Correlated error distribution

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This lecture

- Image space noise correlation using sampler
- Rendering setup
- Sample optimization before rendering
- Scene aware optimization

Reminders

Correlated sampling & image perception

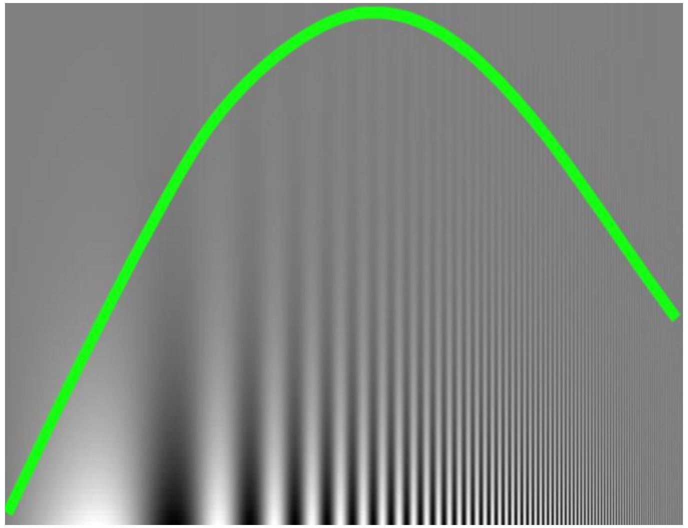
Correlated sampling

- Sample correlation can be used to reduce Monte Carlo error per pixel
  - Use smoothness of the function to distribute samples
  - Uniformly distribute Monte Carlo samples
  - Generally based on error upper bound (KH-inequality)

- Correlation rely on knowledge/assumption about the integration problem
  - Use the random process only for sub-part of the problem

Perception based error

- Mathematical error (MSE, RelMSE) consider all pixel independently
  - Measure the quality of the per pixel estimation
- Image perception is more important
  - All pixel seen at the same time
  - They are not independent
- We are more sensitive to some frequency than other
  - High frequency are naturally filterer by the eye



Motivation

Perceptual error distribution

Image dithering

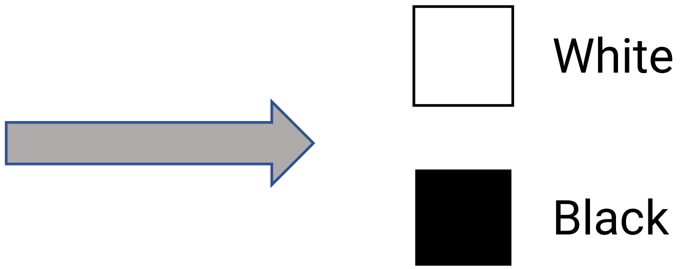


Image dithering

White if (input > dither)

Black else

Image dithering

Threshold

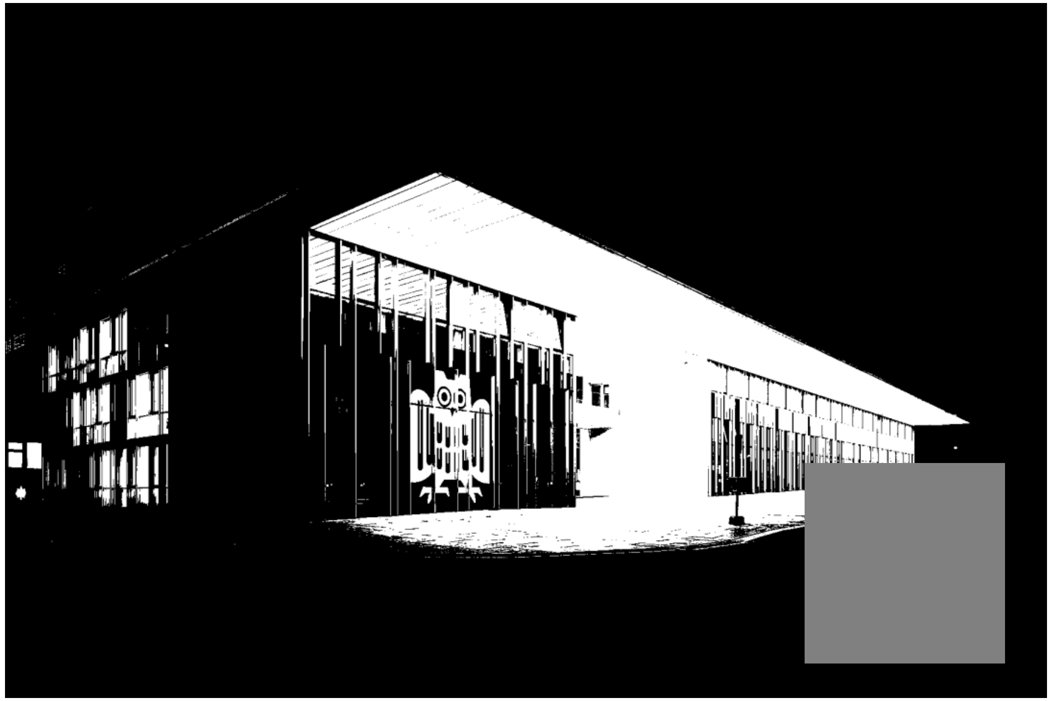


Image dithering

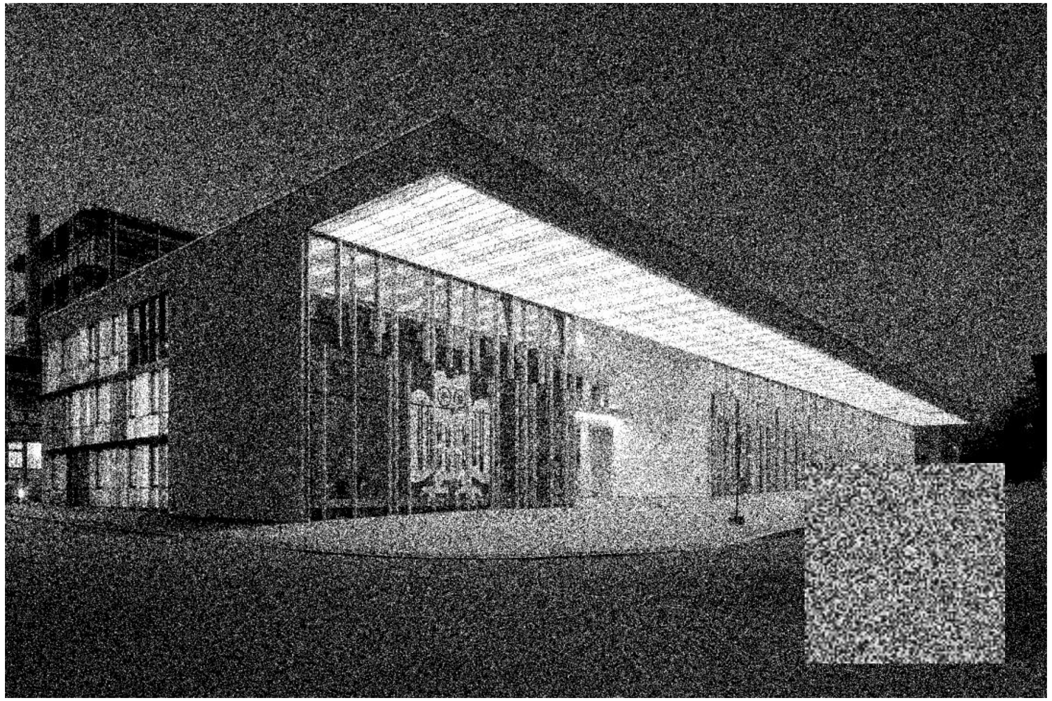


Image dithering

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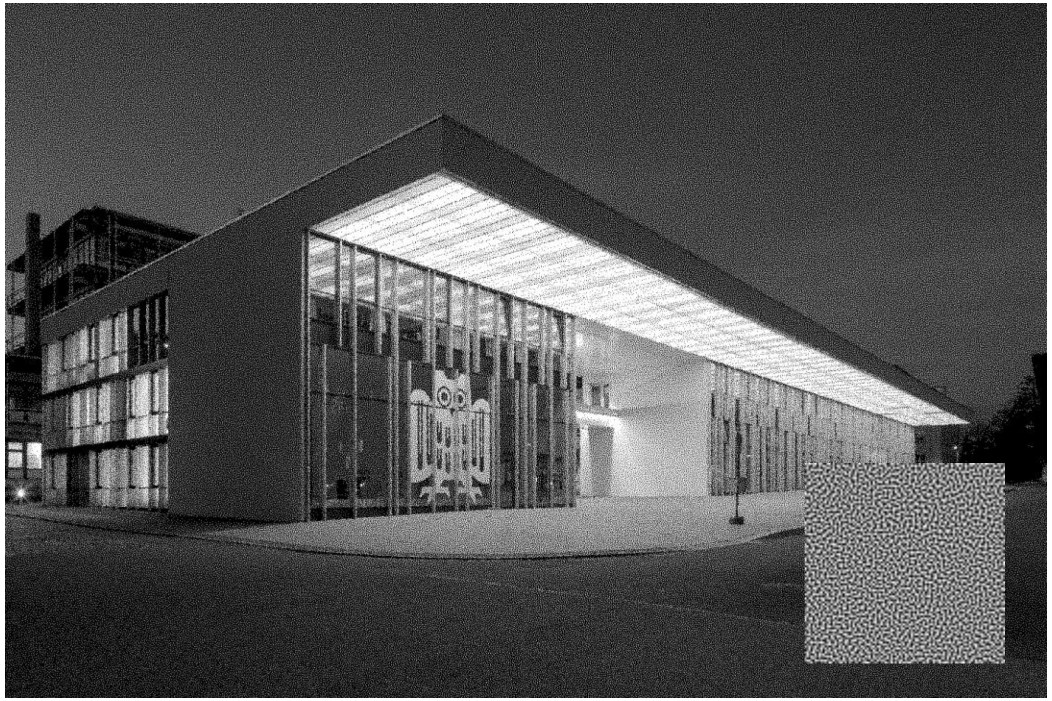

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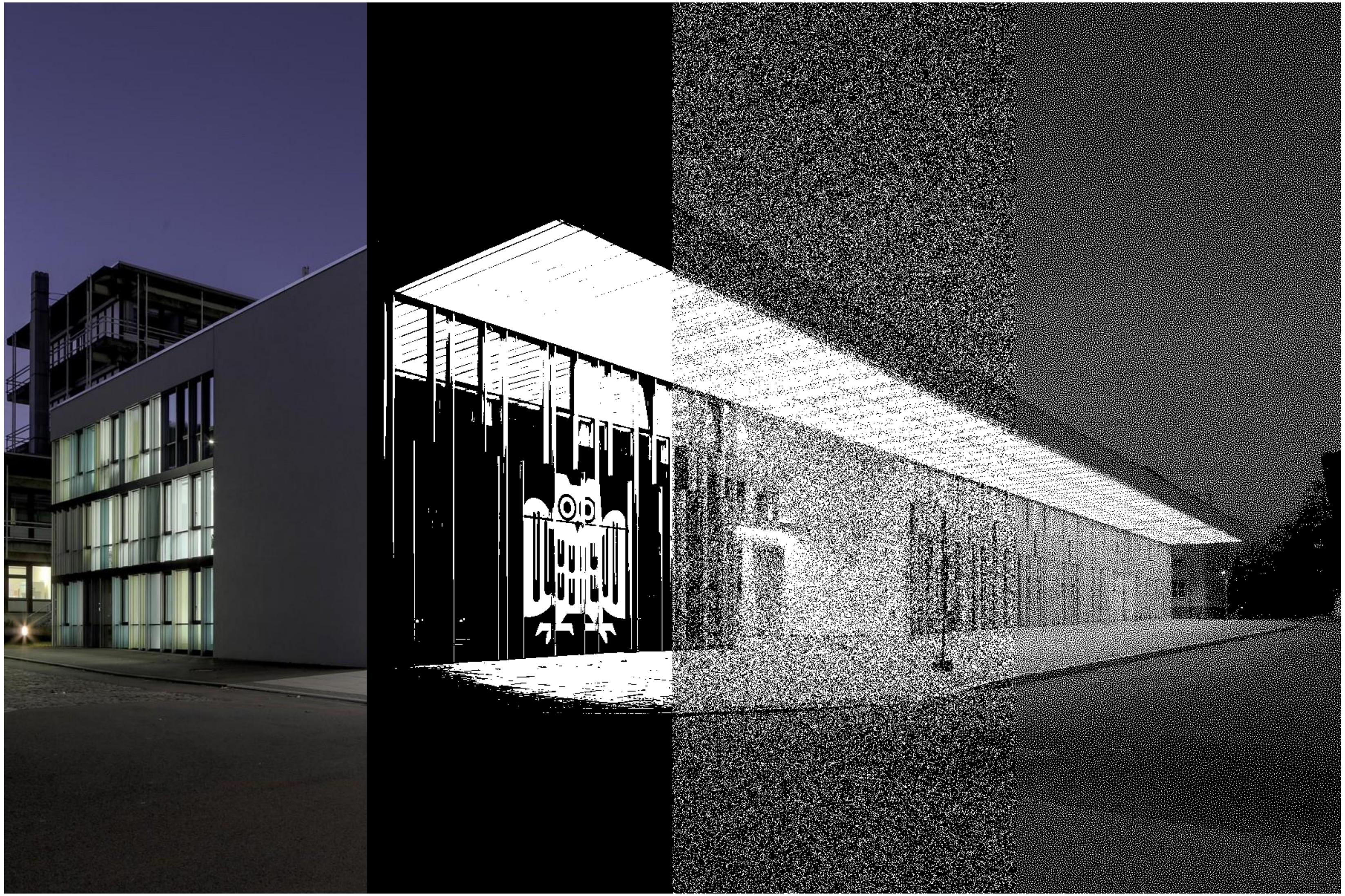
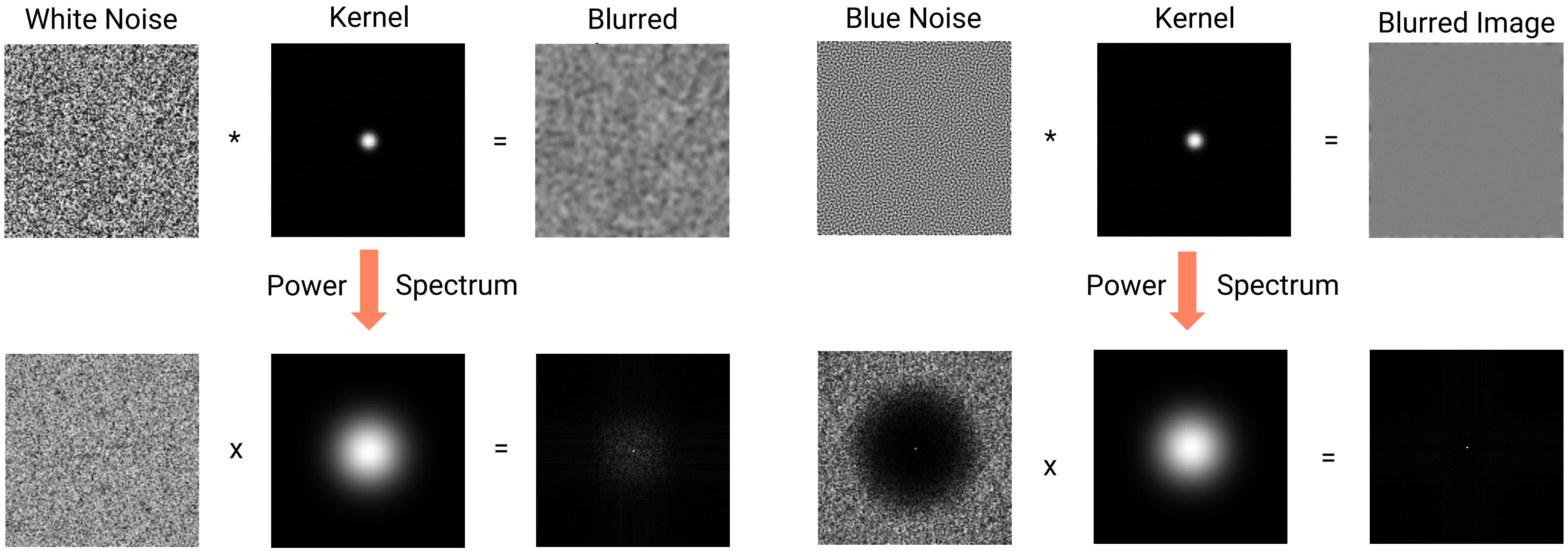


Noise filtering

- Denoising BN image produce less error
- Our eye works as a low pass filter



Better perceptually and numerically!

Dithering for rendering

Consider a rendering where all pixel are simulated independently

- A pixel value only depend on the sample of that pixel
  - Works for forward rendering (pathtracing, direct lightning, …)
  - Doesn’t work for backward, mixed rendering (BDPT, … as light traced samples contribute to all pixel)

- Close pixel render similar function (smooth scene)
- Samples can be interpreted as sequence
  - Generalize to good sample sequence (Rank1, PMJ02, Sobol, …)

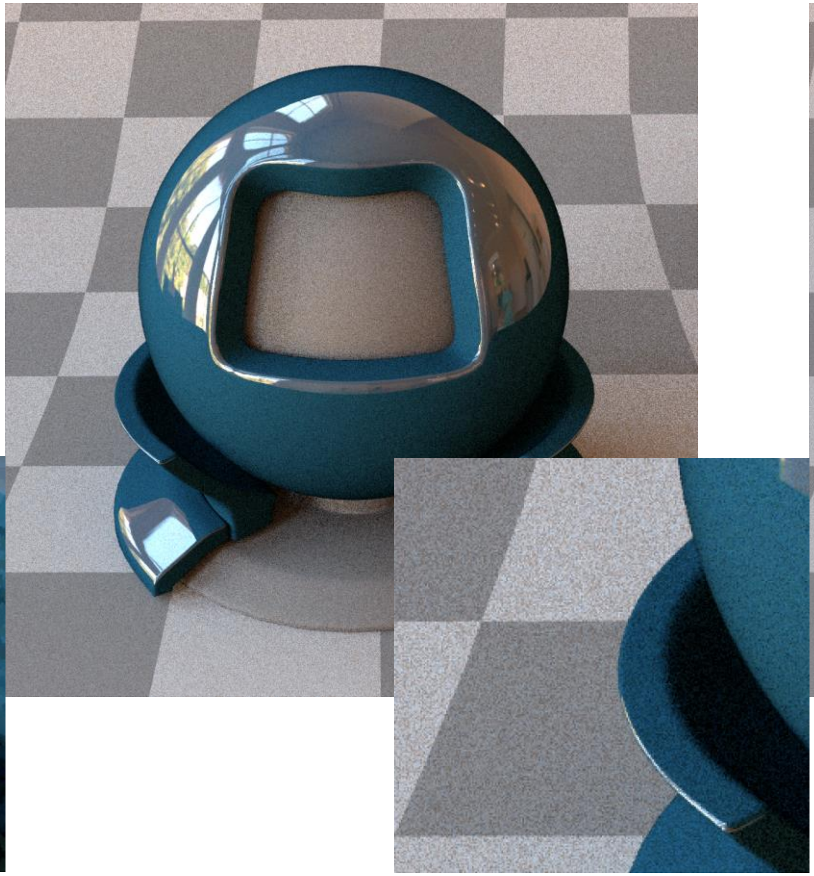
- Is it possible to optimize the sample distribution to improve rendering ?
  - For mean squared error ? No
  - For perception ? Yes

Dithering for rendering

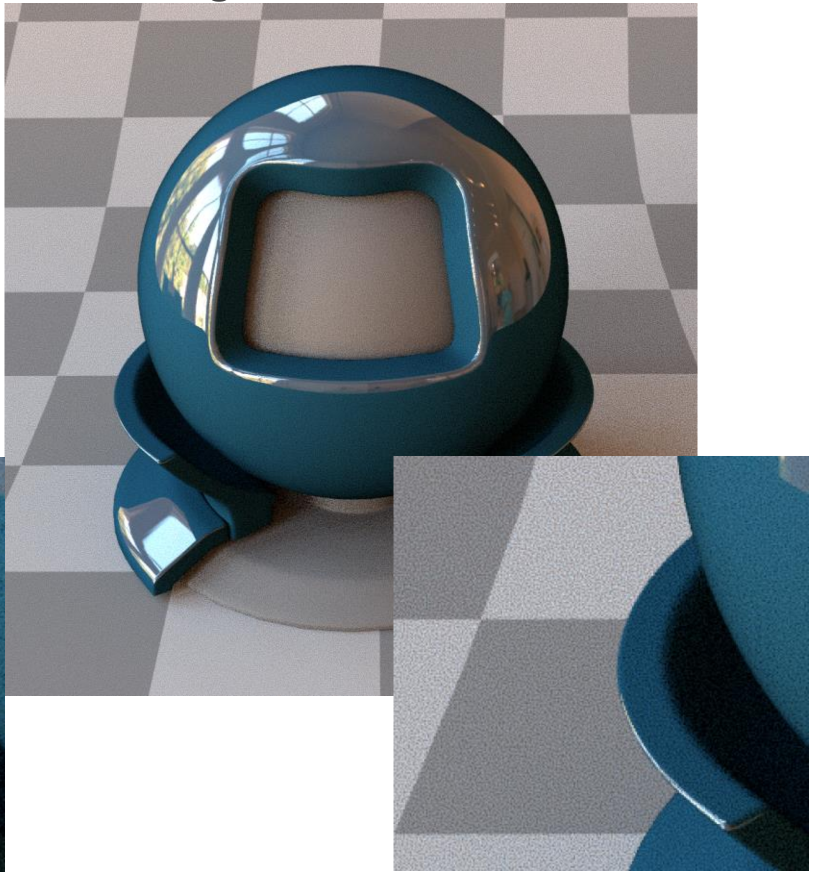
- Can we correlate pixel integration on image plane ?

Positive correlation

No correlation



Negative correlation



Dithering for rendering

What is the desired correlation ?

- Negative correlation is the best correlation
  - It’s based on HVS sensitivity to low frequency
  - High frequency/negative correlation are “blurred” by human eye

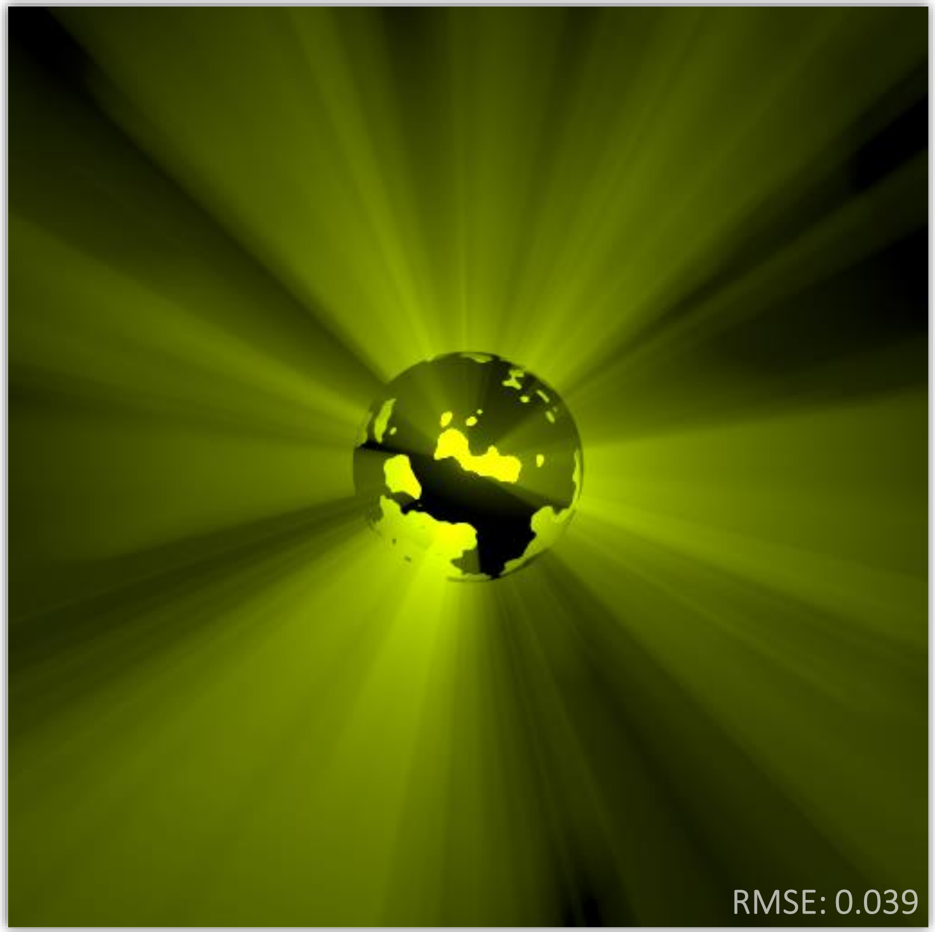
- Positive correlation create artifacts
  - Less noise but some color splat

- Uniform correlation over the image plane
  - Having multiple correlation or quality make them visually unpleasant
  - This looks unnatural

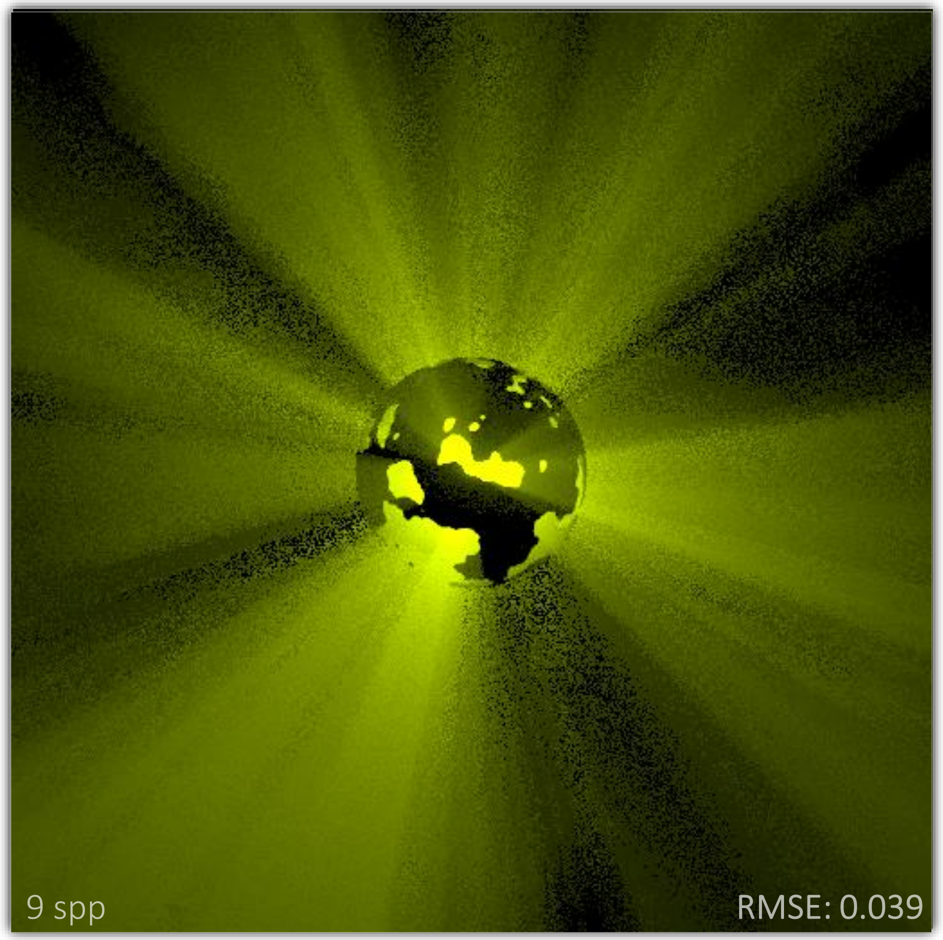
Blue-noise Dithered Sampling

[Georgiev & Fajardo 2016]

Blue noise error distribution



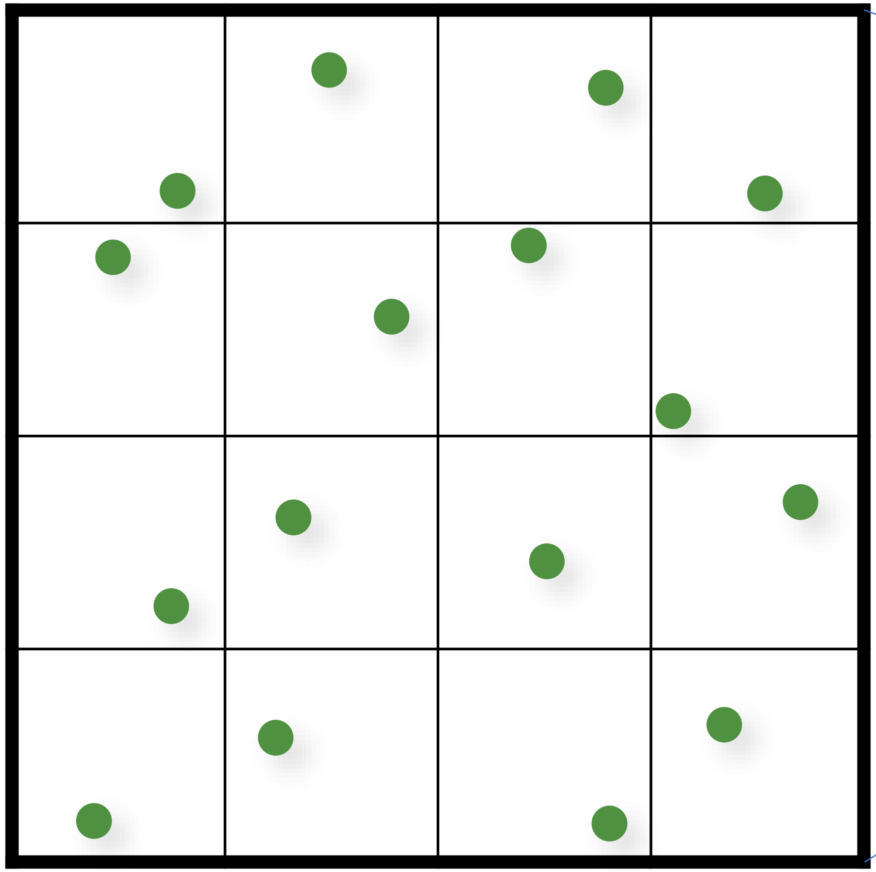
white-noise pixel error distribution

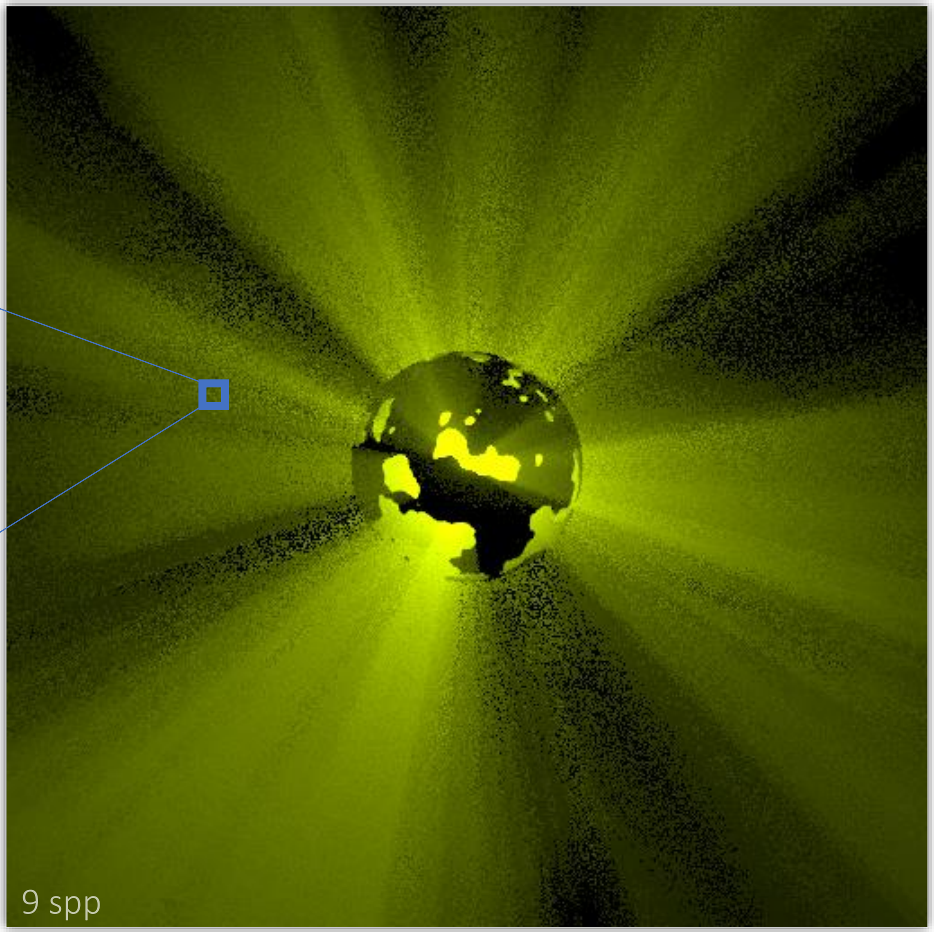


blue-noise pixel error distribution

Blue noise error distribution

Stratified sampling





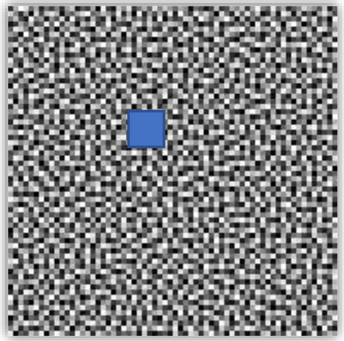
Objectif

- Having similar samples will result in close rendering value
  - Function is smooth

- To get as much different value we want as much different samples as possible
  - While keeping good per pixel quality
  - We expect a correlation between sample distribution and estimate distribution

Sampling with dither masks

1. Create sampling pattern
2. For each pixel
   a. Look-up value from dither mask (tiled over the image)
   b. Use value to offset sample pattern



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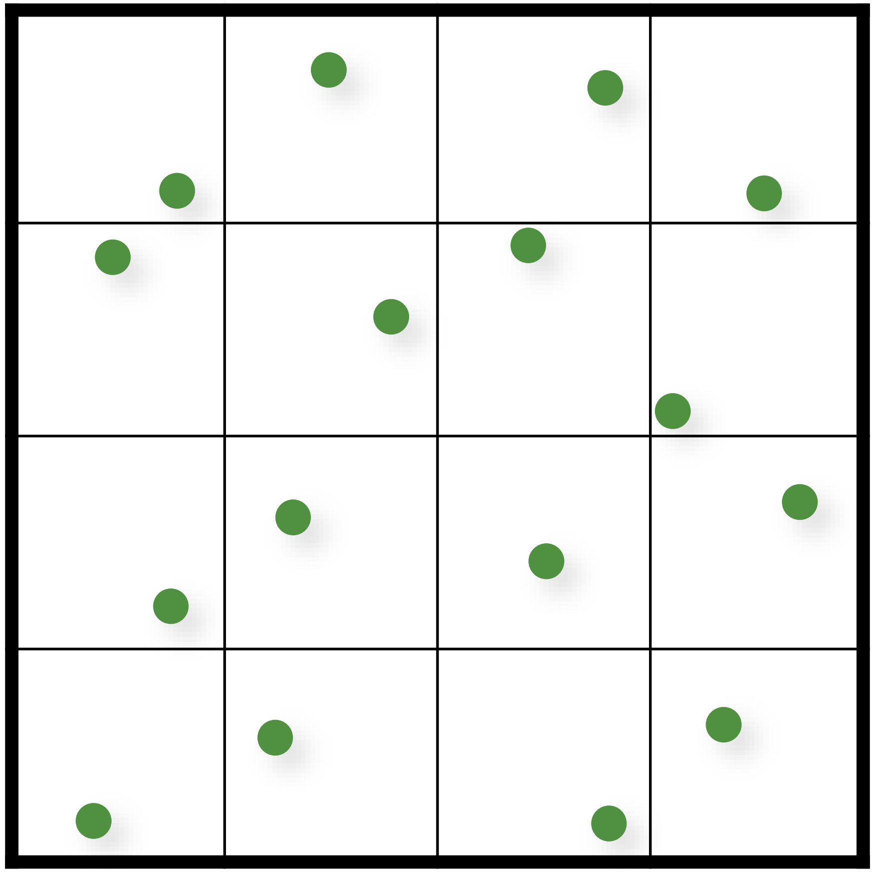

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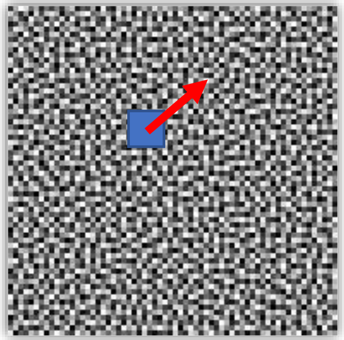

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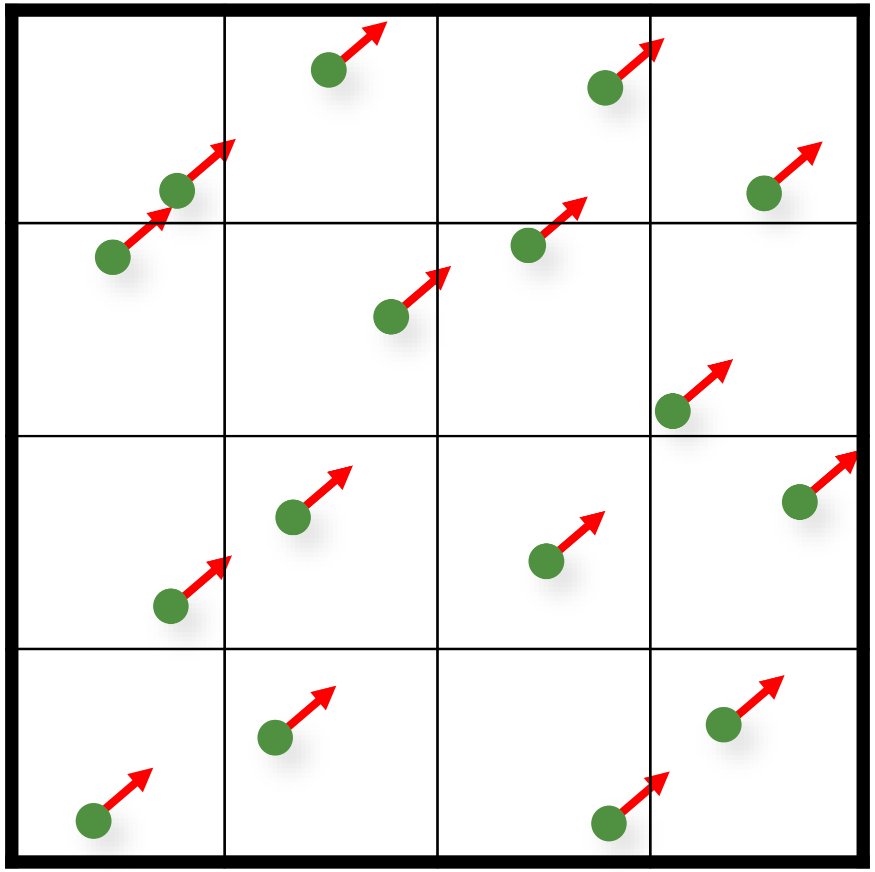

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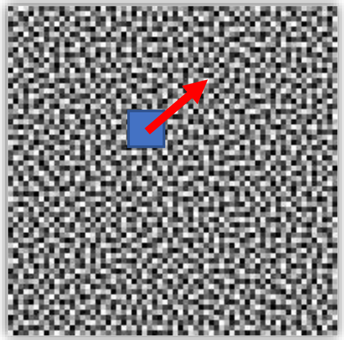

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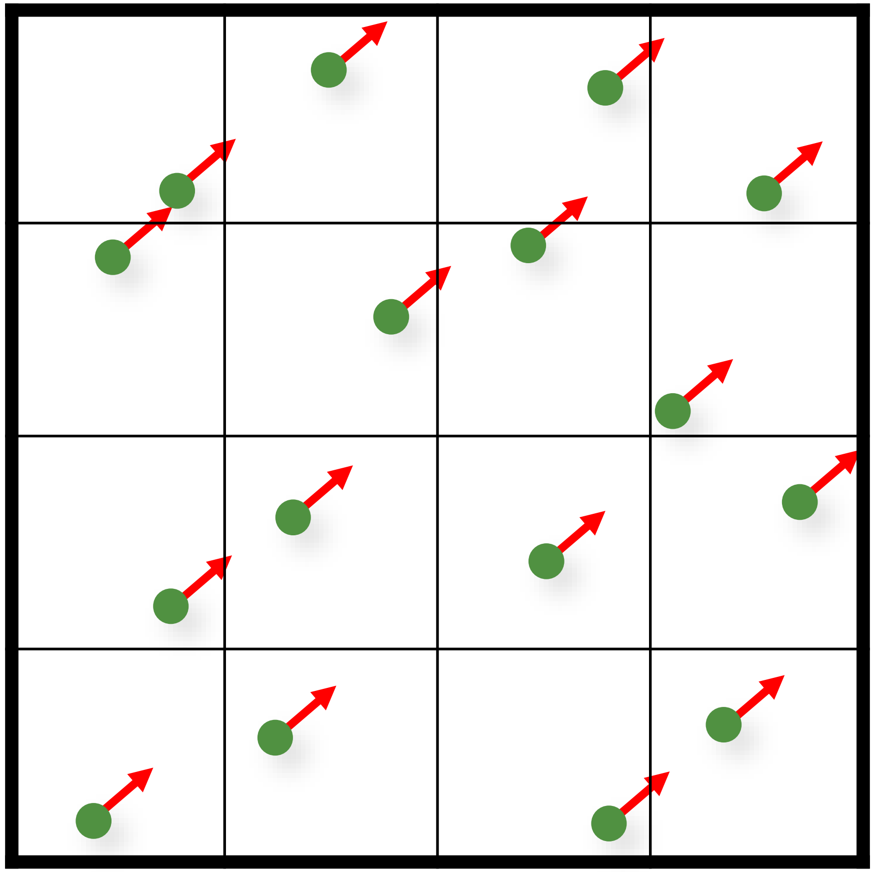

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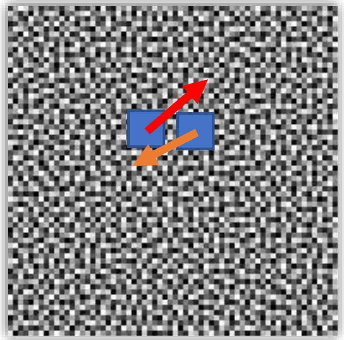

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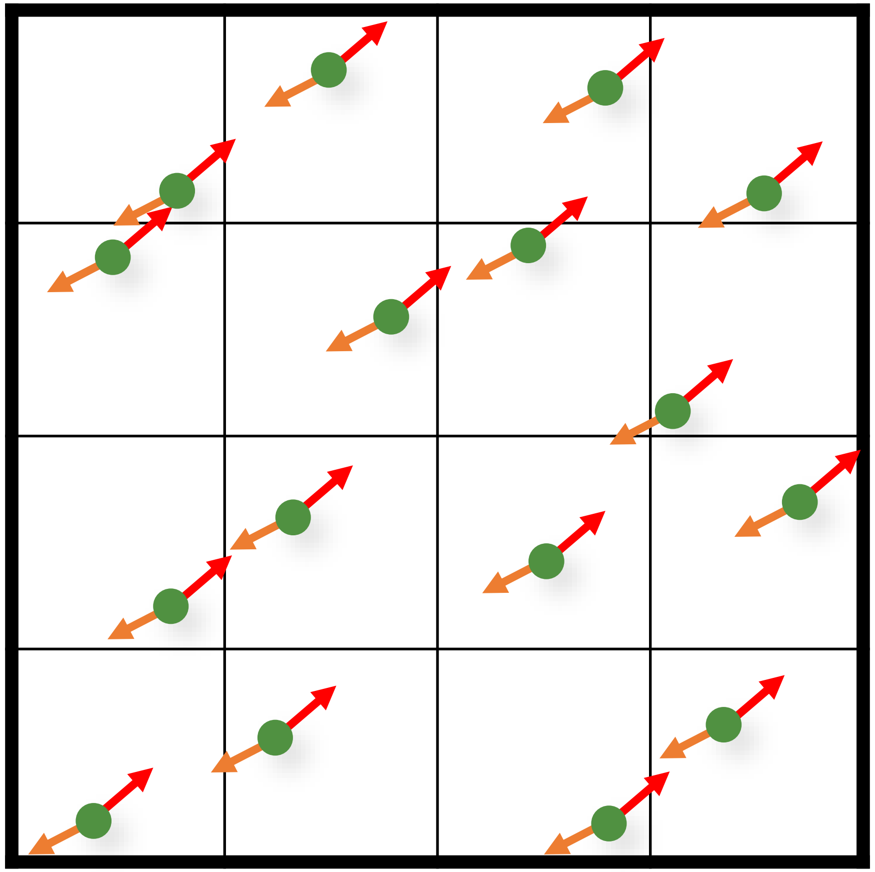

Sampling with dither masks

- Dithered sampling ensure similar offset will result in similar sample-set
  - Similar samples will produce similar rendering if the functions are similar
  - Shift can be added directly inside the renderer
  - Dither mask is a new input to the renderer

- Negative pixel correlation
  - Shift map should be as different as possible
  - Shift map should be “unbiased”
  - It’s difficult to optimize it using SGD based method

Simulated annealing

- Optimize shift map by swapping pixels
  - Initialize with random shift map
  - Try swapping pixel to improve some energy
  - Iterate

- The dither mask can be optimize on a small scale and tile over the image plan
  - Need a toroidal optimization
  - Can create some visual artifact as the noise pattern repeat

Simulated annealing

Pseudo code

$I \leftarrow \mathrm{RandomInitialization}()$
for $k \in \{0, k_{max}\}$ **do**
  $p_x, p_y \leftarrow \mathrm{RandomPixel}()$
  $e \leftarrow \boxed{E(I)}$
  $I_{new} \leftarrow \mathrm{swap}(I, p_x, p_y)$
  $e_{new} \leftarrow \boxed{E(I_{new})}$
  if $e < e_{new}$ and $exp(\frac{e - e_{new}}{c_k}) < \mathrm{rand}(0,1)$ **then**
    $I_{new} \leftarrow \mathrm{swap}(I_{new}, p_x, p_y)$
  end if $I \leftarrow I_{new}$
end for
Return I

Energy term

$$E(I) = \underbrace{\sum_{p \neq q}}_{\text{pixels}} E(p,q) = \sum_{p \neq q} \underbrace{e^{-\frac{\overbrace{\|i_p - i_q\|^2}^{\text{pixel distance}}}{\sigma_i^2}}}_{\text{image-space Gaussian}} \cdot \underbrace{e^{-\frac{\overbrace{\|s_p - s_q\|^{d/2}}^{\text{sample distance}}}{\sigma_s^2}}}_{\text{sample-space Gaussian}}$$

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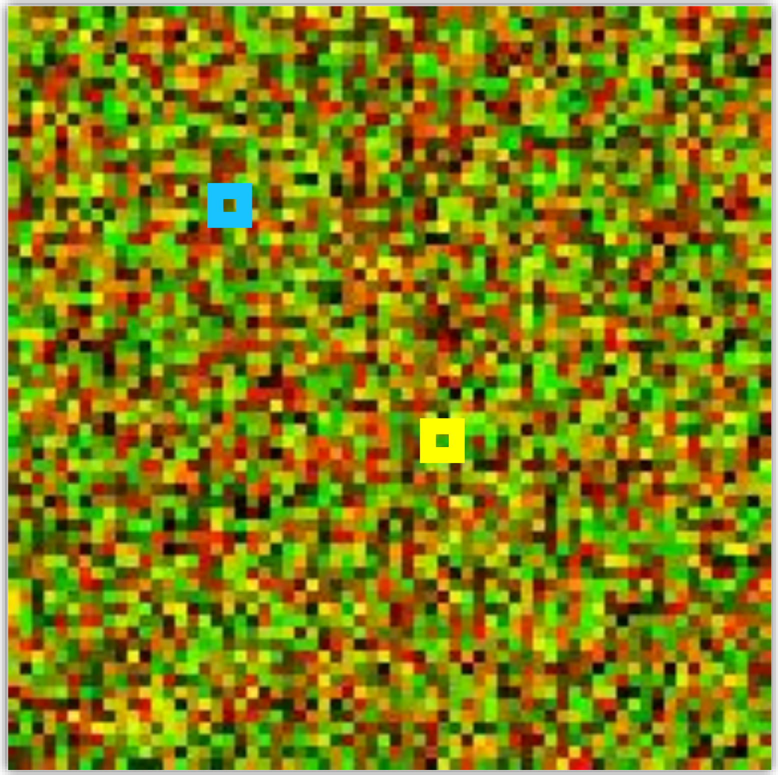

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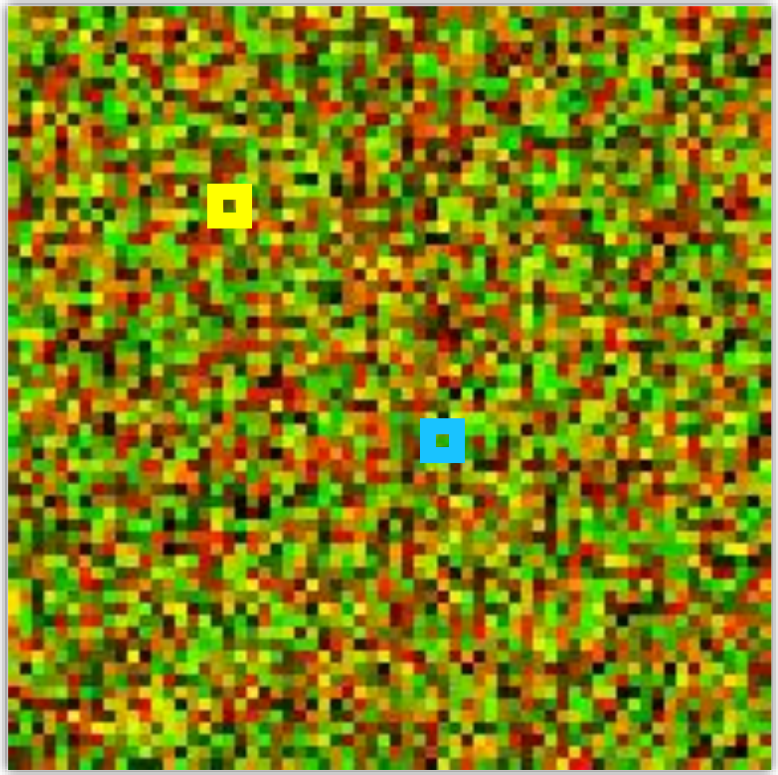


Live demo

Simulated annealing

The sum over every pixel can be simplified

- Spatial gaussian support can be restricted to few pixel
- It is possible to optimize different region of the tile at the same time
- Quality will depend on the approximation of the gaussian

Energy term

$$E(I) = \underbrace{\sum_{p \neq q}}_{\text{pixels}} E(p,q) = \sum_{p \neq q} \underbrace{e^{-\frac{\|i_p - i_q\|^2}{\sigma_i^2}}}_{\substack{\text{image-space} \\ \text{Gaussian}}} \cdot \underbrace{e^{-\frac{\|s_p - s_q\|^{d/2}}{\sigma_s^2}}}_{\substack{\text{sample-space} \\ \text{Gaussian}}}$$

Simulated annealing

Simulated annealing is an importance optimization algorithm

- Works on discreet set (Set of pixel)
- Energy function requires only a point-wise evaluation (no derivative)
- Can converge to the optimal solution under some conditions

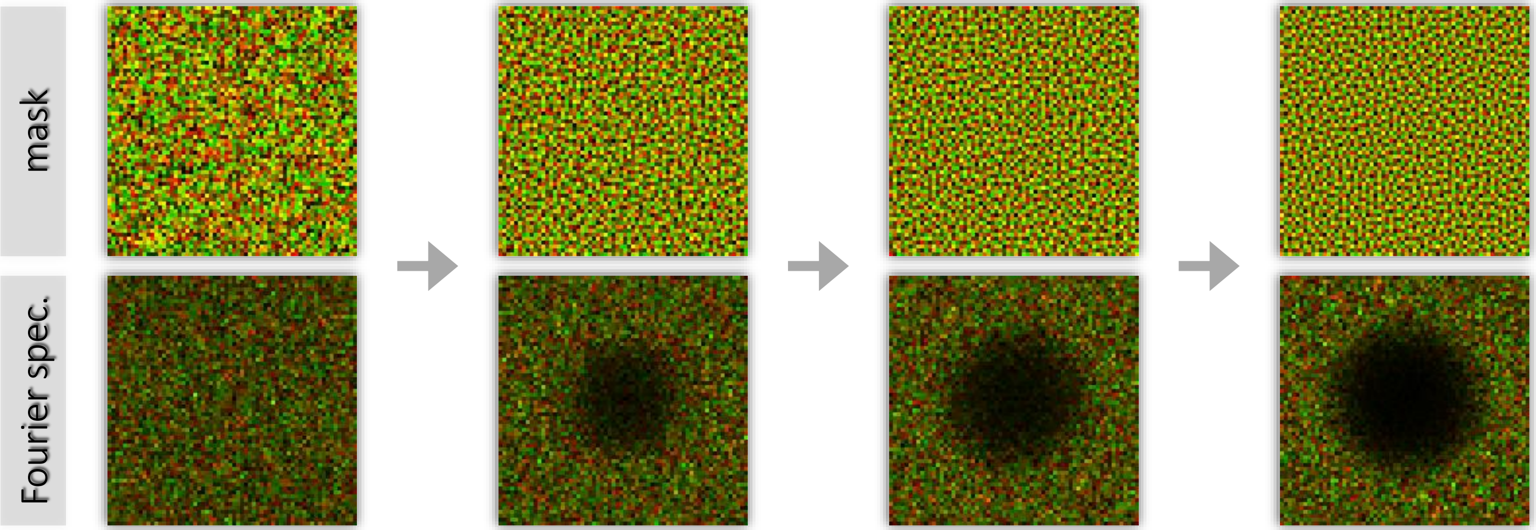
- Slow optimization in general
- Difficult to parallelize on a massive scale

Dither mask construction

1. Generate random dither mask

2. Simulated annealing by random pixel swapping

$$E(M) = \underbrace{\sum_{p \neq q}}_{\text{pixels}} E(p,q) = \sum_{p \neq q} \underbrace{e^{-\frac{\overbrace{\|i_p - i_q\|}^{\text{pixel distance}}{}^{2}}{\sigma_i^2}}}_{\text{image-space Gaussian}} \cdot \underbrace{e^{-\frac{\overbrace{\|s_p - s_q\|}^{\text{sample distance}}{}^{d/2}}{\sigma_s^2}}}_{\text{sample-space Gaussian}}$$



Sampling with dither masks

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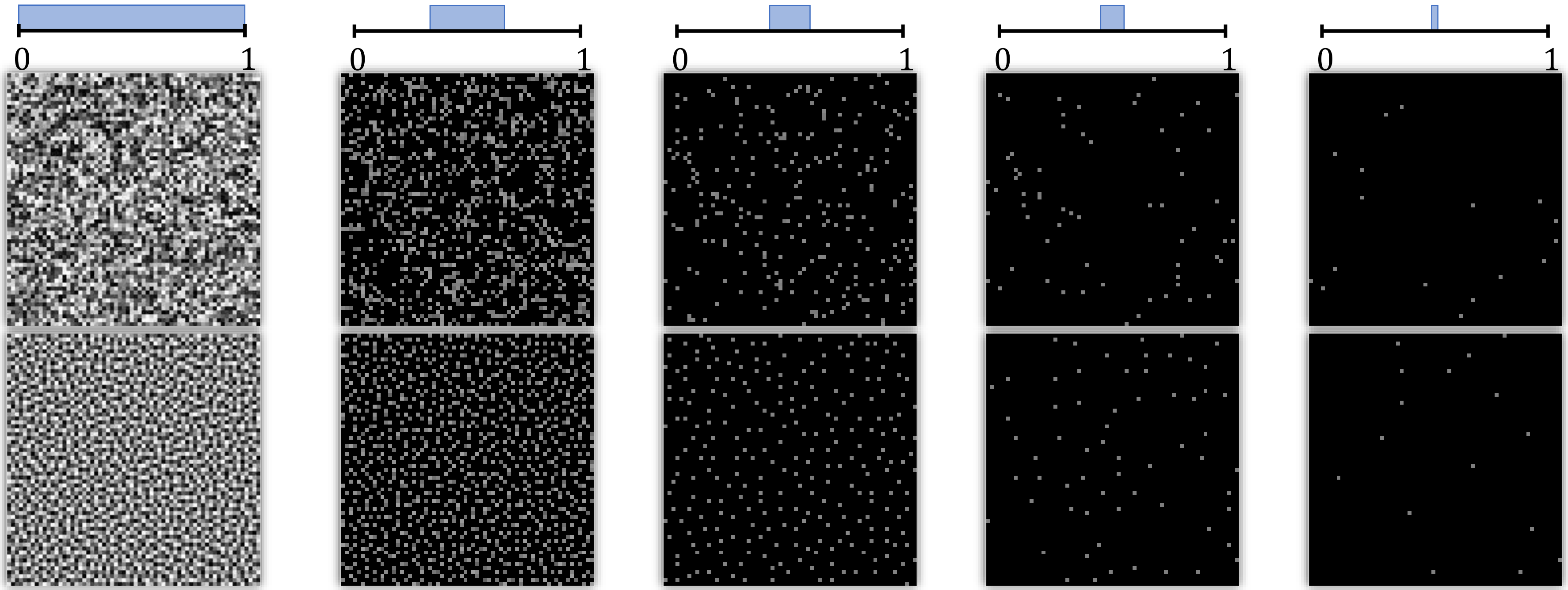
   b. Use value to offset sample pattern (surface, direct sampling, …)



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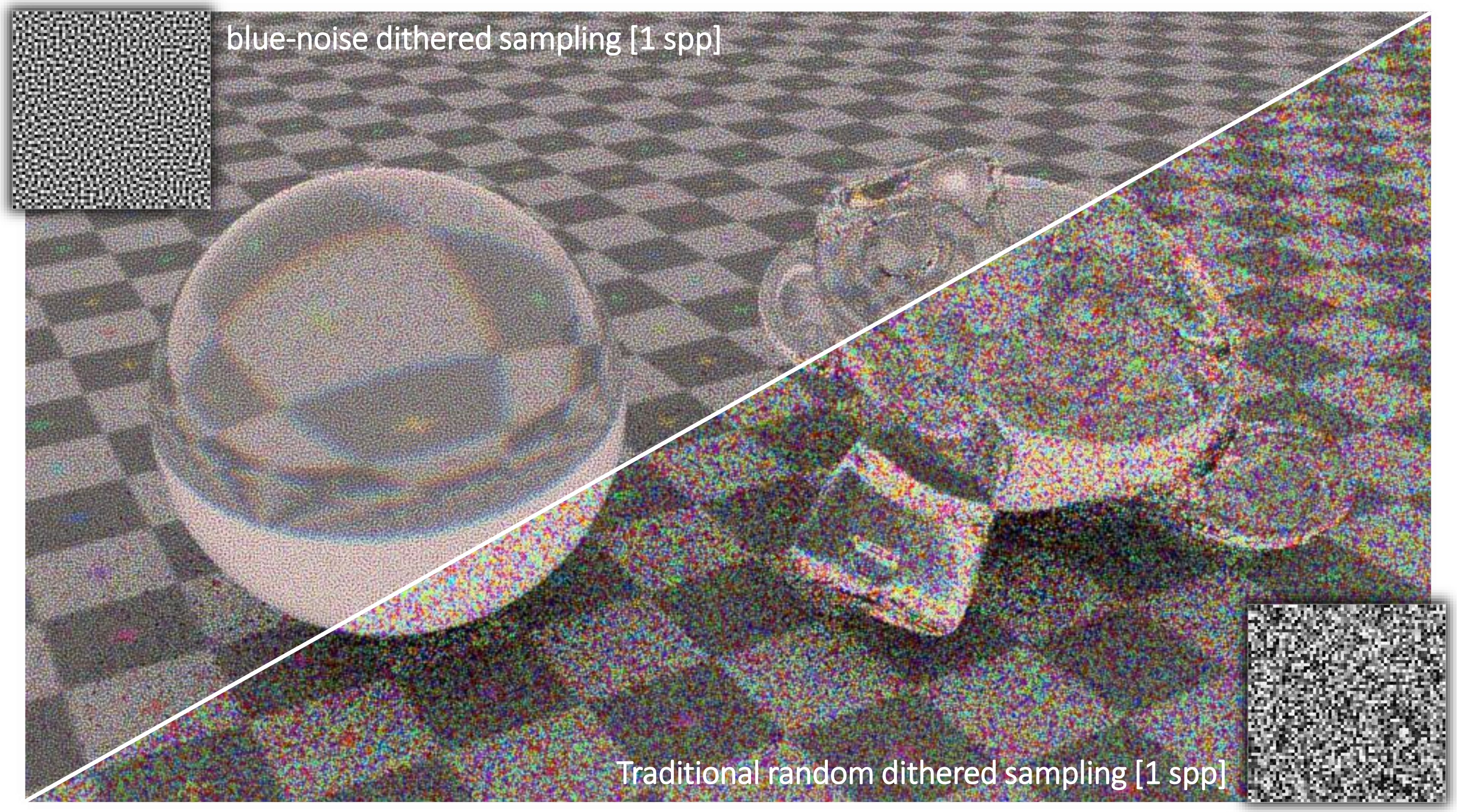
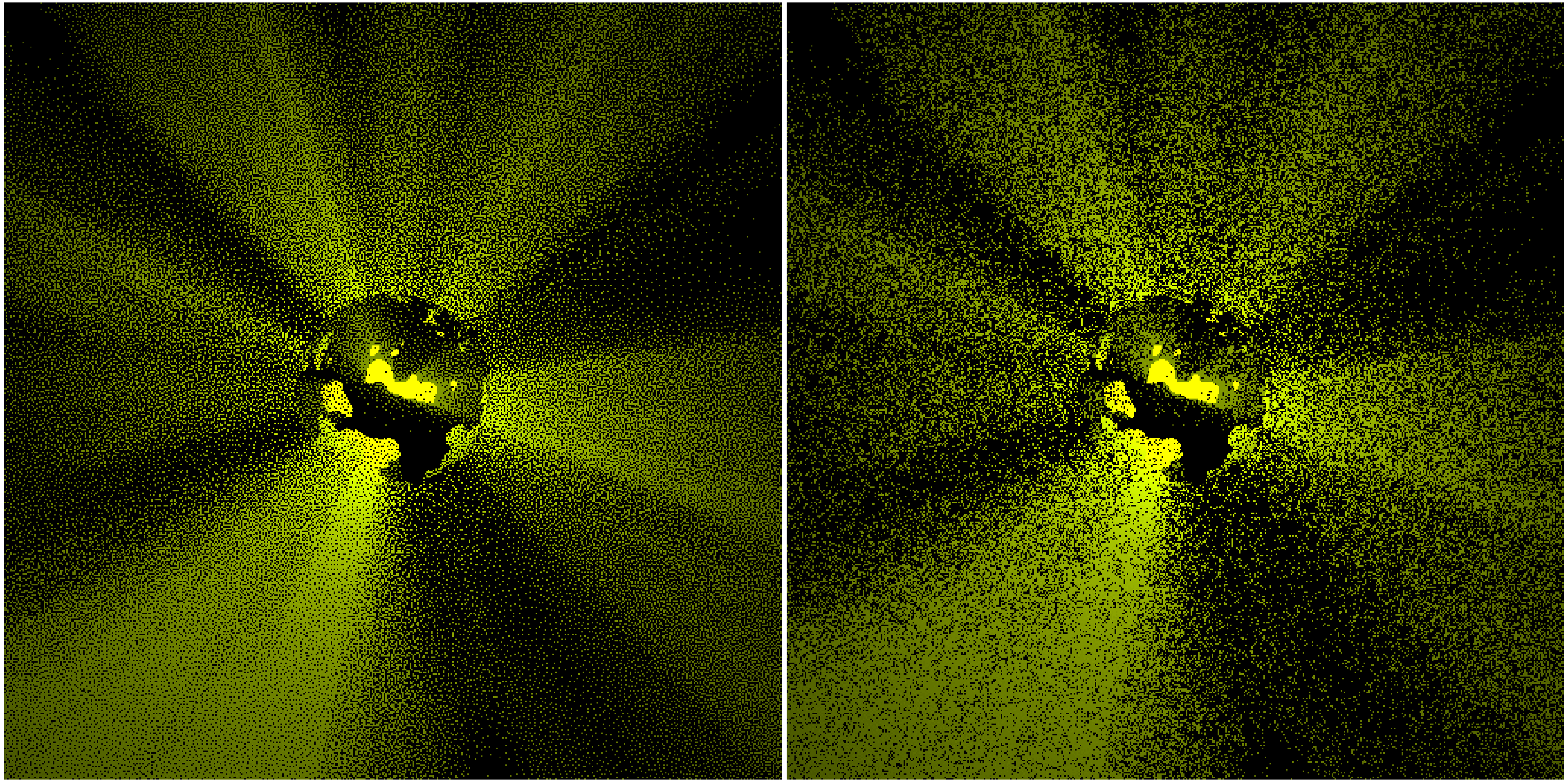


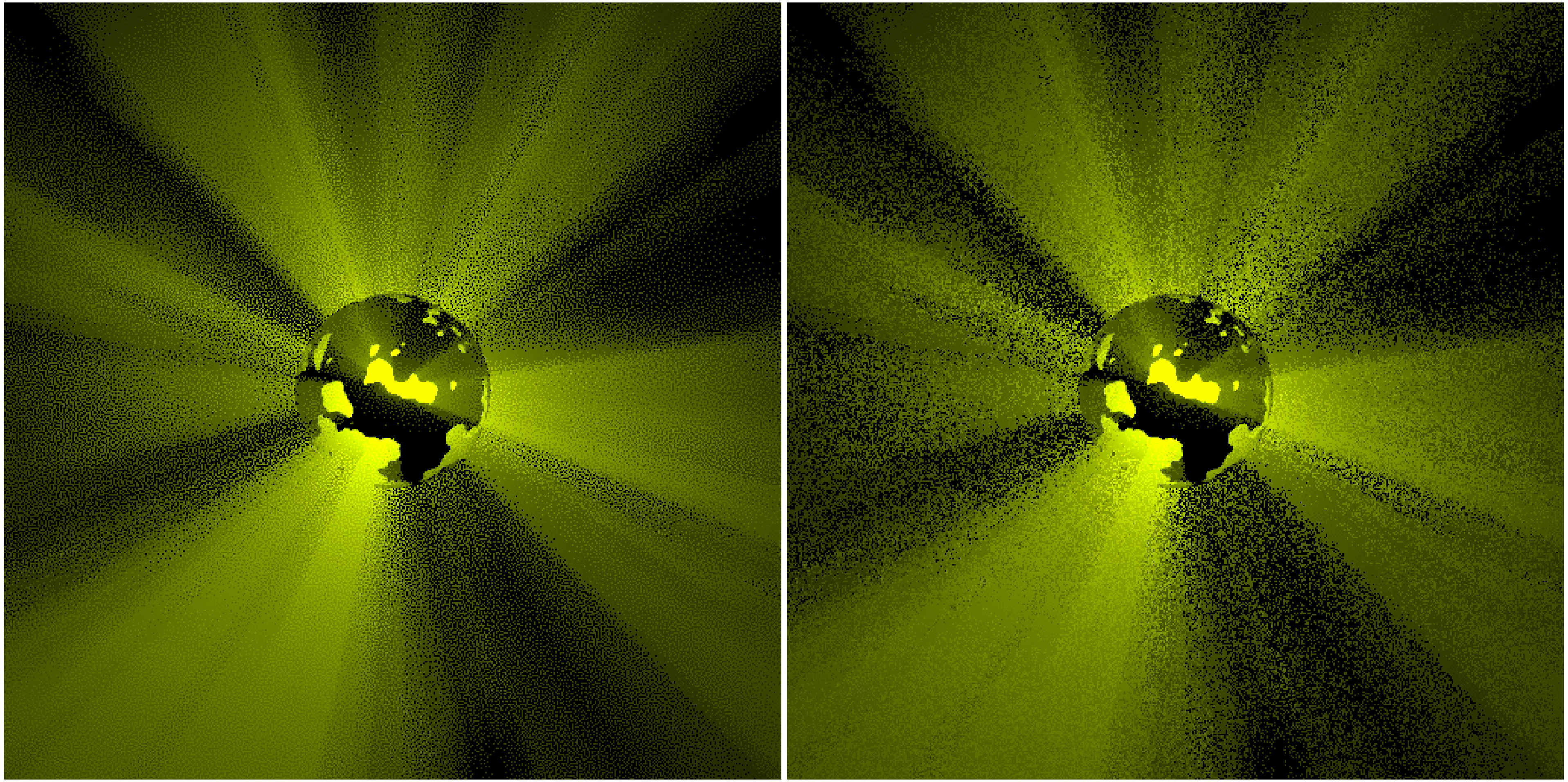
Sampling with dither masks

1 sample per pixel



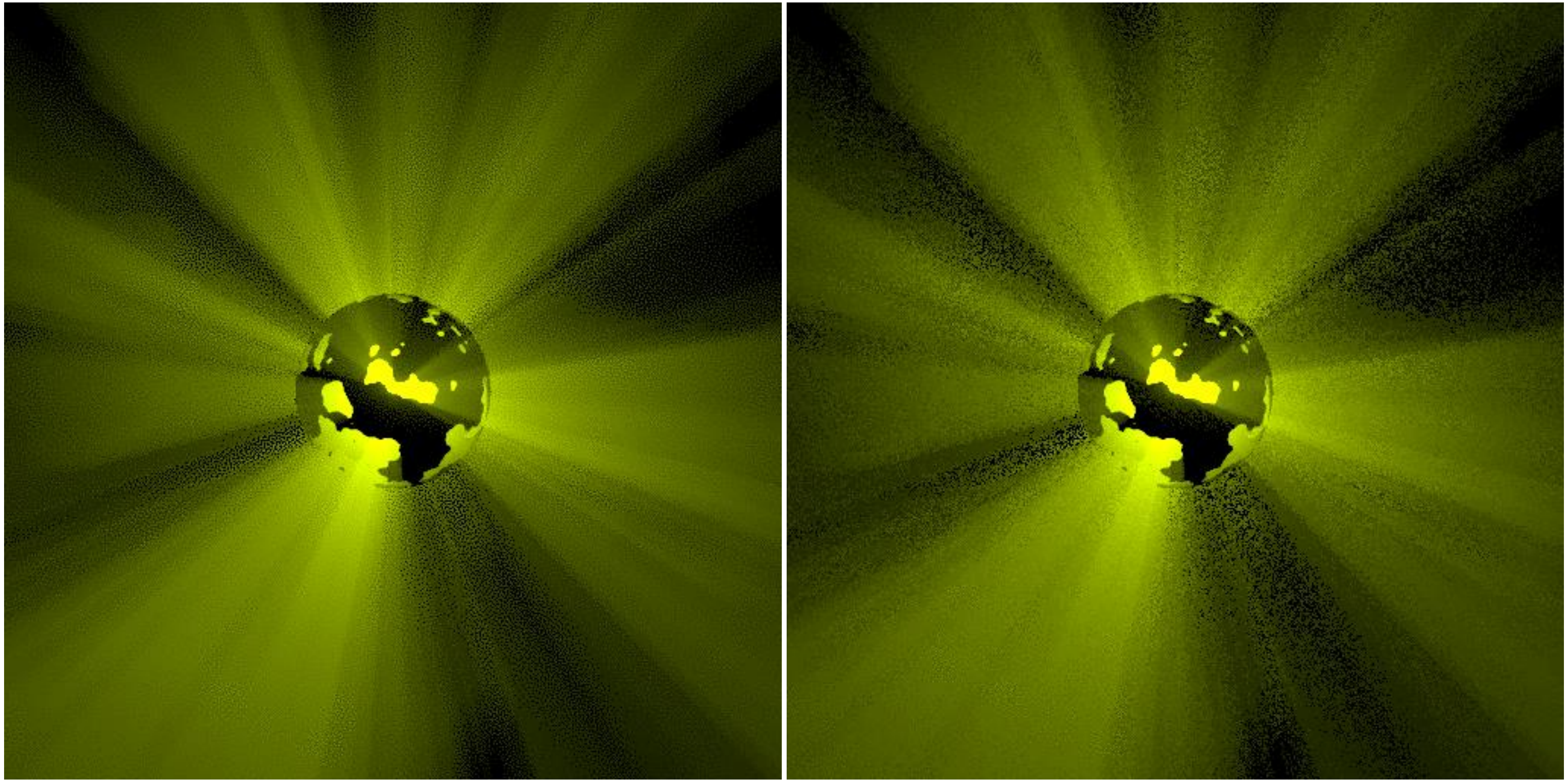
Sampling with dither masks

4 sample per pixel



Sampling with dither masks

9 sample per pixel



Sampling with dither masks

- Connection between halftoning and MC sampling
- Blue correlation > white decorrelation
- Simple, fast method
  - Showed 1D and 2D sampling
- Limitations
  - Improvement only when pixel integrals are correlated
  - Occasional mask tiling artifacts
  - Higher dimensions more difficult
  - Remapping of the samples limit quality

A LOW-DISCREPANCY SAMPLER THAT DISTRIBUTES MONTE CARLO ERRORS AS A BLUE NOISE IN SCREEN SPACE

[Heitz & al 2019]

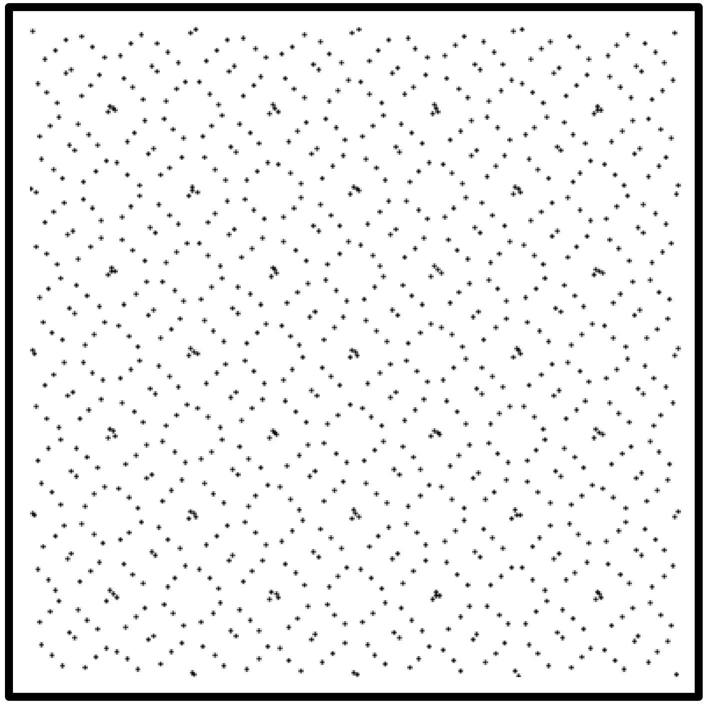
Objectives

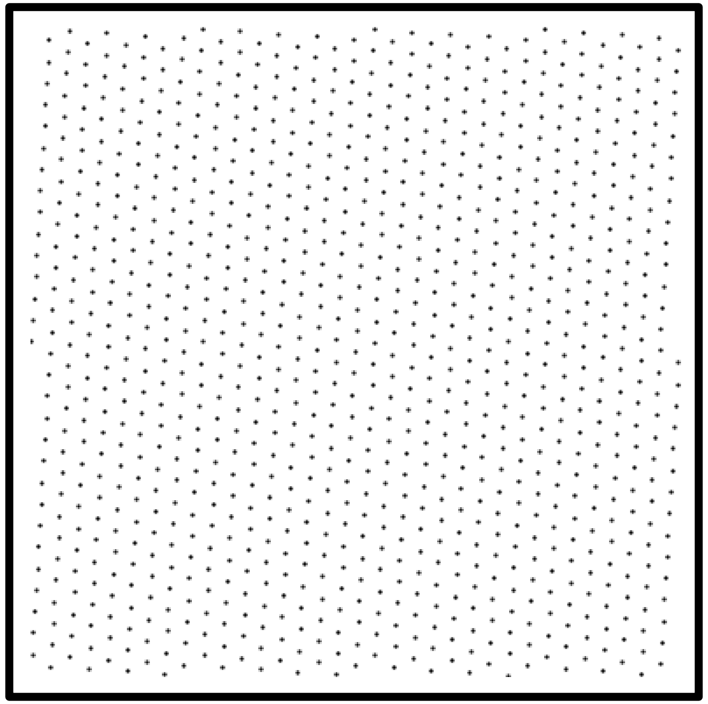
- BNDS limitations
  - Tradeoff between per-pixel sampling quality and error distribution
  - Can be worst than uncorrelated
  - Rely on correlation between sample difference and rendering difference

- Improvements
  - Directly optimize for rendering purpose
  - Ensure worse case falls back to uncorrelated

Sample stratification

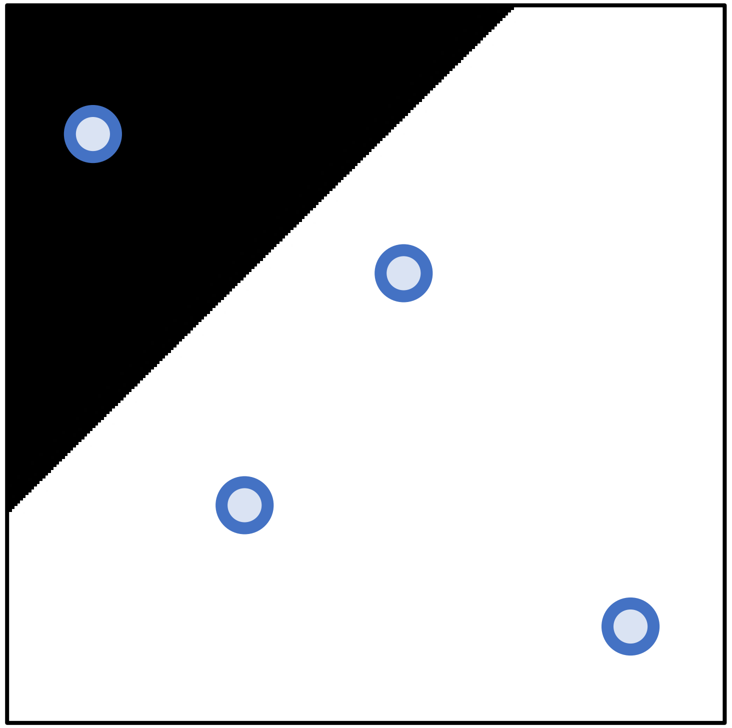
- Use Owen's Scrambling or Rank1 lattice
  - Use a Scrambling Key instead of a shift vector
  - Ensure good integration at each pixel

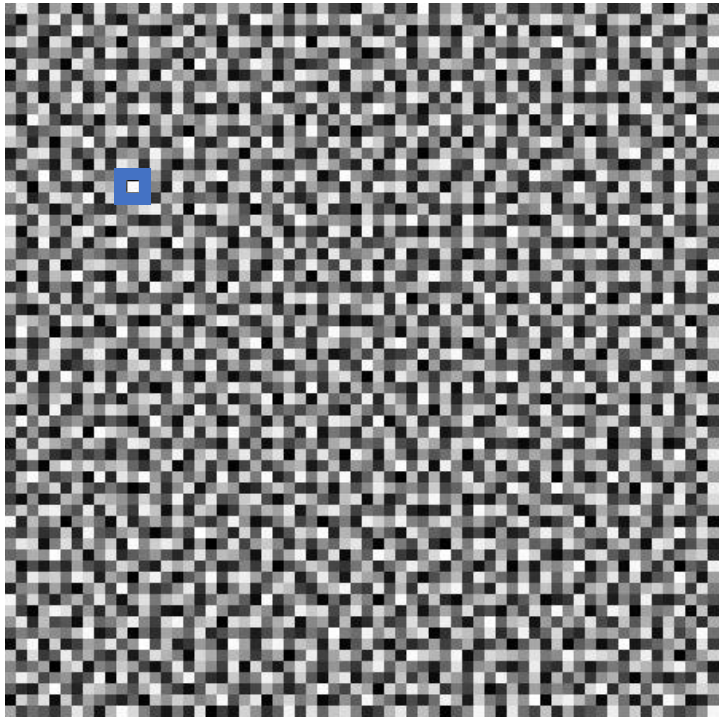




Change the optimization space

- Change from **sample space** optimization to **result space optimization**
  - Assume every pixel render the same integrand
  - But all pixel use a different XOR keys or seed





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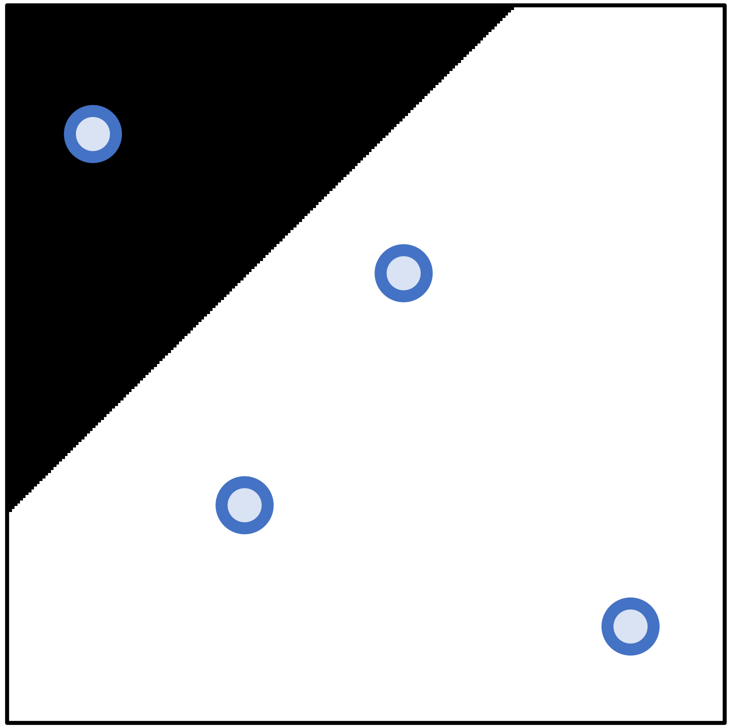

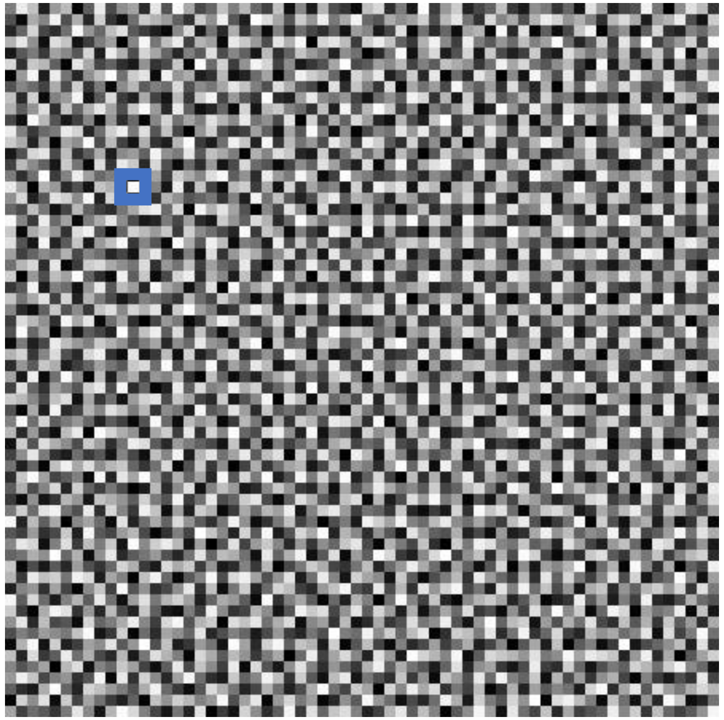

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Optimization setup

- Optimize the distribution of XOR key
  - Define a class of integrands
  - **Oriented Heavisides** define by **θ** and **d**
  - **D** dimensional integrands

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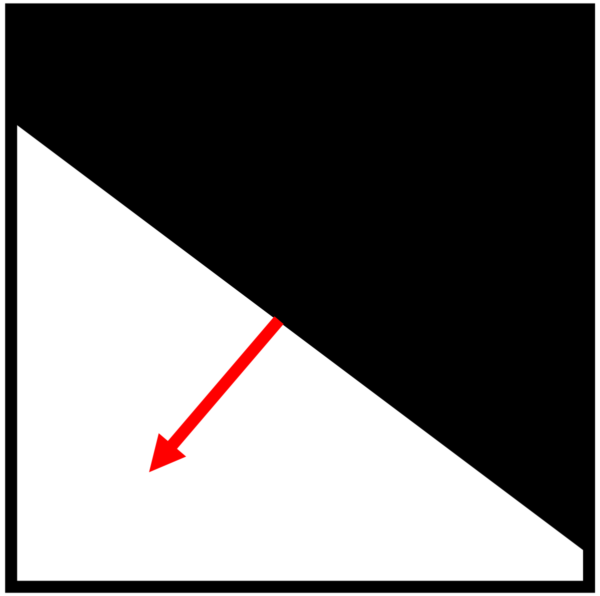
ε_p is a vector of **signed** integration error $(f(x_p) - F)$ on a large set of integrand (typically 65 536 randomized integrands)

The optimization is also done using simulated annealing



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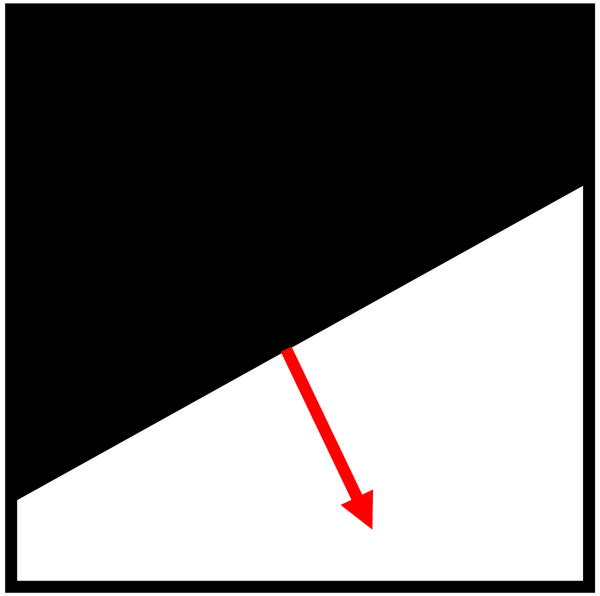
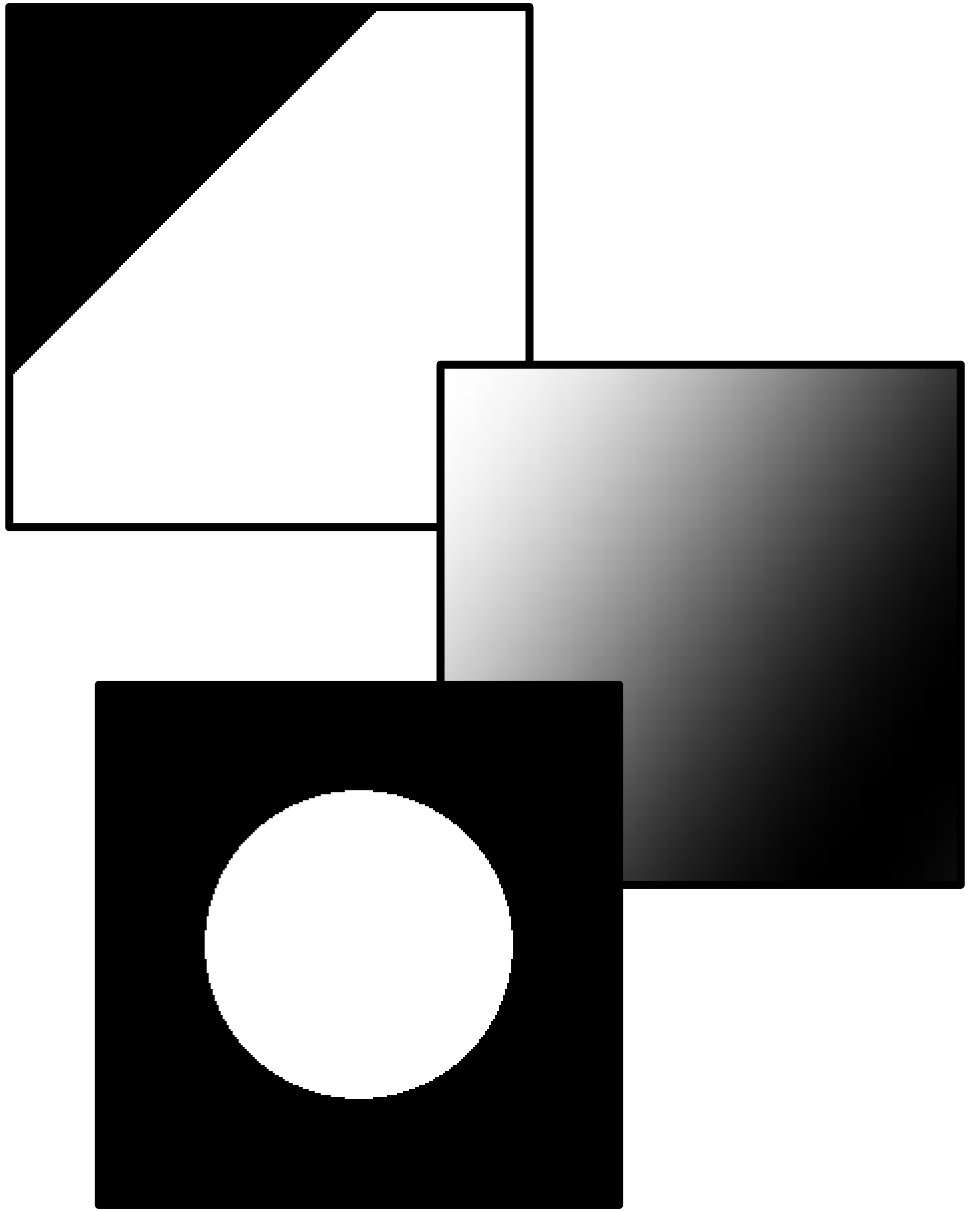


Optimization setup

- Optimize the distribution of XOR key
  - Define a class of integrands
  - **Oriented Heavisides** define by **θ** and **d**
  - **D** dimensional integrands

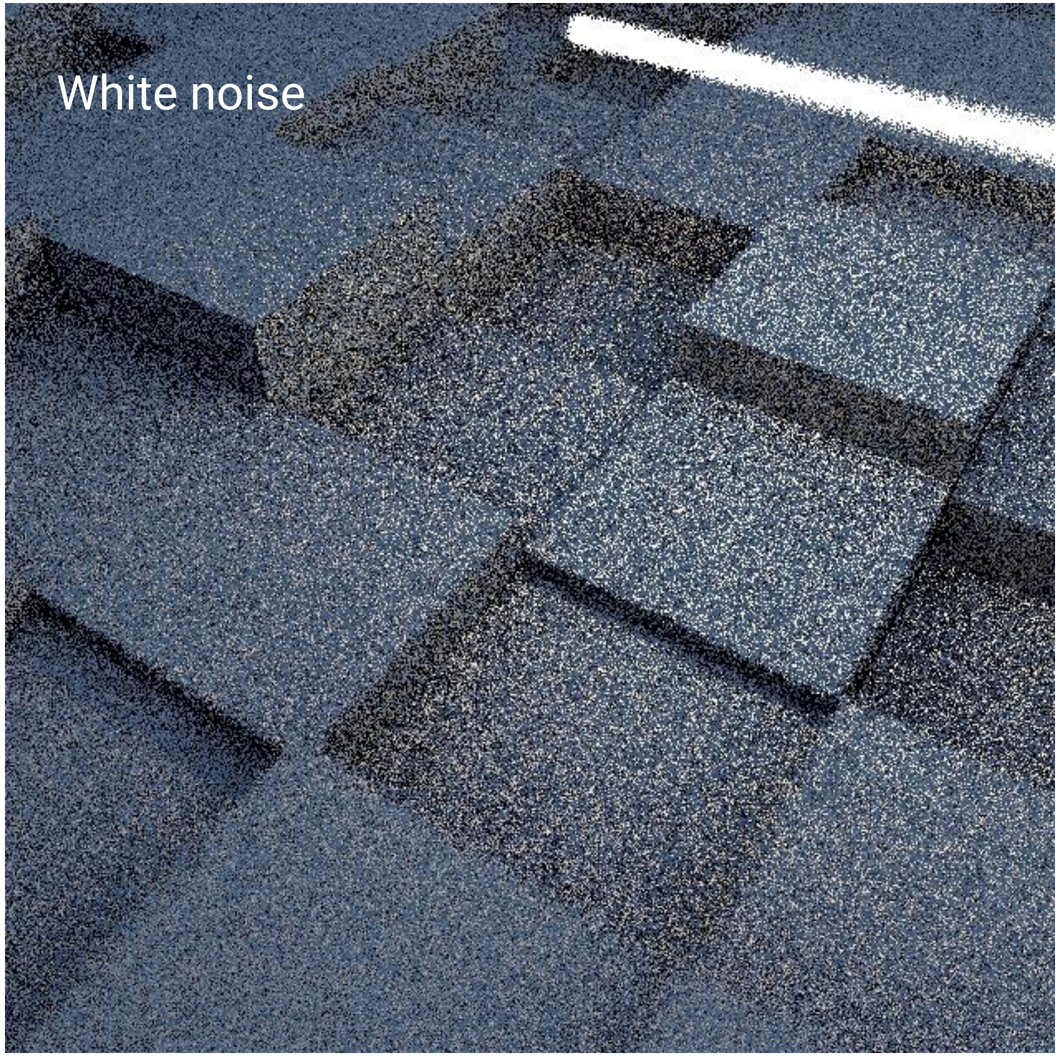
- Robust to other class of integrand
  - Varying orientation
  - Varying smoothness
  - Varying shape

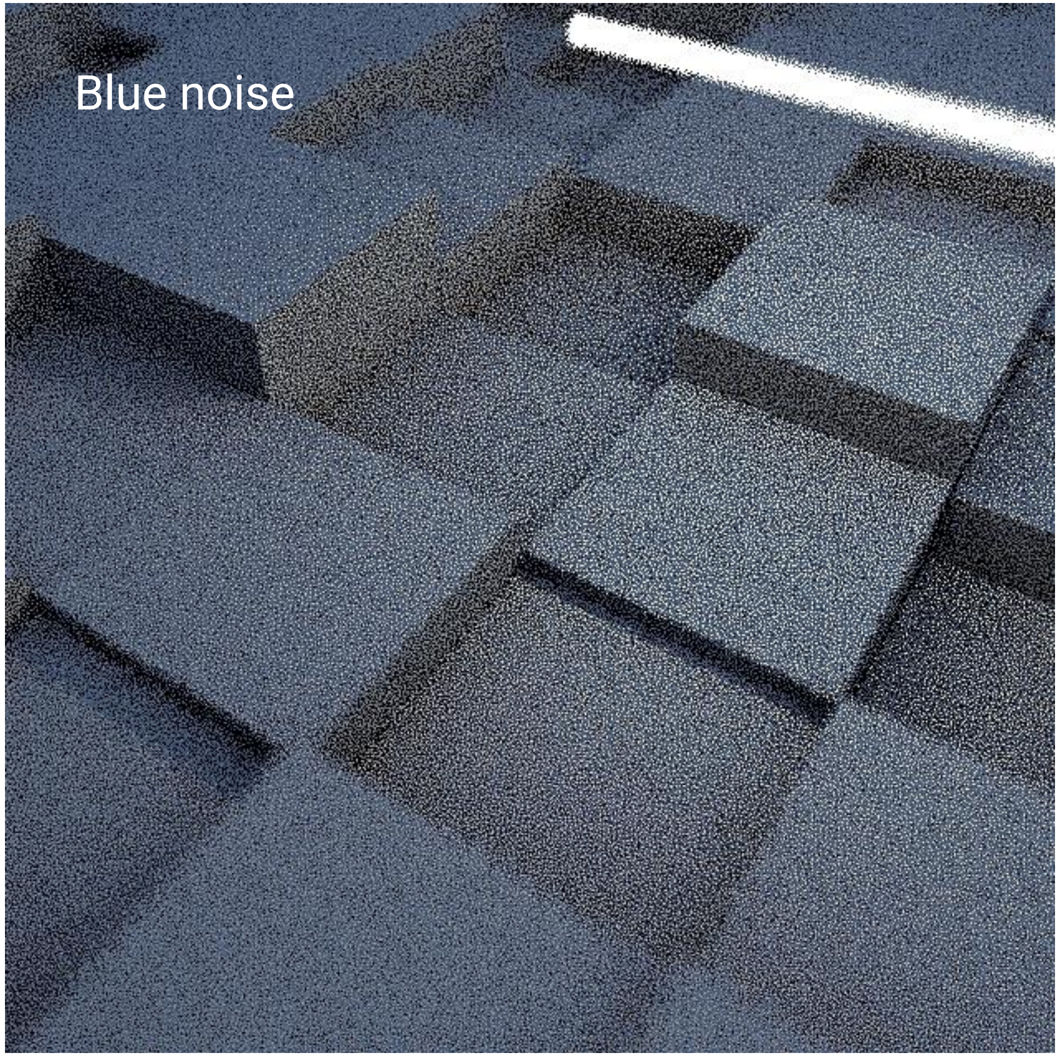


Optimization setup

- Rely on simple function
  - Step function and linear function show properties from rendering (visibility)
  - Act as a form of discrepancy measure
  - Quality of the result depend on the choice of the function class
- Error vector can be simplified
  - Store the distance norm between to sample set (1 value per pair)
- Works with progressive sampling
  - It is possible to optimize the progressive sample set distribution
  - Still limited by the dimensionality

Results





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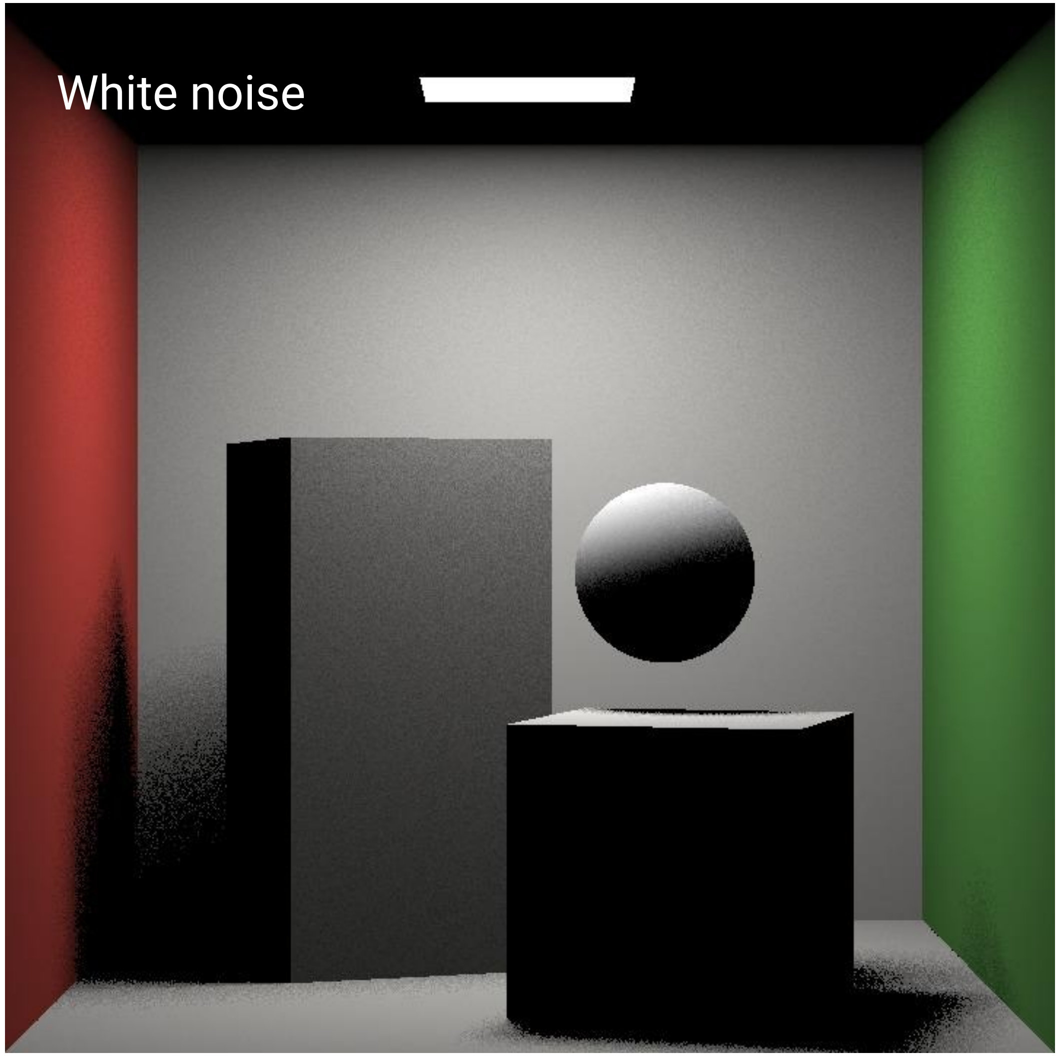
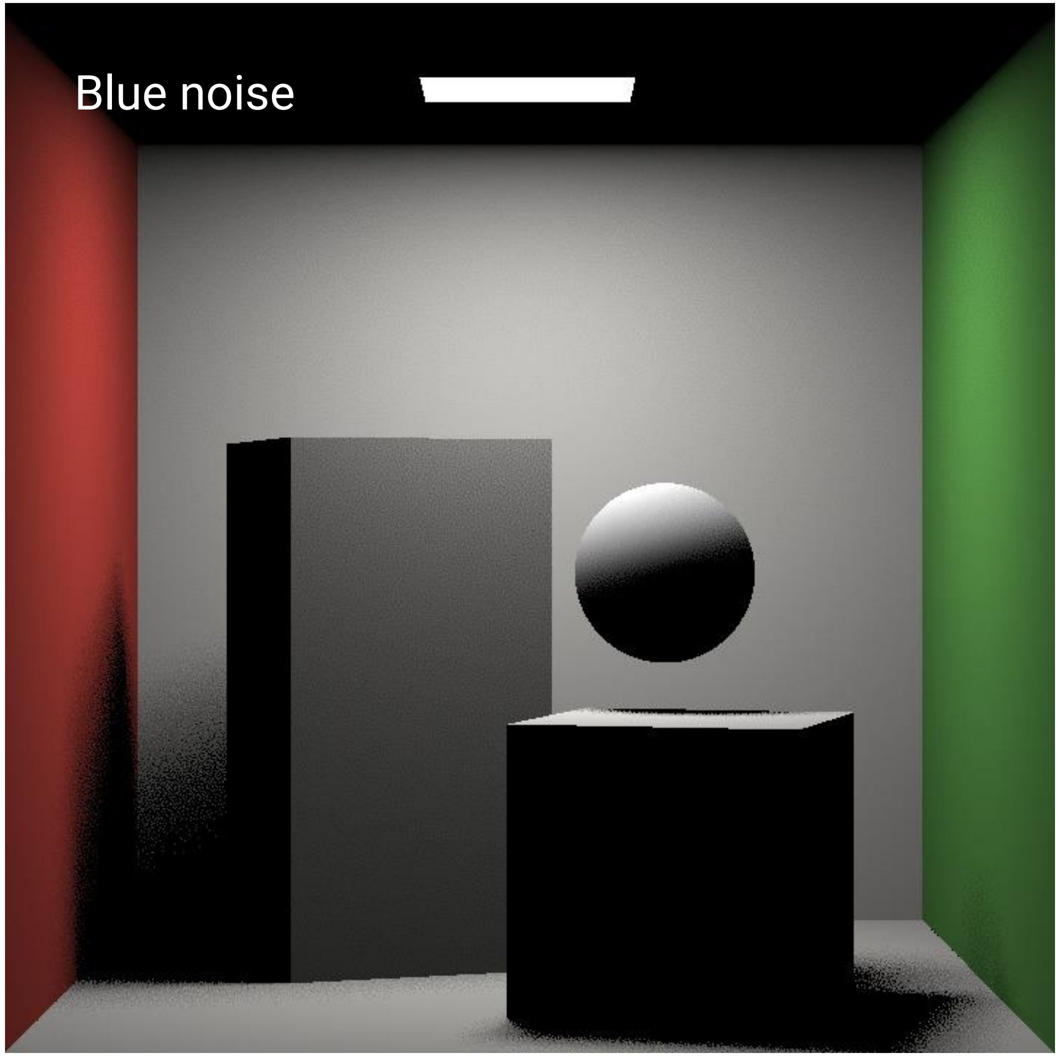
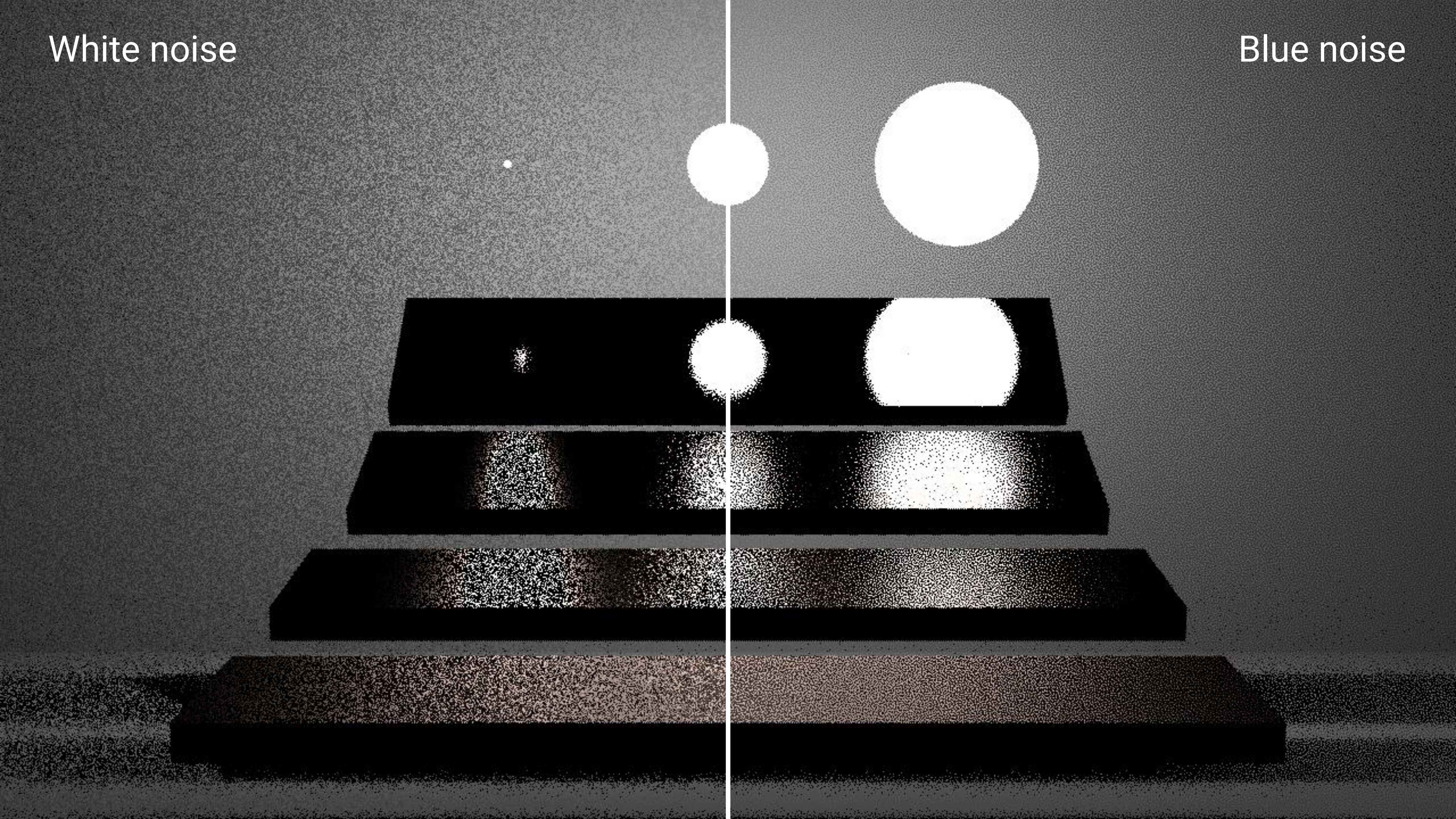





White noise
Blue noise


White noise
Blue noise

Results

- **Share** limitations with BNDS !
  - Can't handle **high-dimensional** integrands
  - Not robust on **complex integrands**

- Still **you should use this method** rather than BNDS
  - Can **only do better**, not worse

Distributing Monte Carlo Errors as a Blue Noise in Screen Space by Permuting Pixel Seeds Between Frames

[Heitz & al 2019]

Objectives

- Previous works precompute a shift map or sample set distribution
  - Not adaptive to the rendering
  - Need one optimization per sample count/dimension

- Scene adaptive sample optimization at runtime
  - Use existing sampler
  - Can achieve higher quality
  - Work at arbitrary sample count/dimensionality
  - No precomputation

Redefining the rendering process

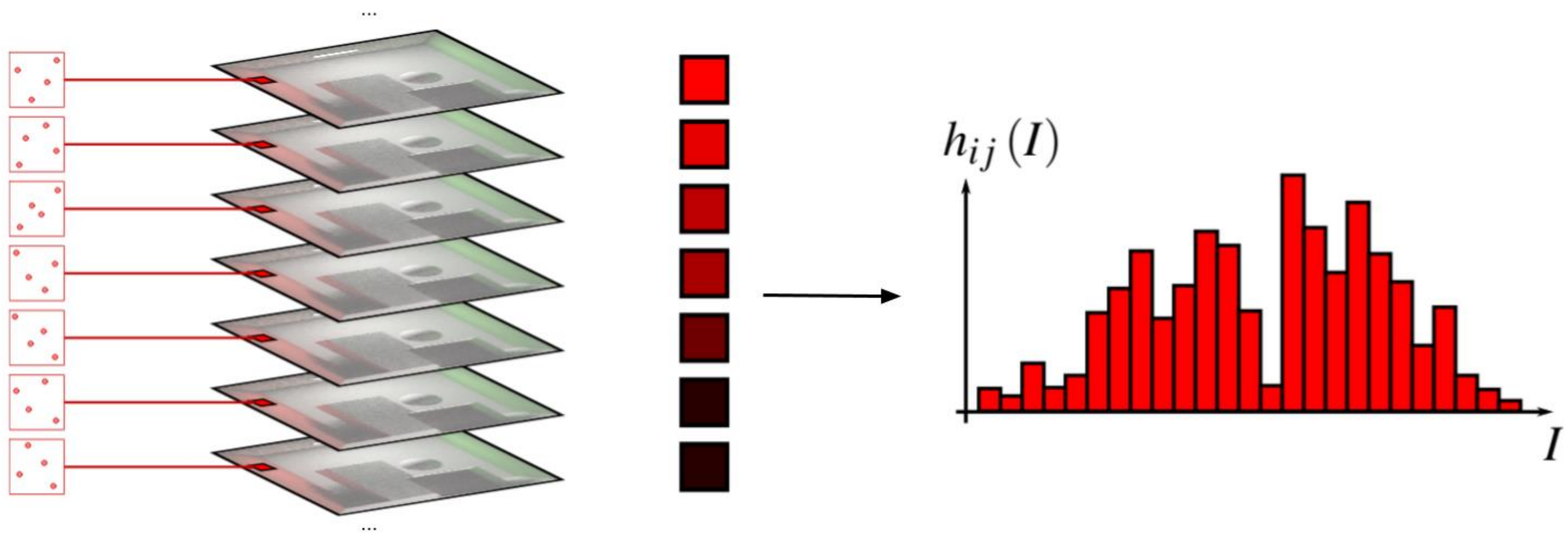
- Integration space is often high dimensional an not smooth
  - Modify the integration process to reduce dimensionality
  - Operate on a smooth space

- Change from Sample space to estimate space
  - Construct the histogram of estimate and sample it
  - The histogram is a discreet representation of the PDF of estimates
  - Histogram is a 1D space independently to the sampling space

$$\int_{\Omega} f(x)dx = \int_0^1 H^{-1}(u)du$$

Histogram of estimate

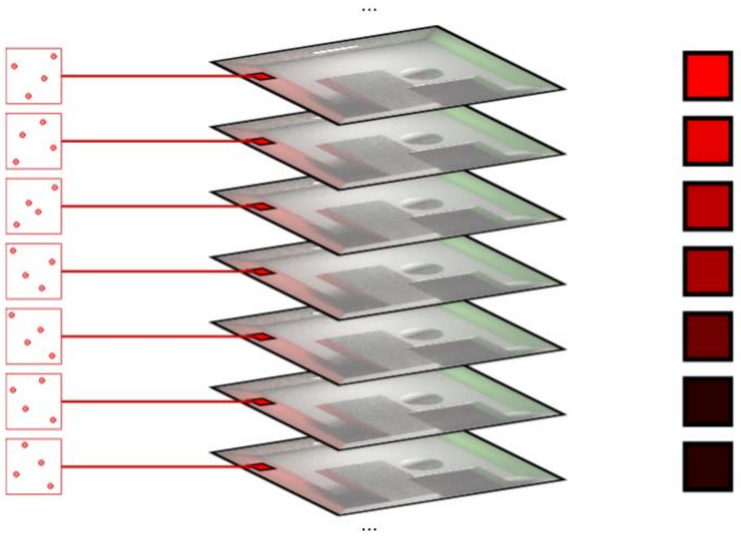
- How to construct the histogram of estimates ?
  - For each pixel, render the image with a large number of independent rendering
  - Rendering can be done with multiple samples
  - It's possible to use Low discrepancy sampler
  - Each rendering can be determined by the seed of the random generator



Histogram of estimate

- Histogram naturally create a smooth space
  - Sample set are ordered by rendering value (monotone function)
  - Estimate the continuous estimate distribution

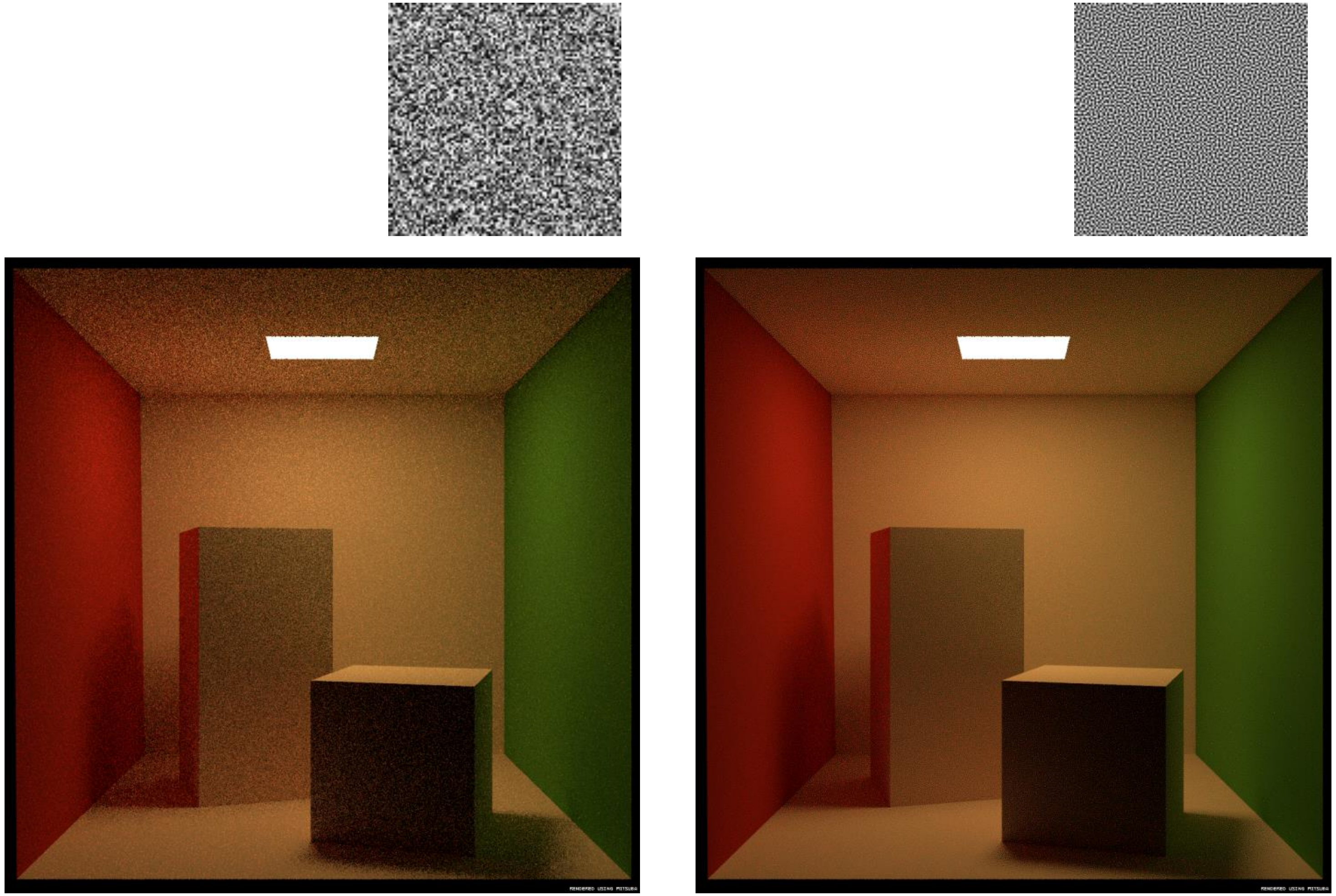
- Simple to construct
  - Rendering with varying sample set
  - Sorting operation based on luminance
  - Once sorted just need to store the seed



Histogram sampling

- Histogram are staircase function
  - It is easily invertible

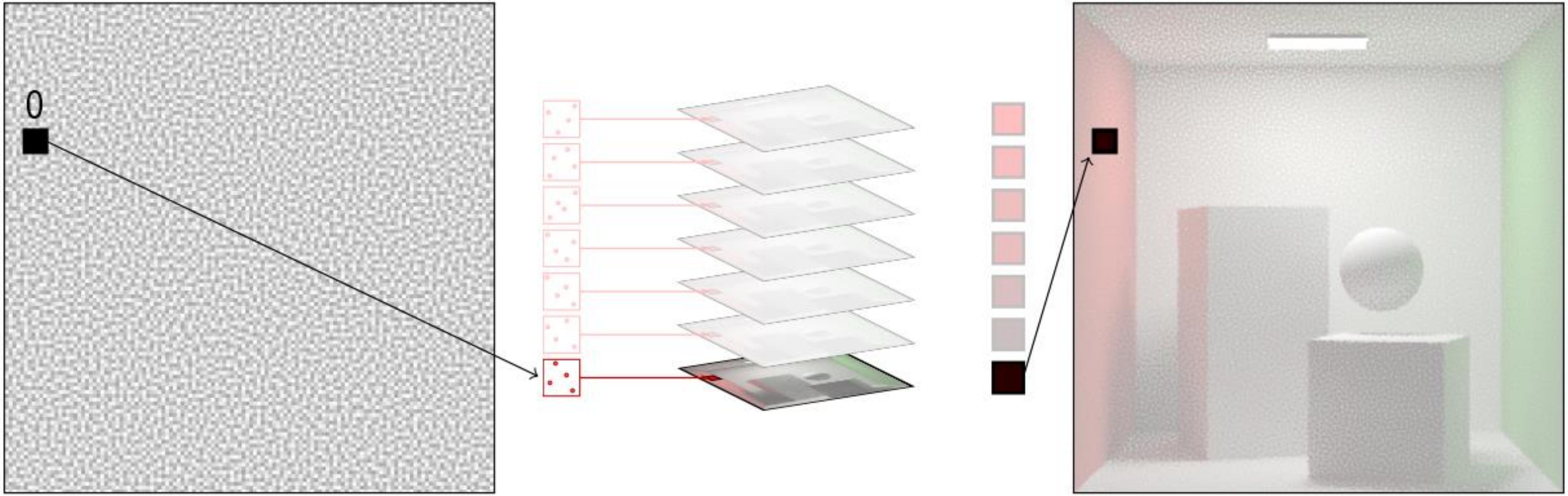
$\int_0^1 H^{-1}(u)du \approx H^{-1}(\xi)$ with $\xi \sim U(0,1)$



ξ can be generated with :
- Random uncorrelated generator
- A dither mask

Histogram sampling

- Use the dithering mask value to select the corresponding seed
  - List of estimate values sorted based on luminance



Histogram sampling

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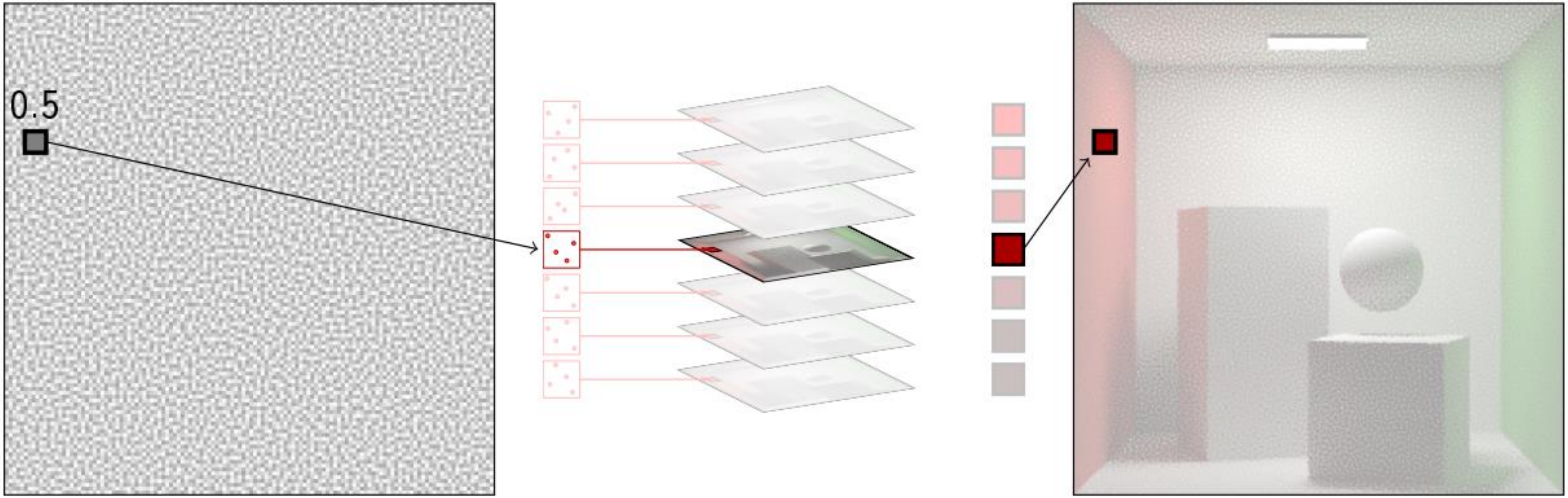

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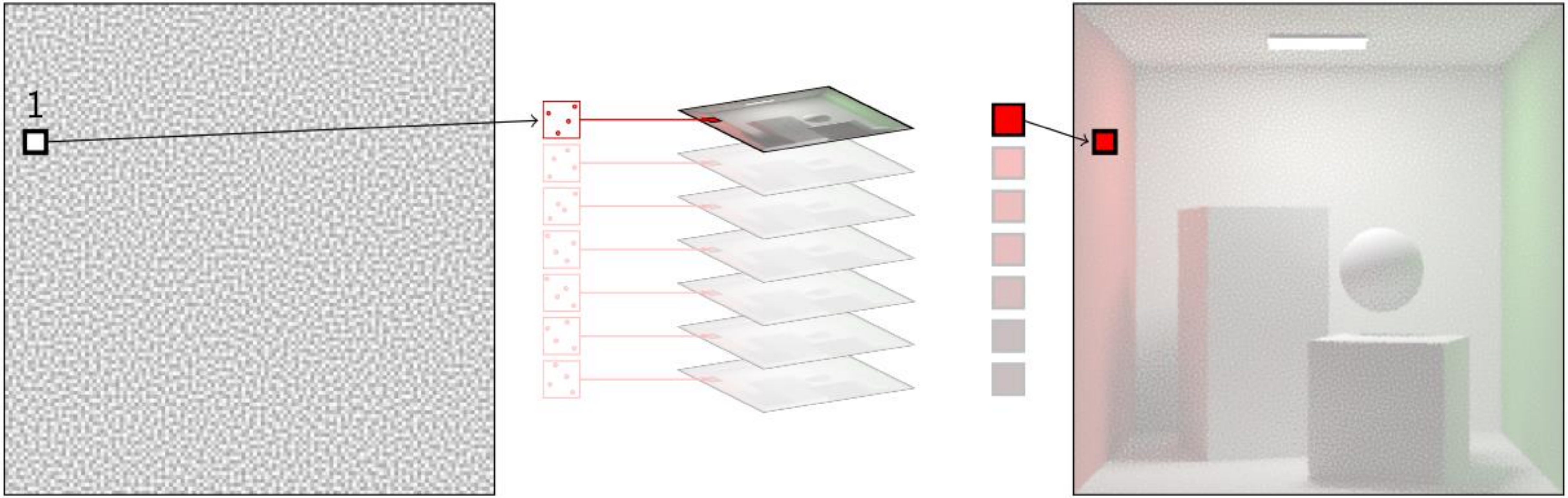
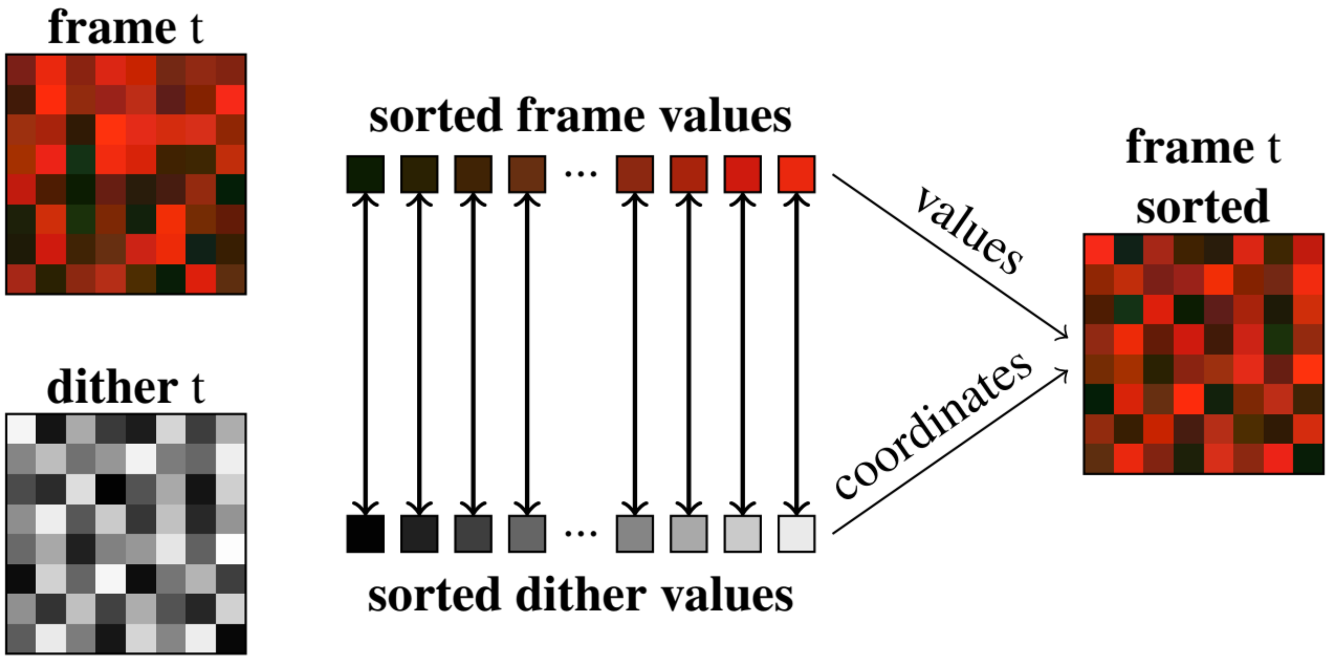


Histogram sampling

- In practice computing the histogram mean multiple rendering
  - Simply average them
- Choosing 1 sample over 4 smartly can produce lower visual error
  - Trade off pixel quality and noise distribution
- Work in high dimensional cases
- Works for all multiple samples per pixel
- Can use Low discrepancy sampler

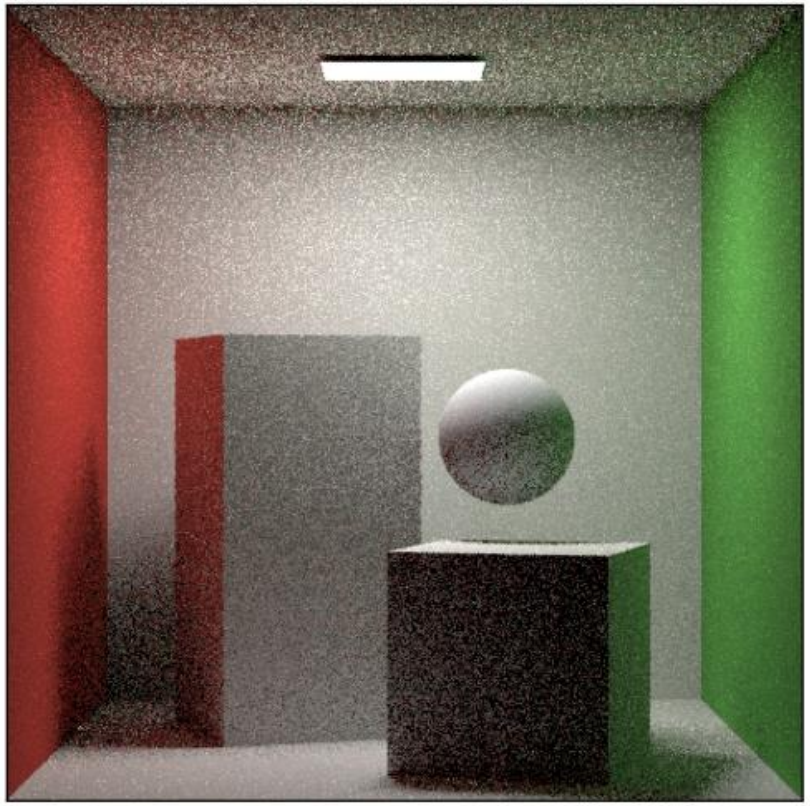
Practical algorithm

- Construct the histogram locally
  - Split the image space in small blocs
  - Use optimal transport to map frame values and dither mask
- Can be combined in a temporal algorithm to avoid re-rendering
- Use the powerful properties of sorting
  - Sorting naturally create a correlated and smooth space



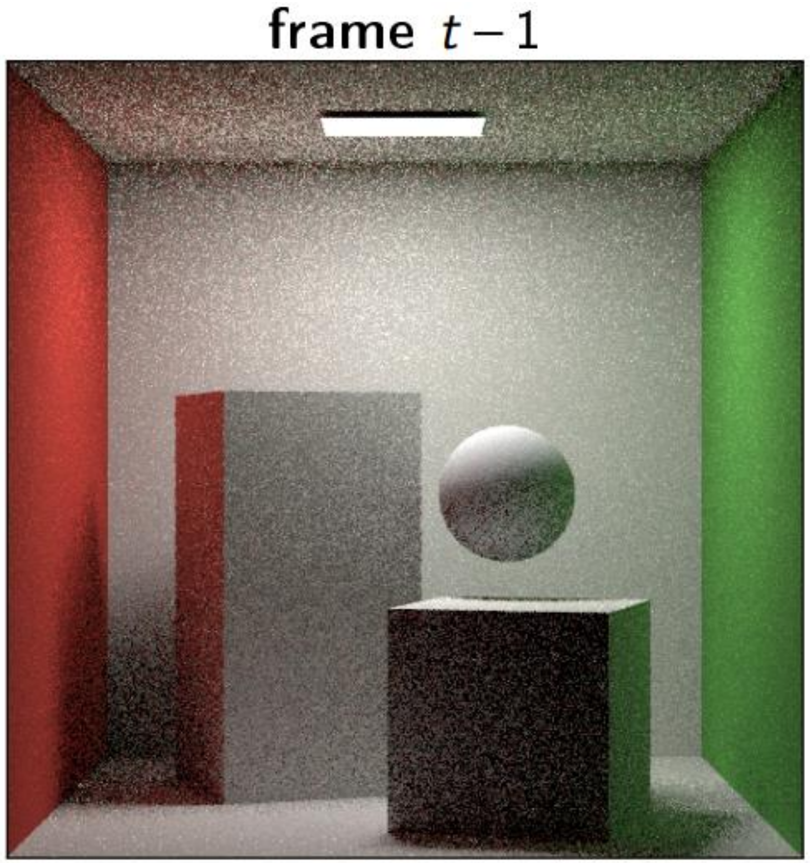
Temporal algorithm

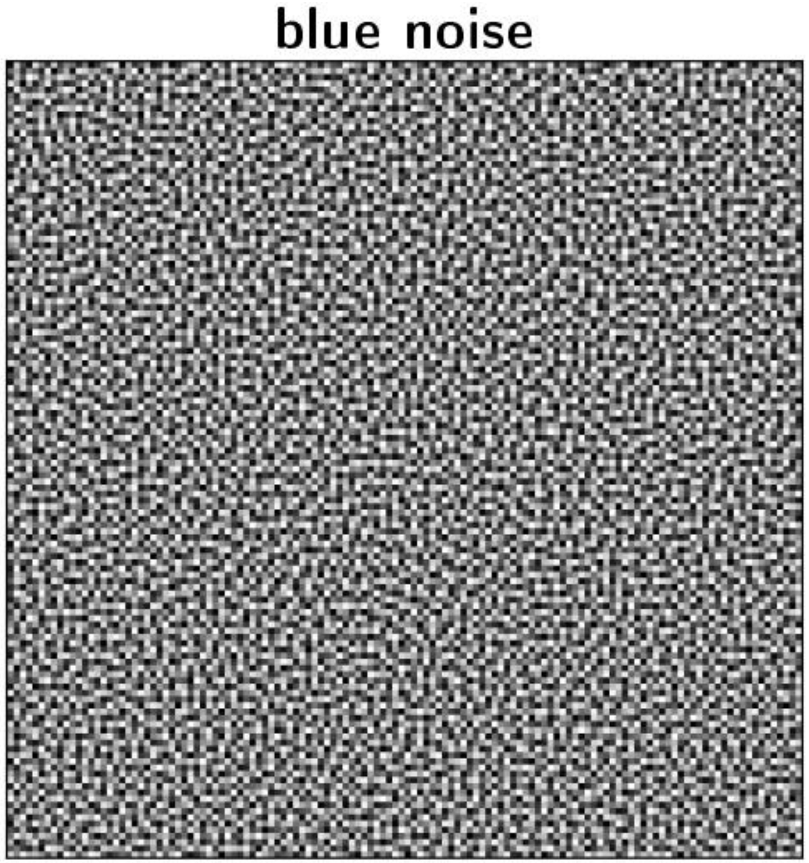
frame $t-1$



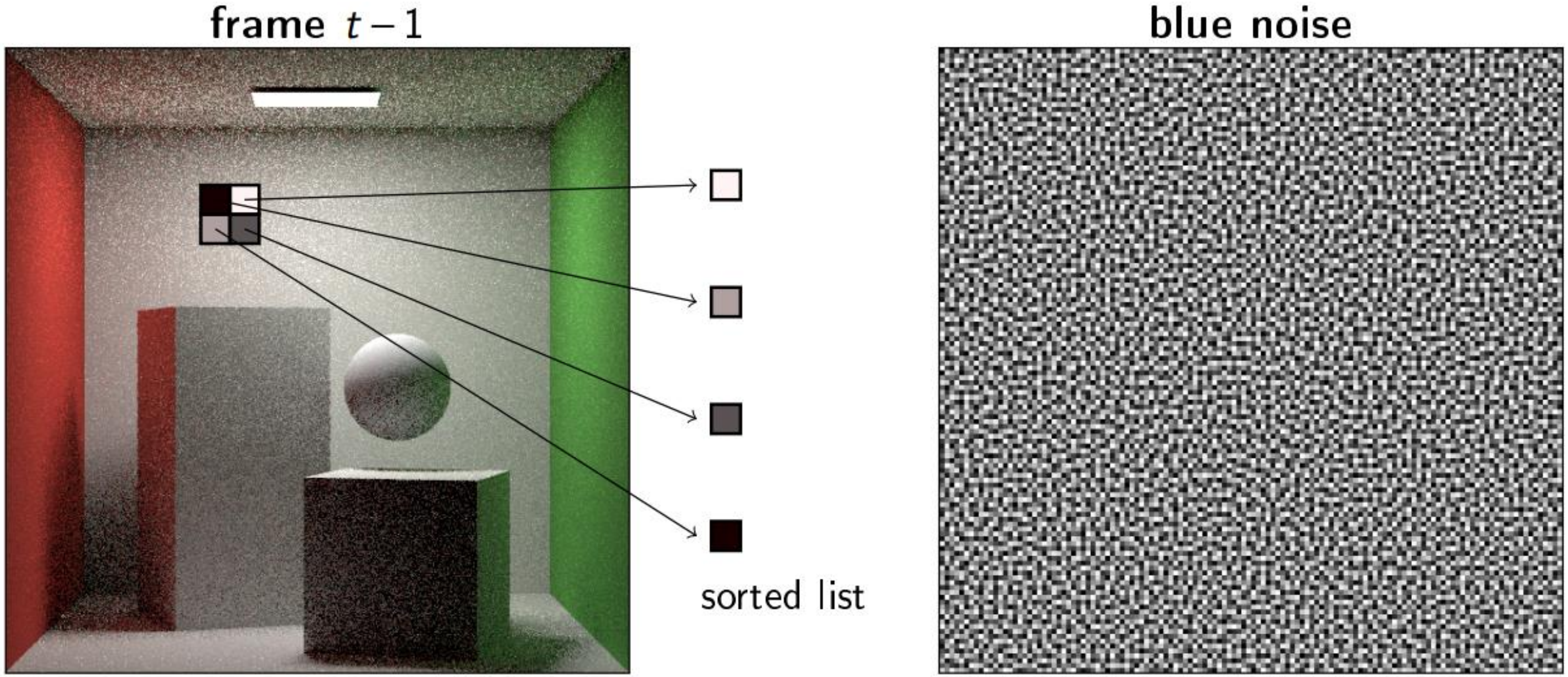
frame t

Temporal algorithm

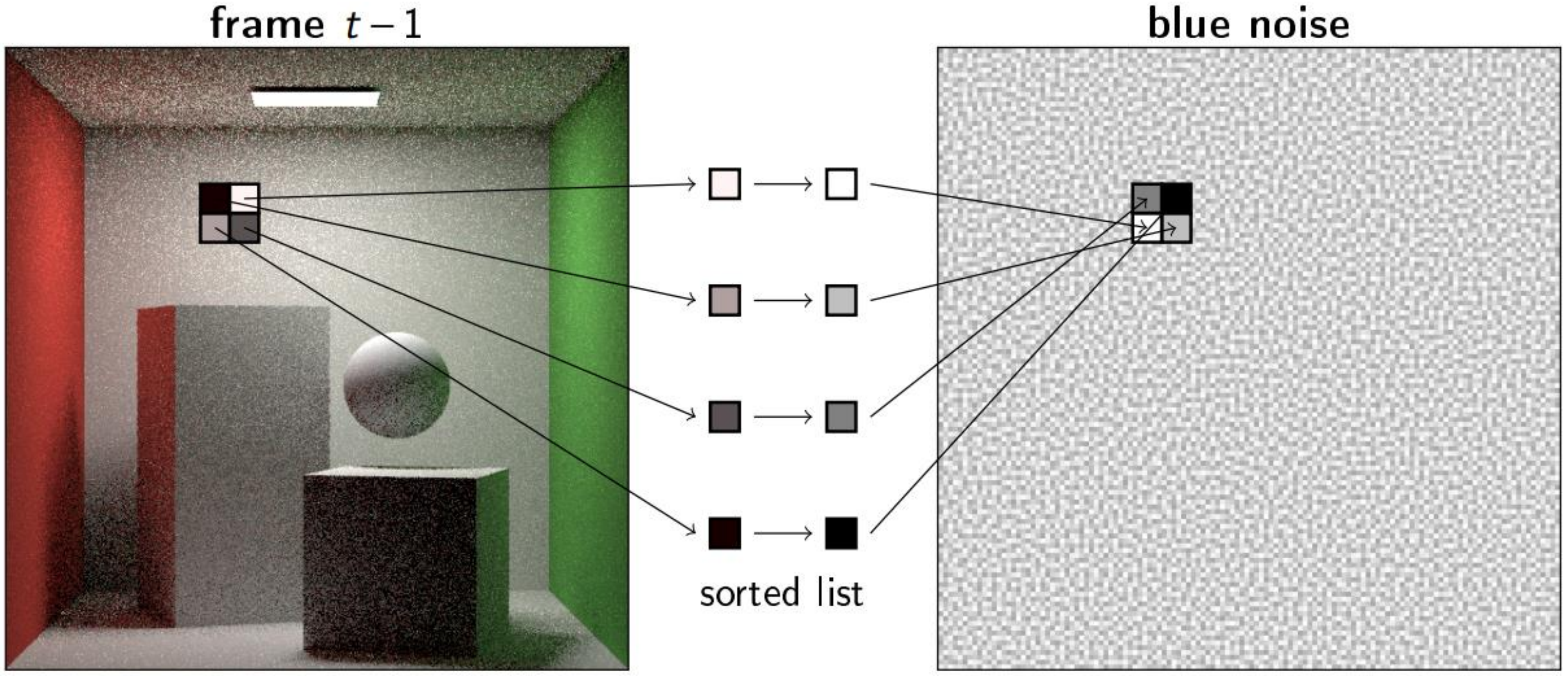




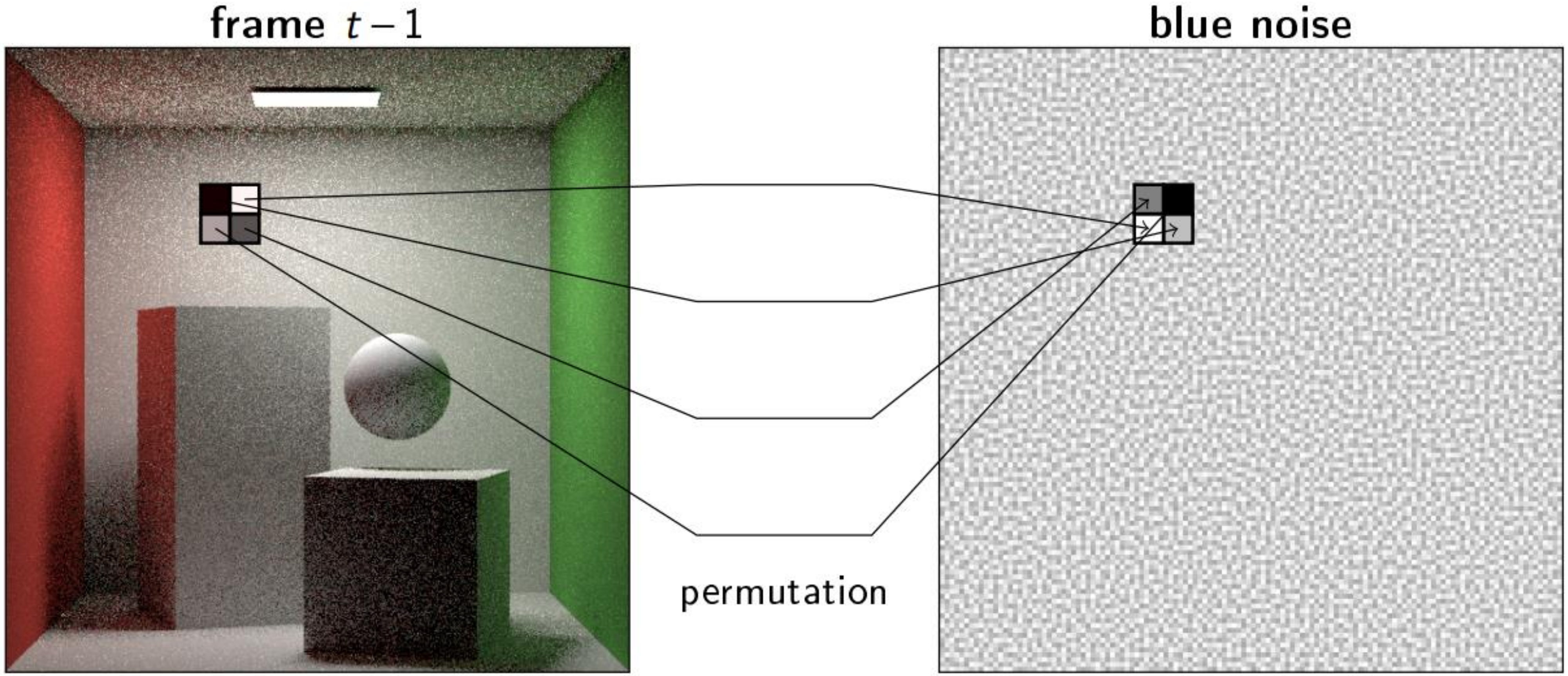
Temporal algorithm



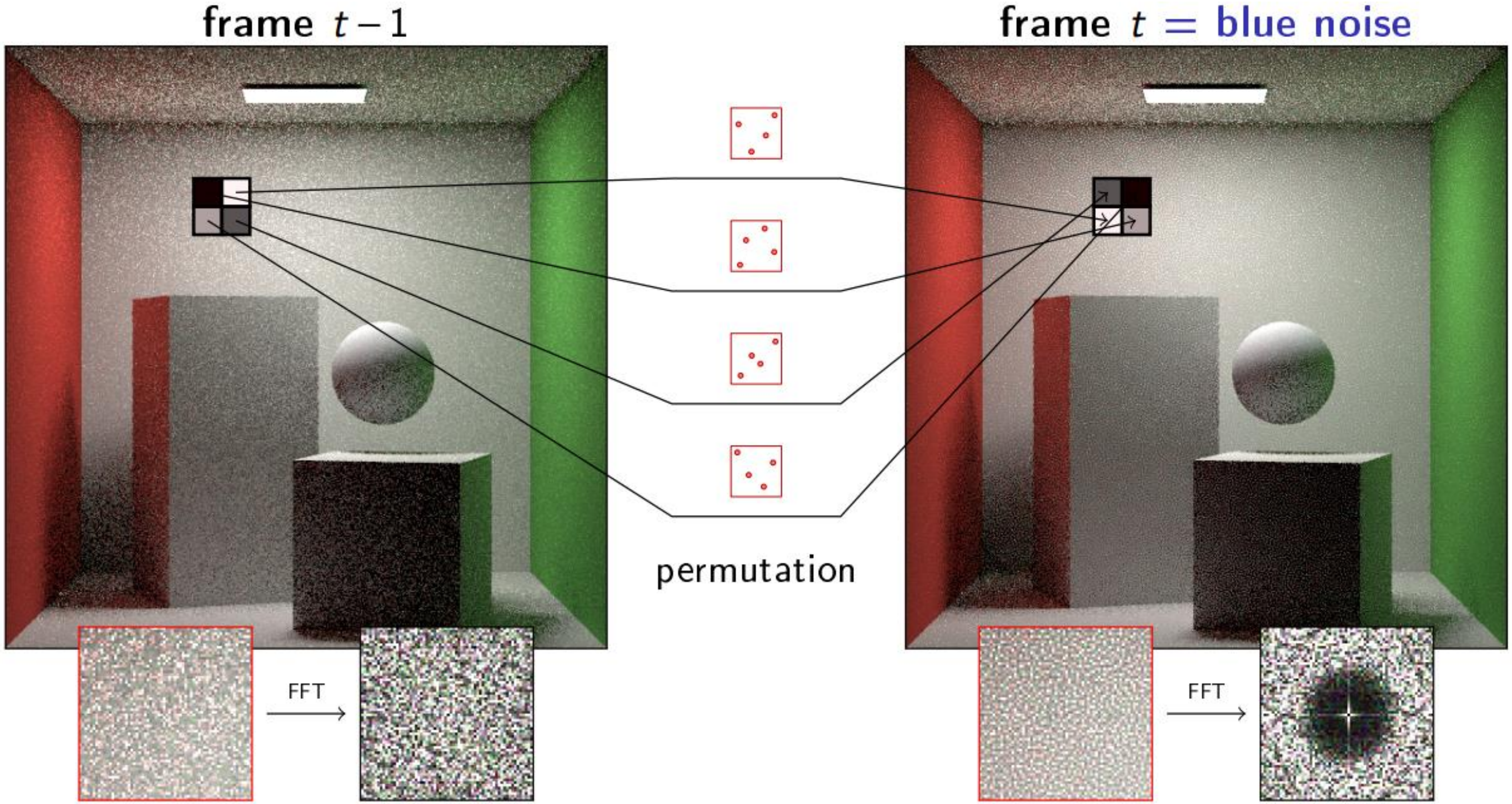
Temporal algorithm



Temporal algorithm



Temporal algorithm

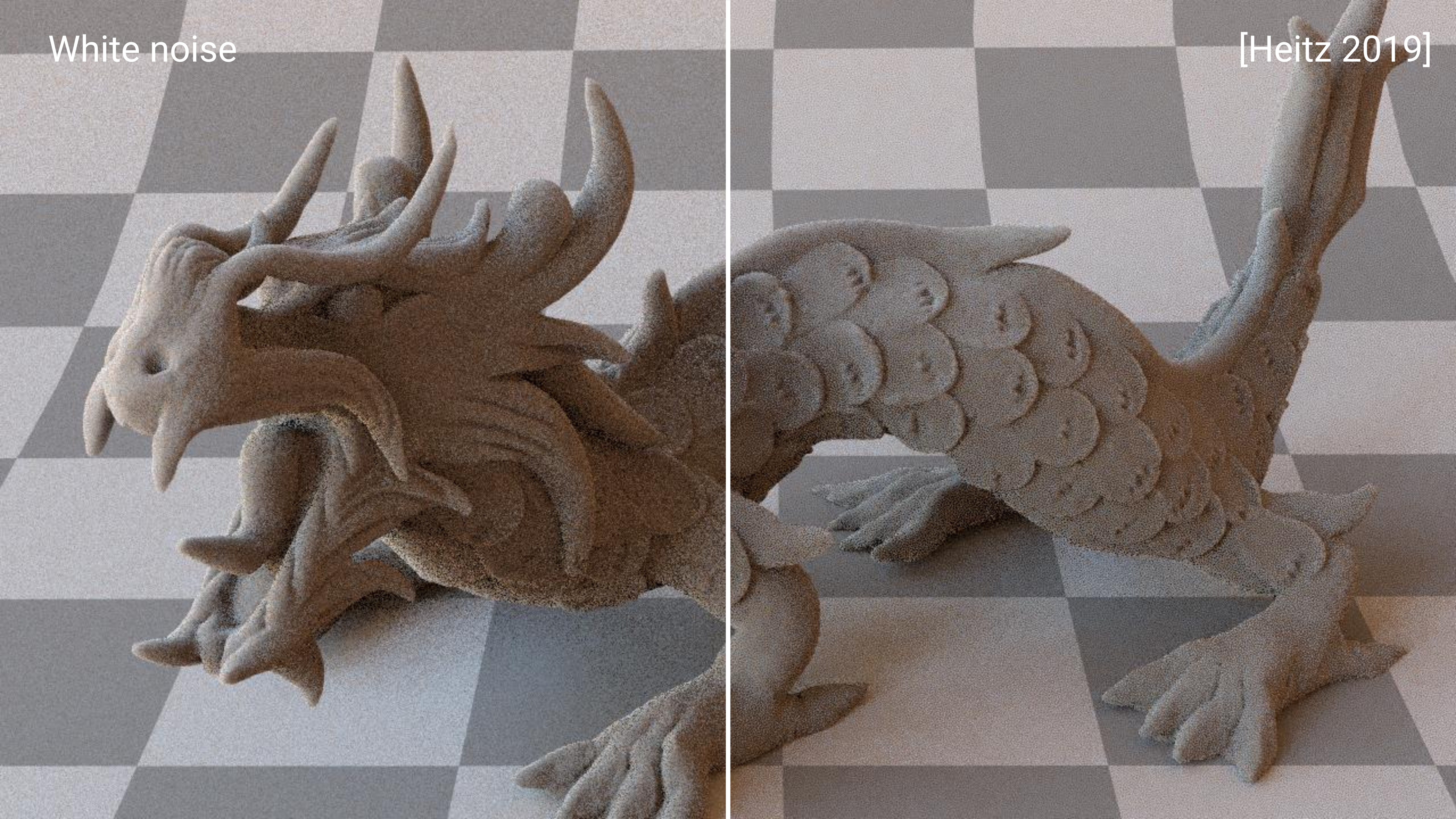


Pseudocode

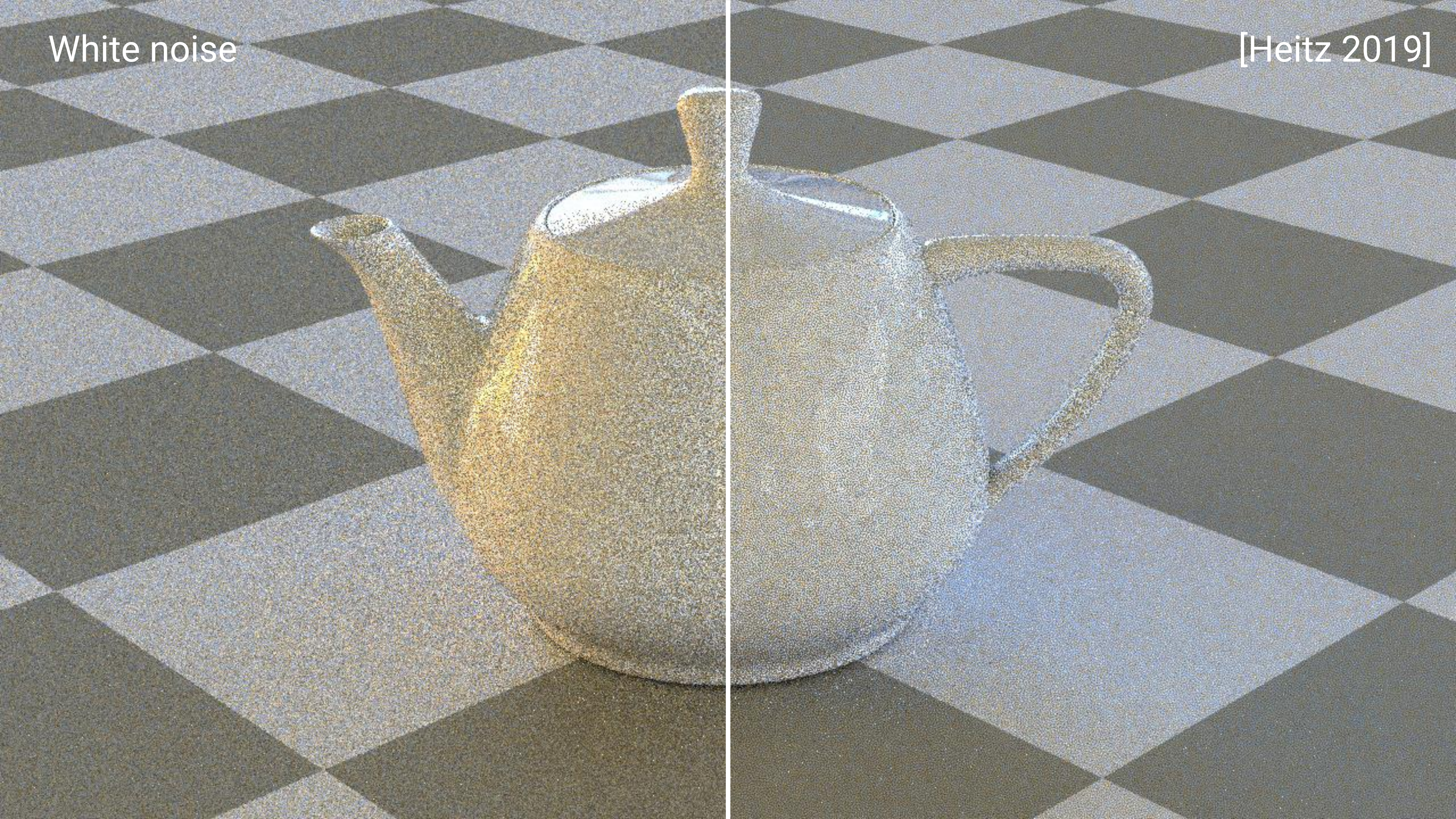
Per bloc :

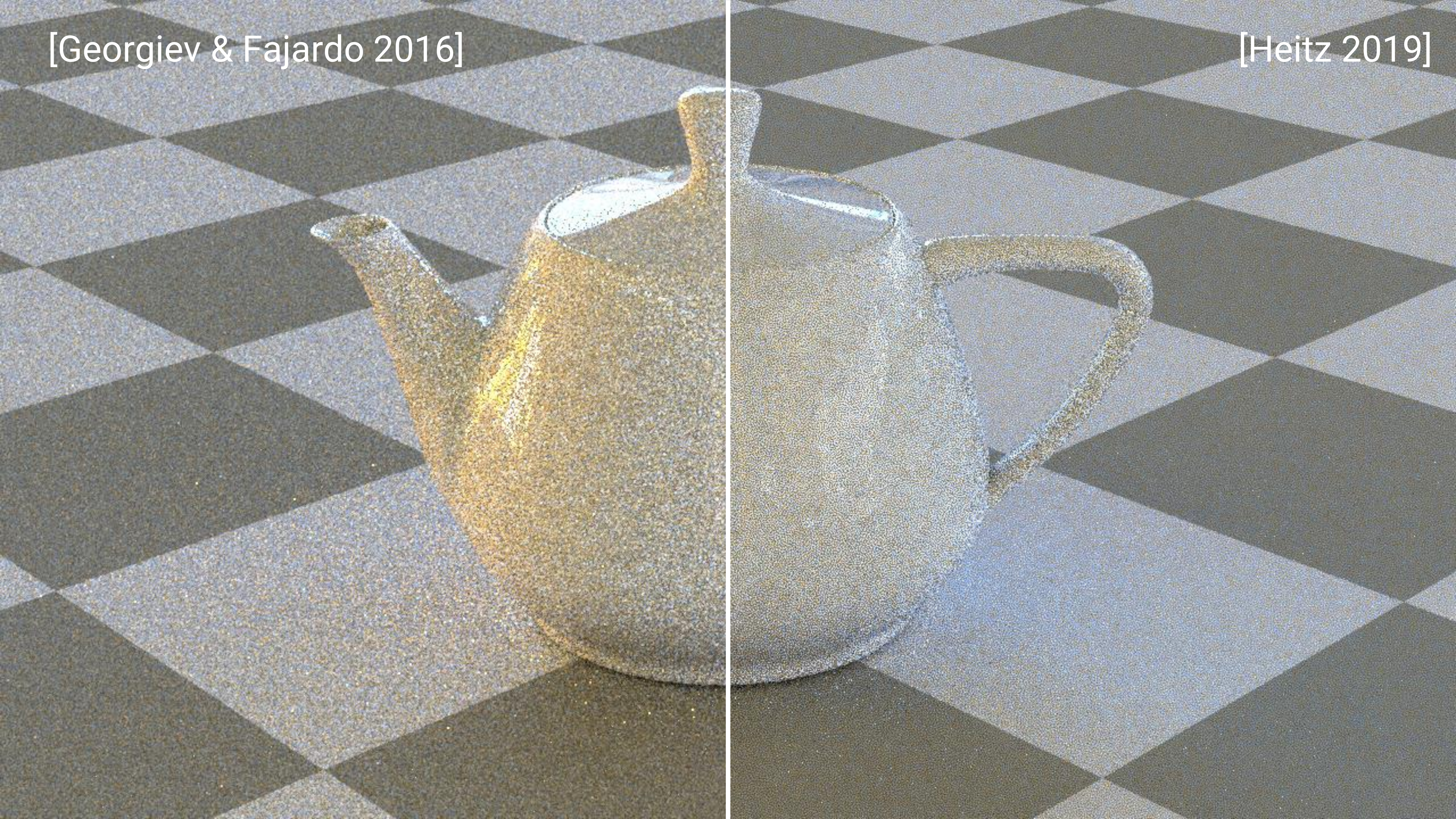
- Sort the dither mask and rendering values
- Assign in order the seed that produced the i-th value to the position of the i-th dithering value
- Render the next frame

Require: I Current rendering, S Current seed, D dither mask
  $S_{t+1} \leftarrow S$
  for each 4×4 bloc b **do**
    $I_b \leftarrow \mathrm{sort}(I[b])$
    $D_b \leftarrow \mathrm{sort}(D[b])$
    for i in $[0..16]$ **do**
      $S_{t+1}(D_b(i)) \leftarrow S(I_b(i))$
    end for
  end for
  Return S_{t+1}


White noise
[Heitz 2019]


[Georgiev & Fajardo 2016]
[Heitz 2019]


White noise
[Heitz 2019]


[Georgiev & Fajardo 2016]
[Heitz 2019]

Results

- Really high quality result in best case
- Scene specific sample distribution
- Works for complex rendering setup
- Low overhead
- Need to render every frames with the same seeds
- Break on edges/texture with significantly worse visual quality
    - Can be improved with some segmentation map but increase cost significantly

Scalable Multi-Class Sampling via Filtered Sliced Optimal Transport

[Salaun & al 2022]

Objectives

- Previous works used arbitrary energy minimization
  - Need provably good energy is needed
  - Lack of theoretical understanding

- This work propose
  - Theorical framework explaining why blue noise error distribution is a good choice
  - General energy term derived from Monte Carlo error
  - An optimization strategy for arbitrary perception metric

Error distribution as sample optimization

- For single pixel it's possible to measure sampling quality with discrepancy
  - Use error upper bound based on sampling quality and function variation
  - This equation is not differentiable and slow to evaluate

$$|I - \frac{1}{n}\sum_{i=1}^{n} f(x_i)| \leq V(f) \cdot D$$

- It is possible to define a similar bound using a differentiable metric (Wasserstein distance)
  - Based on Lipschitz inequality
  - This can be used to optimize pointset [Paulin et al. 2020]

$$\left|\int f(x)\,dx - \frac{1}{n}\sum_{i=1}^{n} f(x^i)\right| \leq \mathrm{Lip}(f)\,.\,W_1(X, 1_\Omega).$$

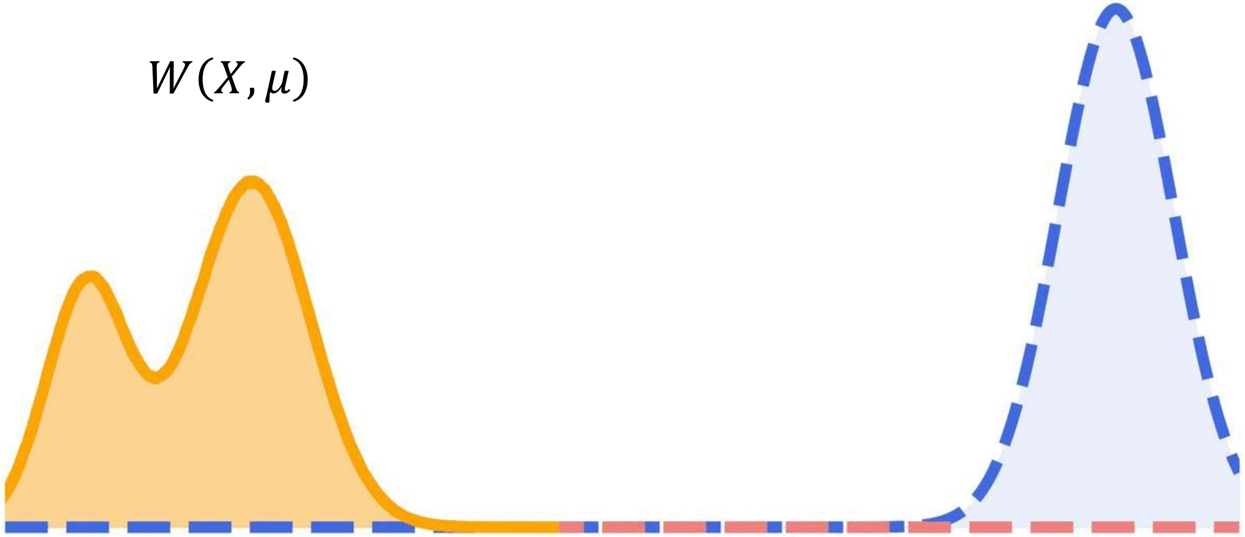
Wasserstein distance

The Wasserstein distance is define by the optimal transport plan minimizing the cost of moving the distribution X to the target distribution μ

- The cost is the total mass displaced per unit distance
- Thinking about 2 piles of sand : Where each sand grain should go to minimize displacement cost ?

- Costly to evaluate even with discreet distribution

$W(X, \mu)$

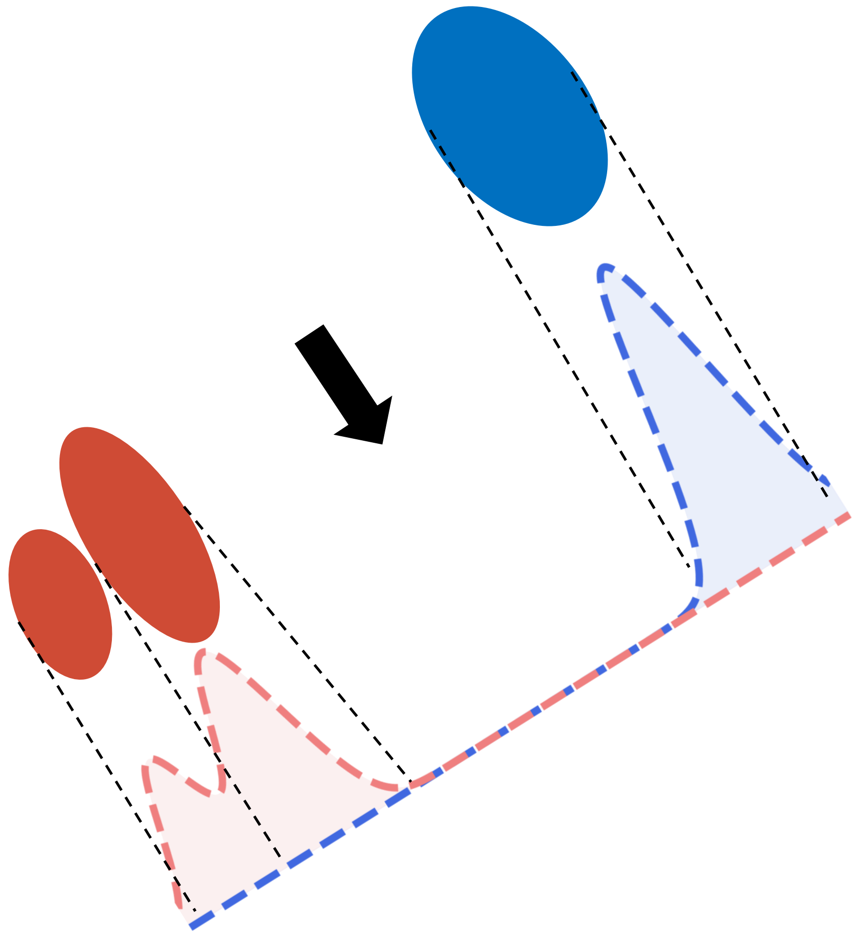


Sliced Wasserstein distance

- Full dimensional Wasserstein distance is often to costly to be used
- Instead we can use an upper bound using the sliced version

$$SW(X, U) = \int_{S^{d-1}} W(X^\theta, U^\theta) d\theta$$

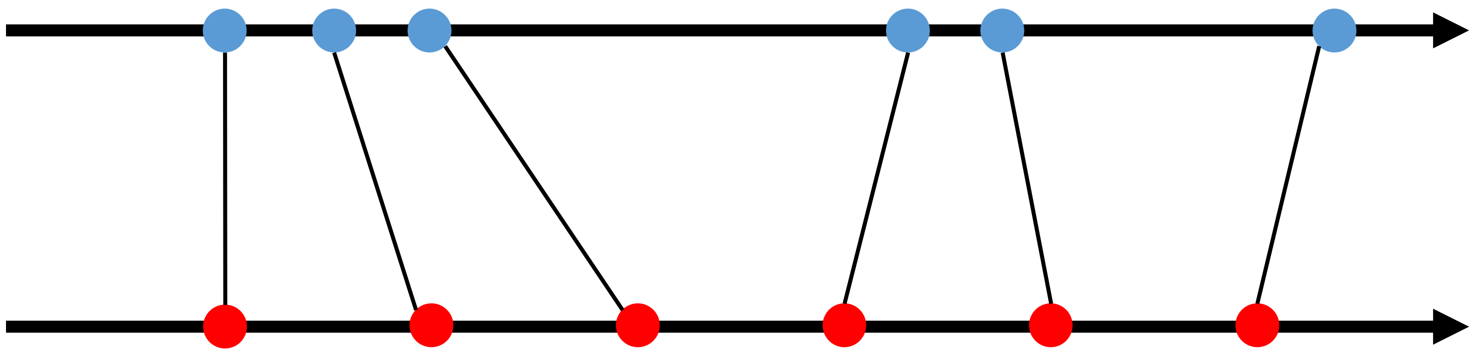
- This integral over all direction can be solved using Monte Carlo estimation



Sliced Wasserstein distance

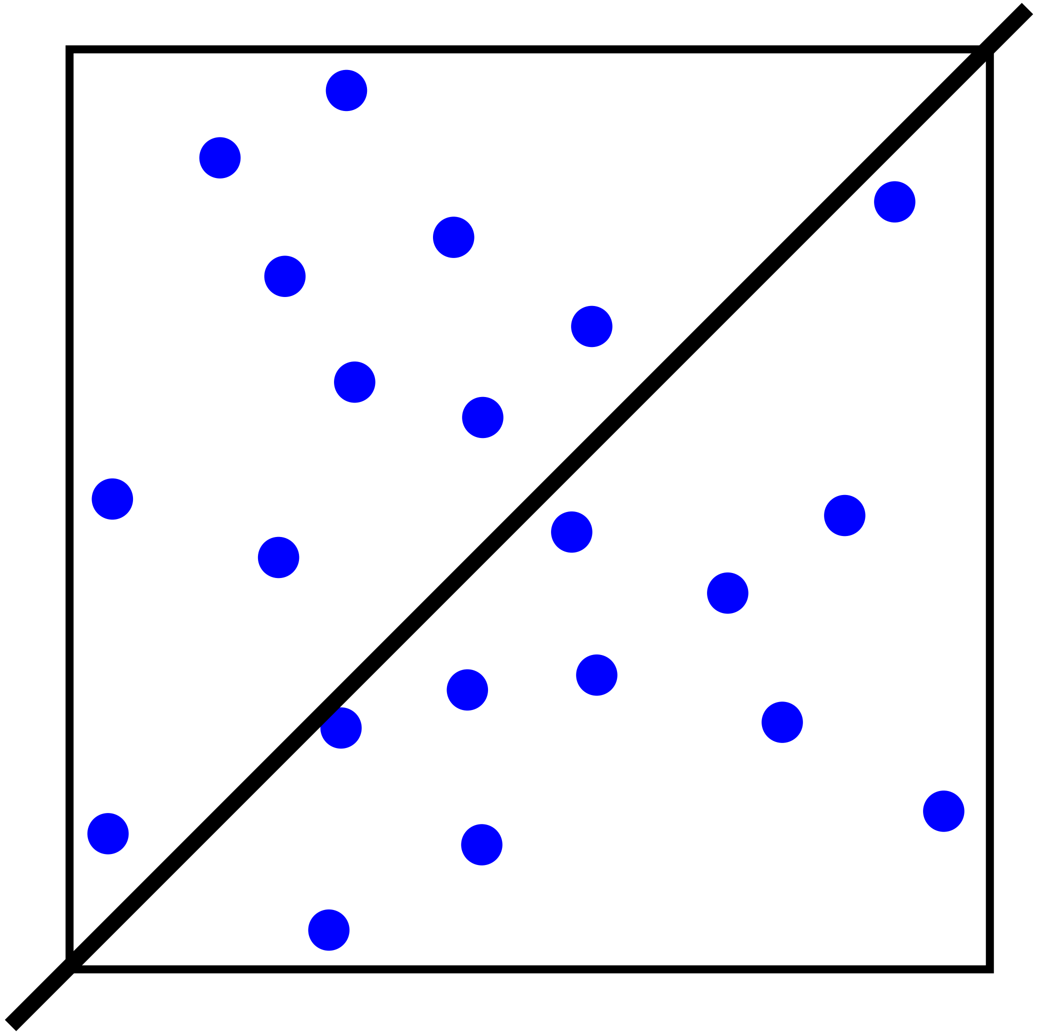
- For each slice we need to solve the optimal transport plan
  - 1D optimal transport between 2 discreet distribution has a simple solution
  - Also work with discretization in semi discreet case

Solving the 1D optimal transport only require sorting points



Sliced Wasserstein distance derivatives

- It is possible to compute the derivatives of the Sliced Wasserstein distance
  - For all sliced, the derivative is the vector for each point to the associated target



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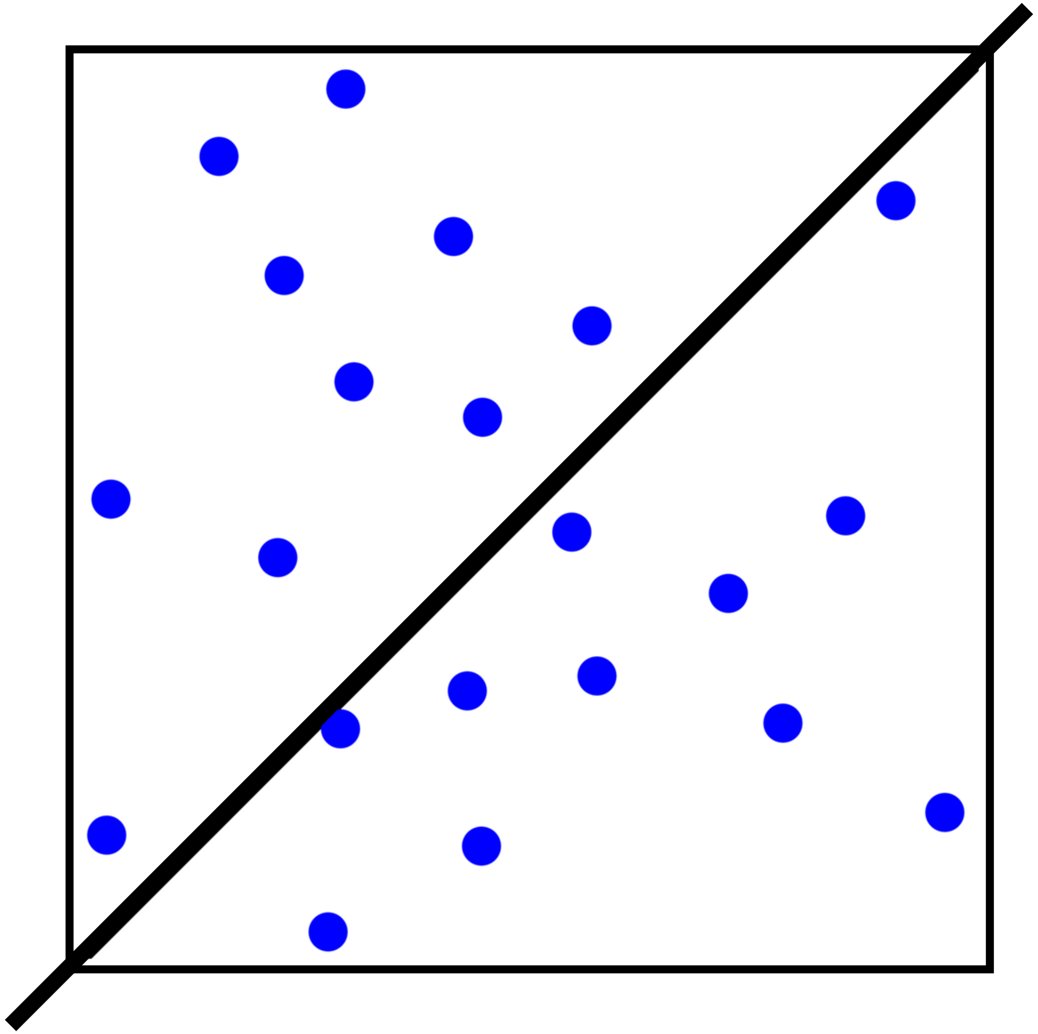

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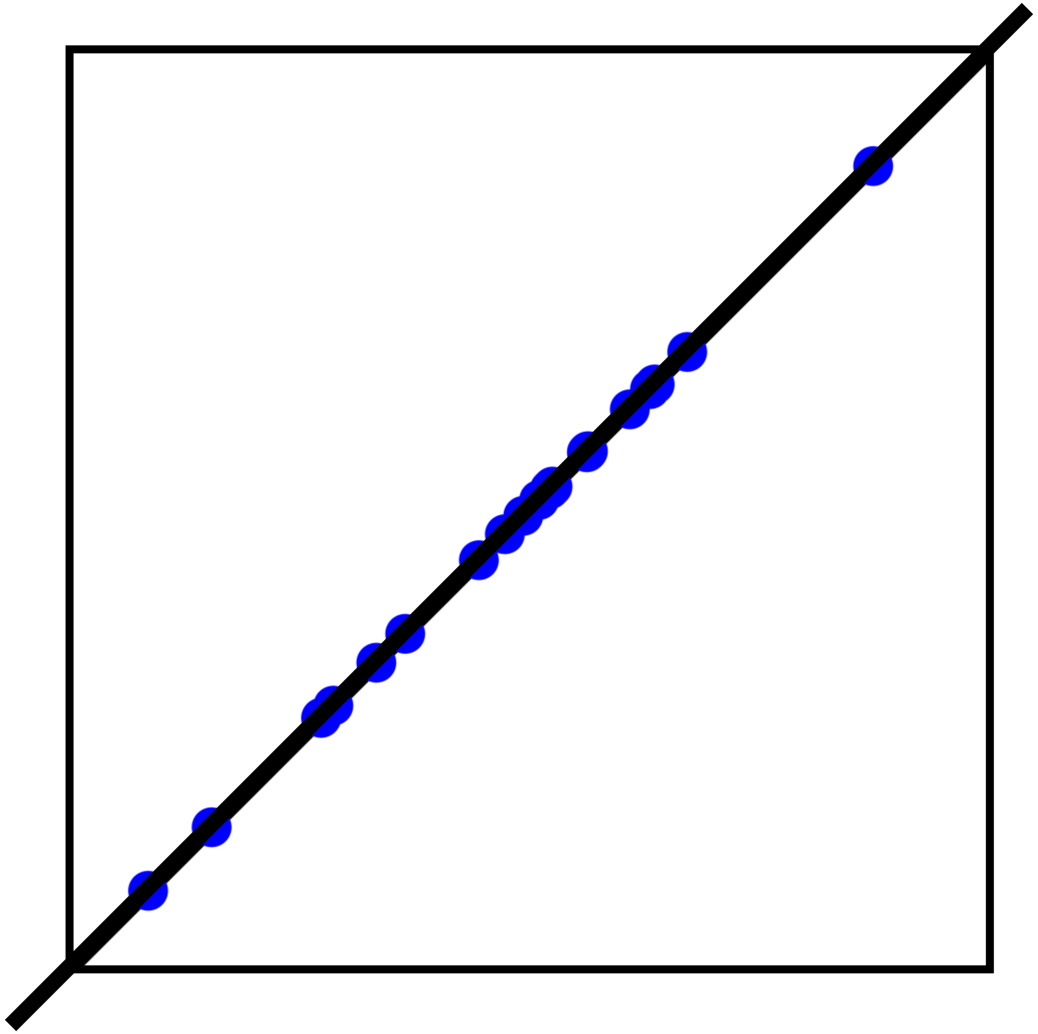
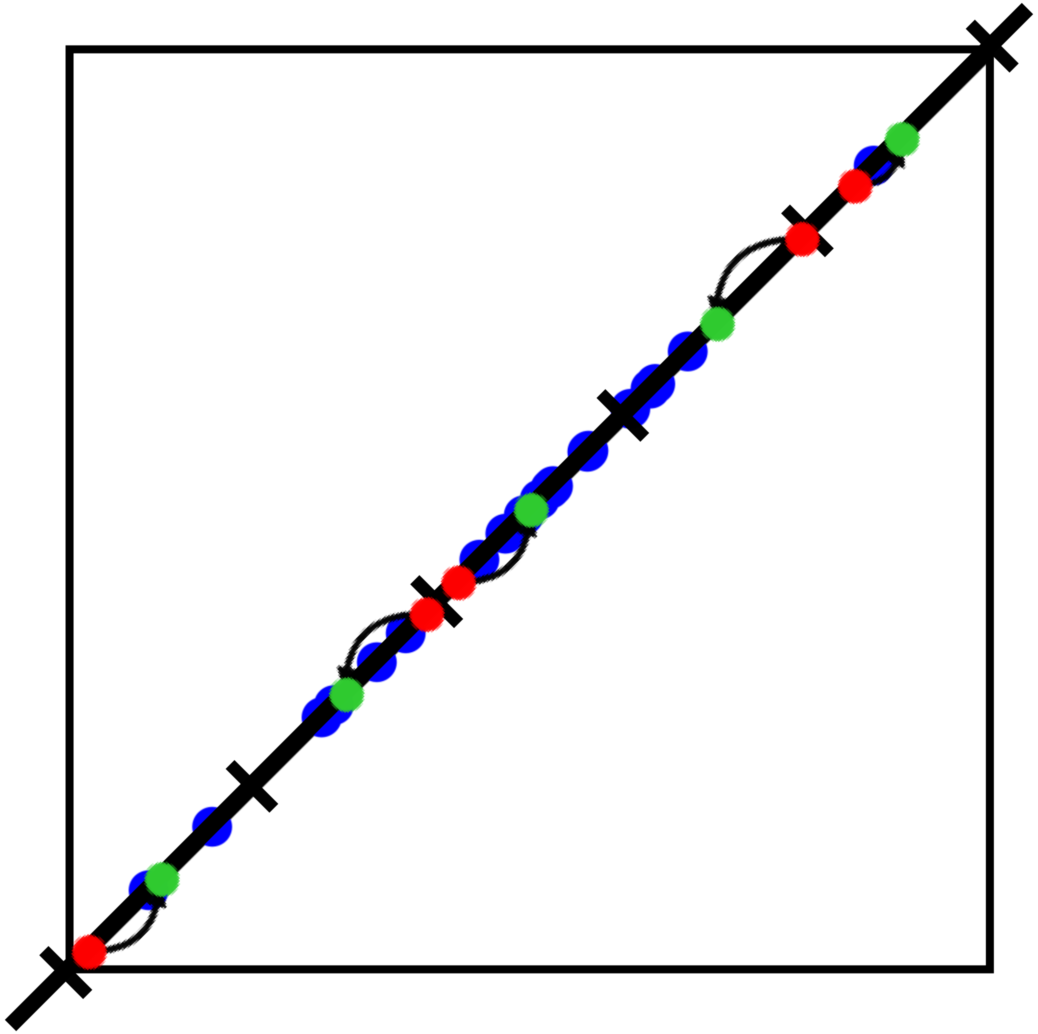


Sliced Wasserstein distance derivatives

- It is possible to compute the derivatives of the Sliced Wasserstein distance
  - For all sliced, the derivative is the vector for each point to the associated target

- Finally use a simple Gradient based optimization to optimize the pointset
  - The gradient are noisy but will get smoothed over iterations



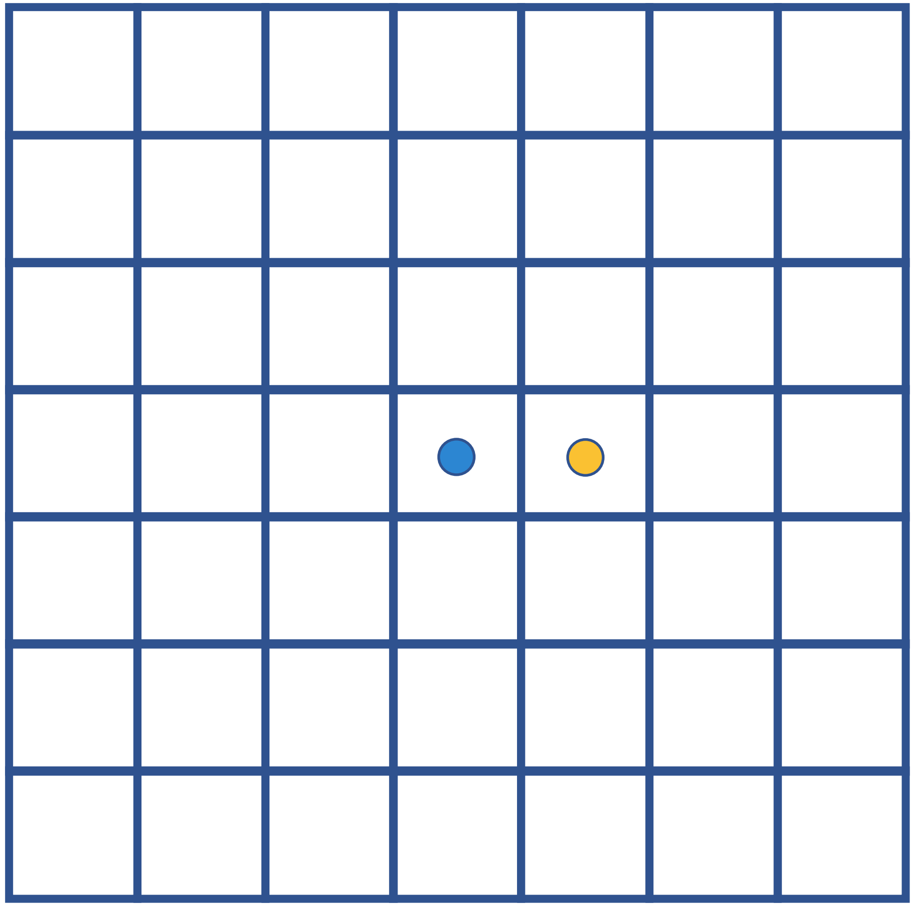
Sliced optimal transport sampling algorithm

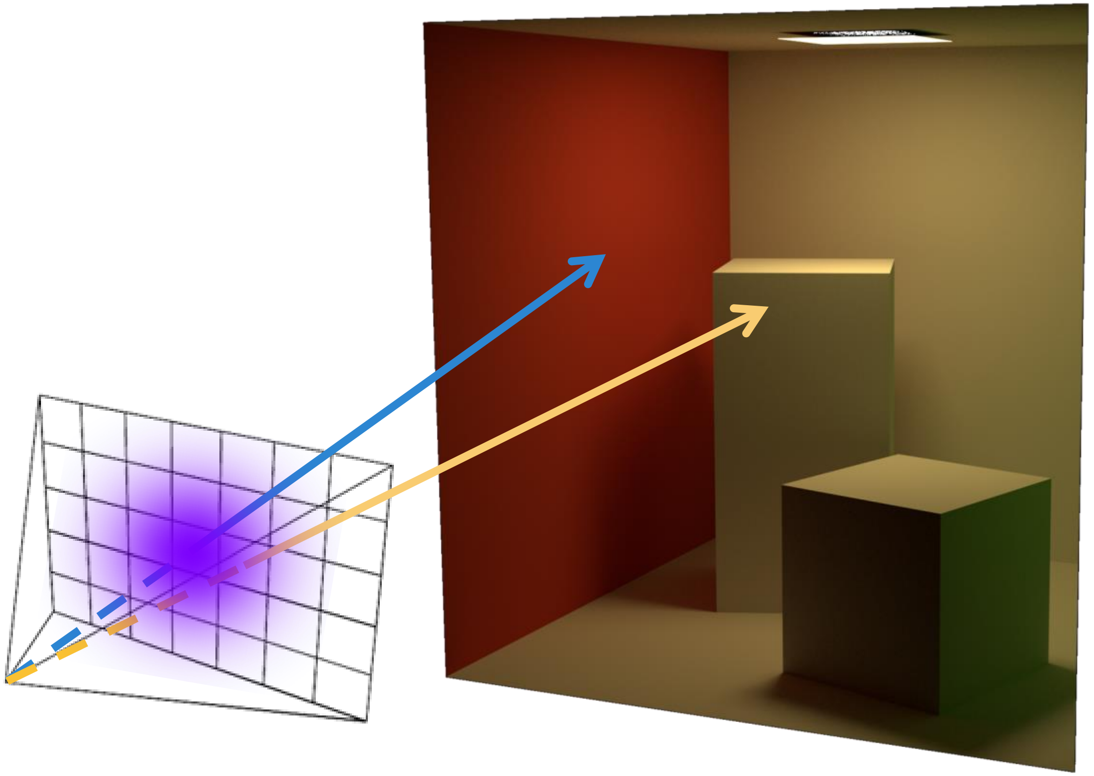
- Randomly regenerate U at each step to avoid overfit to random points
- It's possible to use advanced gradient descent
- Possible to average multiple projection at each step and parallelize it
- Relatively fast convergence but still an optimization task

Example

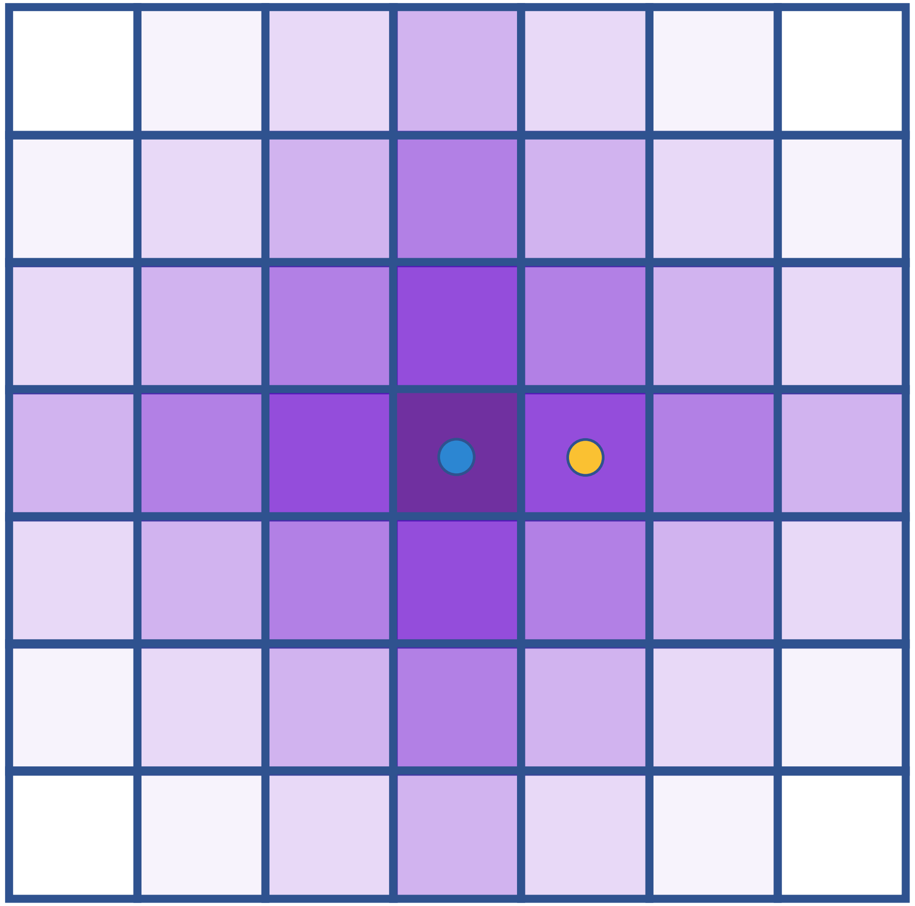
$$
\begin{array}{l}
X \leftarrow \mathrm{rand}(N, D) \\
\textbf{for } i \in \{0, i_{max}\} \textbf{ do} \\
\quad \theta \leftarrow \mathrm{rand\_direction}(D) \\
\quad U \leftarrow \mathrm{rand}(N, D) \\
\quad X^\theta \leftarrow X \cdot \theta \\
\quad U^\theta \leftarrow U \cdot \theta \\
\quad X \leftarrow X + \lambda(U^\theta - X^\theta)\theta \\
\textbf{end for} \\
\textbf{Return } X
\end{array}
$$

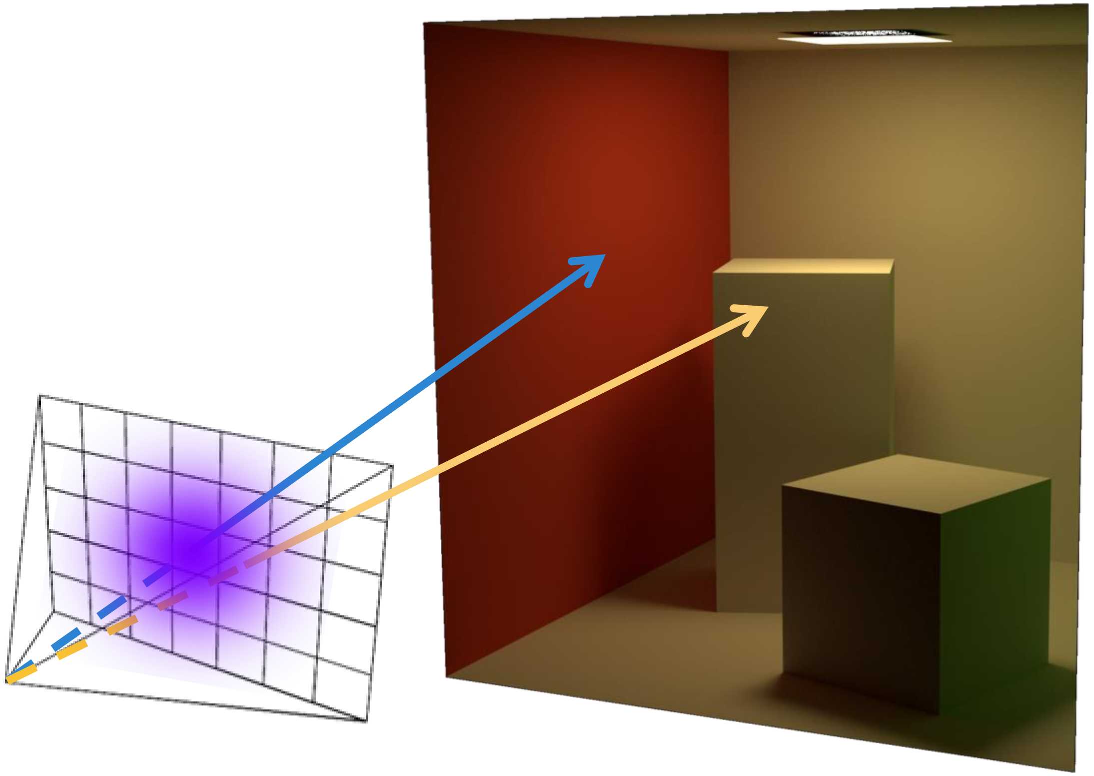
Multi class rendering





Multi class rendering

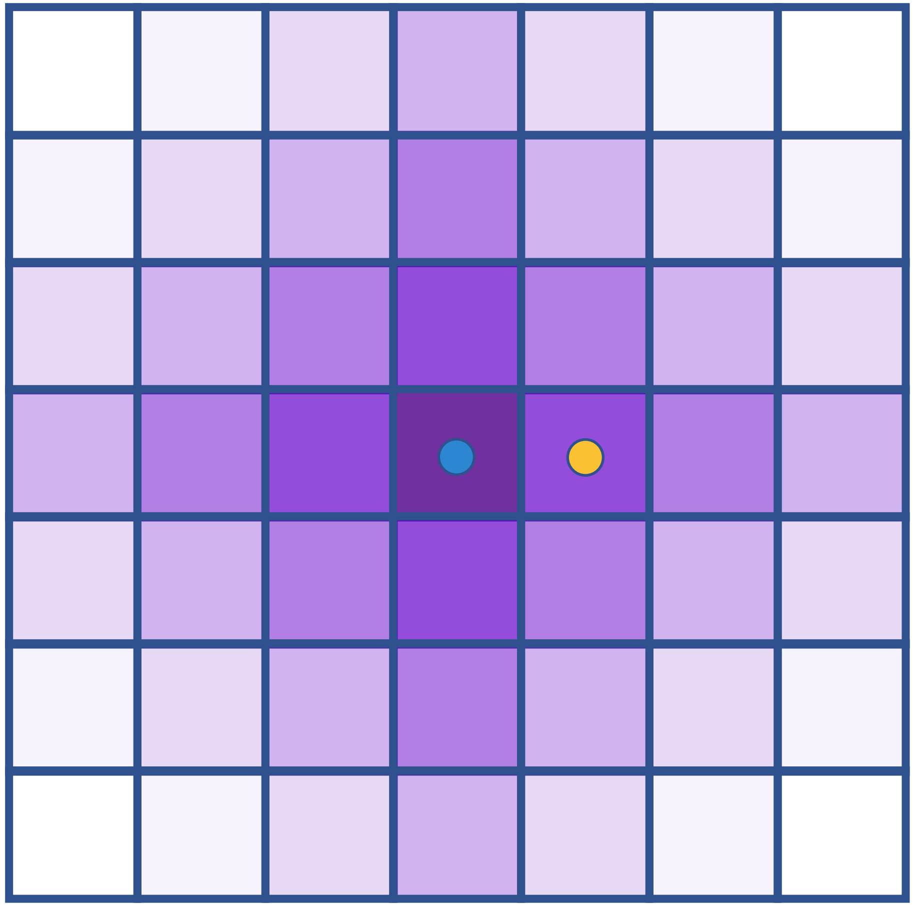


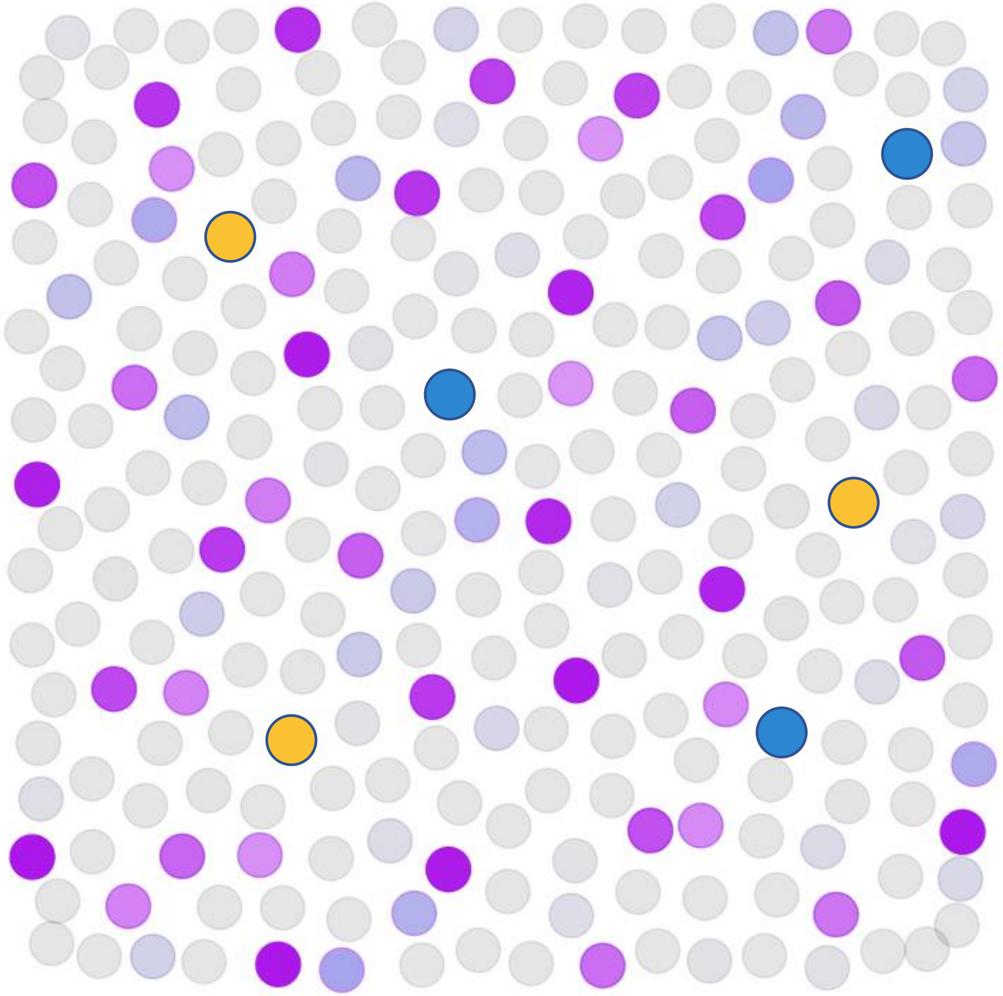


Multi class rendering

- Our eye see pixel together
  - The samples of multiple pixel are seen together
  - Need a commune optimization
  - One sample contribute to multiple pixel integration (into our eye)
    - Some have more contribution than others
- Contrary to previous work we consider pixel integration not independent
  - Samples from multiple pixel can be seen as a single sampling sequence
  - Lot of conflicting objectives

Multi class rendering





Error distribution as sample optimization

- It is possible to define a Multi-class error bound
  - Sum of weighted contribution of neighboring pixels
  - All samples of the image contributes to every other pixel
- It make sample contribution non binary
  - Samples may contribute to the integration with varying factors
  - We introduce Filtering into the Wasserstein distance
  - End up to a barycentric optimization

$$Q_w(X) = \frac{1}{N} \sum_{x_i \in X} \underbrace{w(x_i)}_{\text{image-space Gaussian}} f(x_i) \approx I_w$$

$$\varepsilon_w(X) = |Q_w(X) - I_w|$$

$$\sum_p \varepsilon_{w_p}(X) \leq L_f \sum_p B_1(X, w_p, U) = Loss$$

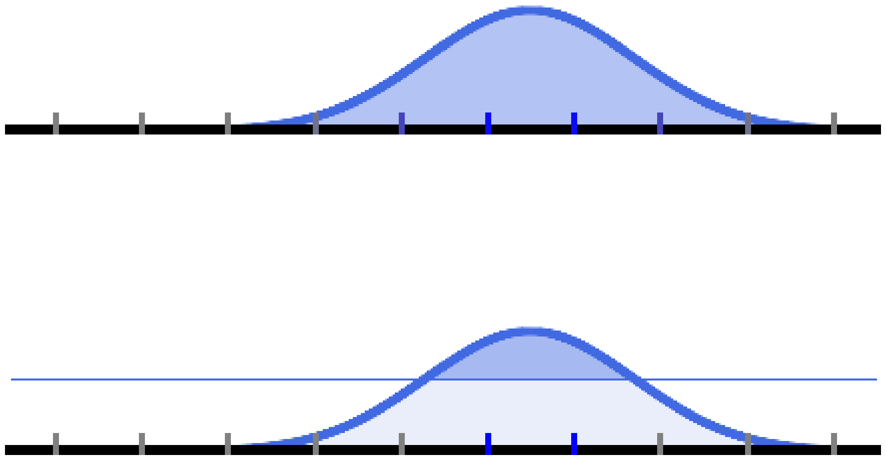
$$B_1(X, w_p, U) = \int_{\mathbb{R}} W(X_{w>z}, U_{w>z}) dz$$

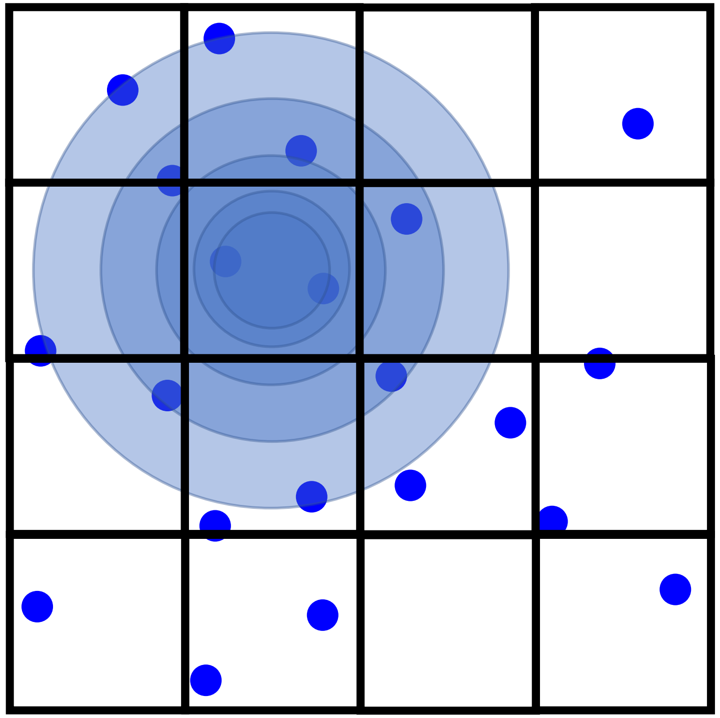
Filtered Wasserstein distance

Single optimization for multiple objectives

$$B_1(X, w_p, U) = \int_{\mathbb{R}} W(X_{w>z}, U_{w>z})dz$$

Decompose a complex optimization into many small problems





Filtered Wasserstein distance

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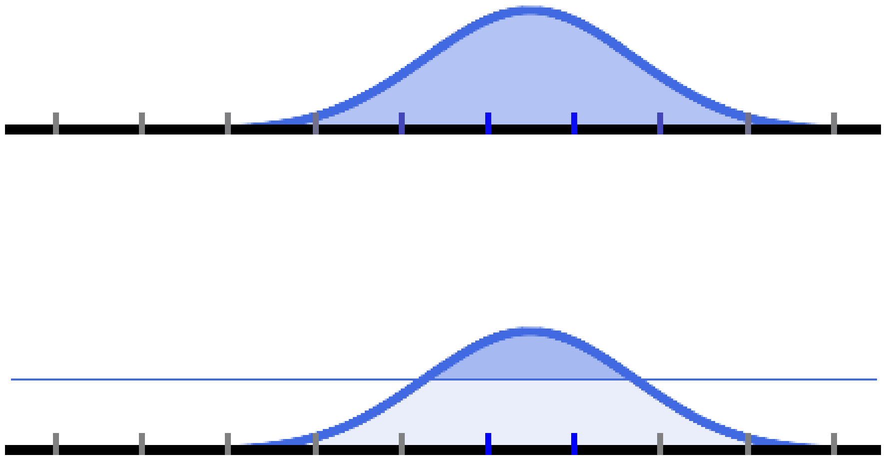
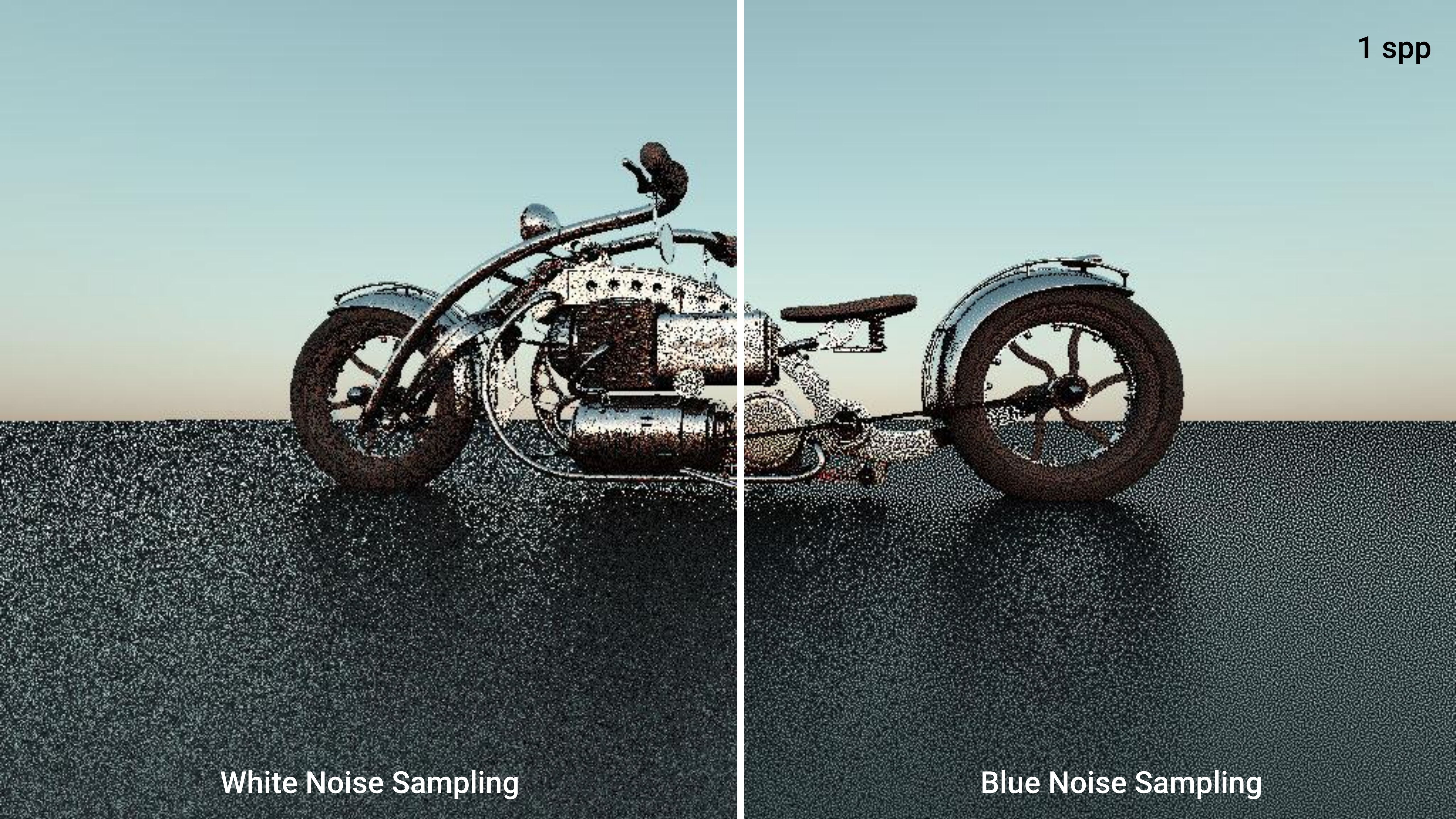


$X \leftarrow \mathrm{rand}(N, D)$
for $i \in \{0, i_{max}\}$ **do**
  $w_i \leftarrow \mathrm{rand\_Kernel}()$
  $z \leftarrow \mathrm{rand}(0, 1)$
  $X_{w_i>z} \leftarrow \mathrm{sample\_selection}(X, w_i, z)$
  $\theta \leftarrow \mathrm{rand\_direction}(D)$
  $U_{w_i>z} \leftarrow \mathrm{rand}(N_{w_i>z}, D)$
  $X^{\theta}_{w_i>z} \leftarrow X_{w_i>z} \cdot \theta$
  $U^{\theta}_{w_i>z} \leftarrow U_{w_i>z} \cdot \theta$
  $X_{w_i>z} \leftarrow X_{w_i>z} + \lambda(U^{\theta}_{w_i>z} - X^{\theta}_{w_i>z})\theta$
end for
Return X


1 spp
White Noise Sampling
Blue Noise Sampling

Results

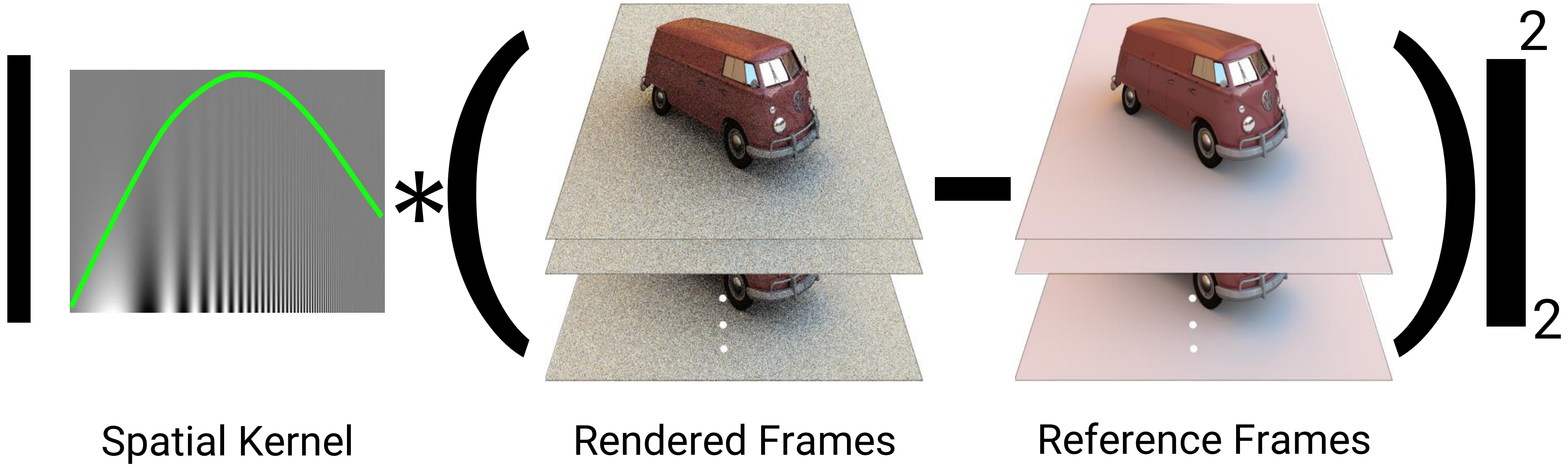
- This is the first gradient based optimization that produce blue noise error distribution
- The blue noise property is a consequence of the minimization of the L1 error bound
  - It mean BN is the expected property because of the perceptual kernel we choose
  - An other kernel could result in other correlation

- The method can be extended to more complex perception models
  - We used the same gaussian as other work for comparison

- Optimization is slow even using a SGD based optimizer
  - The reason is the important cost of computing Wasserstein distance
  - Need to average lot of Wasserstein distance per step to get noise reduction

Temporal extension

It's possible to model the perception in space and time



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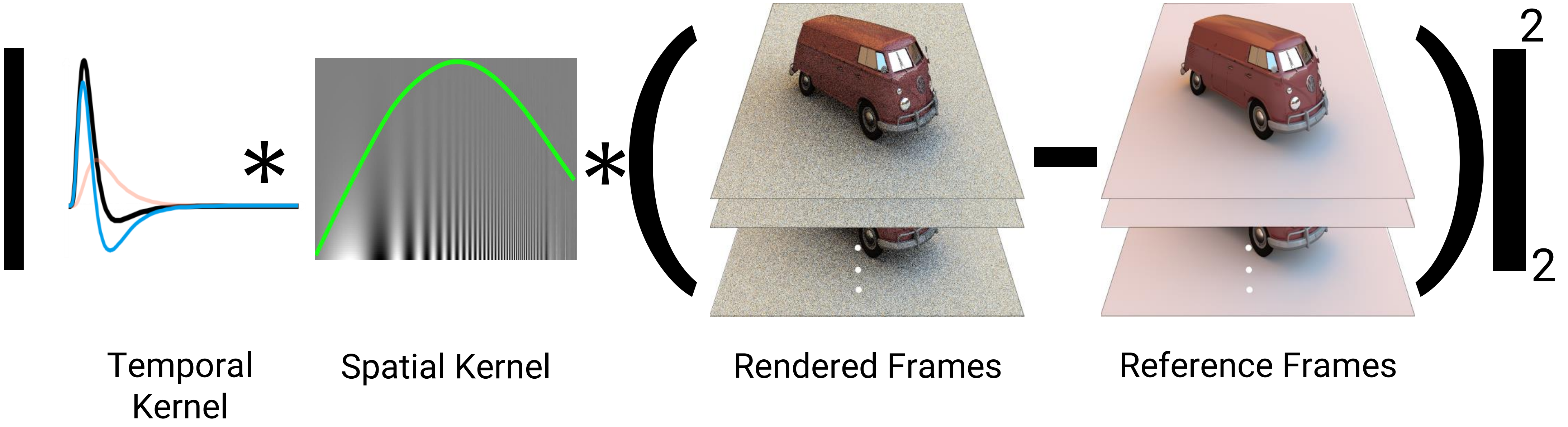
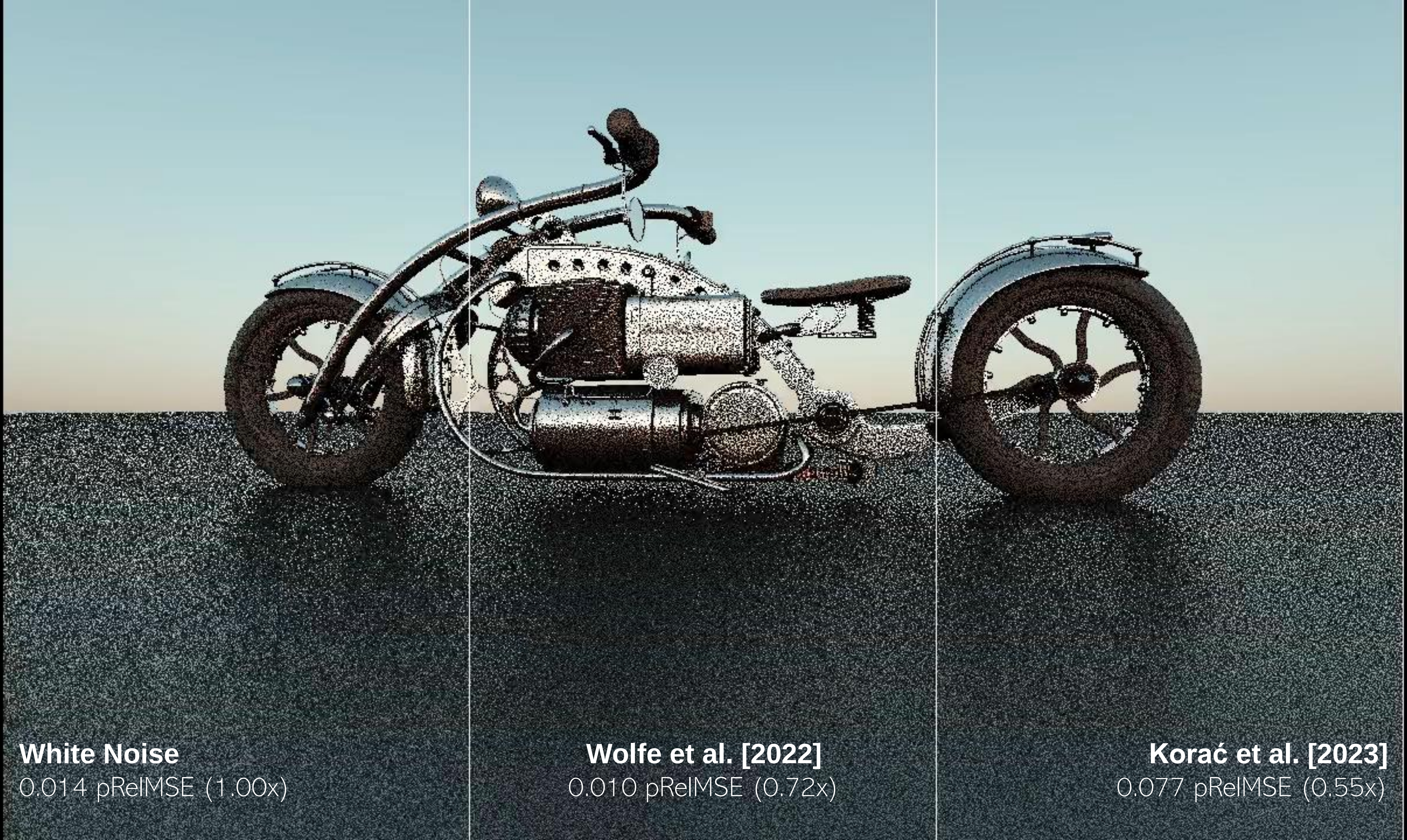
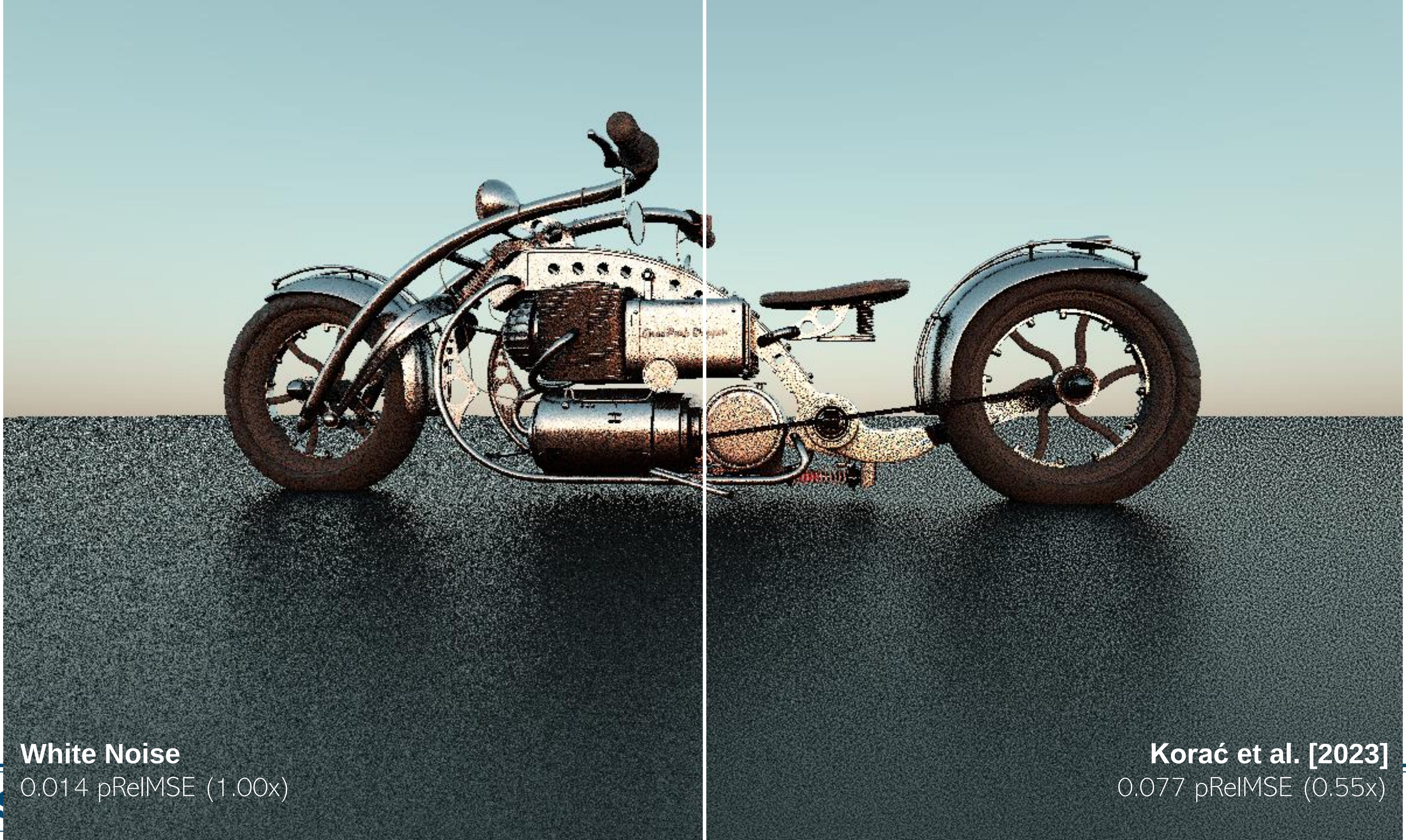




White Noise
0.014 pRelMSE (1.00x)
Wolfe et al. [2022]
0.010 pRelMSE (0.72x)
Korać et al. [2023]
0.077 pRelMSE (0.55x)


White Noise
0.014 pRelMSE (1.00x)
Korać et al. [2023]
0.077 pRelMSE (0.55x)


White Noise
0.0065 pRelMSE (1.00x)
Wolfe et al. [2022]
0.0043 pRelMSE (0.66x)
Korać et al. [2023]
0.0031 pRelMSE (0.48x)


White Noise
0.0065 pRelMSE (1.00x)
Korać et al. [2023]
0.0031 pRelMSE (0.48x)

Temporal extension

- Extension to temporal domain result in Blue noise error distribution
  - Blue noise property is only visible when frames are average or see in an animation
  - Each frame individually looks like uncorrelated noise
  - Result from the energy that aim only for temporal optimization not single frames

- Could be improve with reprojection and scene dependent information
  - All the samples works for multiple scene
  - No reprojection is done
  - Lot of improvement is possible

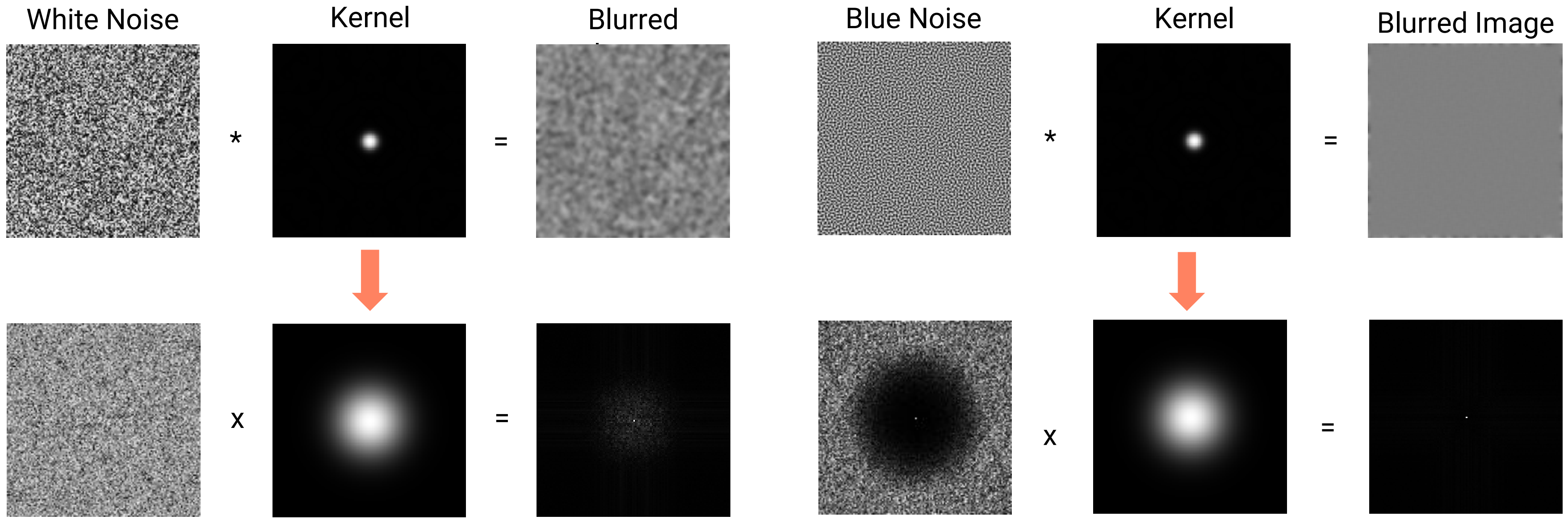
Blue noise error and denoising

Blue noise error and denoising

- All previous method focus on image perception as raw rendering
  - Noise perception only work on noisy rendering
- Denoising also benefit from high frequency noise
  - Simple blur follow exactly the same equations
  - Generally analytic filtering works
  - Neural based denoising require specific attention

Analytic denoising

- Denoising is often composted of low pass filters
  - Even when using bilateral filters
- More generally blue noise error distribution create a good sampling distribution for groups of pixel
  - More information locally



Neural based denoising

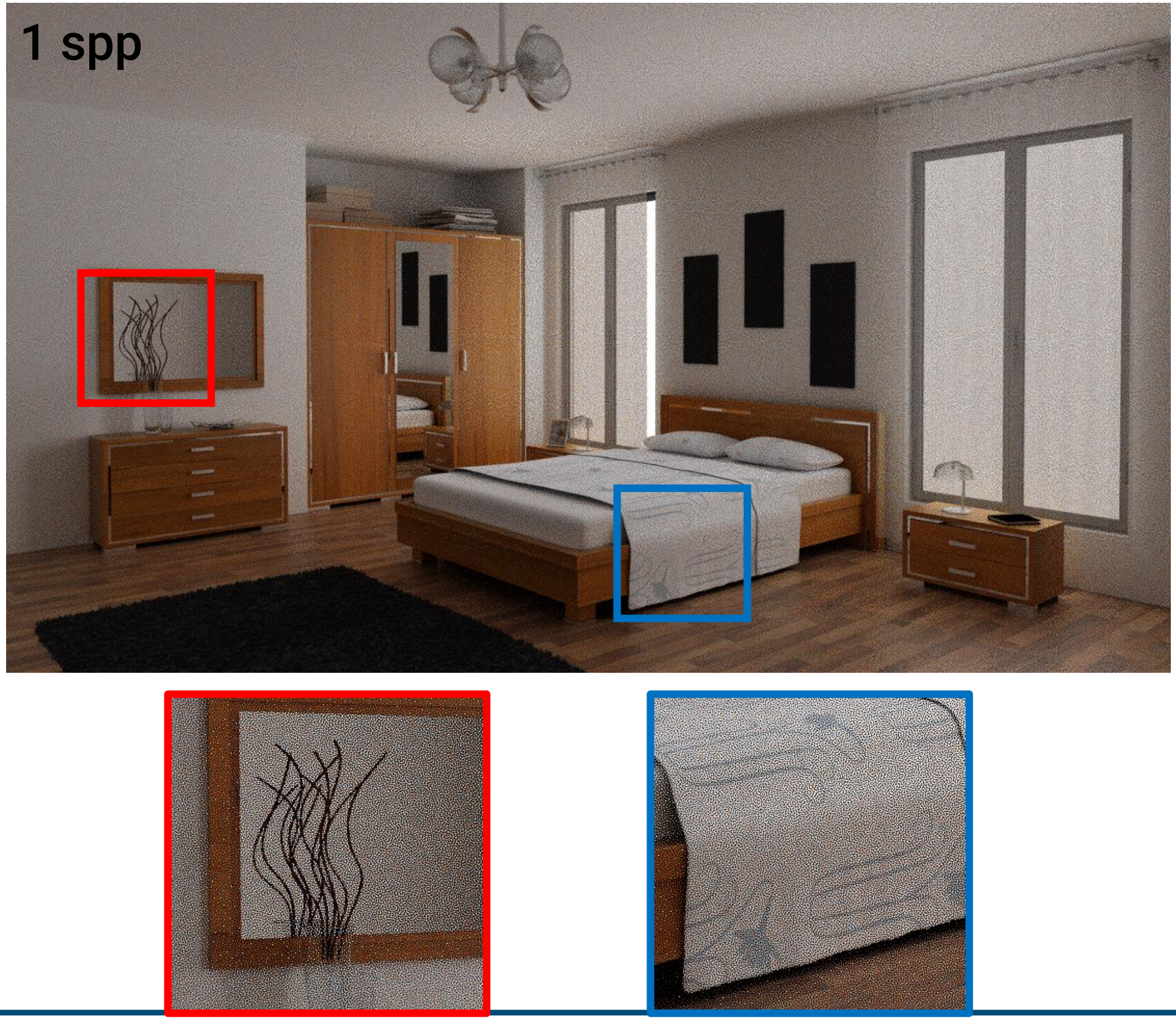
- Neural based denoiser I now the state of the art
  - High efficiency and order of magnitude higher quality
  - Can be guided with G-buffers

- Blue noise distribution works with Neural based denoiser
  - If the network is trained on this type of noise
  - Reduce randomness and better guide information to the denoiser
  - Particularly true with kernel prediction network

- Denoiser train on Blue noise achieve high error is denoising uncorrelated noise
  - Need to ensure Blue noise distribution is actually achieved

White Noise

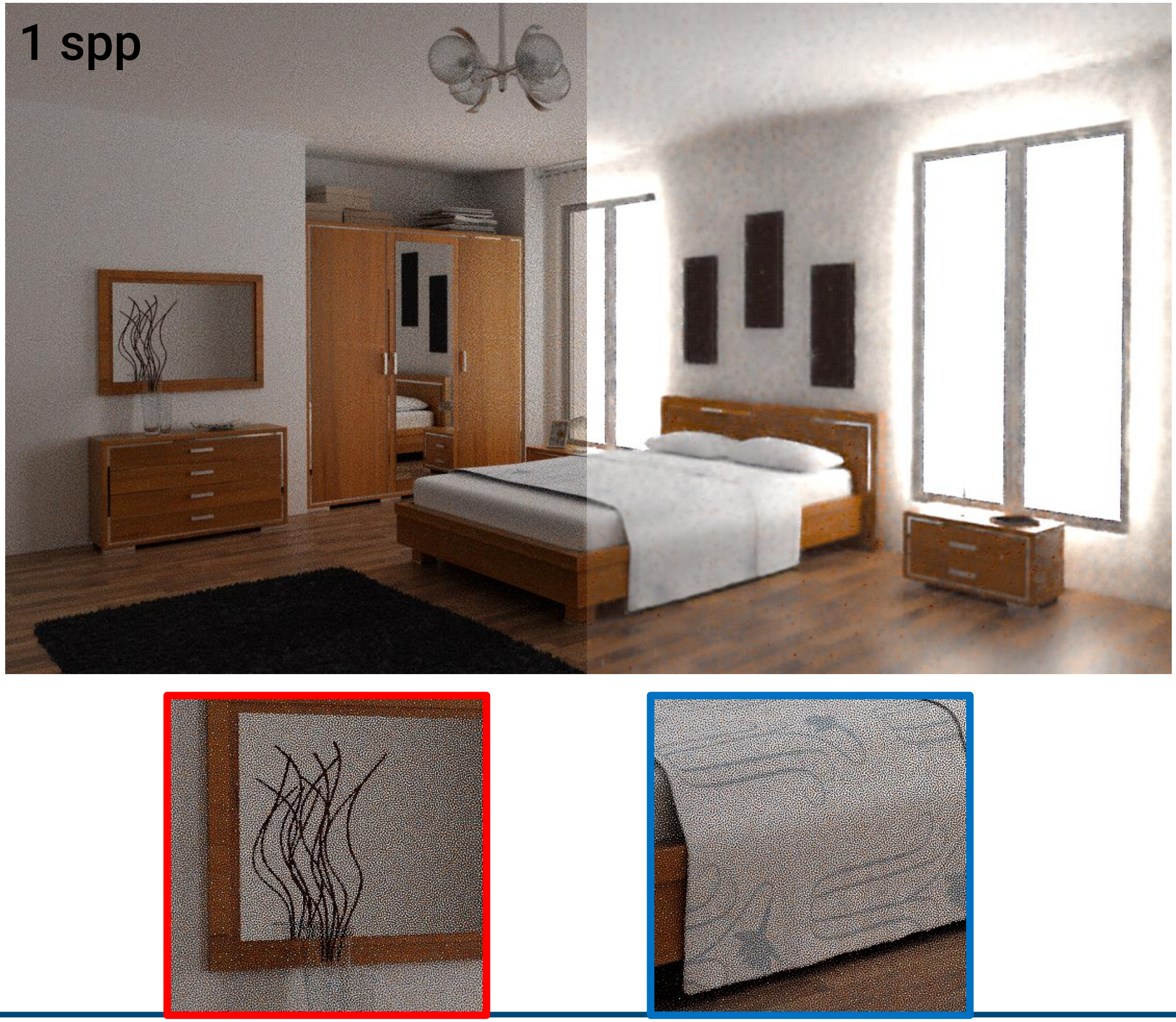
Blue Noise



*using a simple Unet

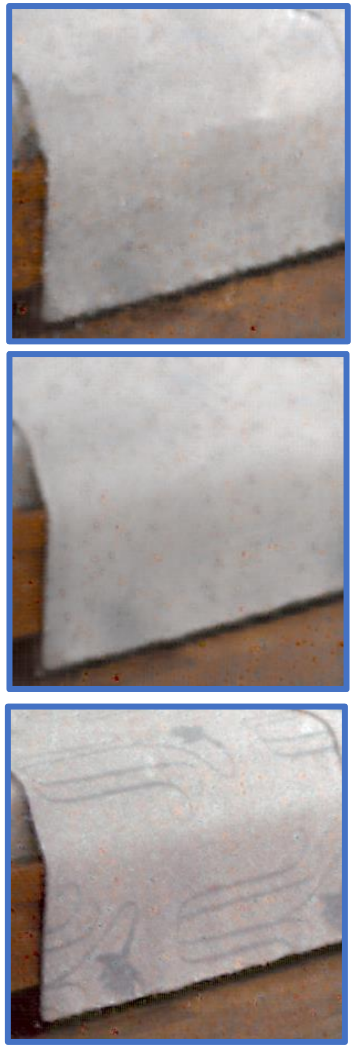
Noisy Input | Denoised Image*

Noisy Input | Denoised Image*



White Noise sampling

Blue Noise sampling



White Noise denoiser*

Blue Noise denoiser*

*simple Unet

White Noise | Blue Noise

Noisy Input | Denoised Image | Noisy Input | Denoised Image

Blue noise error and denoising

- Blue noise error distribution improve rendering
  - For real time and offline rendering
  - Adapt sampling for denoising
- Each problem need tailored algorithm
  - There is no perfect algorithm
  - Match strengths and weakness with your problem
- Low Blue noise quality can be still better than no correlation at all

Summary

What have we learned today?

Summary

- Blue noise error distribution is an important axis of improvement
  - Simply include the perception into the sampling process
  - Good coordination with denoising

- Require few conditions
  - Smooth integrand in screen space
  - Low dimension for apriori method
  - Temporal stability for a posteriori