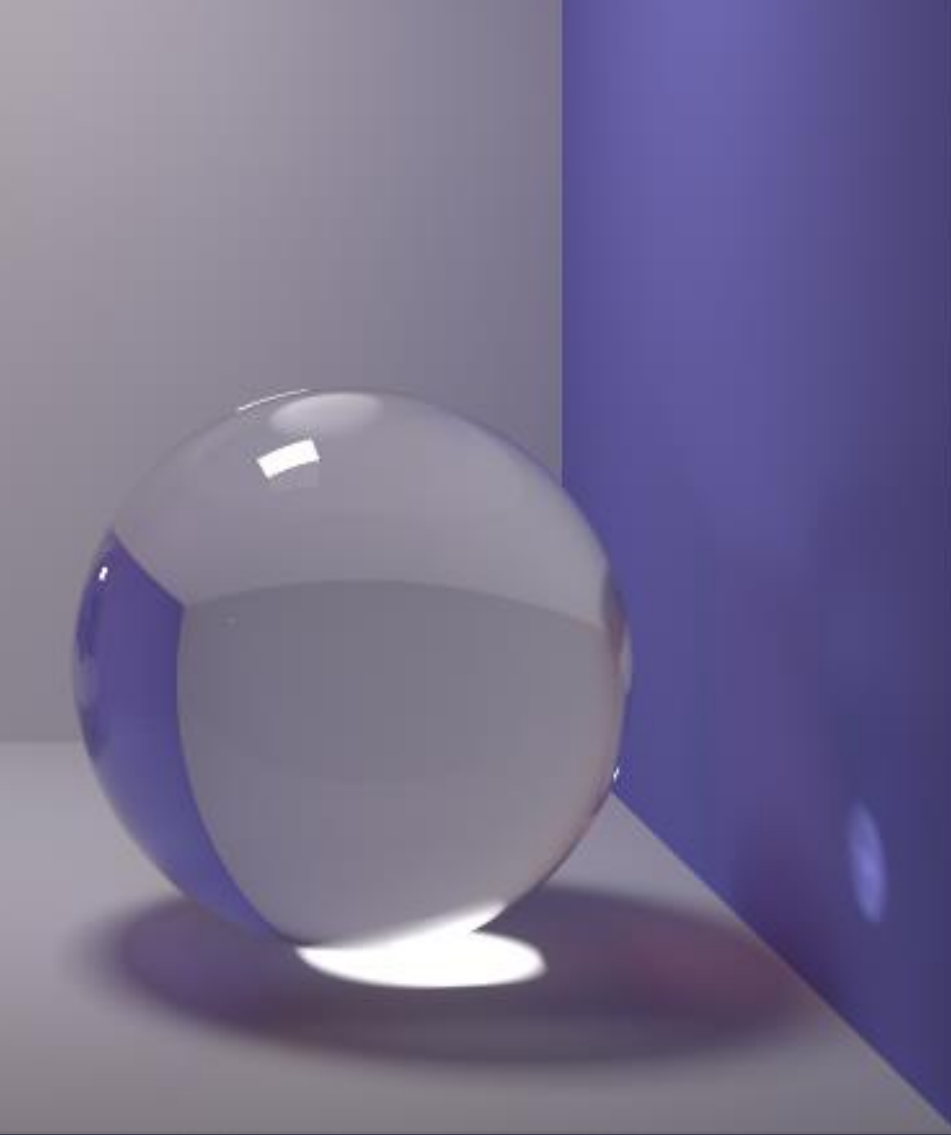




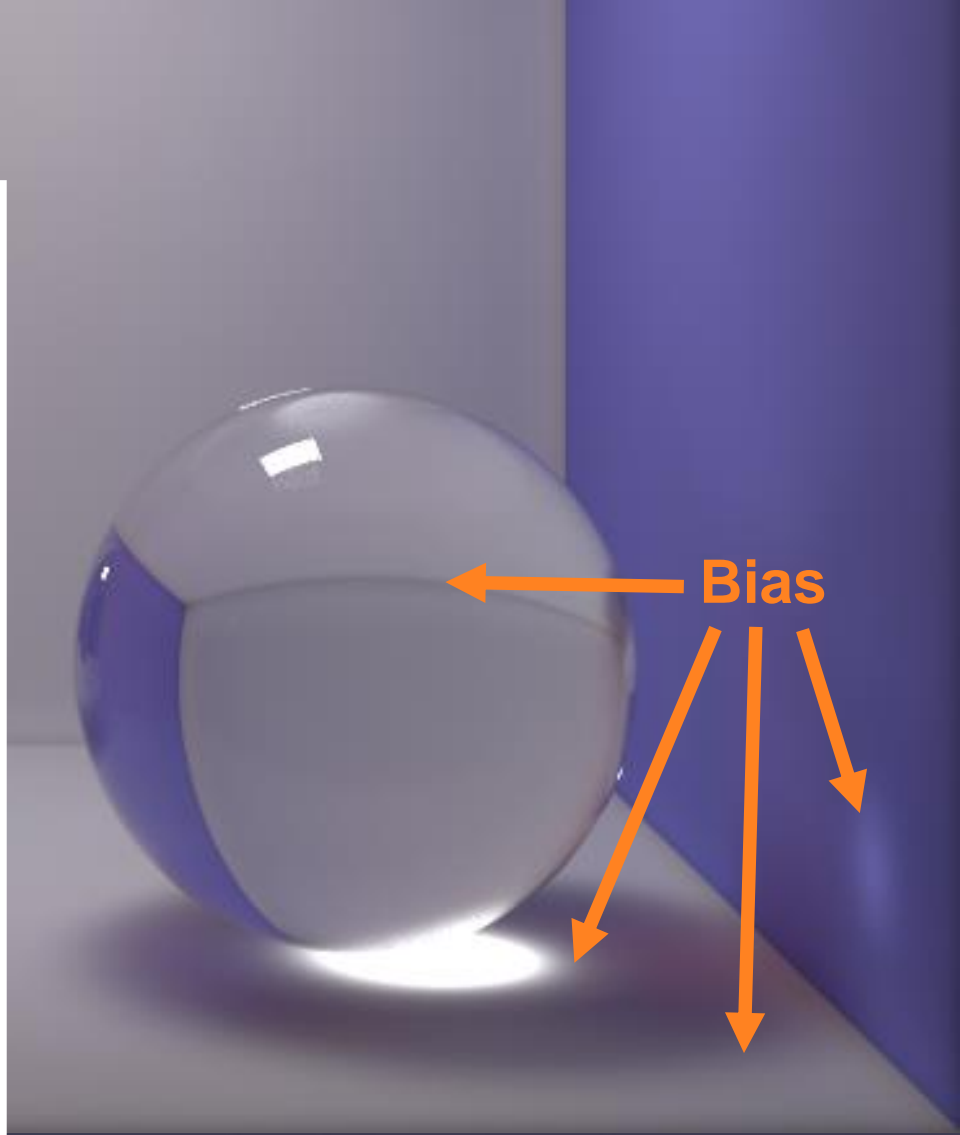
Ground Truth



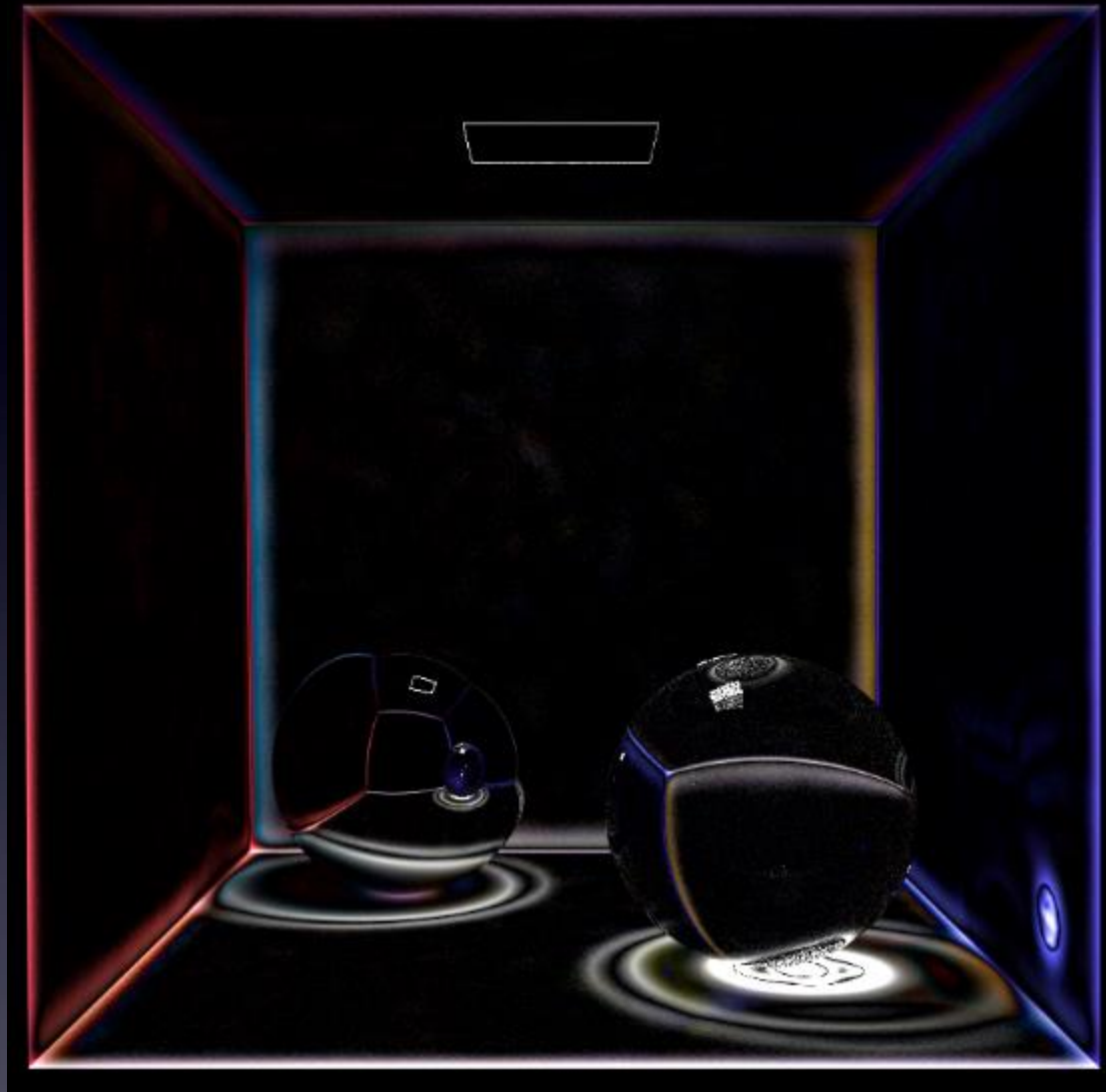
Photon Mapping



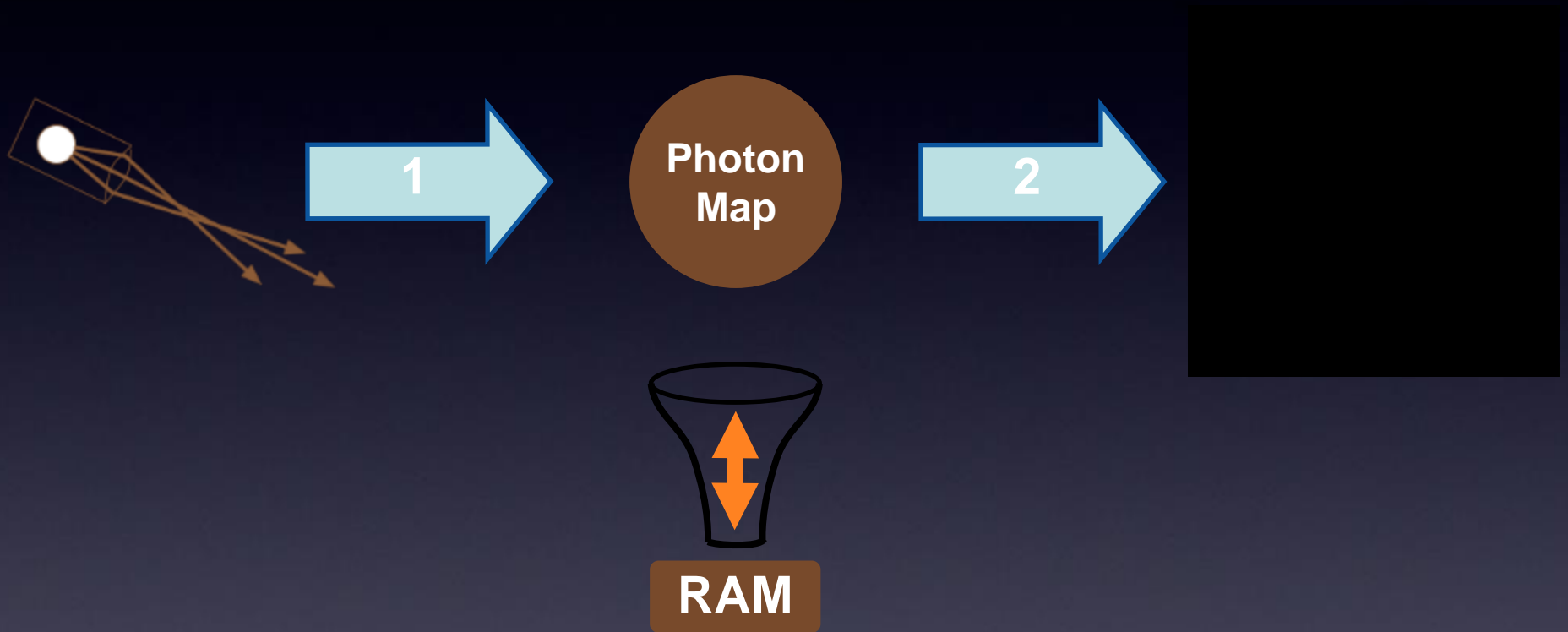
Ground Truth



Photon Mapping



Memory Bottleneck



Progressive Photon Mapping

Hachisuka et al. (2008)

Progressive Photon Mapping

Toshiya Hachisuka
UC San Diego

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The University of Nottingham

Henrik Wann Jensen
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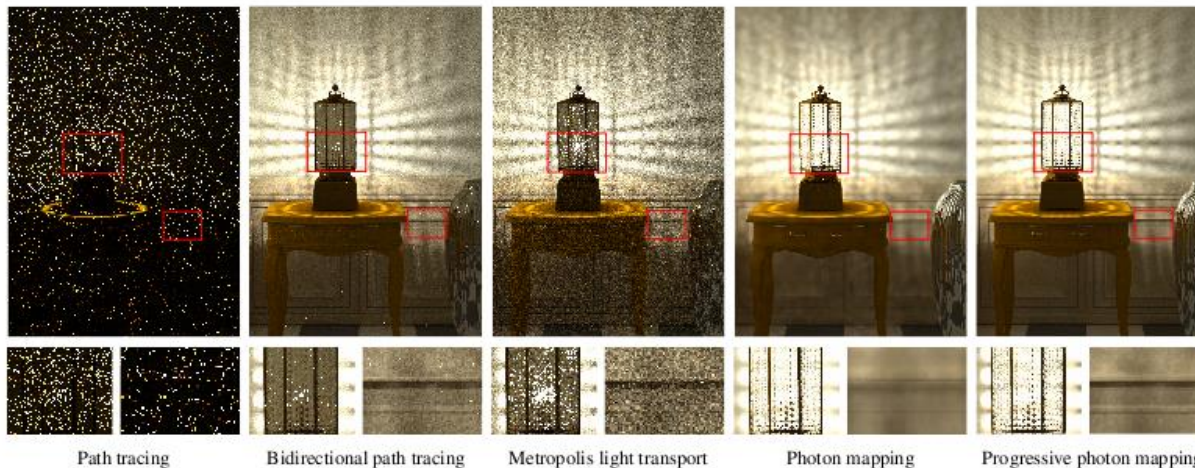


Figure 1: A glass lamp illuminates a wall and generates a complex caustics lighting pattern on the wall. This type of illumination is difficult to simulate with Monte Carlo ray tracing methods such as path tracing, bidirectional path tracing, and Metropolis light transport. The lighting seen through the lamp is particularly difficult for these methods. Photon mapping is significantly better at capturing the caustics lighting seen through the lamp, but the final quality is limited by the memory available for the photon map and it lacks the fine detail in the illumination. Progressive photon mapping provides an image with substantially less noise in the same render time as the Monte Carlo ray tracing methods and the final quality is not limited by the available memory.

Progressive Photon Mapping

First algorithm for computing *all* types of light transport with arbitrary accuracy

Progressive Photon Mapping

- New formulation of photon mapping
 - Robust for *any* light path including SDS path
 - Arbitrary accuracy using finite memory
 - New progressive radiance estimation algorithm
 - Easy to implement

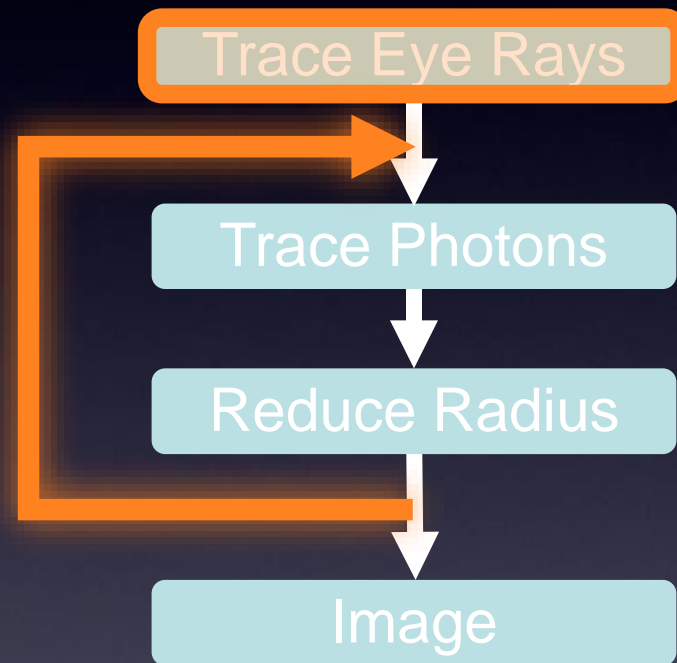
Progressive Photon Mapping

- Multi-pass method
 - Initial pass:
 - points generation for radiance estimates
 - Refinement pass:
 - photon tracing
 - progressive radiance estimate

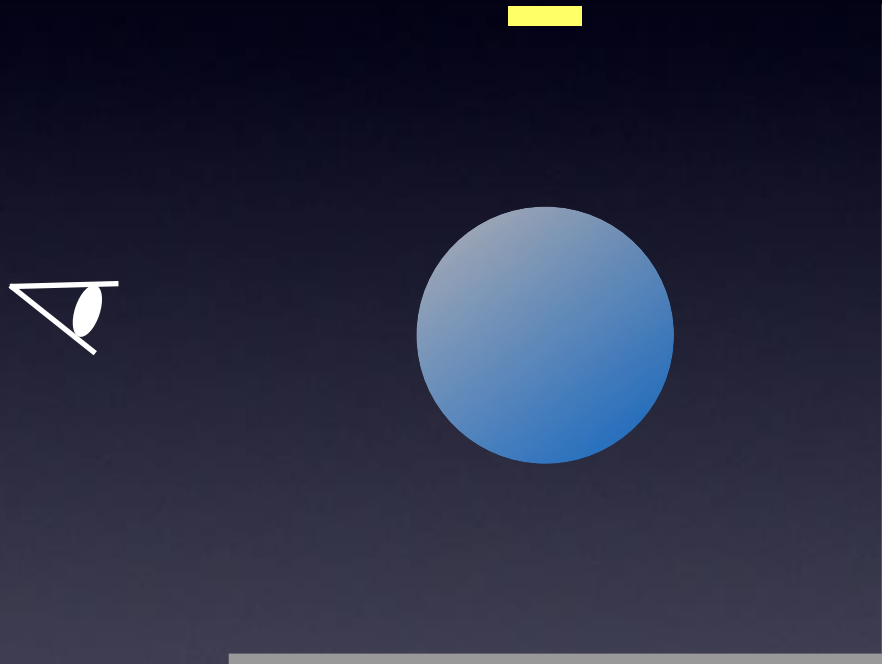
Key Idea

- Progressive radiance estimation
 - New density estimation algorithm
 - Converges to the correct value

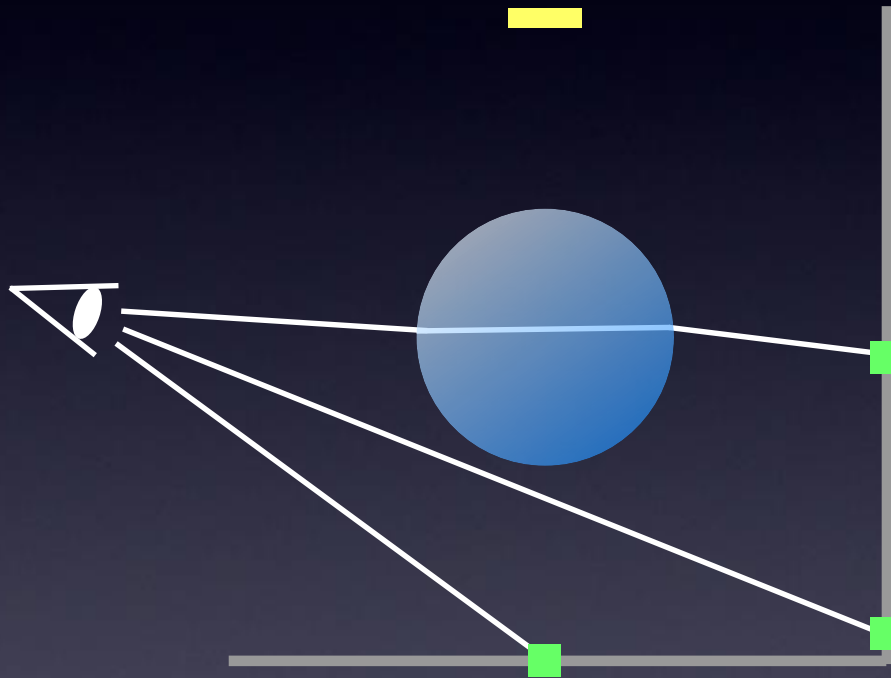
Progressive Photon Mapping



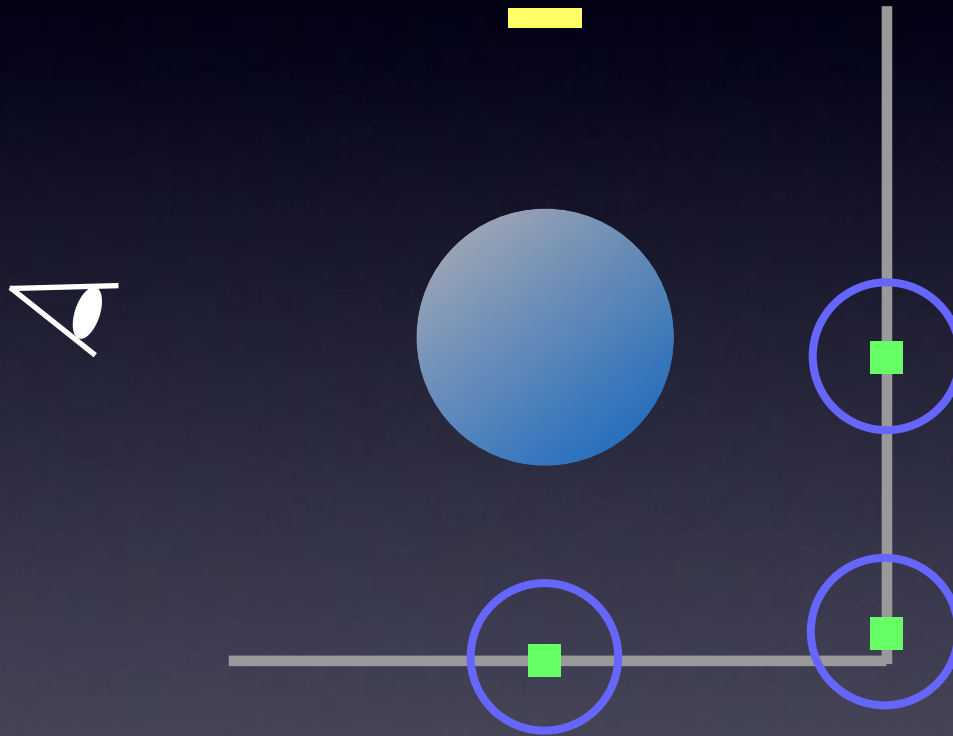
Progressive Photon Mapping - Initial Pass



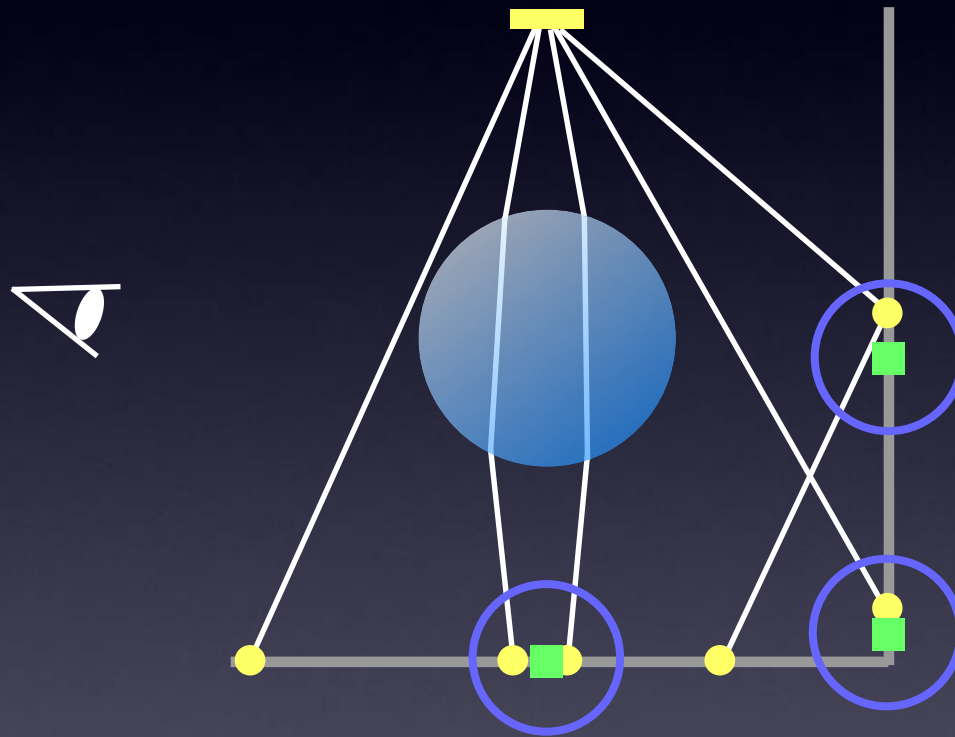
Progressive Photon Mapping - Initial Pass



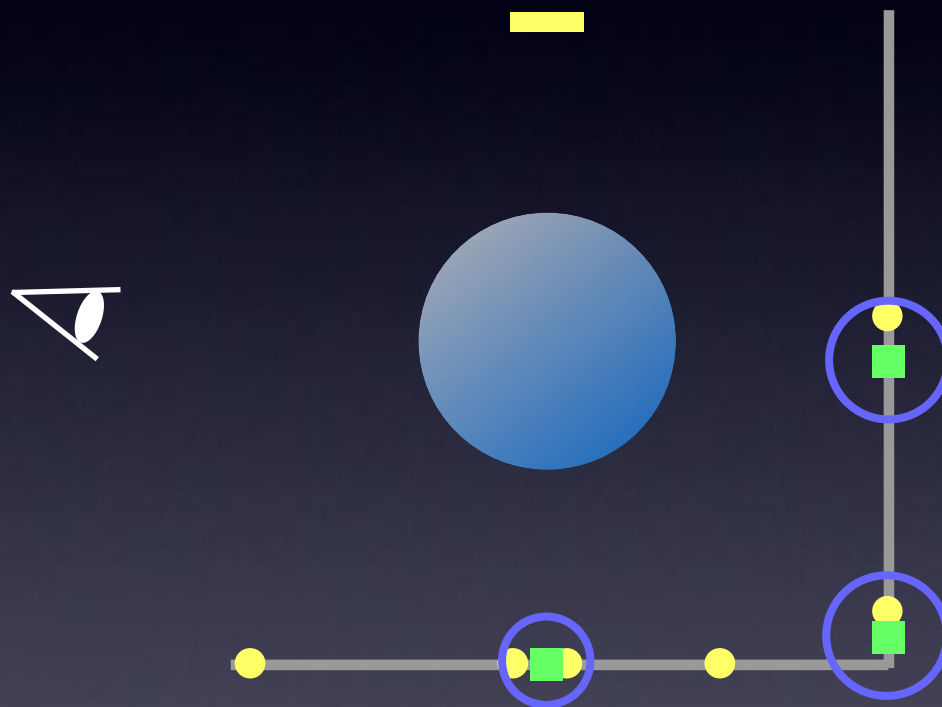
Progressive Photon Mapping - Initial Pass



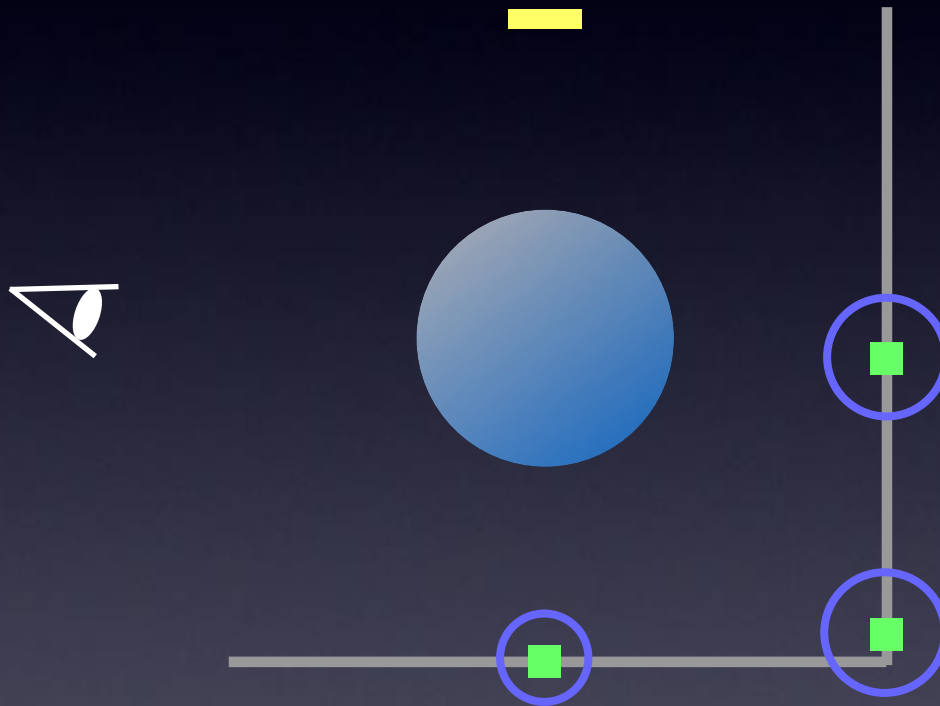
Progressive Photon Mapping - 1st Refinement Pass



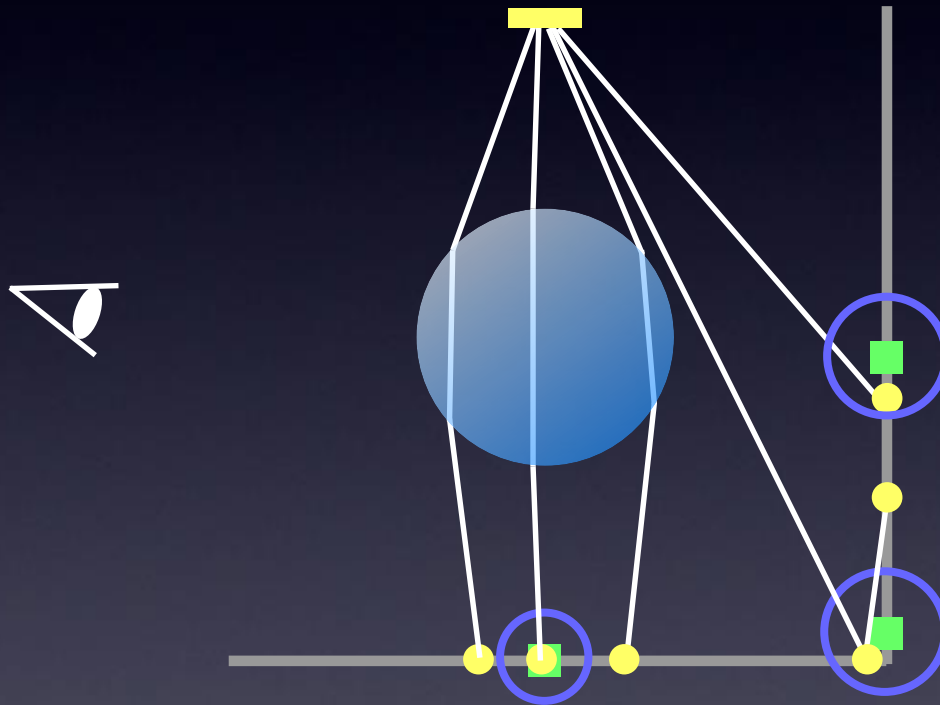
Progressive Photon Mapping - 1st Refinement Pass



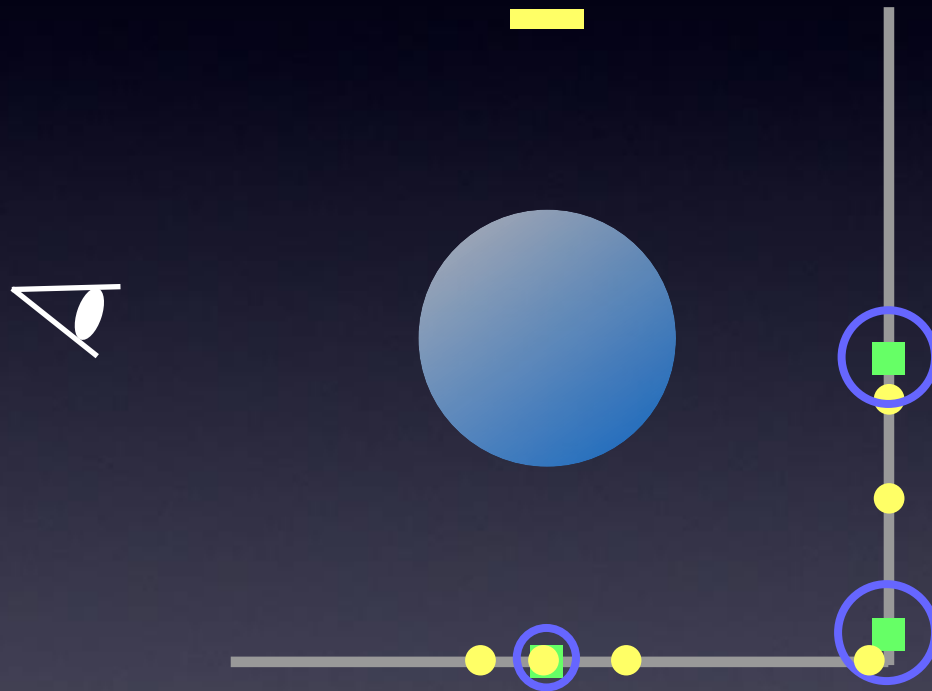
Progressive Photon Mapping - 1st Refinement Pass



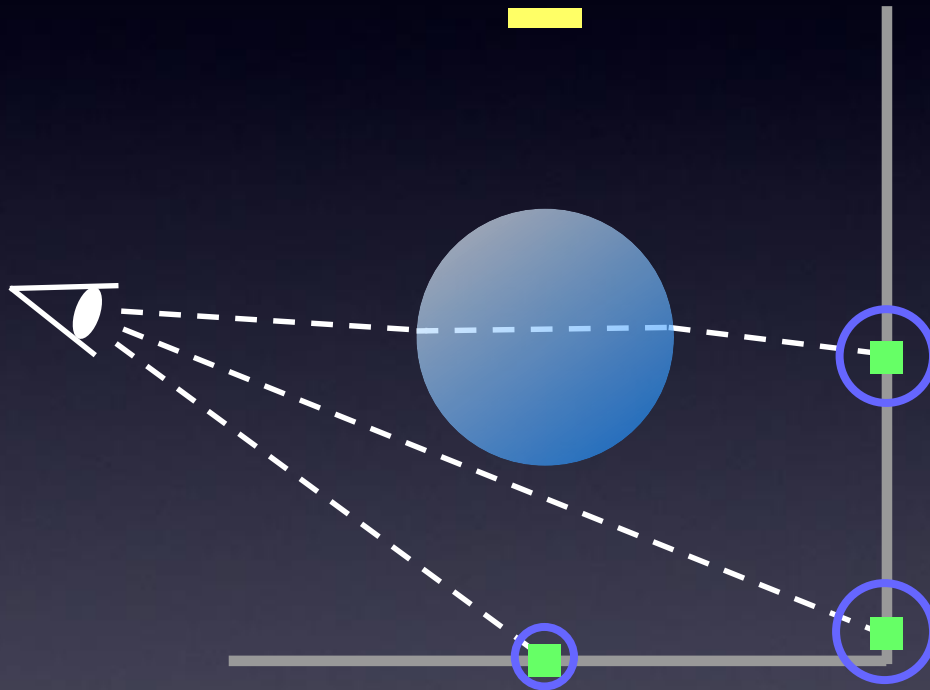
Progressive Photon Mapping - 2nd Refinement Pass



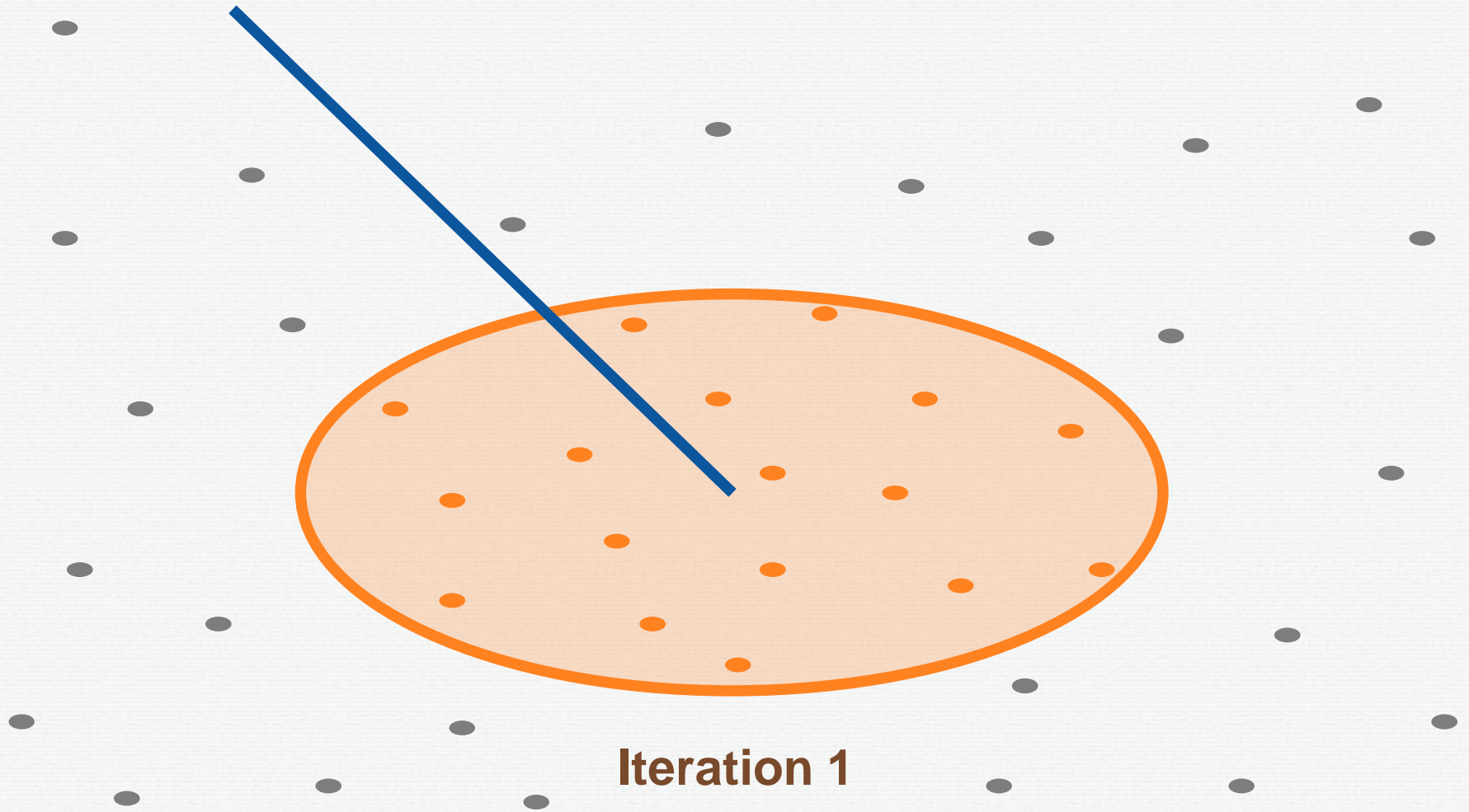
Progressive Photon Mapping - 2nd Refinement Pass



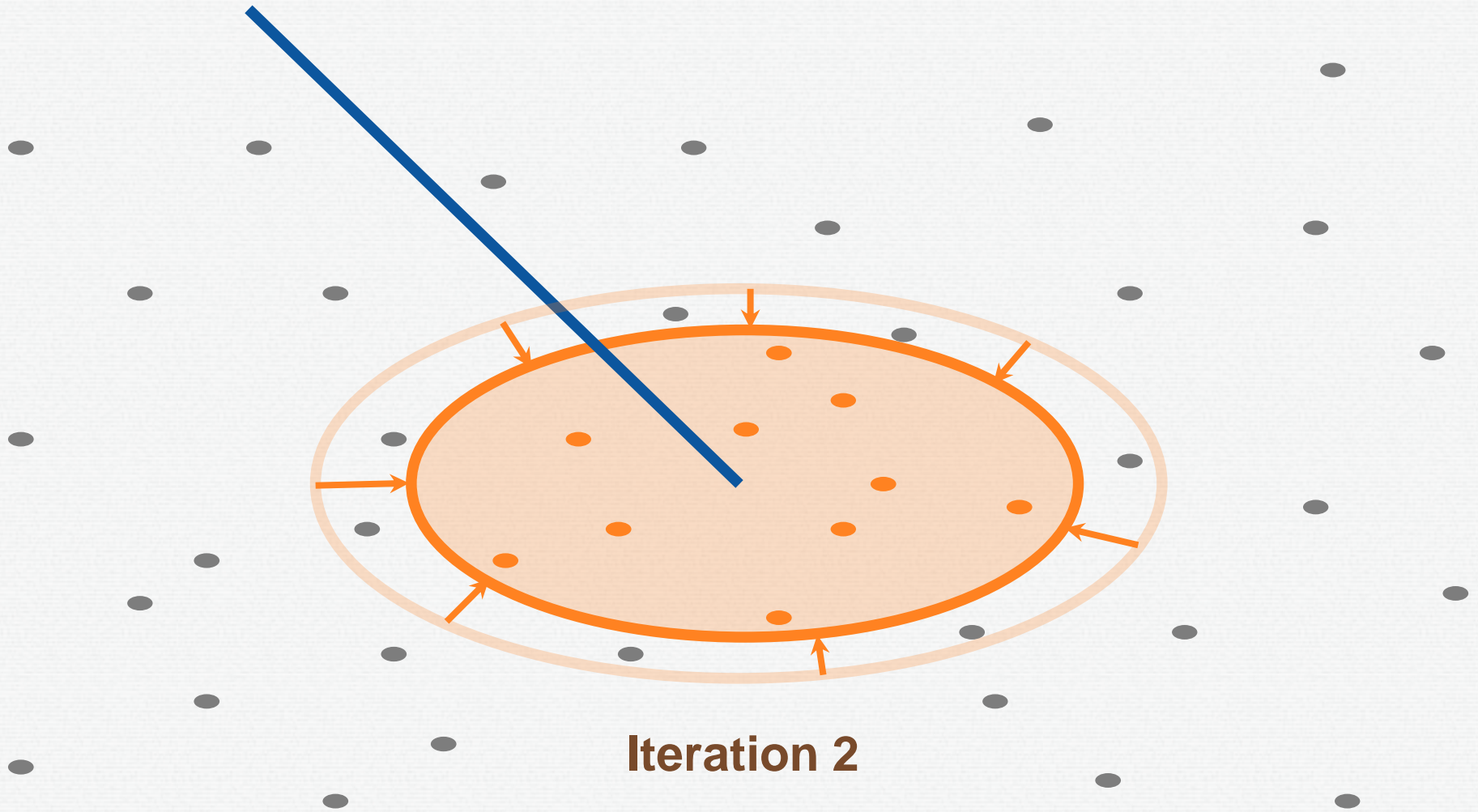
Progressive Photon Mapping - Rendering



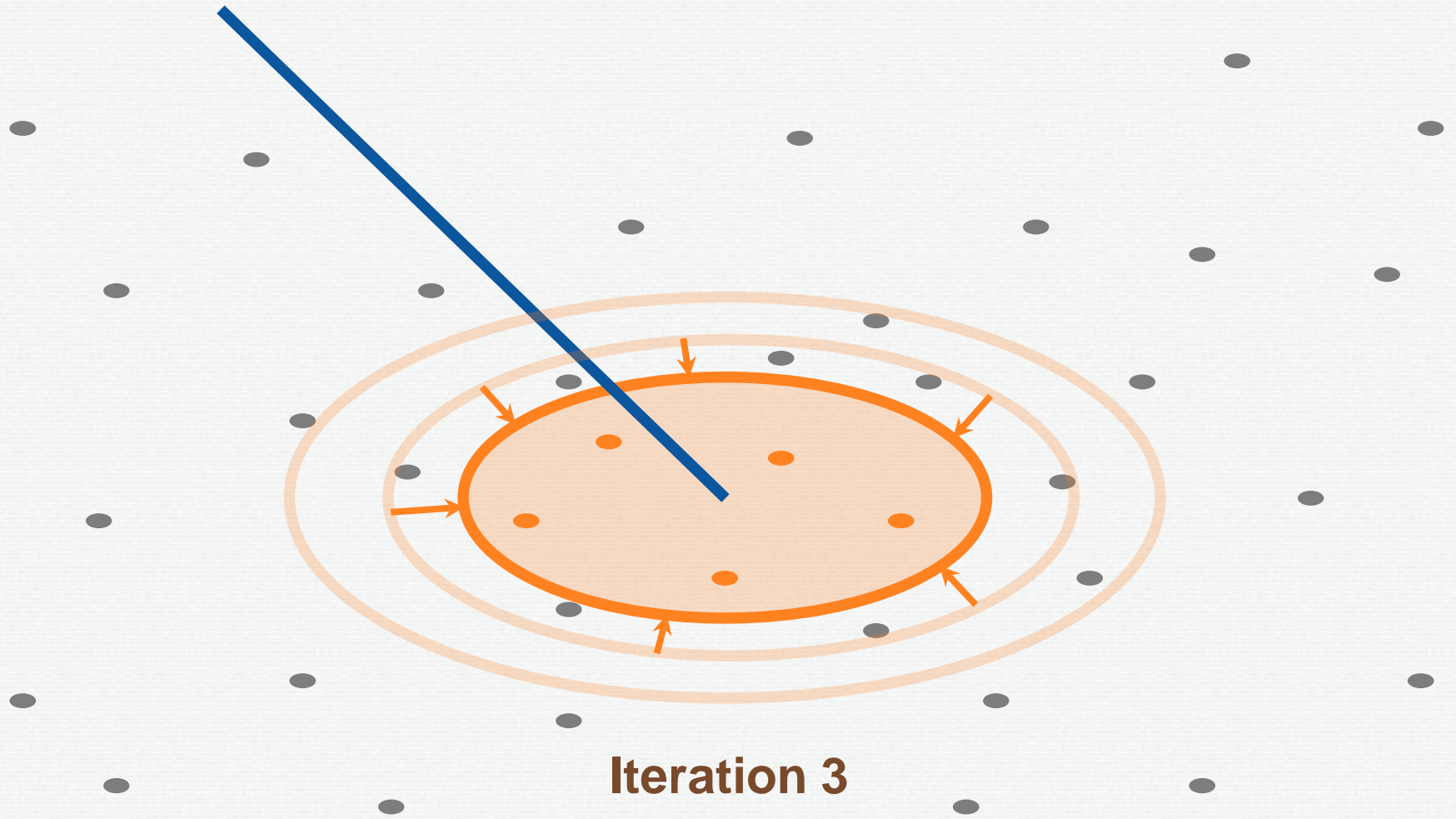
Radius Reduction



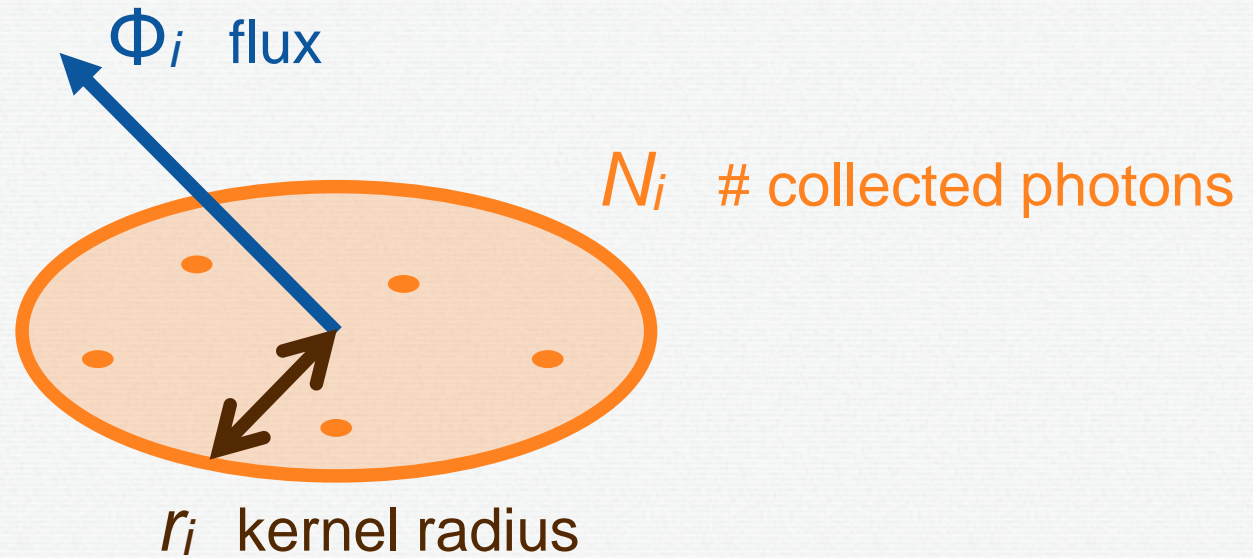
Radius Reduction



Radius Reduction



Locations with Statistics



Radius Reduction

$$\frac{r_{i+1}^2}{r_i^2} = \frac{N_i + \alpha M_i}{N_i + M_i}$$

totally collected photons



currently collected photons



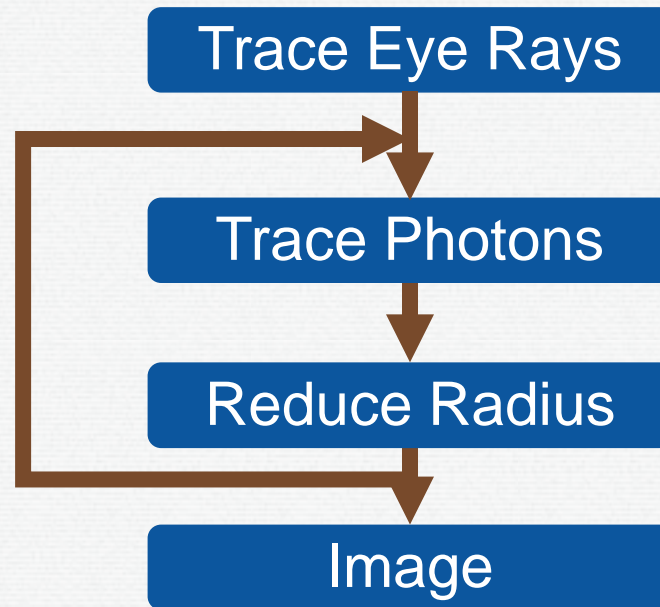
Stochastic PPM

Hachisuka & Jensen (2009)

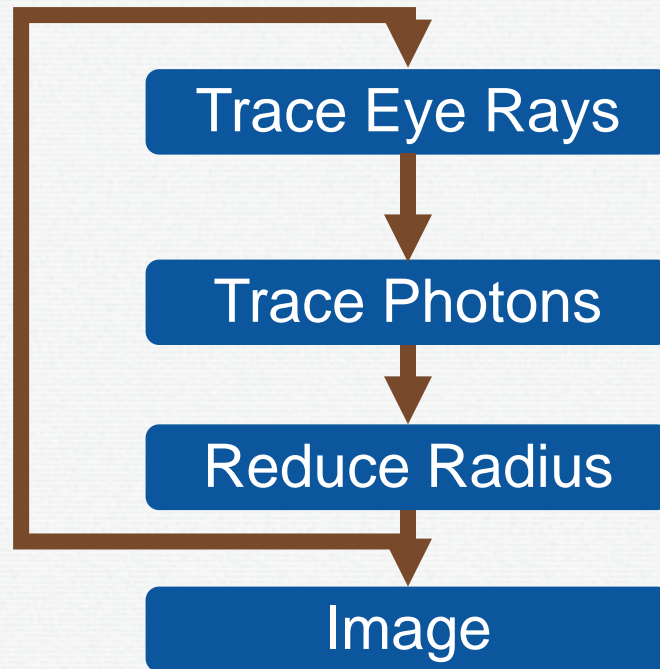


- ★ Glossy reflections
- ★ Depth of field
- ★ Motion blur

Stochastic PPM



Stochastic PPM





SIGGRAPH2011

Progressive Photon Mapping: A Probabilistic Approach

Claude Knaus and Matthias Zwicker

University of Bern

cgg
computer graphics group

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^b
UNIVERSITÄT
BERN

Our Probabilistic Approach

- New derivation using probabilistic perspective
- No local statistics
- Parallelization
- Convergence analysis
- Arbitrary radiance estimation kernels
- Easy to generalize

Radiance Estimation

$$L(x) \approx \frac{1}{N_e} \sum_{i=1}^{N_s} k_r(x_i - x) \gamma_i$$

Stored Photons

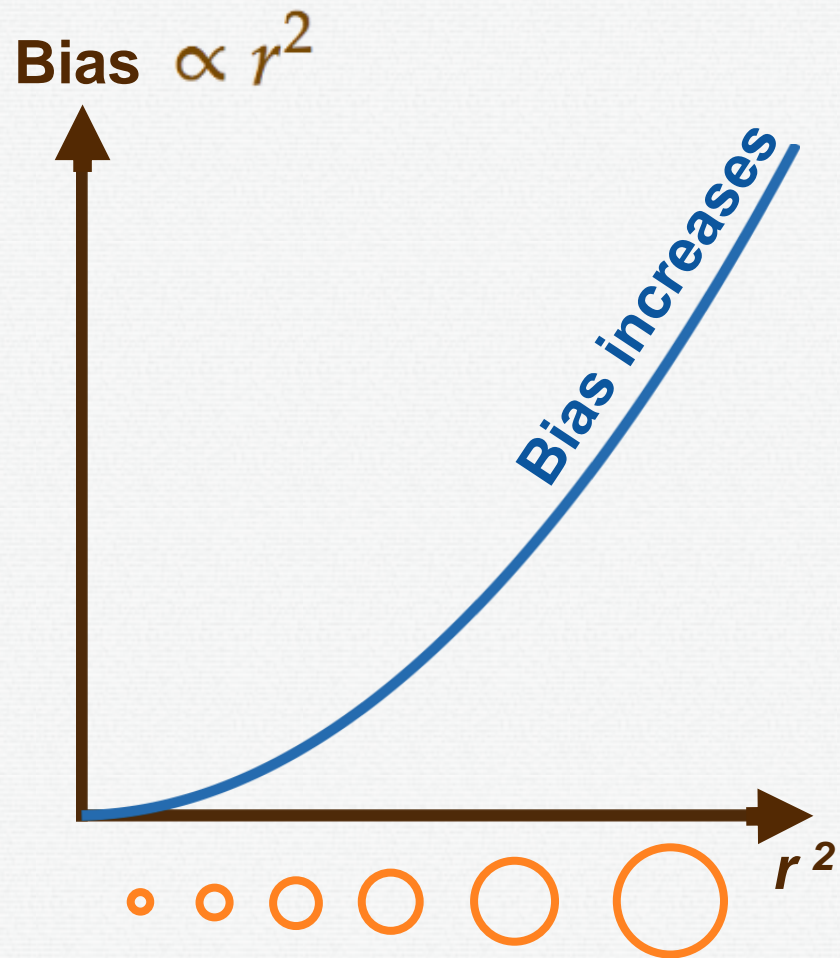
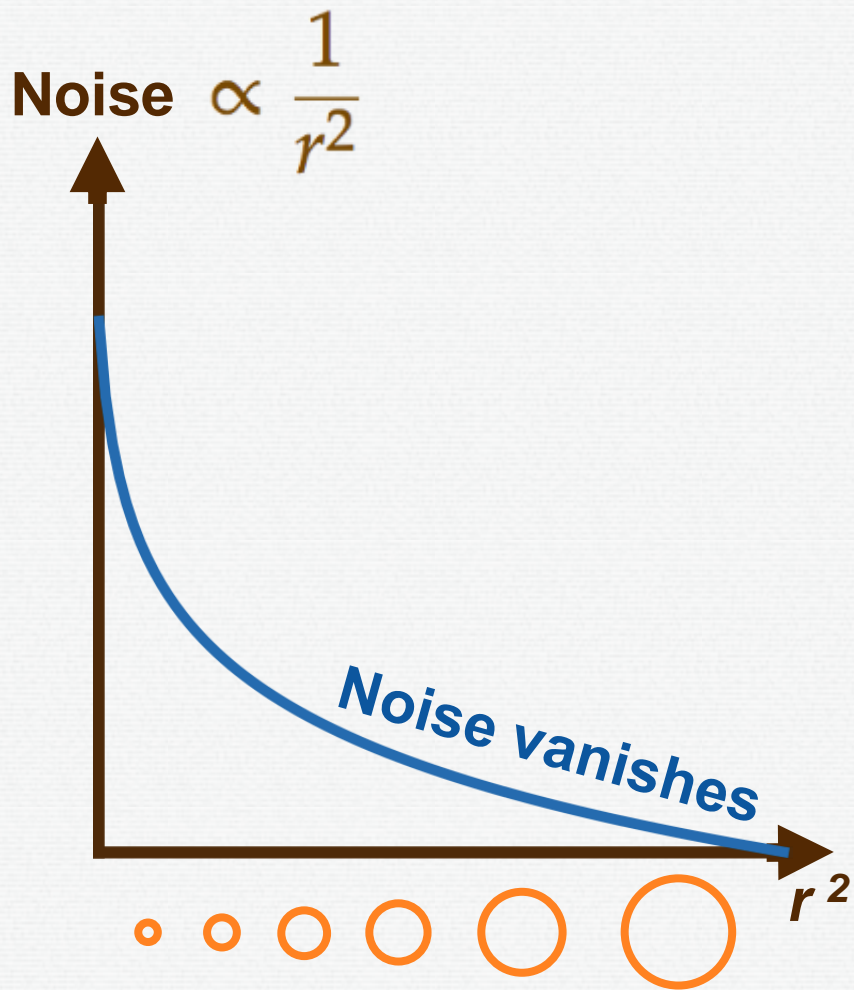
Emitted Photons

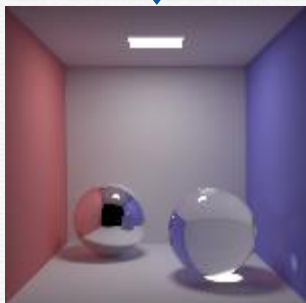
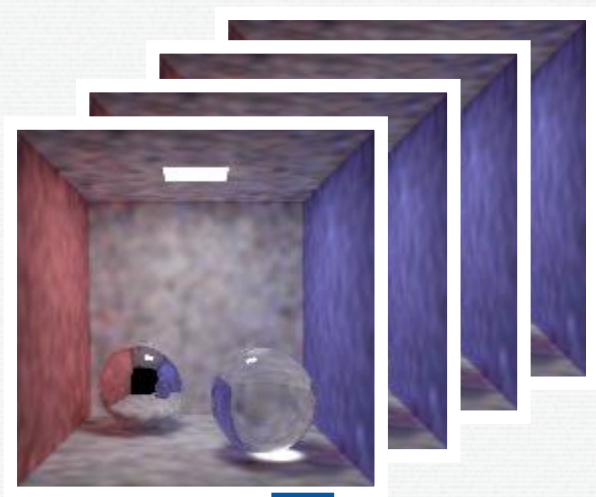
Kernel with Radius r

Photon Position

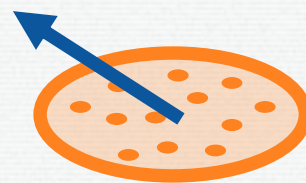
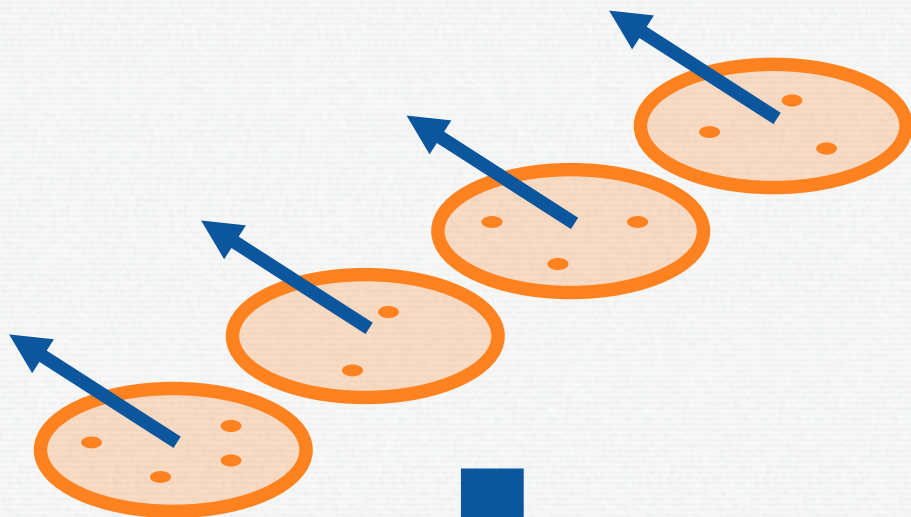
Photon Power

The diagram illustrates the radiance estimation equation $L(x) \approx \frac{1}{N_e} \sum_{i=1}^{N_s} k_r(x_i - x) \gamma_i$. It includes five labels with blue arrows pointing to specific parts of the equation: '# Stored Photons' points to the upper limit N_s of the summation; '# Emitted Photons' points to the denominator N_e ; 'Kernel with Radius r ' points to the kernel function $k_r(x_i - x)$; 'Photon Position' points to the variable x_i in the kernel; and 'Photon Power' points to the power variable γ_i .



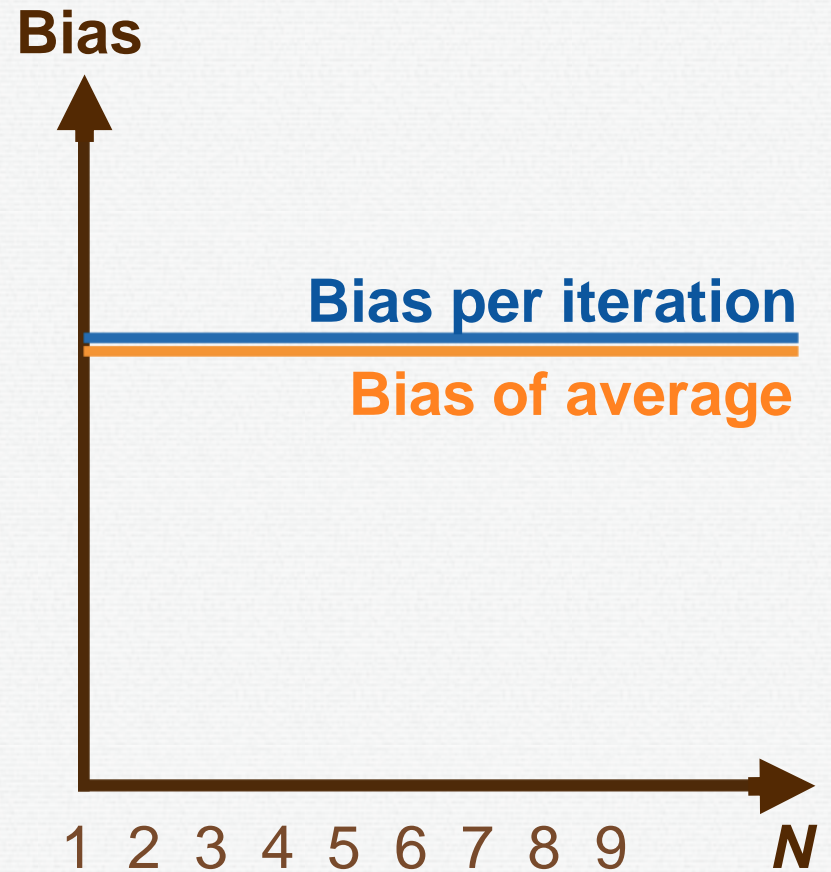
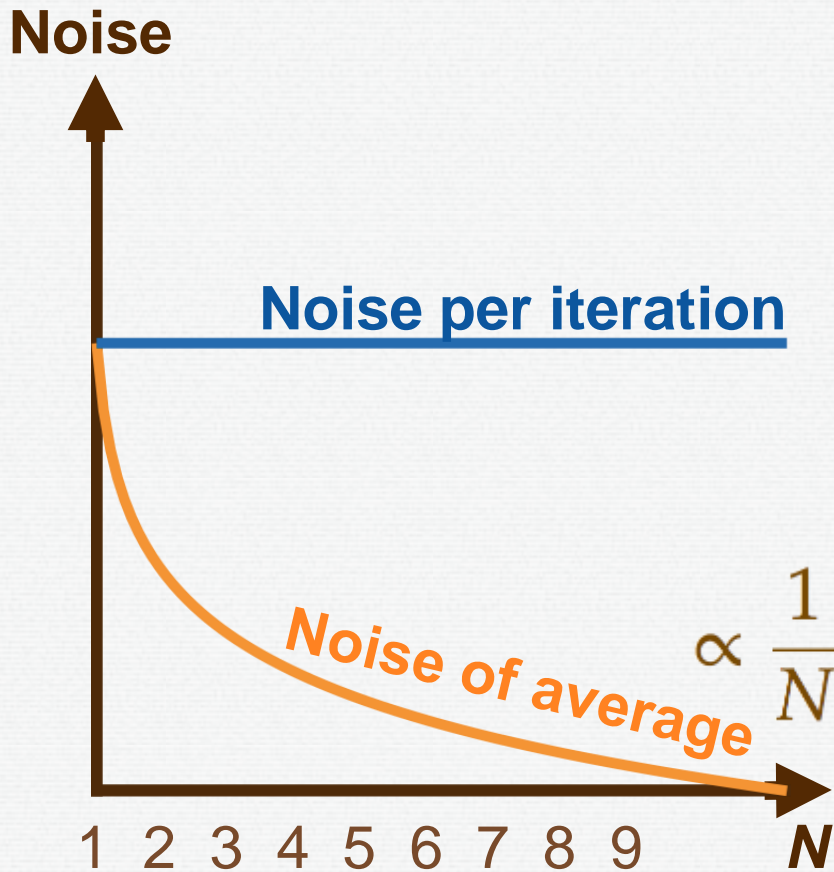


Averaged Image

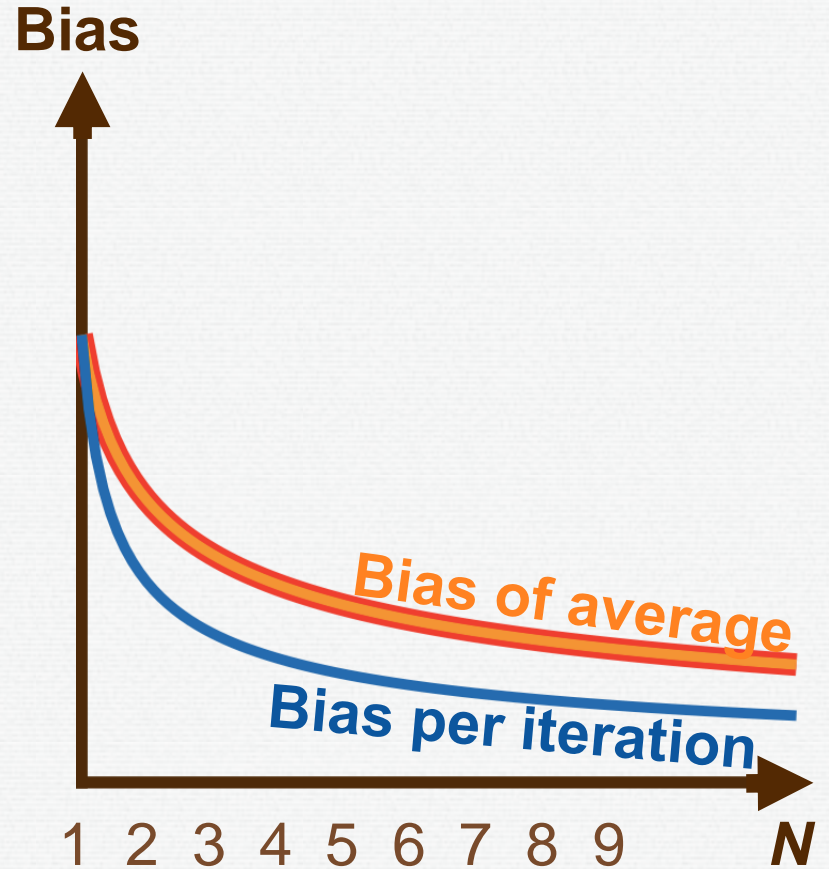
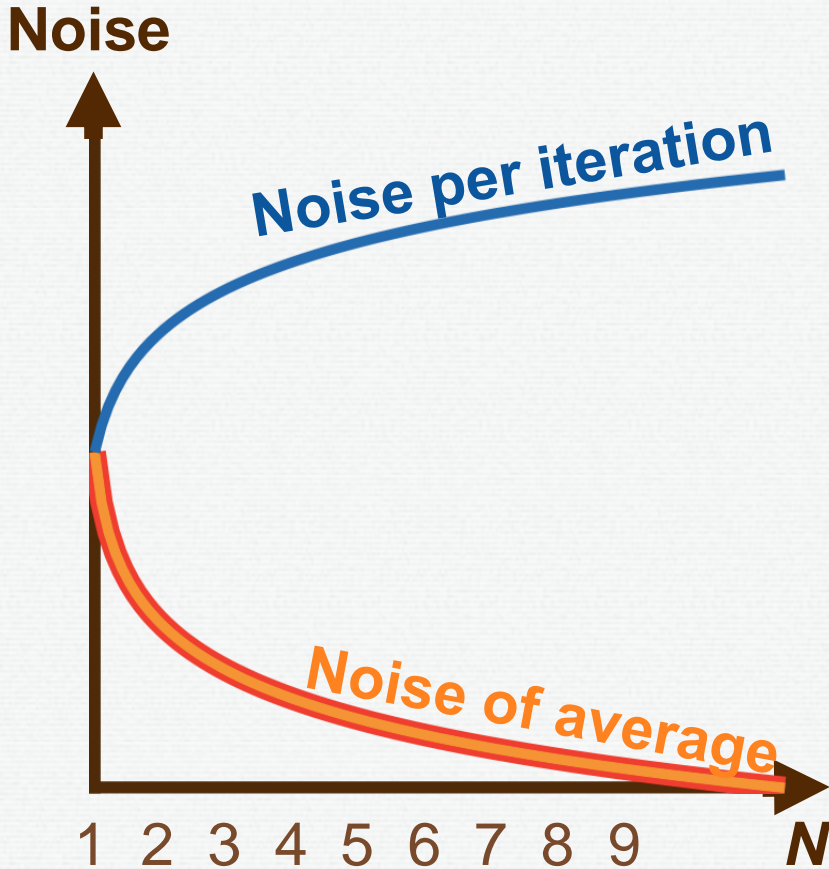


Averaged Radiance Estimate

Averaged Radiance Estimates

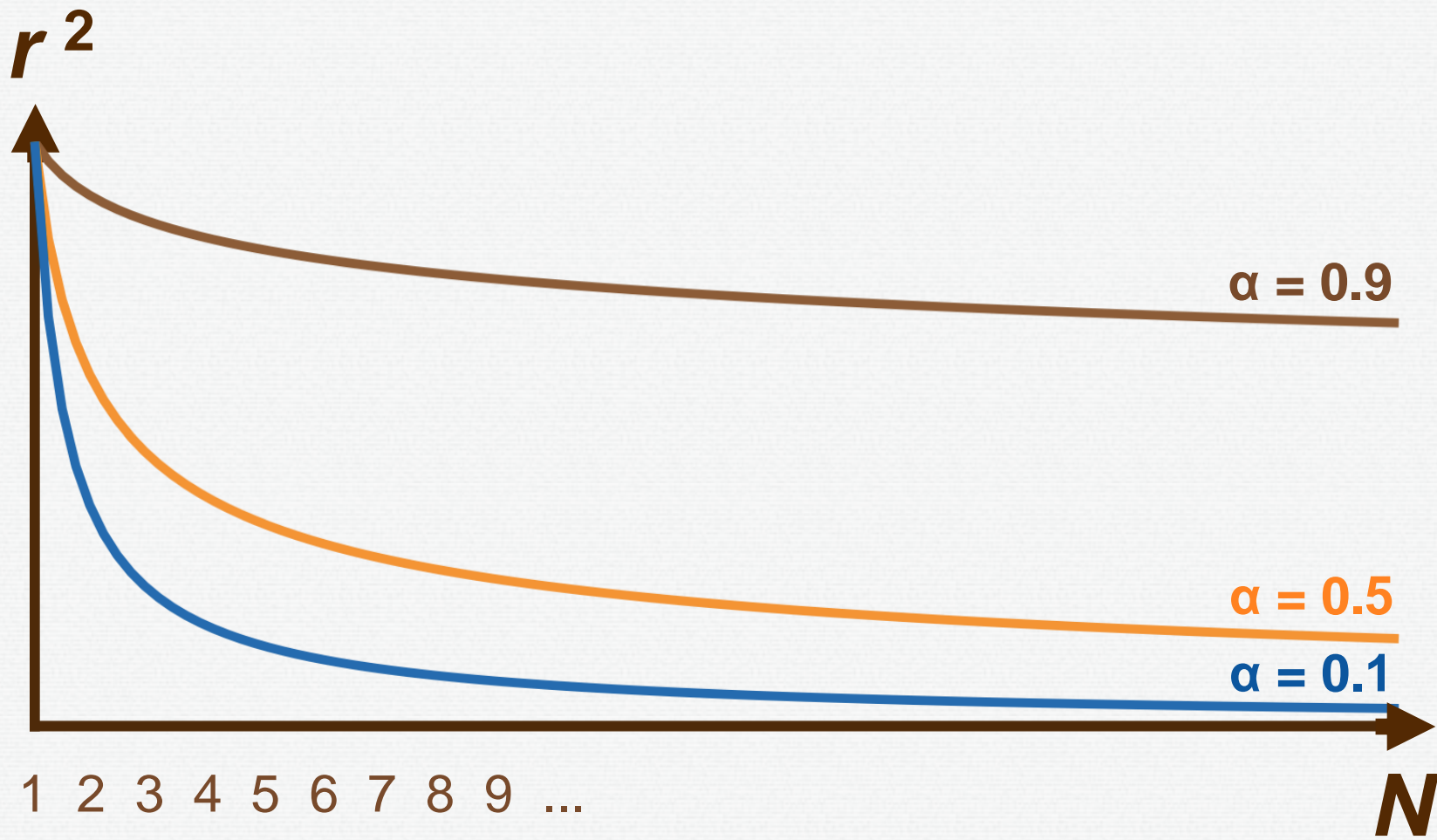


Averaging + Radius Reduction



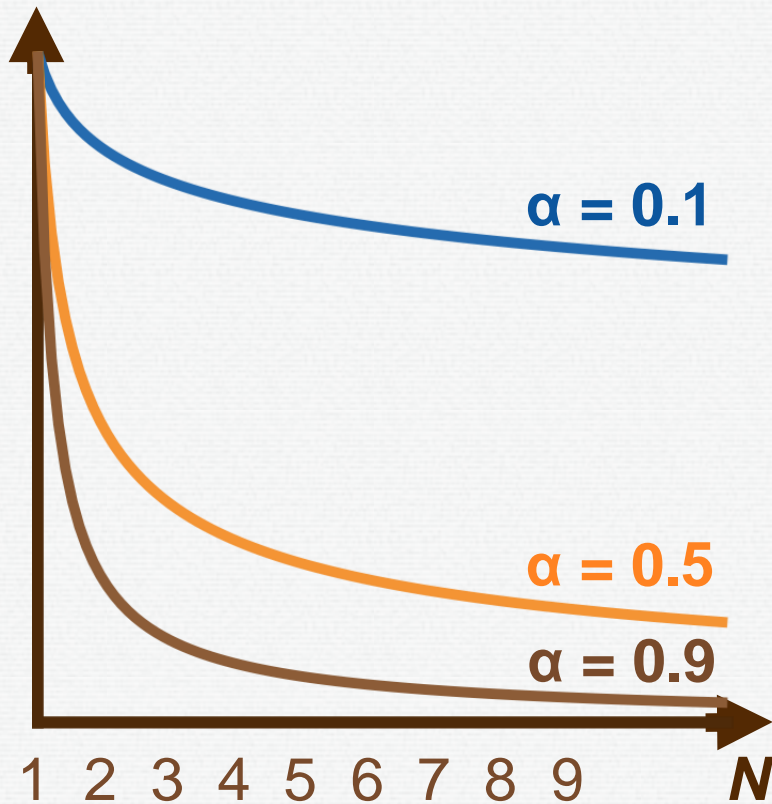
Radius Sequence

$$\frac{r_{i+1}^2}{r_i^2} = \frac{i + \alpha}{i + 1}$$

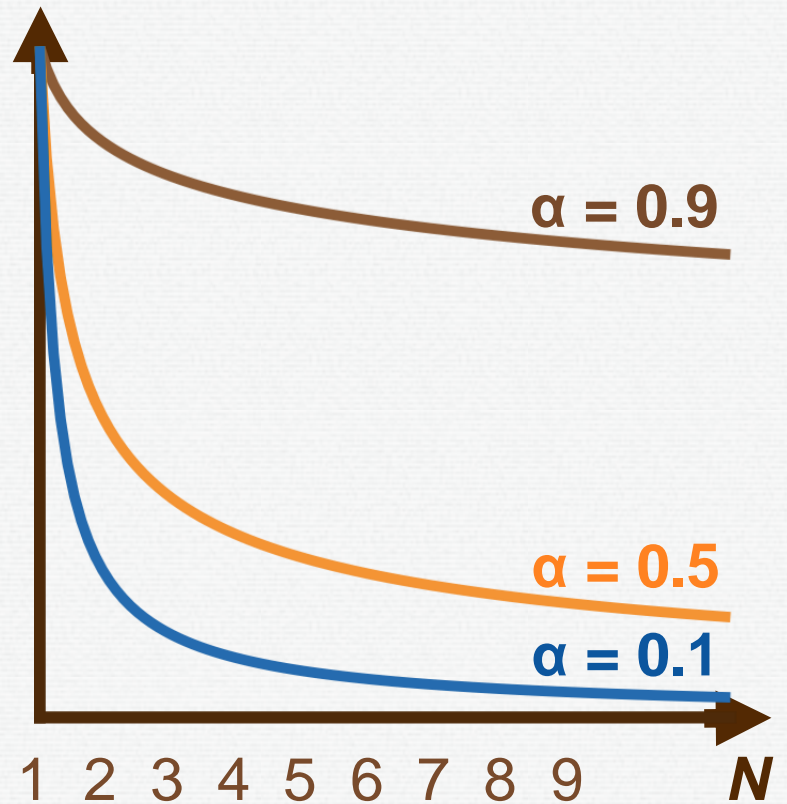


Asymptotic Convergence

Noise of average $\propto 1/N^\alpha$

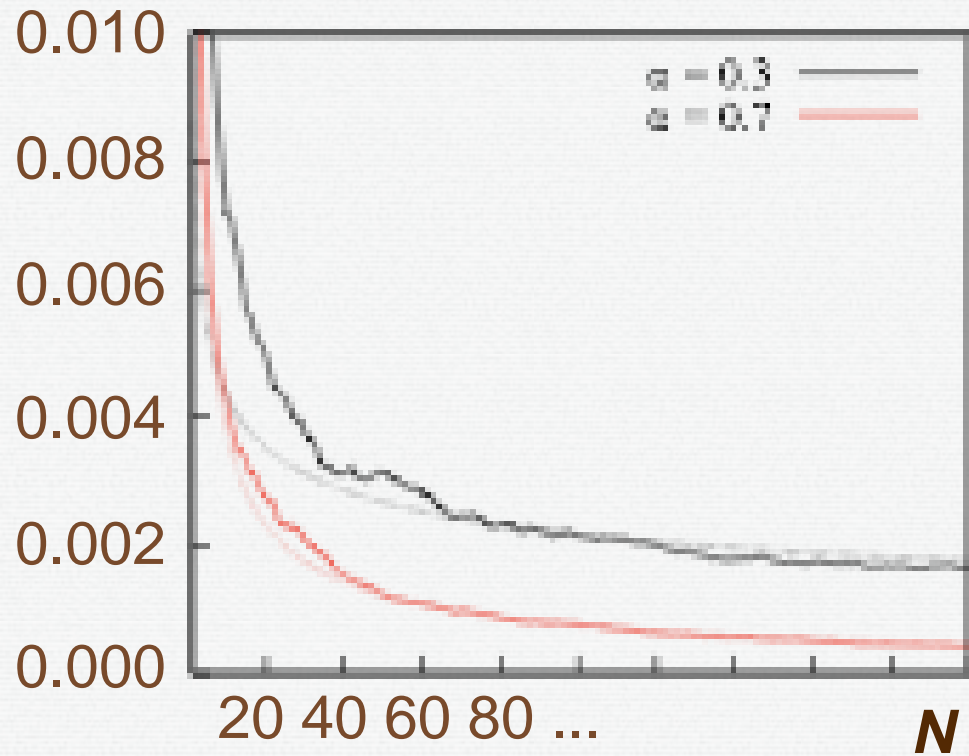


Bias of average $\propto 1/N^{1-\alpha}$

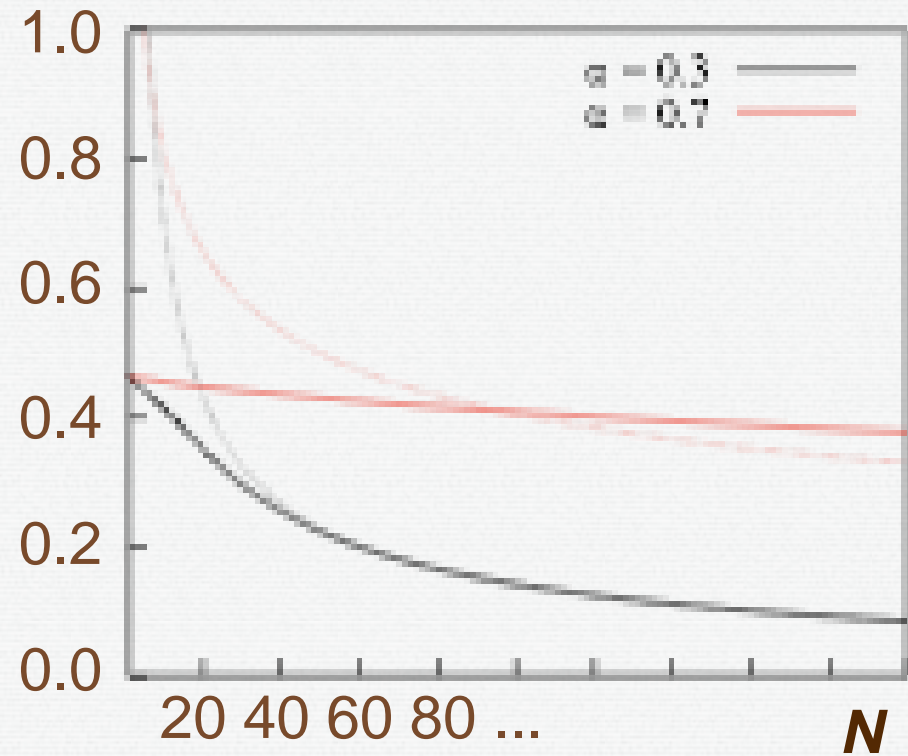


Empirical Validation

Noise of average



Bias of average



No Statistics Needed

PPM Radius Update Rule

$$\frac{r_{i+1}^2}{r_i^2} = \frac{N_i + \alpha M_i}{N_i + M_i}$$


Local Statistics

Our Radius Sequence

$$\frac{r_{i+1}^2}{r_i^2} = \frac{i + \alpha}{i + 1}$$

No Local Statistics!

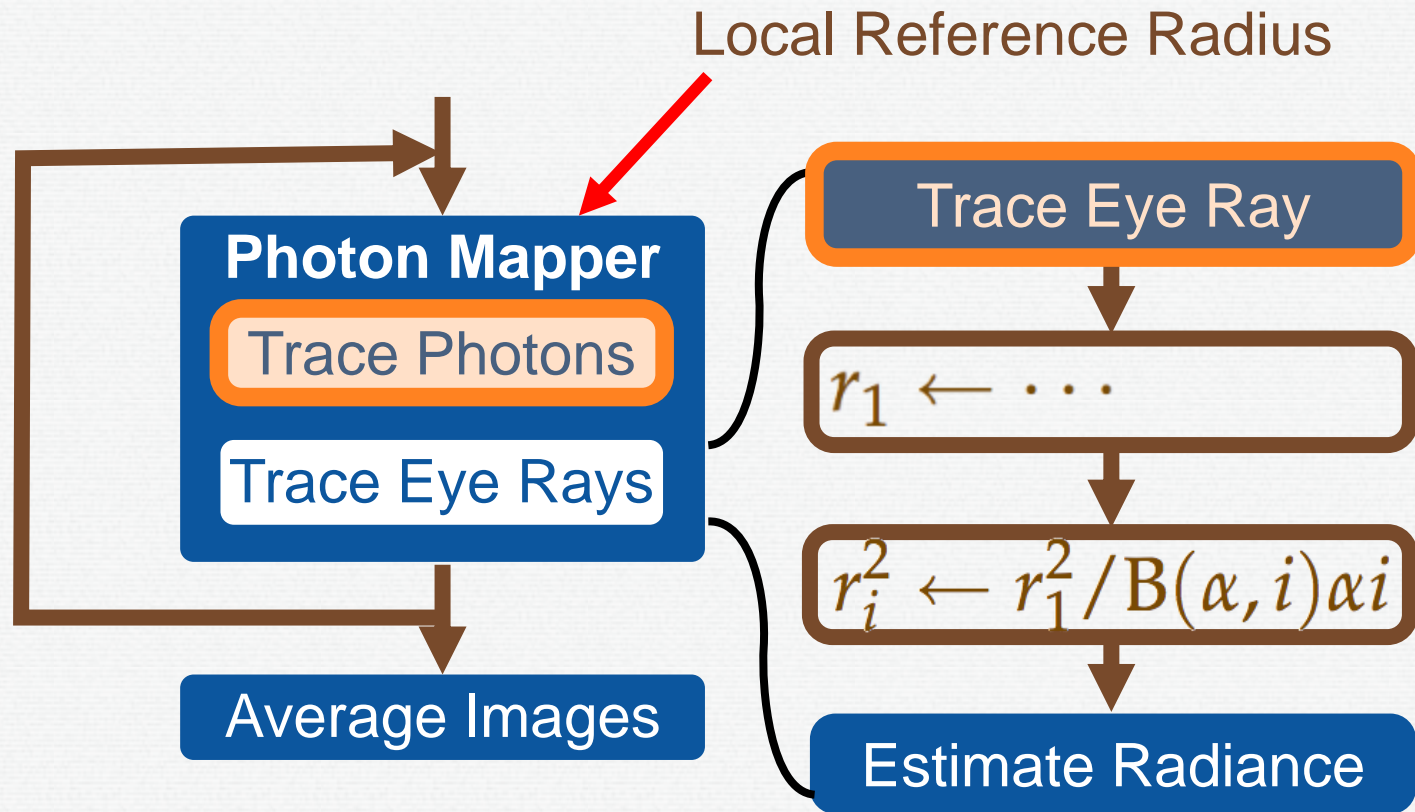
Radius Sequence (Explicit)

$$r_i^2 = \frac{r_1^2}{B(\alpha, i) \alpha i}$$

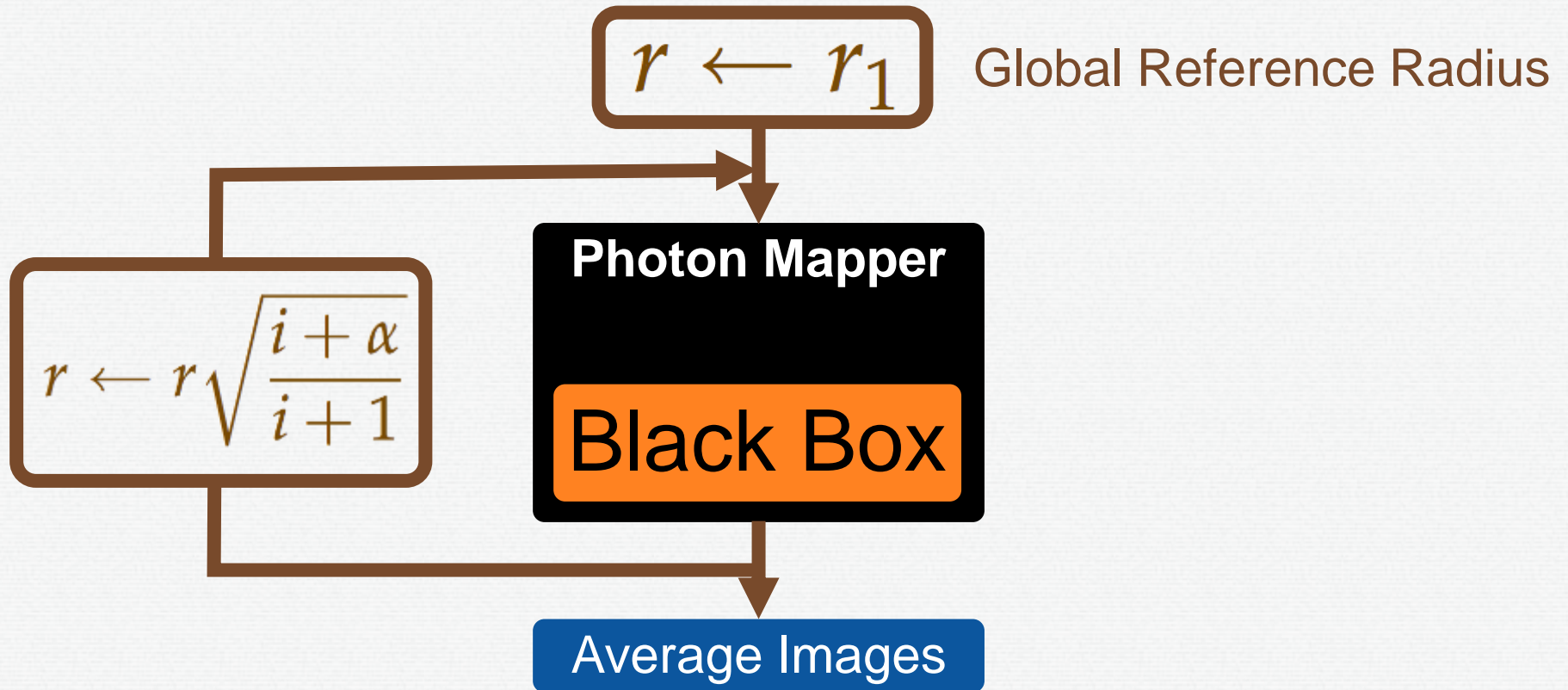
Reference Radius

Beta Function

Our Algorithm



Script





1

2

3

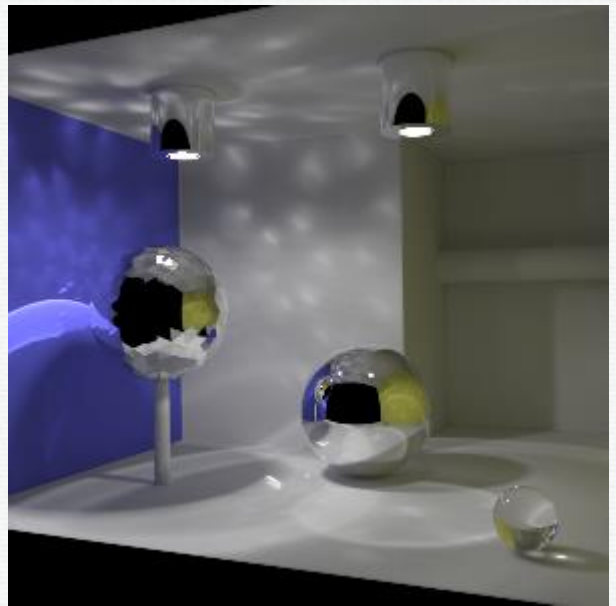
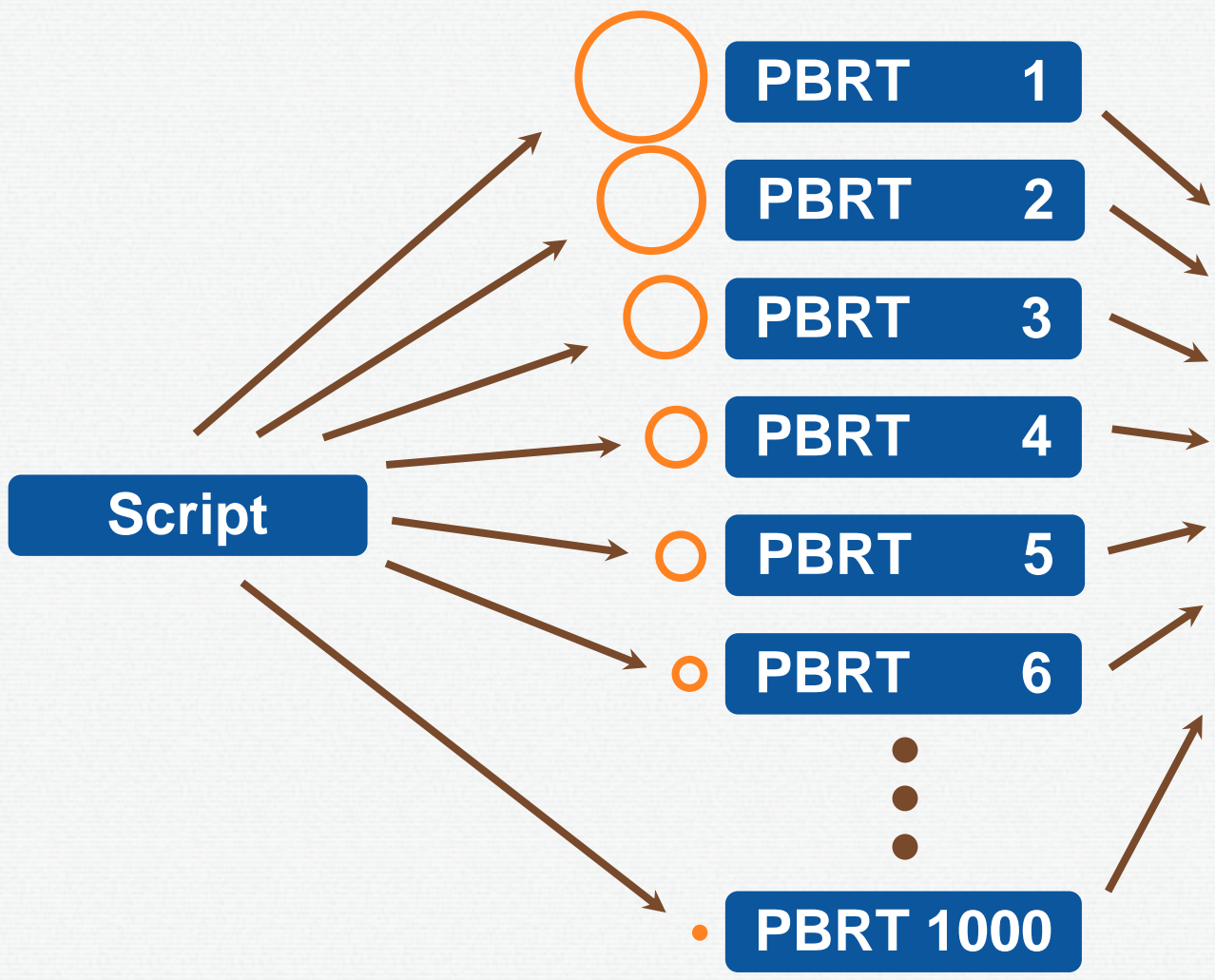
4

5

6



1000



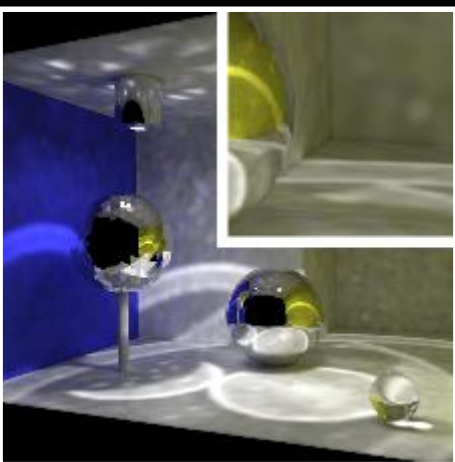


Image 1

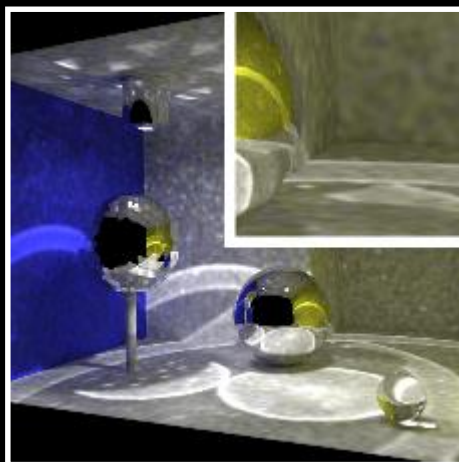


Image 10

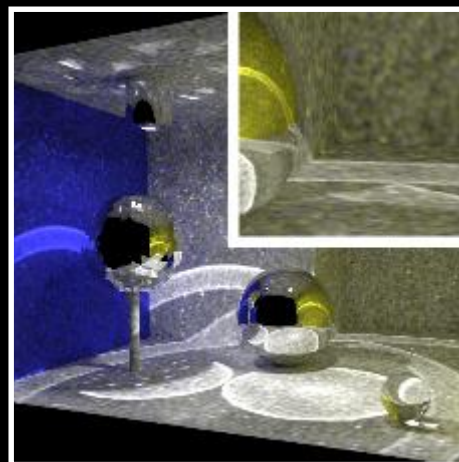


Image 100

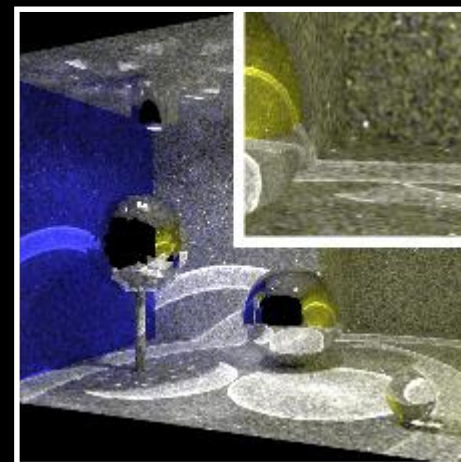
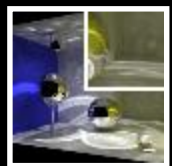
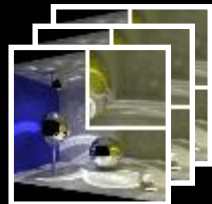
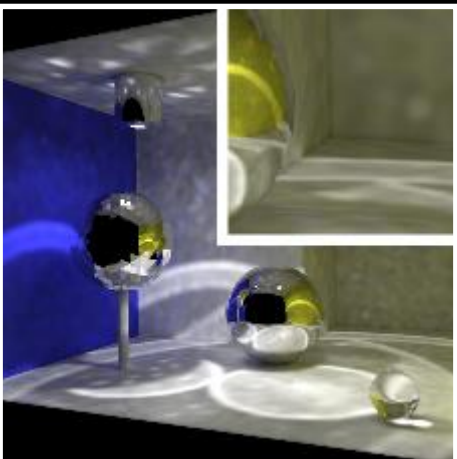


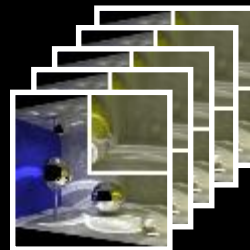
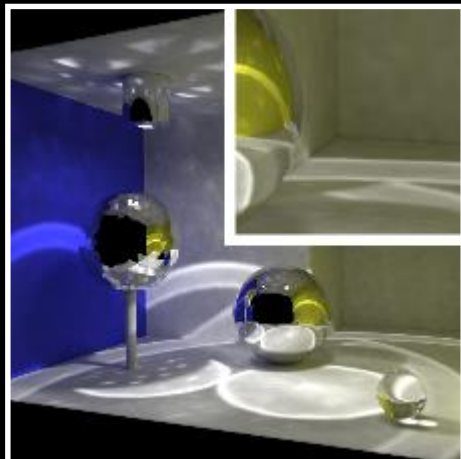
Image 1000



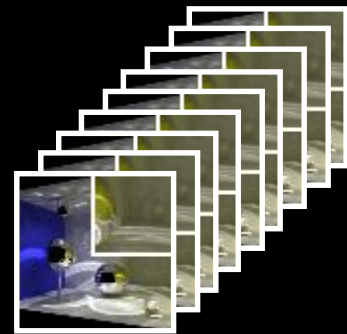
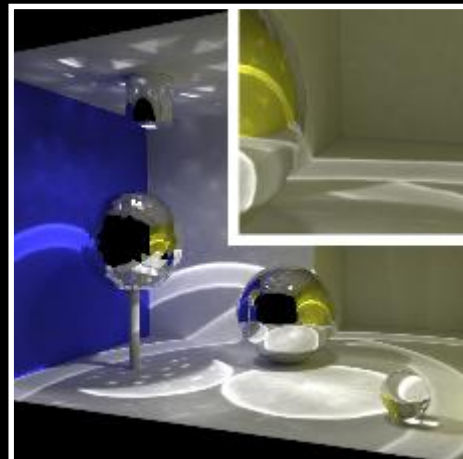
1



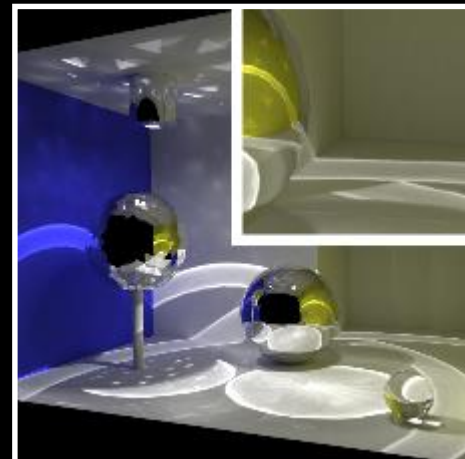
1—10



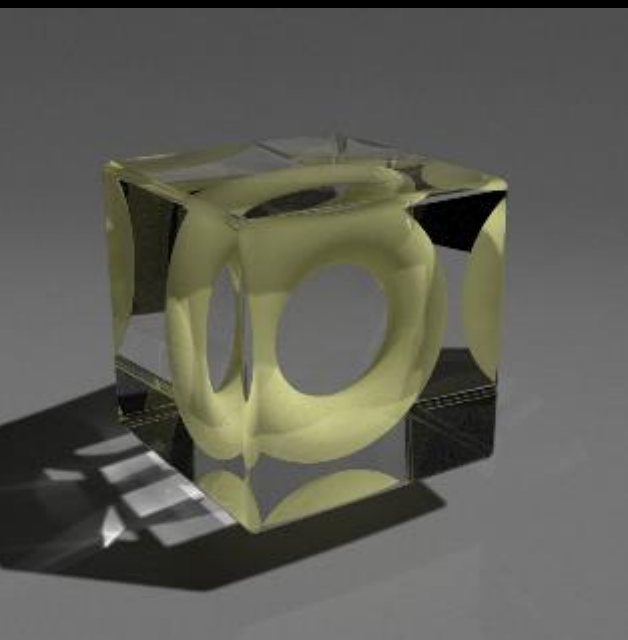
1—100



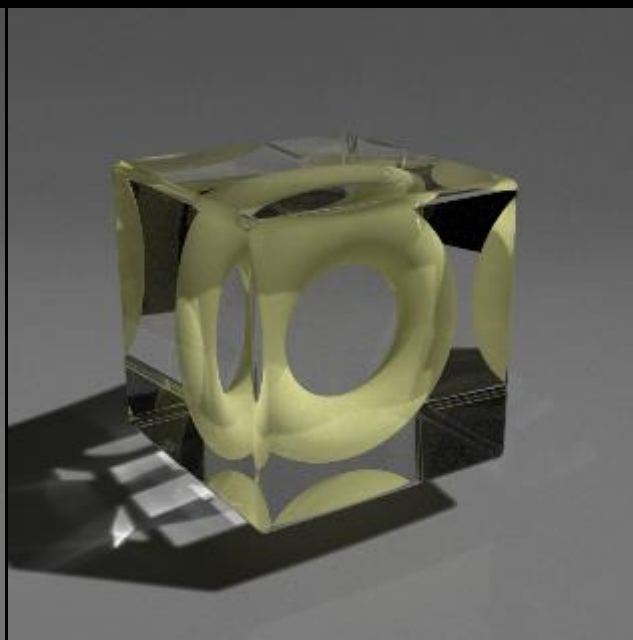
1—1000



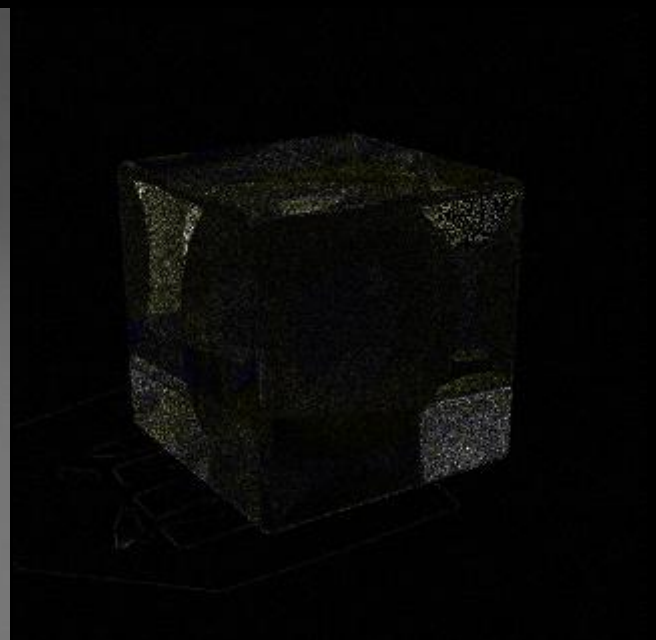
Stochastic PPM



Our method

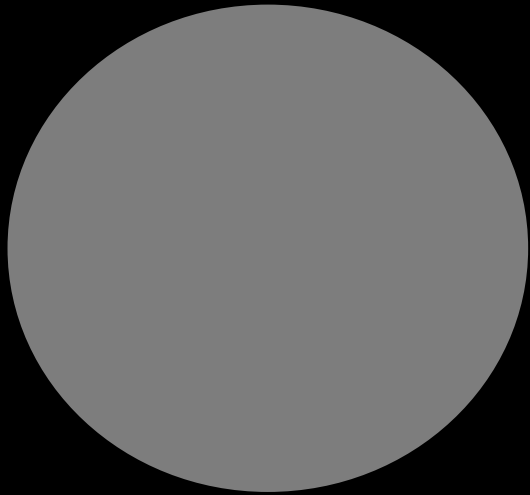


20x Difference

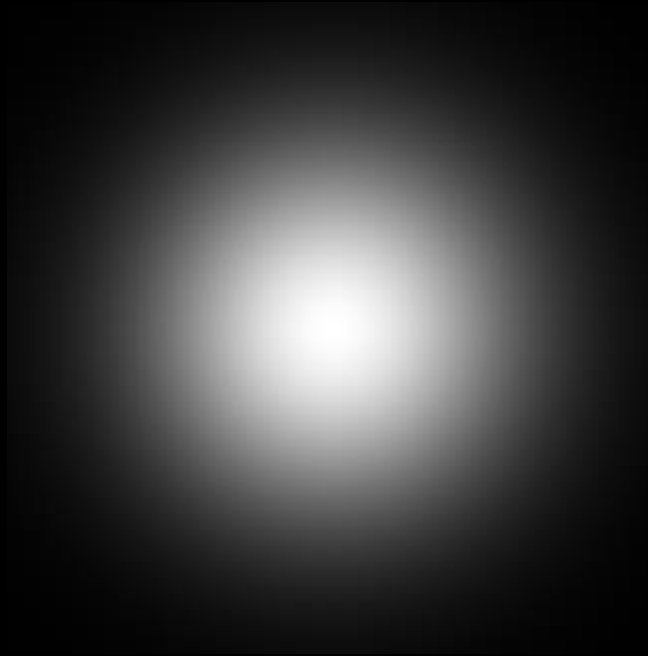


Scene courtesy of Toshiya Hachisuka

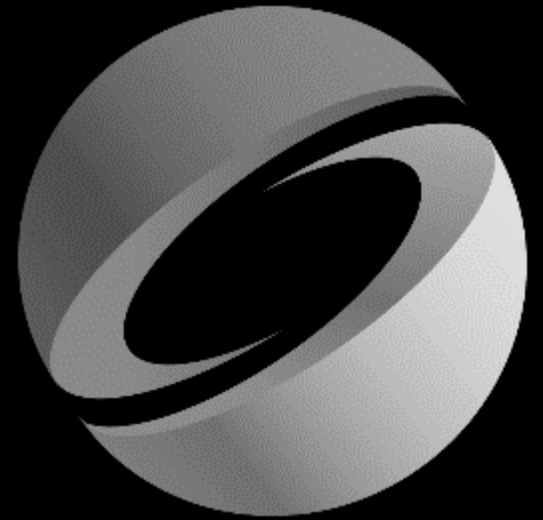
Arbitrary Kernels



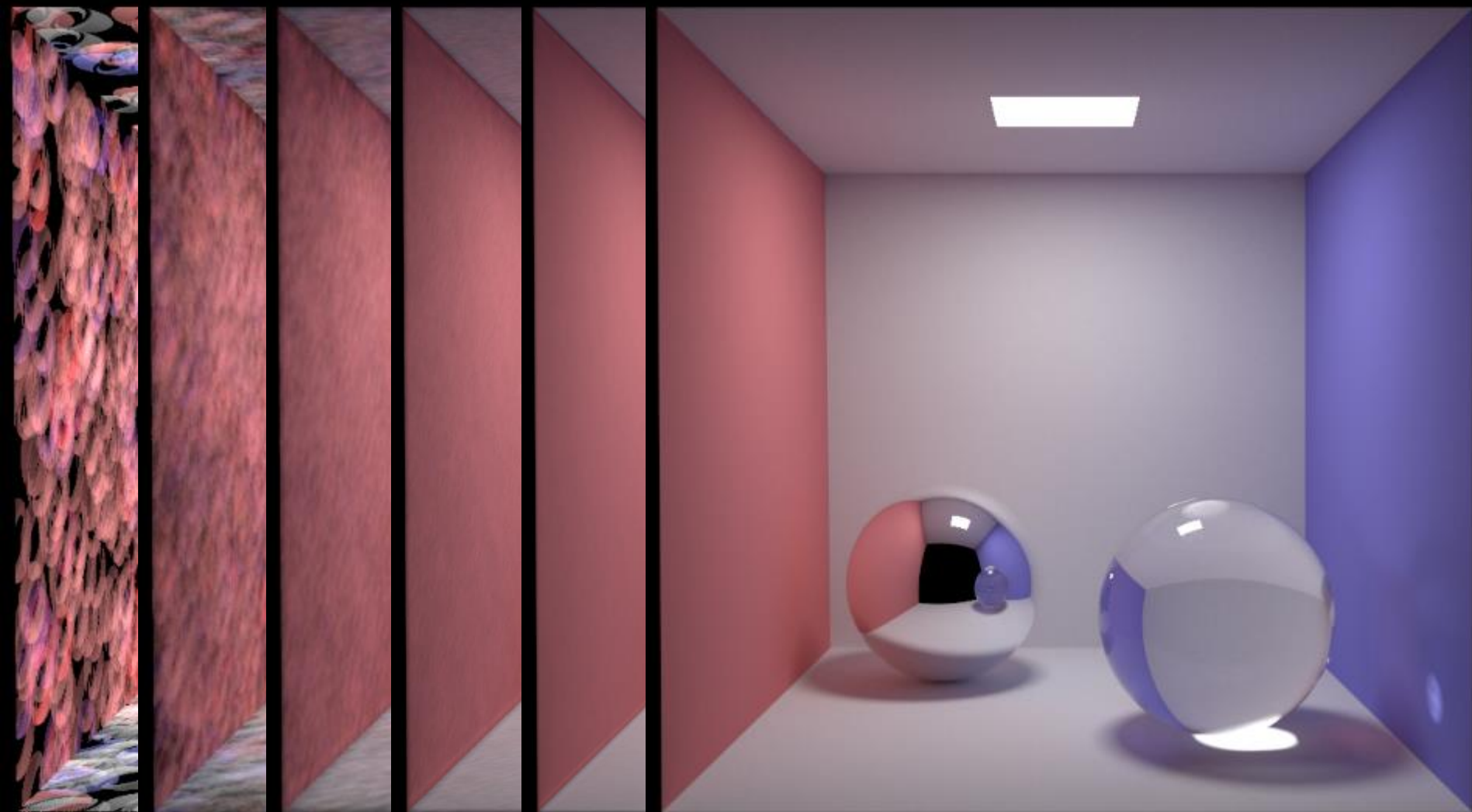
Box



Gaussian



SIGGRAPH



Stochastic Effects



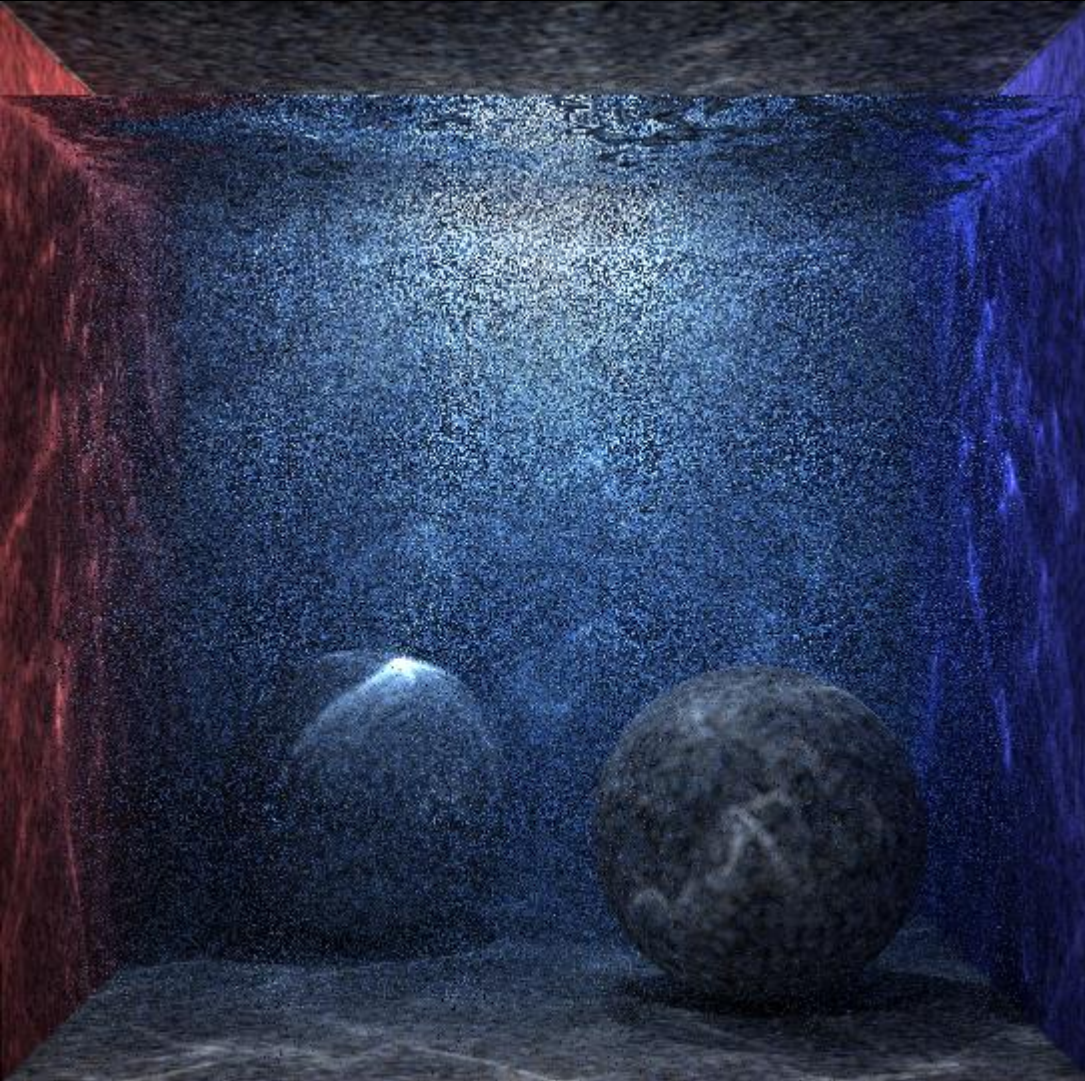
Scene courtesy of Toshiya Hachisuka

Participating Media

$$\frac{r_{i+1}^2}{r_i^2} = \frac{i + \alpha}{i + 1}$$

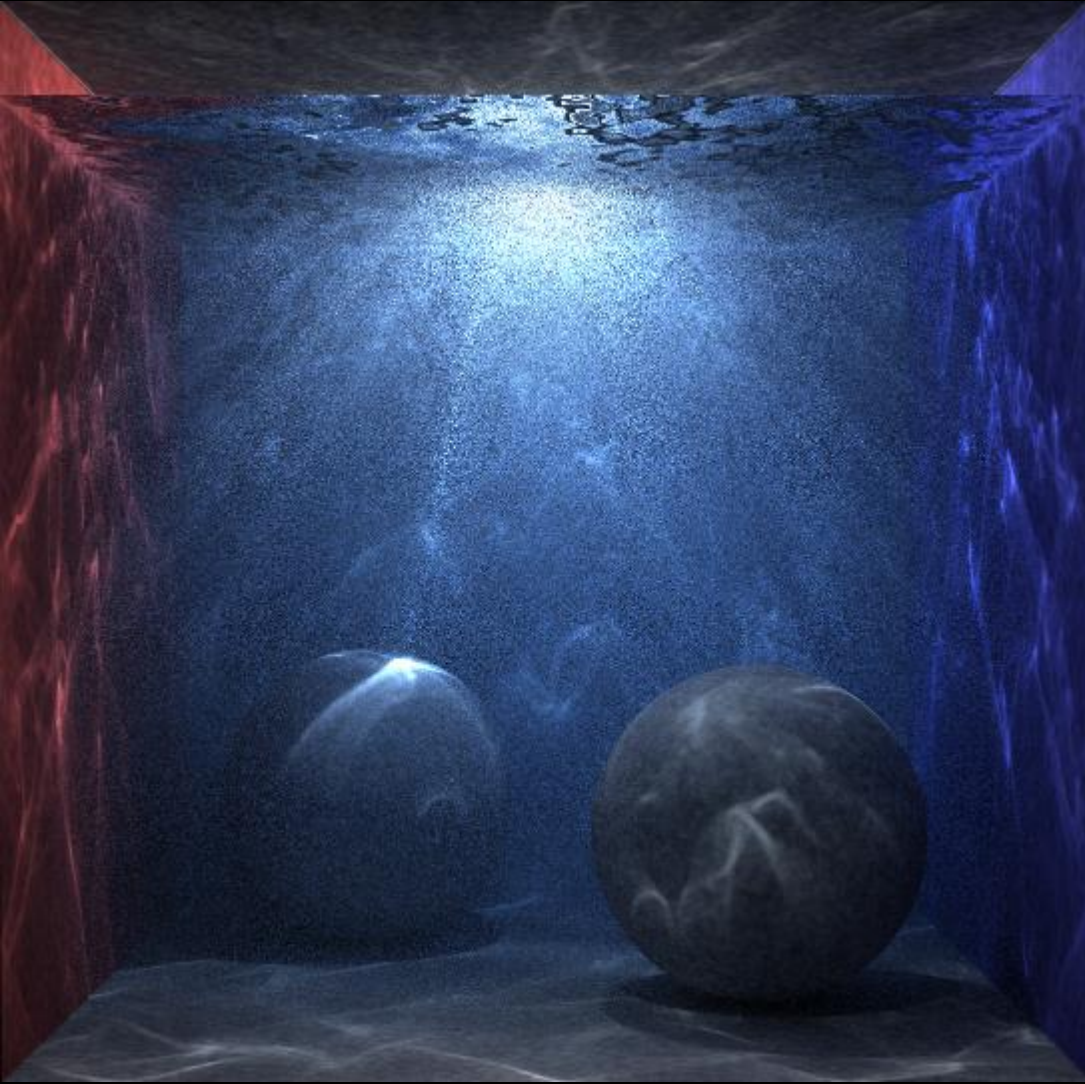
Participating Media

$$\frac{r_{i+1}^3}{r_i^3} = \frac{i + \alpha}{i + 1}$$



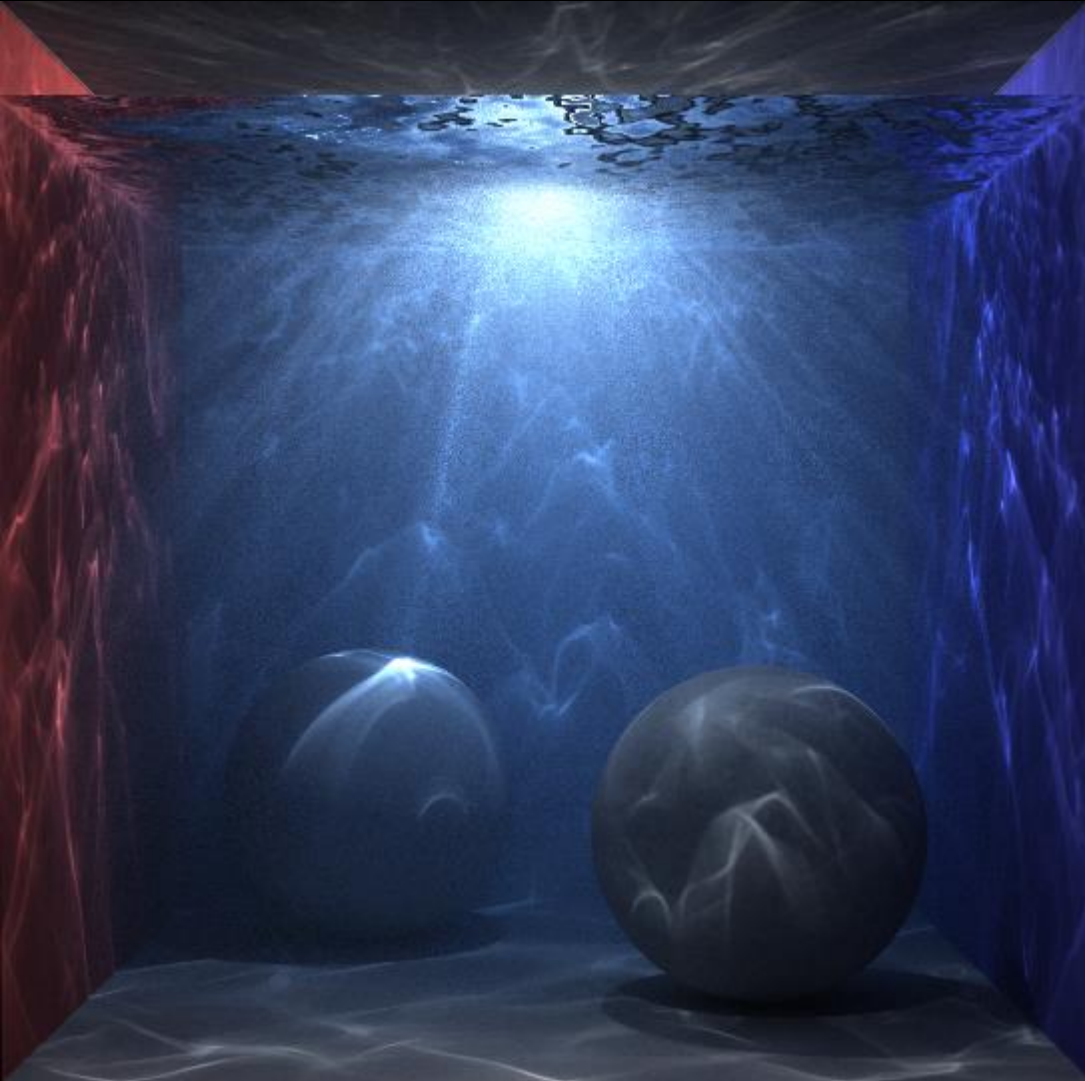
1 iteration

2 million photons



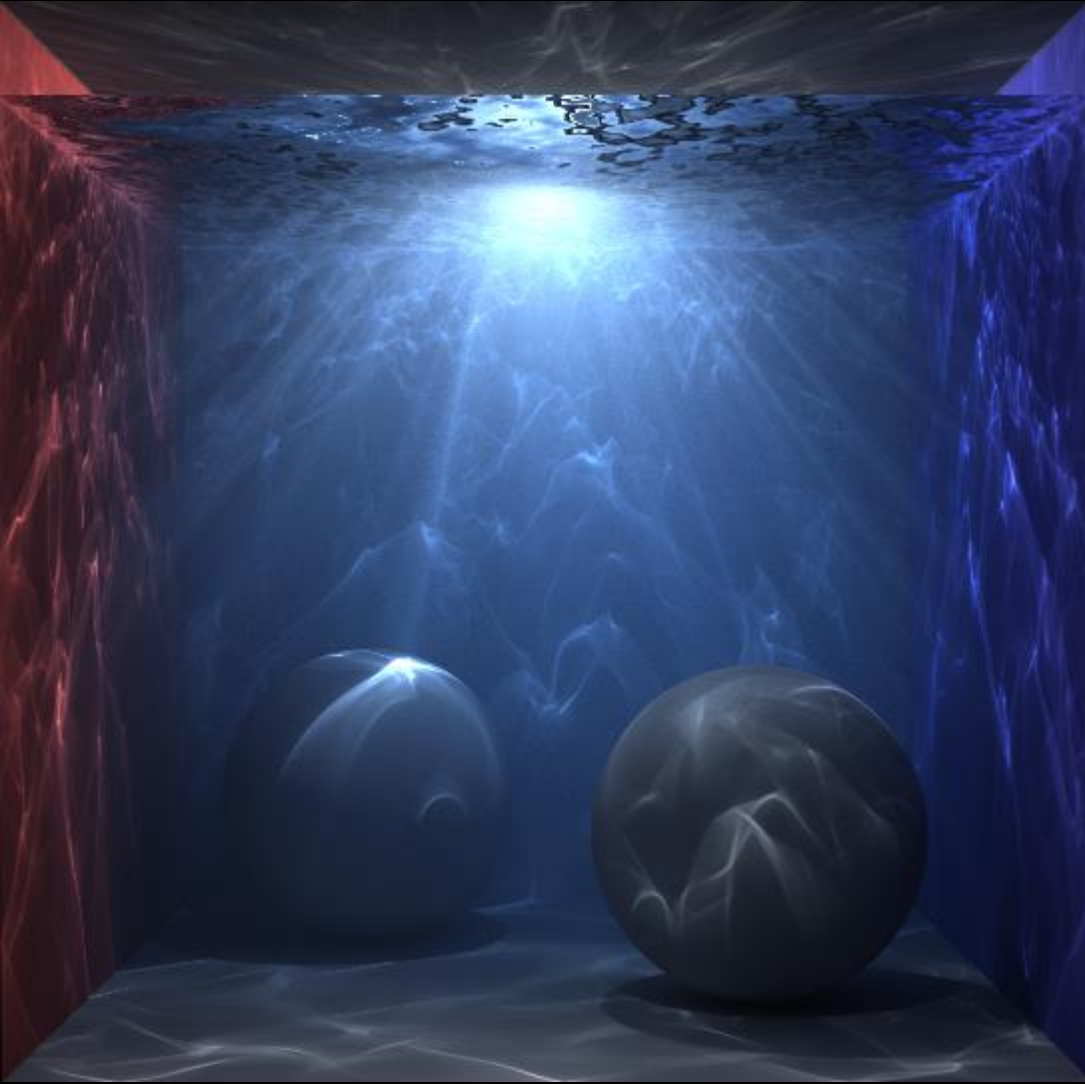
10 iterations

20 million photons



100 iterations

200 million photons



1000 iterations

2 billion photons

Conclusions

- Probabilistic analysis
- Asymptotic convergence
- No local statistics
- Parallelization
- Arbitrary kernels
- Participating media



Combining Photon Mapping and Bidirectional Path Tracing



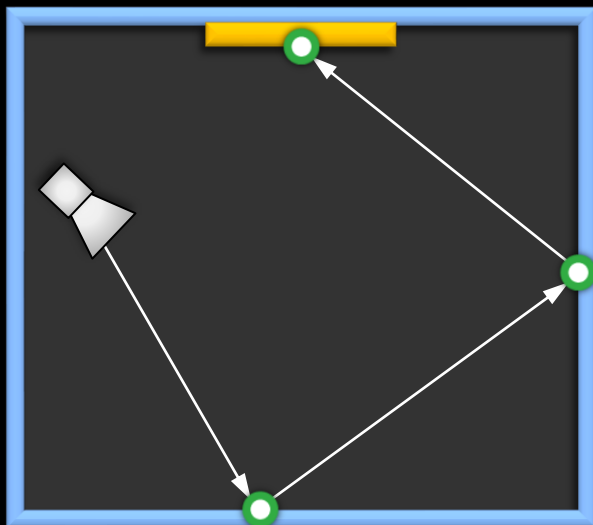
Bidirectional path tracing (30 min)



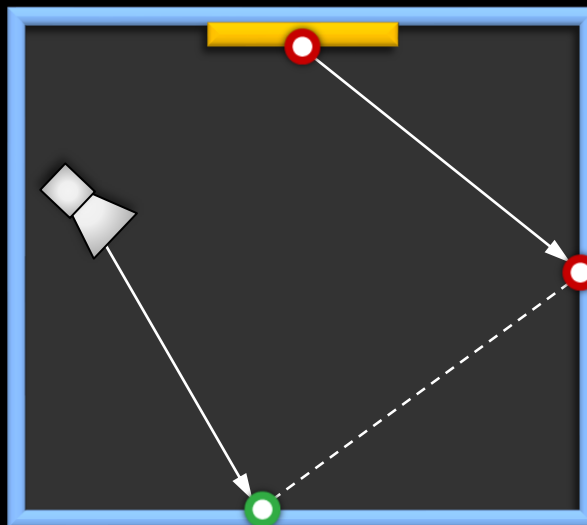
Stochastic progressive photon mapping (30 min)



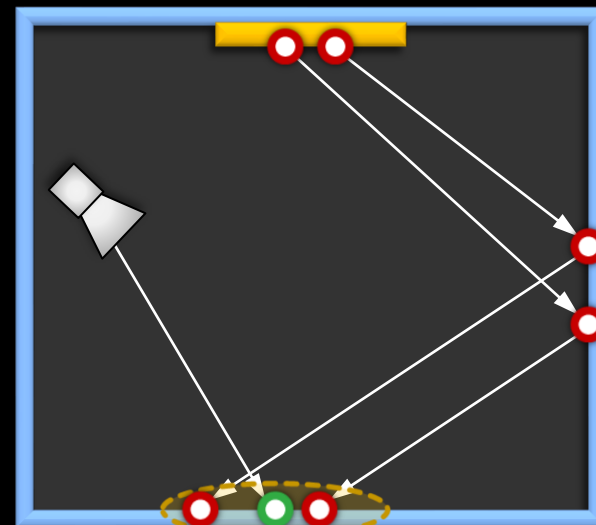
Combined algorithm (30 min)



Unidirectional sampling



Vertex connection



Density estimation

Bidirectional path tracing

Photon mapping

- ▶ **BPT & PM:** different solutions to the same problem
 - ▶ If we ignore bias in PM
- ▶ Want to combine
 - ▶ Best of both
 - ▶ Automatically
- ☹ Problem: Different mathematical frameworks
 - ▶ **BPT:** Monte Carlo integration
 - ▶ **PM:** Density estimation

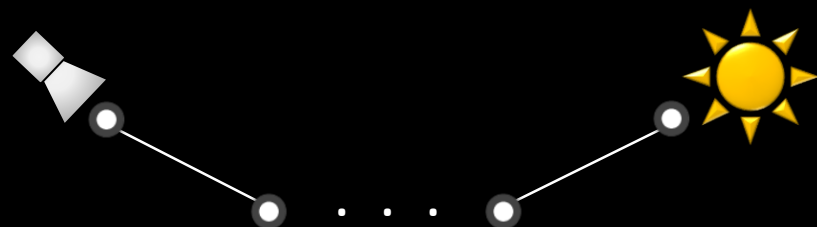
☹ Problem: Different mathematical frameworks

😊 Solution: Cast both in the same framework

- ▶ Path integral framework [Veach 1997]
- ▶ Multiple importance sampling
- ▶ New insight

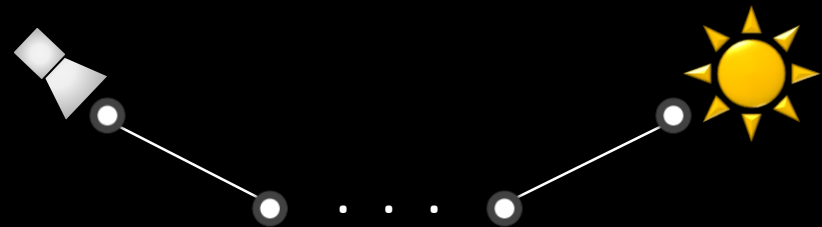
$$I_j = \int_{\Omega} f_j(\bar{\mathbf{x}}) d\mu(\bar{\mathbf{x}})$$

$$\langle I_j \rangle = \frac{f_j(\bar{\mathbf{x}})}{p(\bar{\mathbf{x}})}$$



- ▶ Multiple importance sampling [Veach and Guibas 1995]
 - ▶ Balance heuristic for n techniques

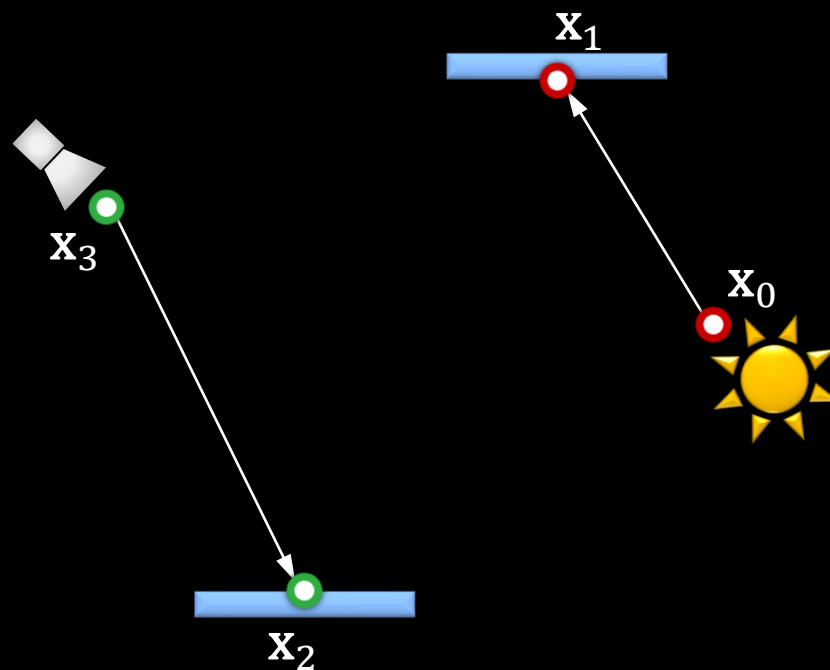
$$w_j(\bar{\mathbf{x}}) = \frac{p_j(\bar{\mathbf{x}})}{\sum_{k=1}^n p_k(\bar{\mathbf{x}})}$$



- ▶ Need to:
 - 1) Find a common definition of a **path**
 - ▶ In a common space
 - 2) Derive path **probability density function** (pdf)
 - ▶ With common units

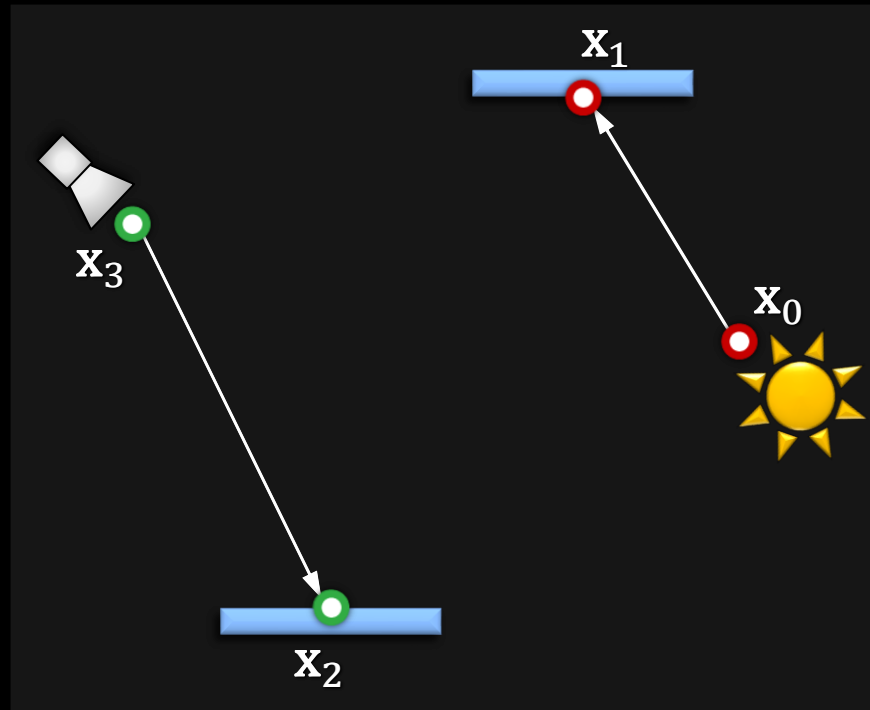
Bidirectional path sampling

- Light vertex
- Camera vertex



Bidirectional path sampling

- Light vertex
- Camera vertex

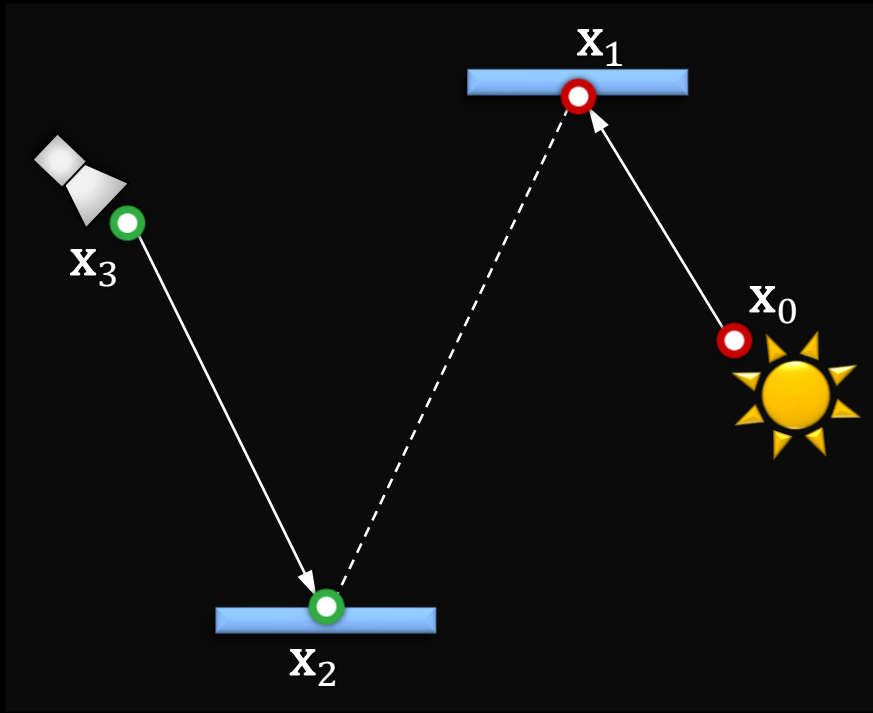


Bidirectional path tracing

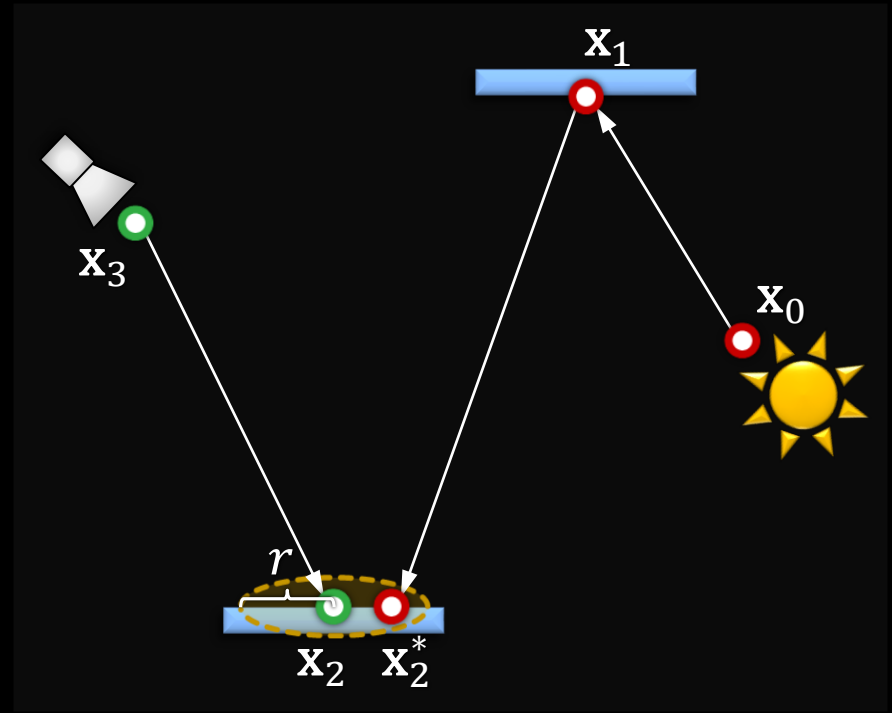
Photon mapping

Bidirectional path sampling

- Light vertex
- Camera vertex



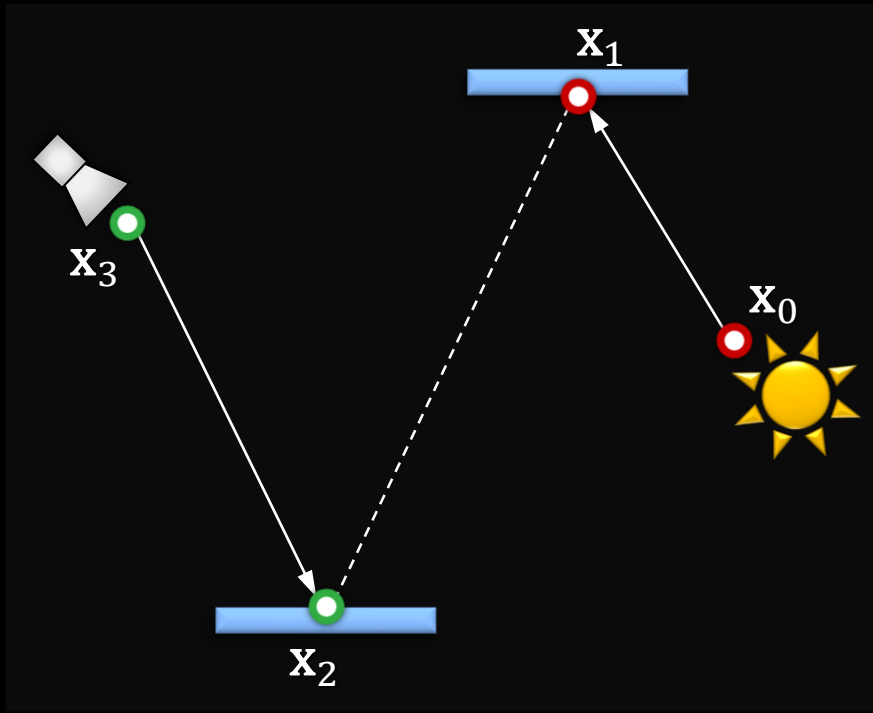
Bidirectional path tracing



Photon mapping

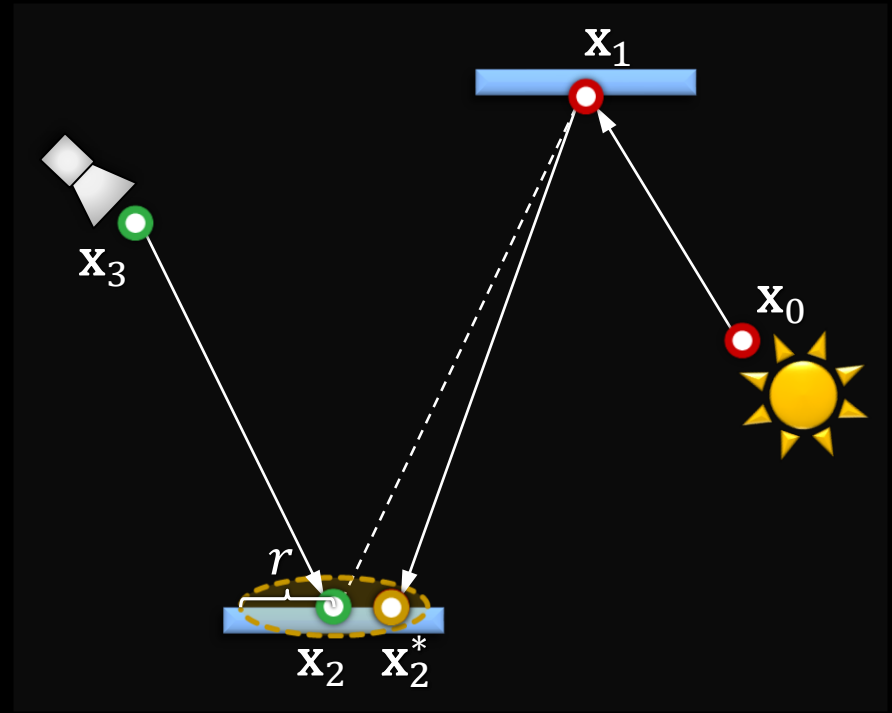
Vertex merging [Georgiev et al. 2012]

- Light vertex
- Camera vertex



Vertex connection

$$p_{VC}(\bar{x}) = p(x_0)p(x_0 \rightarrow x_1) p(x_3)p(x_3 \rightarrow x_2)$$

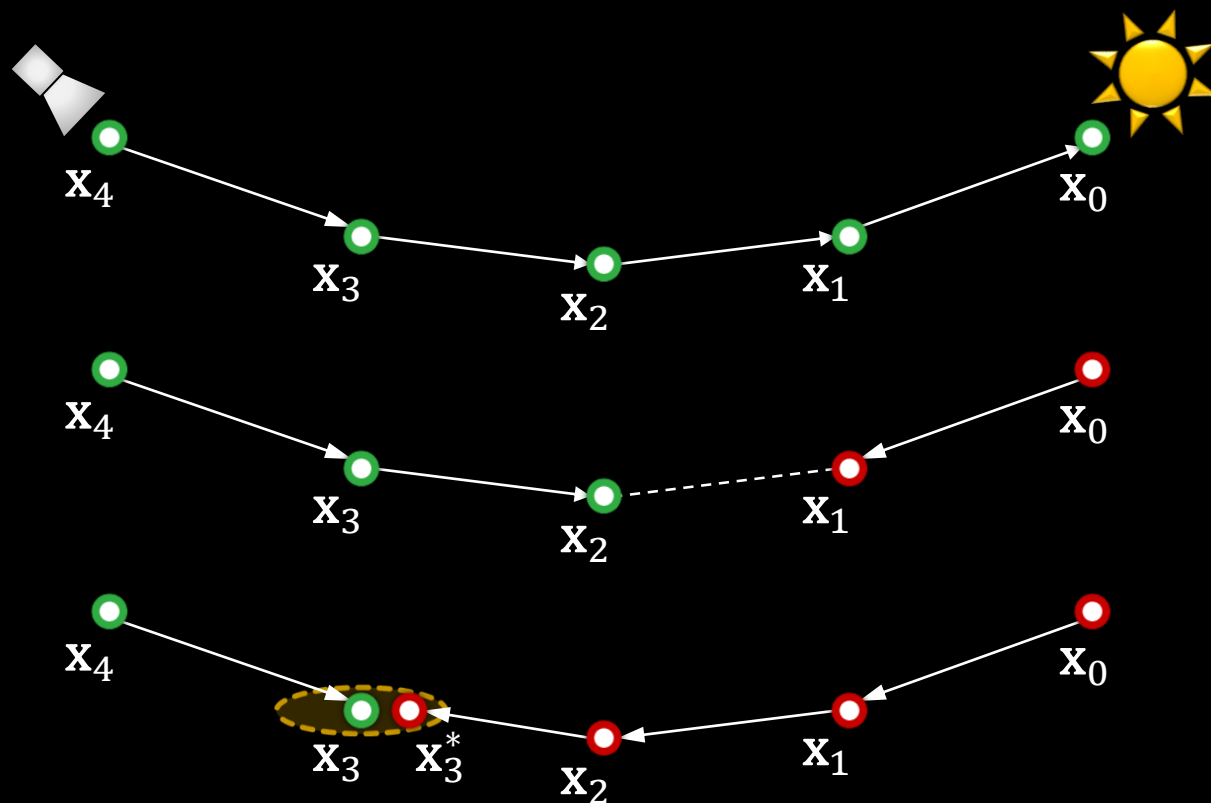


Photon mapping

$$p_{VM}(\bar{x}) \approx p(x_0)p(x_0 \rightarrow x_1) p(x_3)p(x_3 \rightarrow x_2) \int_{|r| \leq r} p(x_2 \rightarrow x_2^*)$$

Sampling technique summary

- Light vertex
- Camera vertex



Unidirectional 2 ways

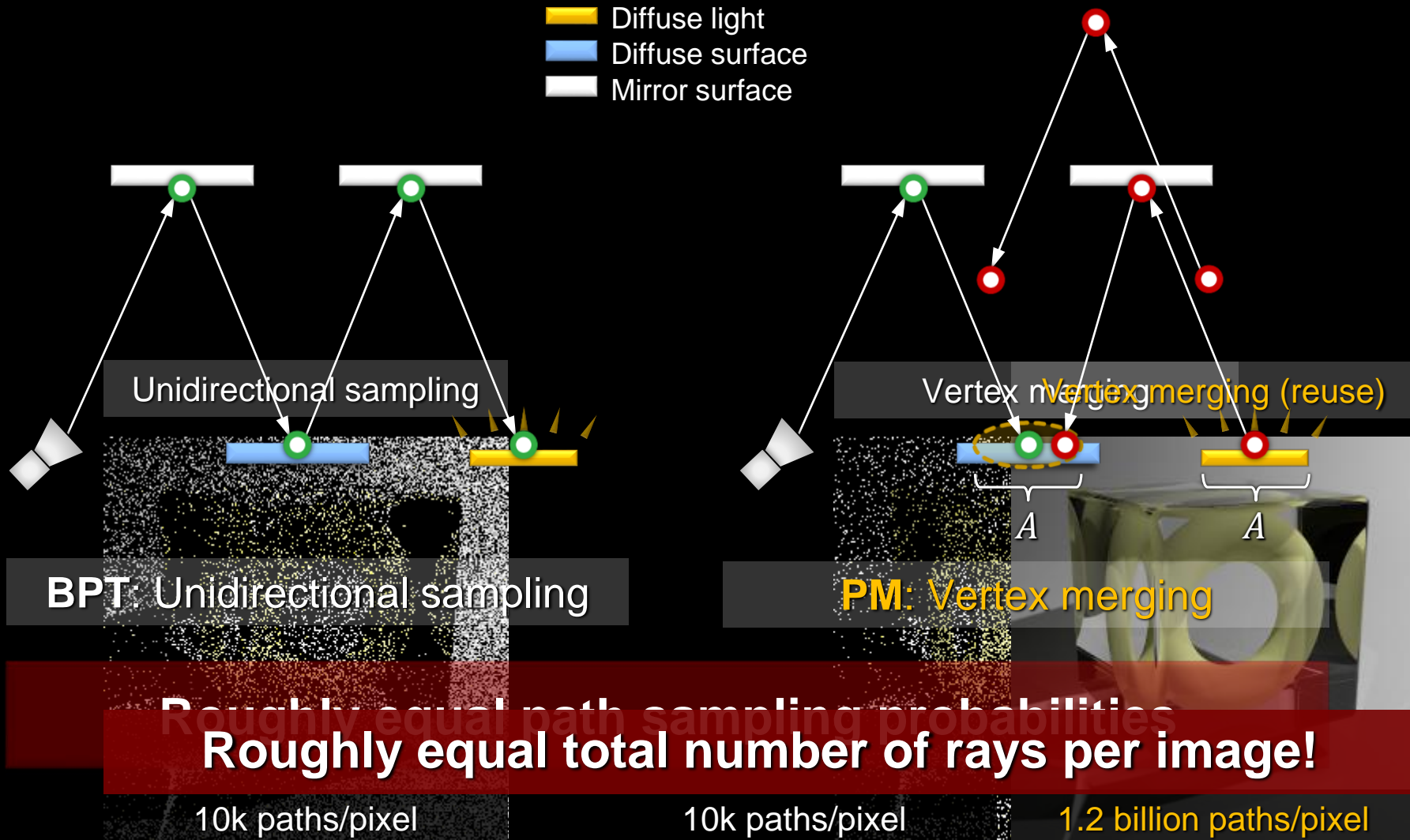
Vertex connection 4 ways

Photon mapping 5 ways

Total 11 ways

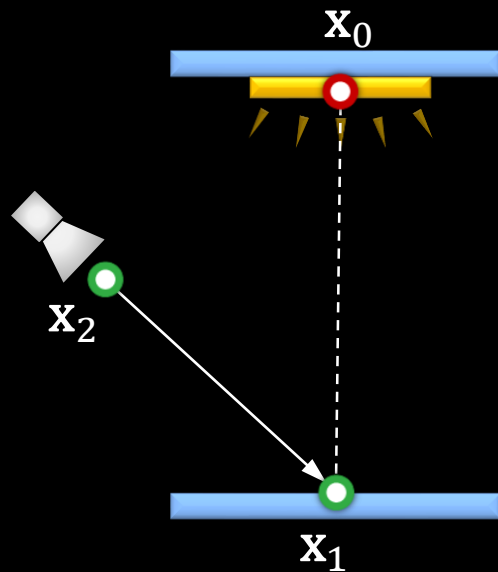
Technique comparison

- Diffuse light
- Diffuse surface
- Mirror surface

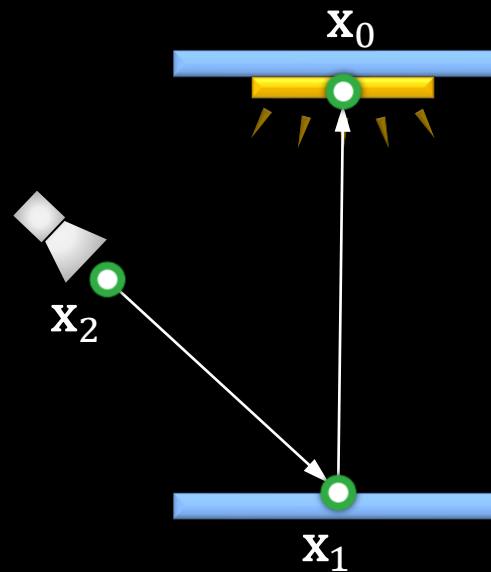


Technique comparison

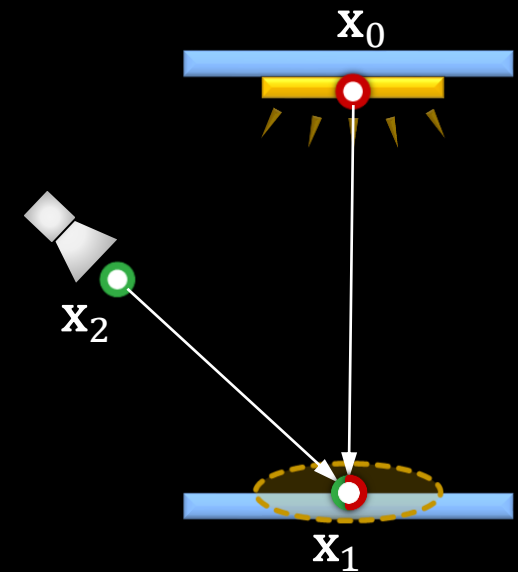
- Diffuse light
- Diffuse surface



Vertex connection
(VC)



Unidirectional
sampling (US)



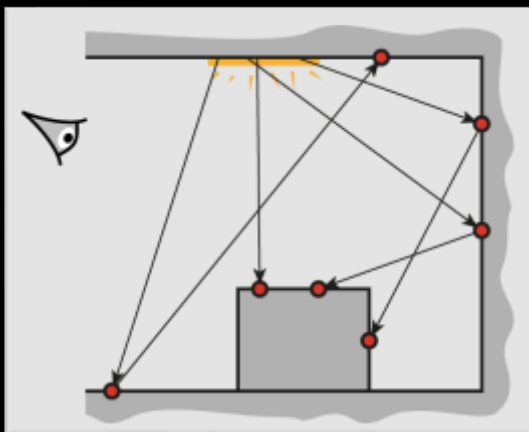
Vertex
merging (VM)

Roughly equal sampling densities

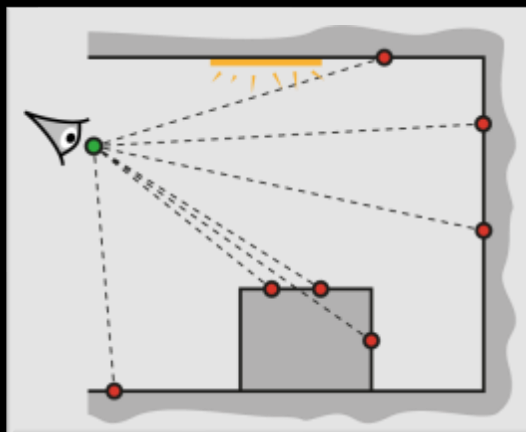
$$p_{US} \approx p_{VM} \approx \frac{p_{VC}}{100,000}$$

Combined algorithm

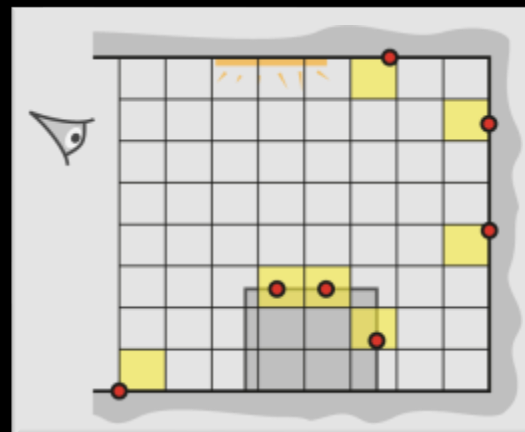
Stage 1: Light sub-path sampling



a) Trace sub-paths

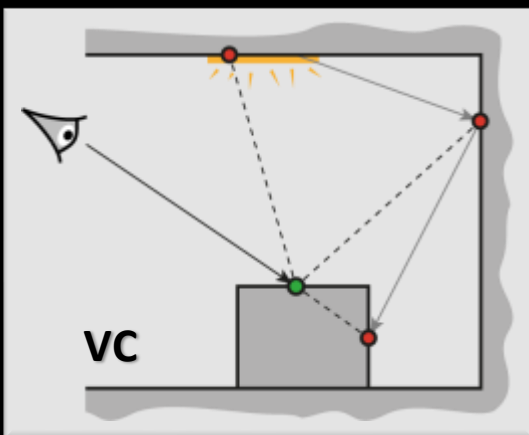


b) Connect to eye

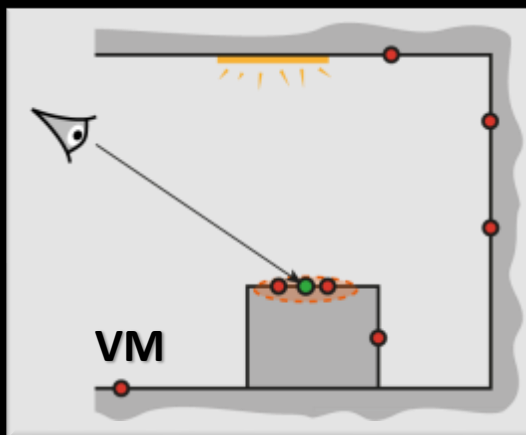


c) Build search structure

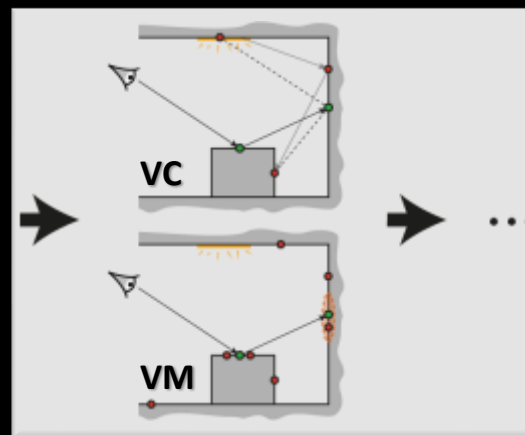
Stage 2: Eye sub-path sampling



a) Vertex connection



b) Vertex merging



c) Continue sub-path



Bidirectional path tracing (30 min)



Stochastic progressive photon mapping (30 min)



Combined algorithm (30 min)





Bidirectional path tracing (30 min)

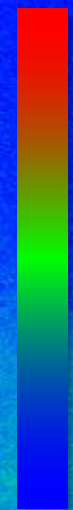


Stochastic progressive photon mapping (30 min)



Combined algorithm (30 min)

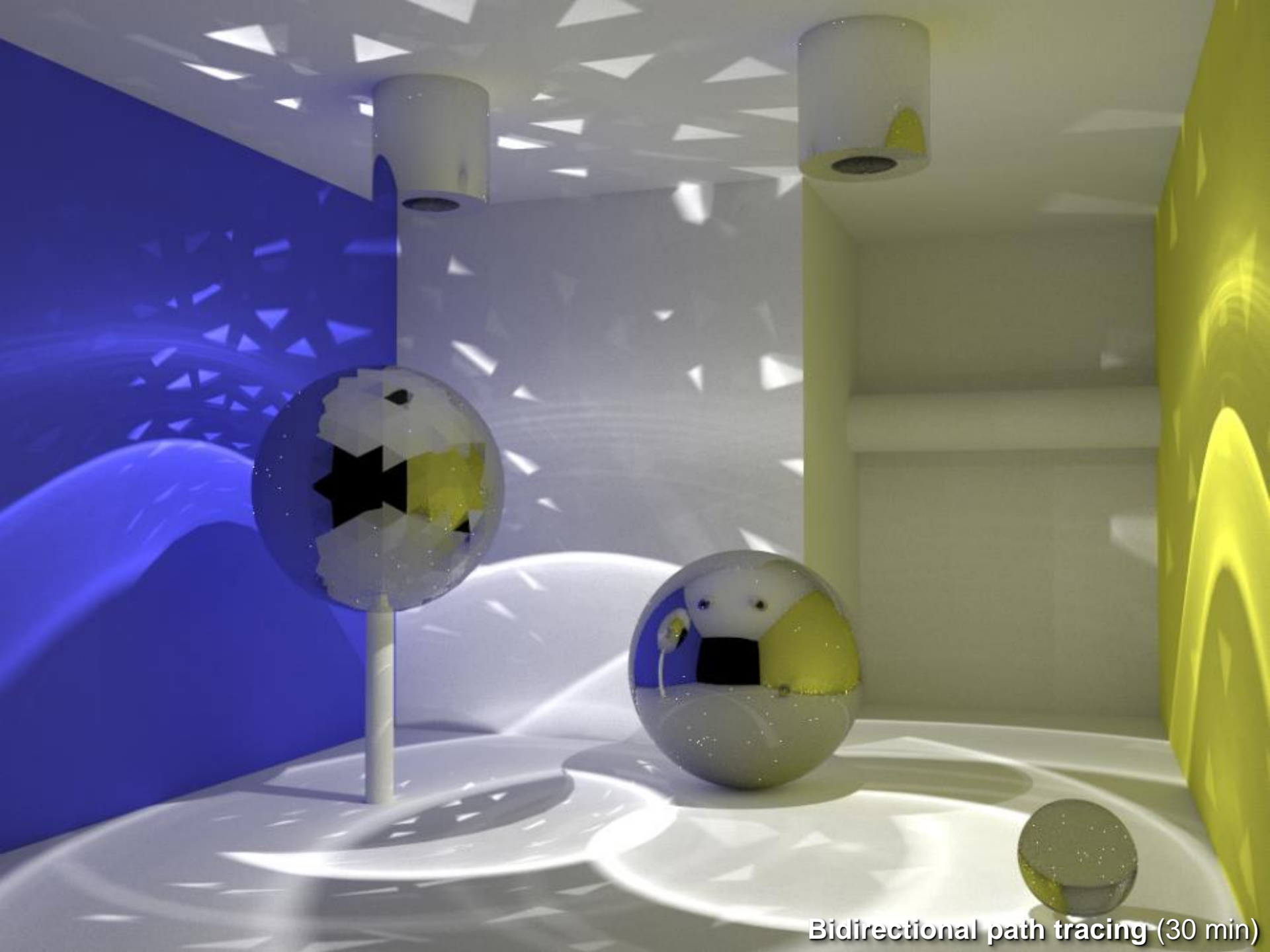
PM



BPT



Relative technique contributions



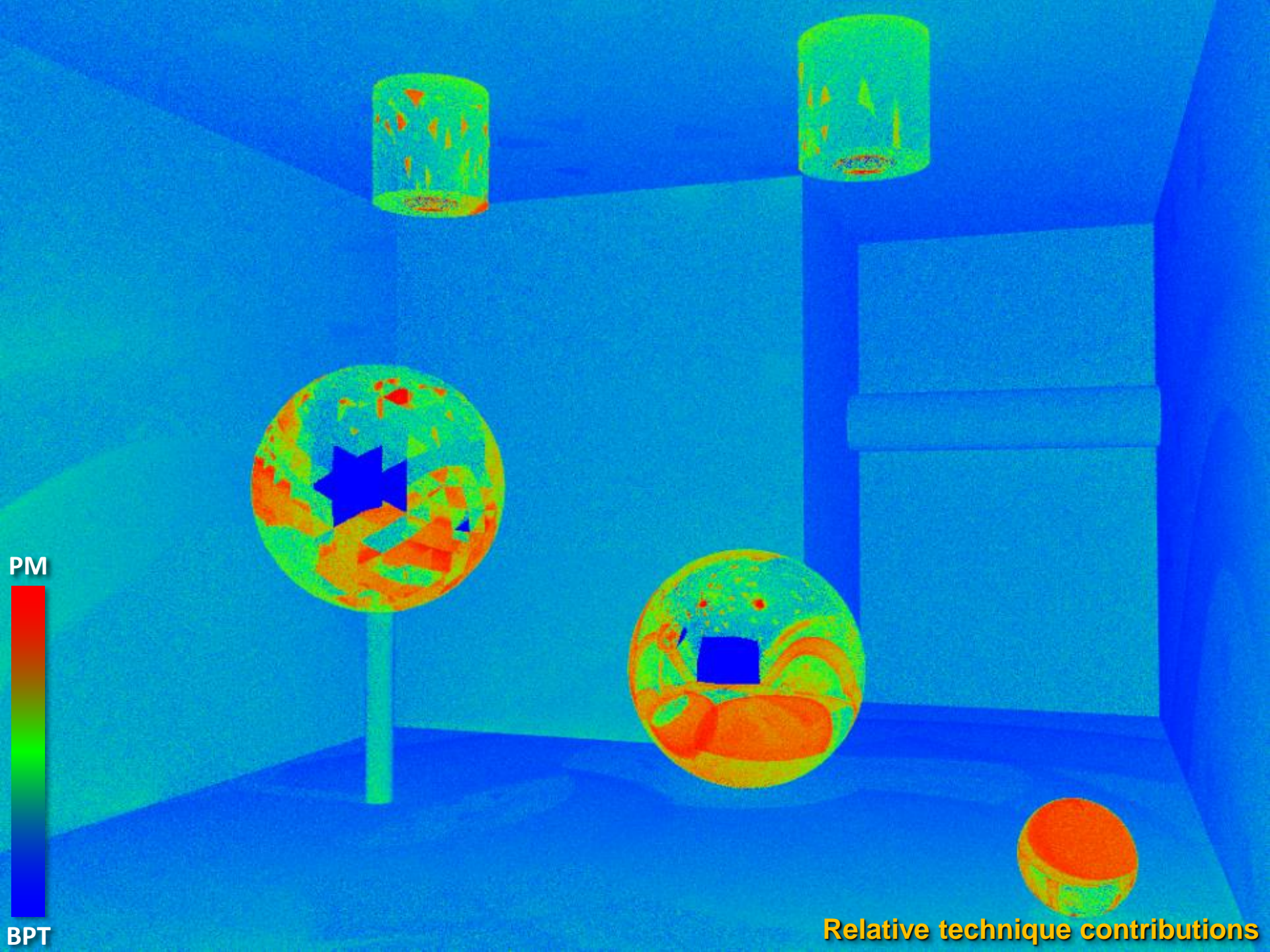
Bidirectional path tracing (30 min)



Stochastic progressive photon mapping (30 min)



Combined algorithm (30 min)



- ▶ **A path space extension for robust light transport simulation**
[Hachisuka et al. 2012]
 - ▶ Paper, supplemental analysis [\[http://cs.au.dk/~toshiya/\]](http://cs.au.dk/~toshiya/)

- ▶ **Light transport simulation with vertex connection & merging**
[Georgiev et al. 2012]
 - ▶ Paper, tech. report, image comparisons [\[http://www.iliyan.com\]](http://www.iliyan.com)

Wrap up

- ▶ Two approaches
 - ▶ Same result
- ▶ Error convergence
 - 👍 BPT: $O(N^{-0.5})$
 - 👎 PPM: $O(N^{-0.33})$
 - 👍 Combined: $O(N^{-0.5})$
- ▶ Remaining challenges

