Spectral Raytracing

Realistic Image Synthesis - 2021

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Outline

- Motivation
- Properties
- Rendering Equation
- Display
- Sampling
- Upsampling

Why use Spectral Raytracing?

- Physical plausibility
- Precise
- Correct wavelength dependent IORs
- Dispersion for free
- Blackbody curves
- Energy analysis

Why NOT use Spectral Raytracing?

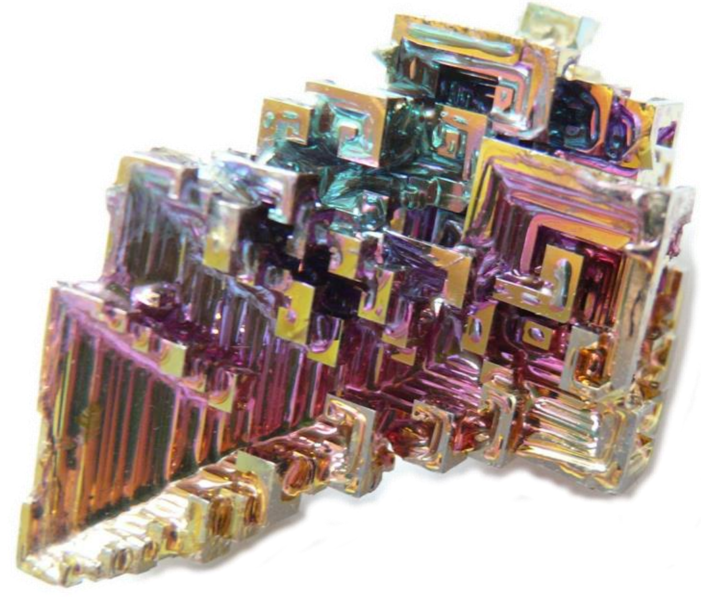
- Asset pipeline requires preprocessing
- Spectral data not as common as RGB data
- Stylized rendering is difficult
- Can get complicated

Spectral Renderer

- Manuka (Weta Digital)
- Thea Render (Altair)
- Mitsuba (Tizian & Jakob et al.)
- ART (Tobler & Wilkie et al.)

© Weta Digital

Spectral Properties



Electromagnetic Spectrum

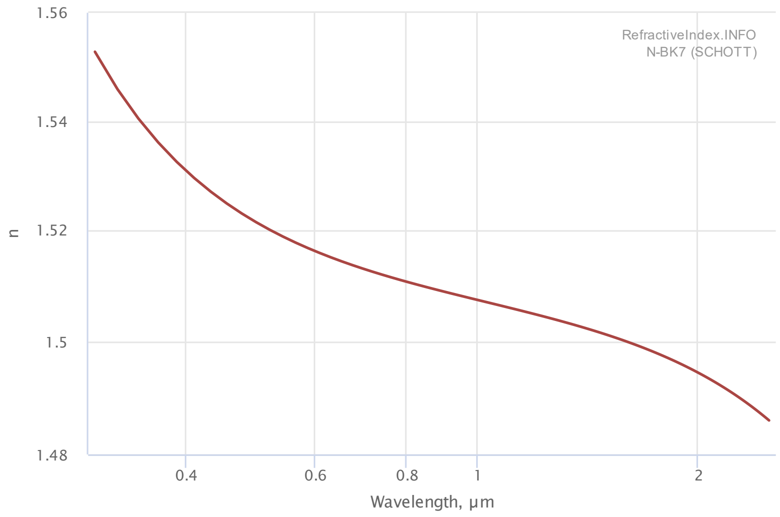
- $\lambda = \frac{v}{f} = \frac{c}{nf}$
- v Speed of light in medium
  - Vacuum $v = c = 299792458\, {}^{m}/_{s}$
- n Refractive index of medium
  - Vacuum $n = 1$
- f Frequency
- λ Wavelength
  - Usually given in nanometer
  - Nonlinear as wavelength depends on surrounding medium

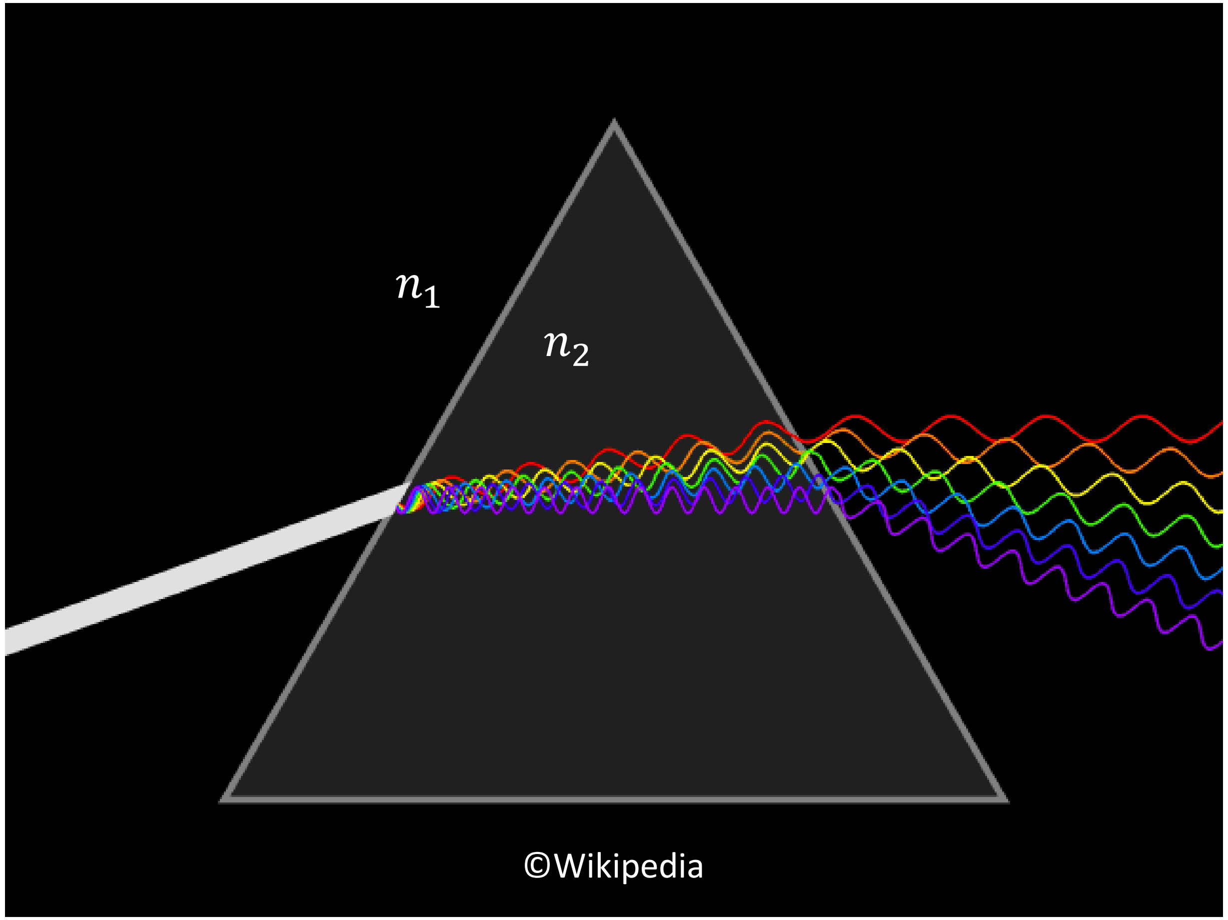
Visible Spectrum

- $\sim 380nm\ - \sim 750nm$
- Range visible by the human perception
- Wavelength around the size of bacteria, but normally scenes are given in larger scales
- This makes it possible to neglect a lot of wave-specific effects
  - Especially ⇒ no interference or diffraction

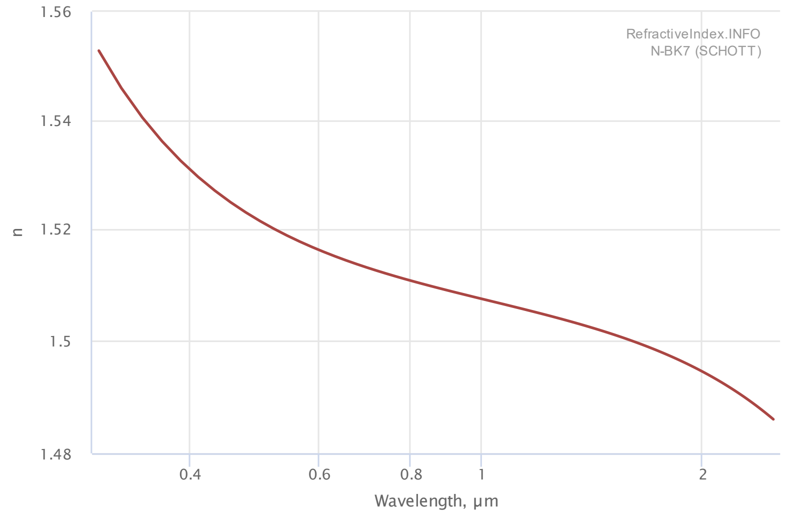
Wavelength Dependency

- Reflection & refraction on a medium interface is wavelength dependent
- Equally true for conductive and dielectric interfaces
- Light bundle split into the composing parts is called dispersion





Sellmeier Equation



- $n^2(\lambda) = 1 + \sum_i \frac{B_i \lambda^2}{\lambda^2 - C_i}$
- Material can be characterized with simple coefficients instead of a lookup table
- Developed 1872 by Wilhelm Sellmeier
- Alternatively, Cauchy's equation can be used

- E.g., BK7:
  - $n^2(\lambda) = 1 + \frac{1.03961212\,\lambda^2}{\lambda^2 - 0.00600069897} + \frac{0.231792344\,\lambda^2}{\lambda^2 - 0.0200179144} + \frac{1.01046945\,\lambda^2}{\lambda^2 - 103.560653}$

Rendering Equation

Also called Light Transport Equation…

Rendering Equation

- $L_o(x, \omega_o) = L_e(x, \omega_o) + \int_\Omega f(x, \omega_i, \omega_o) L_i(x, \omega_i) \cos\theta \, d\omega_i$
  - $L_o(x, \omega_o)$ - Outgoing radiance
  - $L_e(x, \omega_o)$ - Emitting term
  - $L_i(x, \omega_i)$ - Incoming radiance
  - $f(x, \omega_i, \omega_o)$ - BSDF

- Integral over hemisphere
- No wavelengths to be found

RGB Rendering Equation

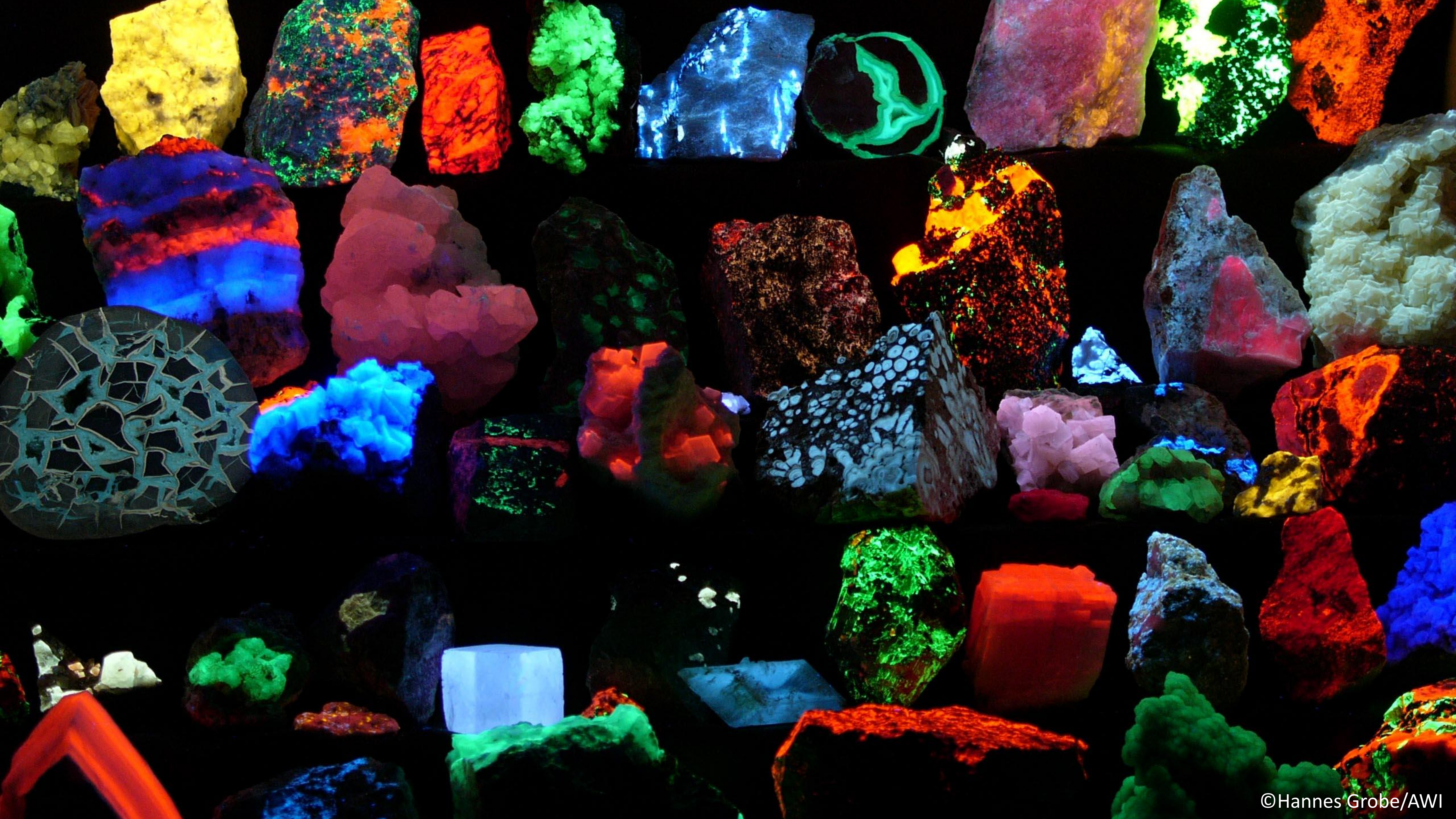
- $\begin{pmatrix} L_{o,R} \\ L_{o,G} \\ L_{o,B} \end{pmatrix}(x, \omega_o) = \begin{pmatrix} L_{e,R} \\ L_{e,G} \\ L_{e,B} \end{pmatrix}(x, \omega_o) + \int_\Omega \begin{pmatrix} f_R \\ f_G \\ f_B \end{pmatrix}(x, \omega_i, \omega_o) \begin{pmatrix} L_{i,R} \\ L_{i,G} \\ L_{i,B} \end{pmatrix}(x, \omega_i) \cos\theta \, d\omega_i$
    - Three values associated with one ray
    - R,G,B $\neq$ wavelength
    - Not physical, but practical

Spectral Rendering Equation

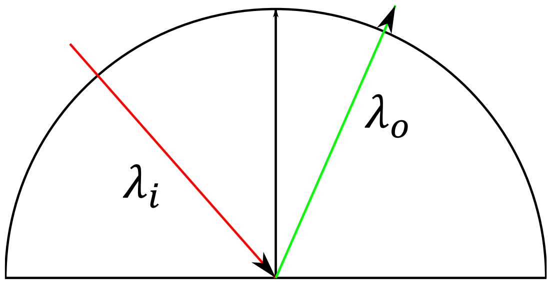
- Do not use RGB but wavelengths
- Associate each ray with a wavelength λ

- $L_o(x, \omega_o, \lambda) = L_e(x, \omega_o, \lambda) + \int_\Omega f(x, \omega_i, \omega_o, \lambda) L_i(x, \omega_i, \lambda) \cos\theta \, d\omega_i$

- What about fluorescence?

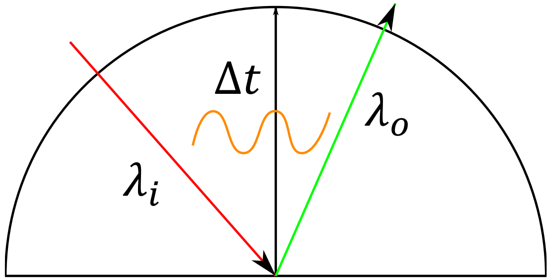


Fluorescence



- Emission of photon with λ_o after absorbing photon of different wavelength λ_i
- Rule of thumb: $\lambda_o > \lambda_i$
  - Exceptions exists!
- Common case: Ultraviolet light triggers visible light
  - Render with extended visible spectrum, but display only visible part
- <u>All</u> photon absorption and follow-up emission takes time
  - ⇒Phosphorescence

Phosphorescence



- Same as fluorescence but time is spent between emission at t_o and absorption at t_i
- This is the case for every emission & absorption event!
  - BUT: $\Delta t = t_o - t_i \ll \epsilon$ for many practical materials
- ϵ depends on the problem case, but we can argue *rendering is not quantum mechanics*, therefore set ϵ high.
- If $\Delta t \ll \epsilon$ we call it fluorescence, phosphorescence otherwise

Fluorescence Rendering Equation

- Incoming wavelength λ_i
- Outgoing wavelength λ_o
- Extend integration by spectral dimension

- $L_o(x, \omega_o, \lambda_o) = L_e(x, \omega_o, \lambda_o) + \int_\lambda \int_\Omega f(x, \omega_i, \omega_o, \lambda_i, \lambda_o) L_i(x, \omega_i, \lambda_i) \cos\theta \, d\omega_i d\lambda_i$

Phosphorescence Render Equation

- Incoming time t_i
- Outgoing time t_o
- Extend by time domain

- $L_o(x, \omega_o, \lambda_o, t_o) = L_e(x, \omega_o, \lambda_o, t_o) + \int_0^{t_o} \int_\lambda \int_\Omega f(x, \omega_i, \omega_o, \lambda_i, \lambda_o, t_i, t_o) L_i(x, \omega_i, \lambda_i, t_i) \cos\theta \, d\omega_i \, d\lambda_i \, dt_i$

- Very impractical
- Many materials have a very insignificant $\Delta t = t_0 - t_1 \ll \epsilon$
  - Handling fluorescent as a special case is important

Display

Display spectral images

- Common monitors are only capable of displaying RGB
- We need a map from $\lambda \to RGB$
- Already exists as a standard!
- After mapping to RGB, tone mapping must be applied

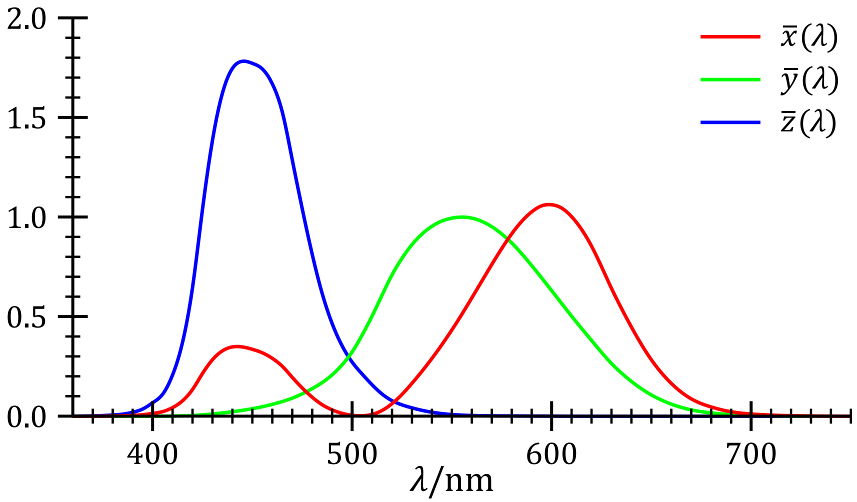
CIE XYZ Mapping

- Based on CIE 1931 color space or its successors
- The resulting CIE XYZ triplet can be transformed to sRGB or other color spaces
- The three-color matching curves are given as measured data

$$X = \int_\lambda L(\lambda)\bar{x}(\lambda)d\lambda$$

$$Y = \int_\lambda L(\lambda)\bar{y}(\lambda)d\lambda$$

$$Z = \int_\lambda L(\lambda)\bar{z}(\lambda)d\lambda$$



Energy Visualization

- Energy of a single photon in Joule: $E_p = \frac{hc}{\lambda}$
  - h - Planck's constant
  - c - Speed of light
  - λ - Wavelength at vacuum
- Total energy at sensor:
  - $E = \int_{\lambda} \frac{L(\lambda)hc}{\lambda} d\lambda$
- Electronvolts might be used instead of Joule

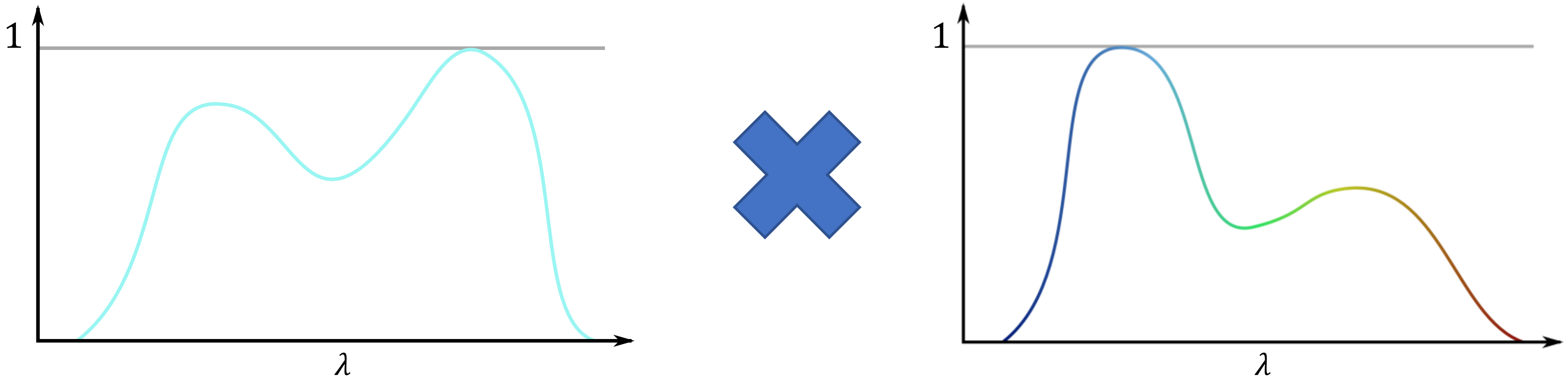
Spectral Sampling

Camera Wavelength Sampling

- Estimator: $\langle I \rangle = \frac{1}{N} \sum_{i=1}^{N} \frac{L(X_i, \lambda_i)}{p(X_i, \lambda_i)}$
- Uniform sampling within the visible spectrum?
- Not the best solution as the scene consists of inhomogeneous set of colors
- Alternatively, use data known at the start of the rendering process!

Response - Camera Wavelength Sampling

- Construct histogram based on the camera response curve and the CIE color matching curve
- Sample the resulting histogram
- Does not adapt to scene

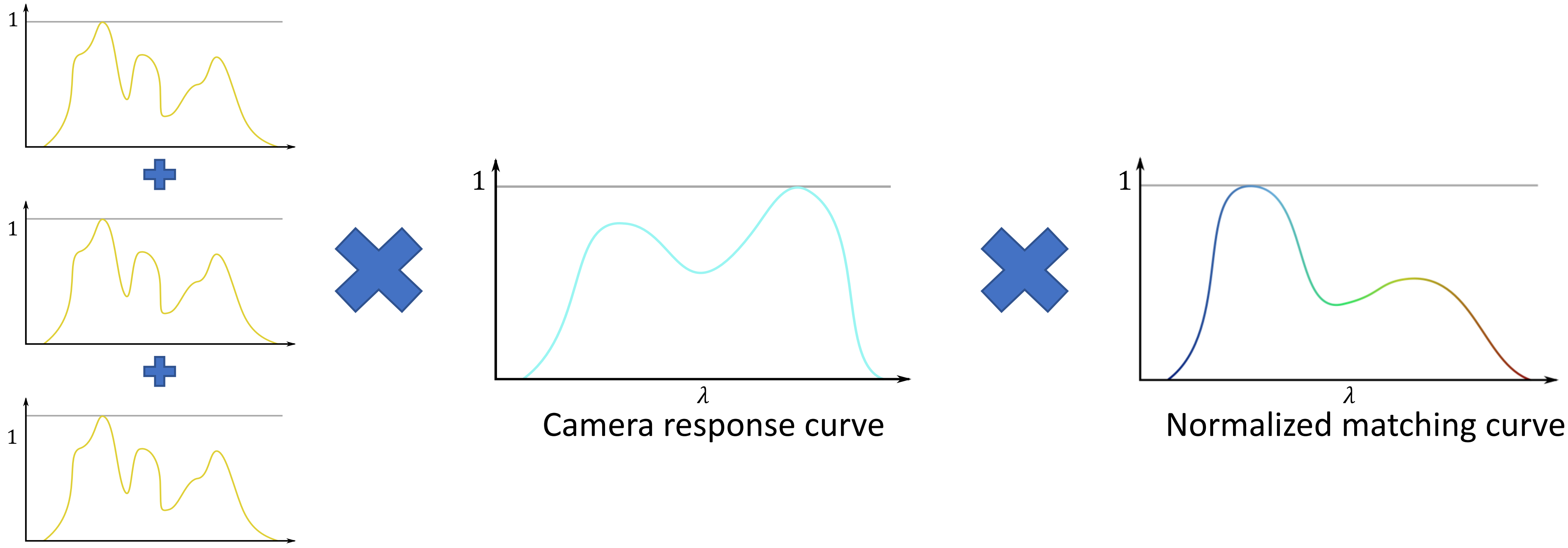


Camera response curve

Normalized matching curve

SPD - Camera Wavelength Sampling

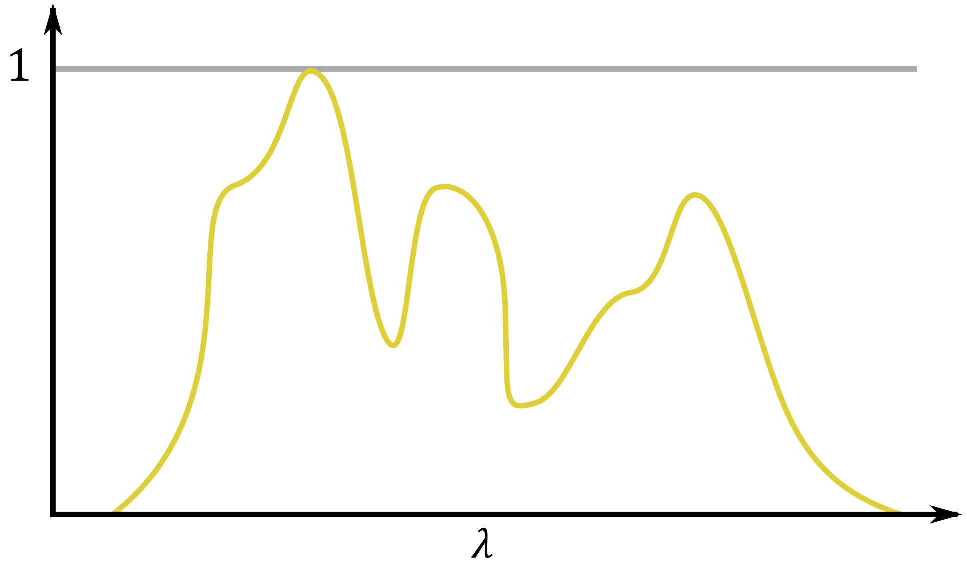
- Same as before, but take SPD(*Spectral power distribution*) of lights into account
- Will not work for scenes containing fluorescent materials
- Keeping track of the relative power is crucial. Local normalization of SPDs does not work. The SPDs must be normalized globally
- SPD can not be spatial varying, else use average



Spectral power distributions

Light Wavelength Sampling

- Sample using the respective SPD
- SPD can be spatial and temporal varying

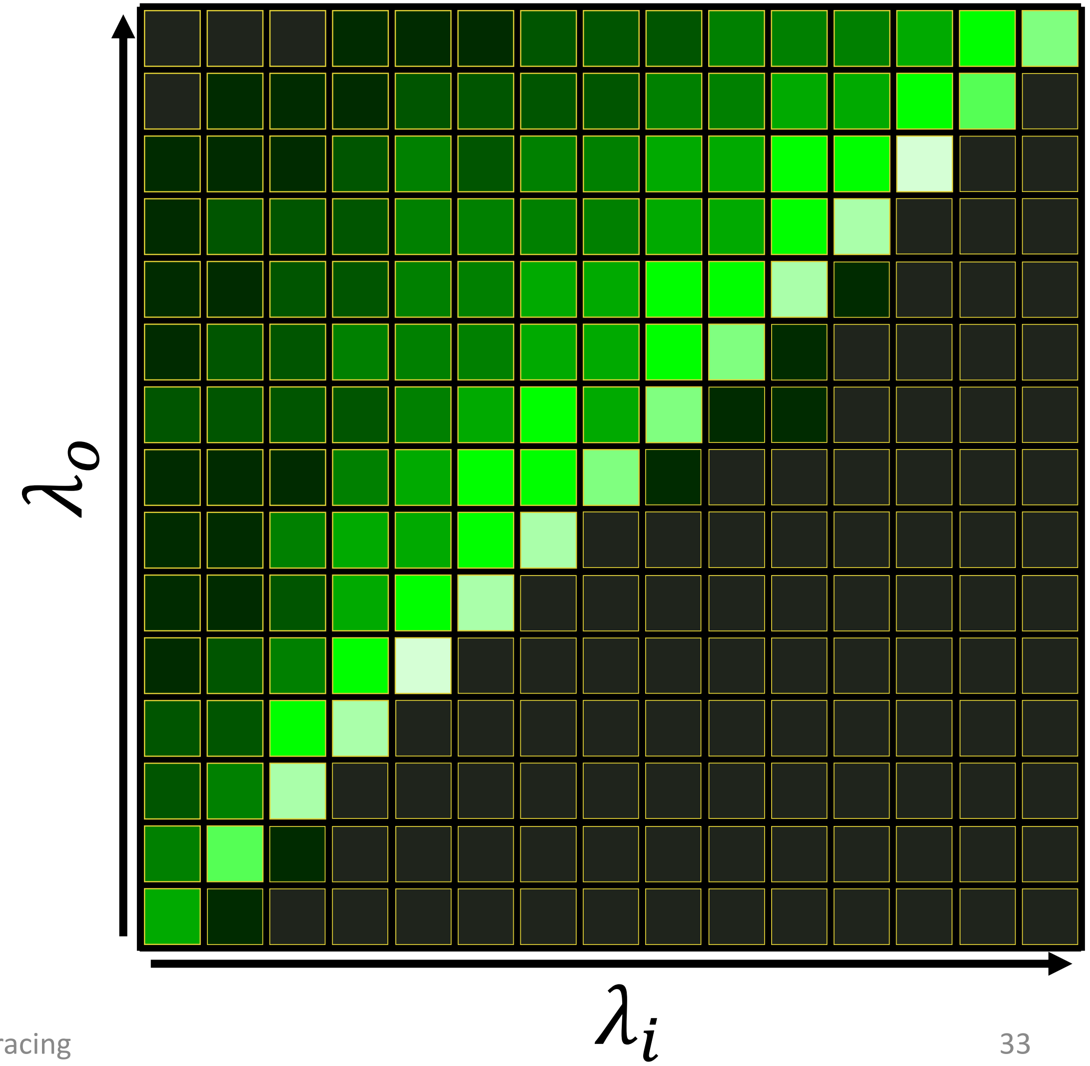


Fluorescent Sampling

- Sample outgoing wavelength based on BSDF/Medium
- Only sample wavelengths if its fluorescent!
  - Changing wavelength can be costly, and quite a lot of materials in scenes are not fluorescent at all!
- In practice the wavelength is sampled independently from the direction $p(\omega, \lambda) = p(\lambda)p(\omega)$
  - A combined approach might be more efficient, but also more complex to implement
- Common methods use a discrete re-radiation matrix

Re-radiation Matrix

- Diagonal entries represent non-fluorescent probability
- A standard piecewise-constant 1D sampler can be used to randomly select λ_o based on λ_i

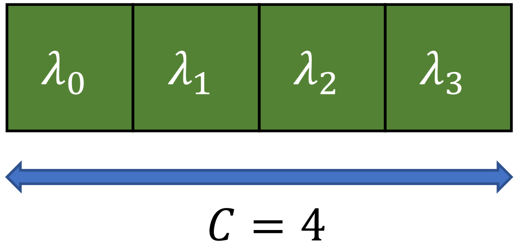


Hero Wavelength Sampling*

* *Wilkie, A., Nawaz, S., Droske, M., Weidlich, A. and Hanika, J. (2014), Hero Wavelength Spectral Sampling. Computer Graphics Forum, 33: 123-131.* https://doi.org/10.1111/cgf.12419

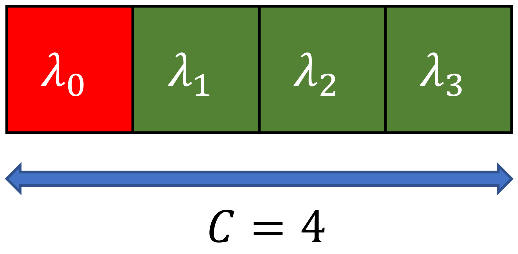
General Idea

- Associate each path a wavelength package of size C
- $\langle I \rangle = \frac{1}{N}\frac{1}{C}\sum_{i=1}^{N}\sum_{j=1}^{C}\frac{L(X_i,\lambda_i^j)}{p(X_i,\lambda_i^j)}$
- $p\left(X_i, \lambda_i^j\right) = \sum_{k=1}^{C}\frac{p\left(\lambda_i^k\right)p(X_i|\lambda_i^k)}{C}$



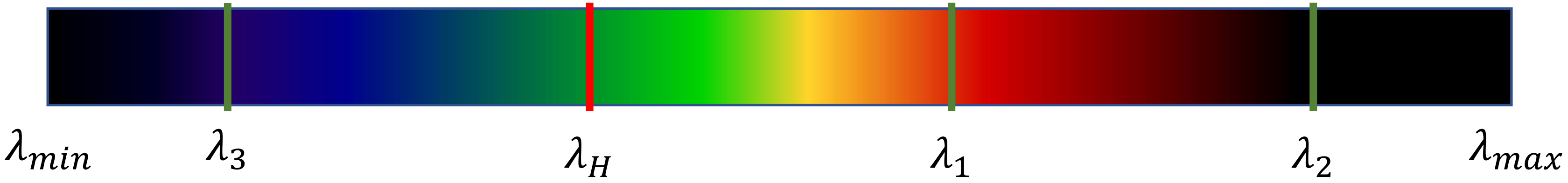
The Hero

- One wavelength is the *hero*
- Only this *hero* wavelength is used to sample the actual path



HWS: Sample Method

- Randomly select $\lambda_H \sim p(\lambda_H)$
- Set other wavelengths in the current package according to:
  - $\lambda_i = \left(\lambda_H - \lambda_{min} + \frac{i}{C}(\lambda_{max} - \lambda_{min})\right) \mathbf{mod}(\lambda_{max} - \lambda_{min}) + \lambda_{min}$
- As *shifting* is a deterministic operation we can simplify:
  - $p\left(X_i, \lambda_i^j\right) = p(\lambda_i^H) p(X_i | \lambda_i^H)$

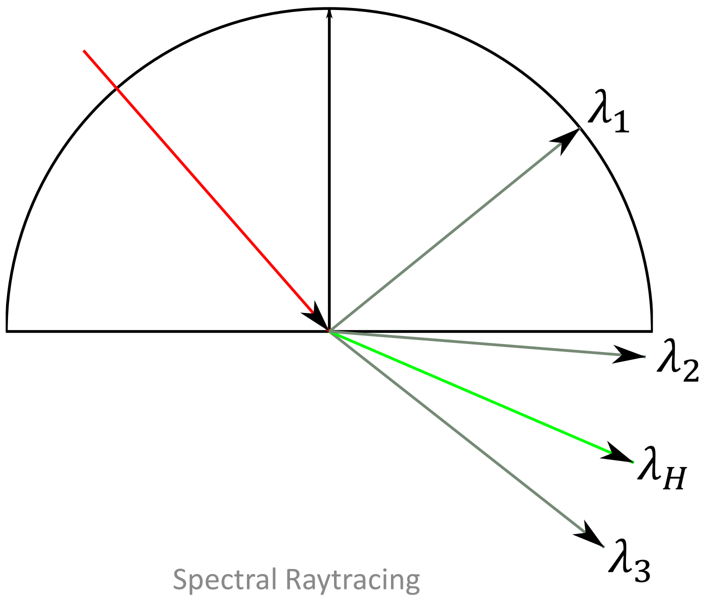


HWS: Multiple Importance Sampling

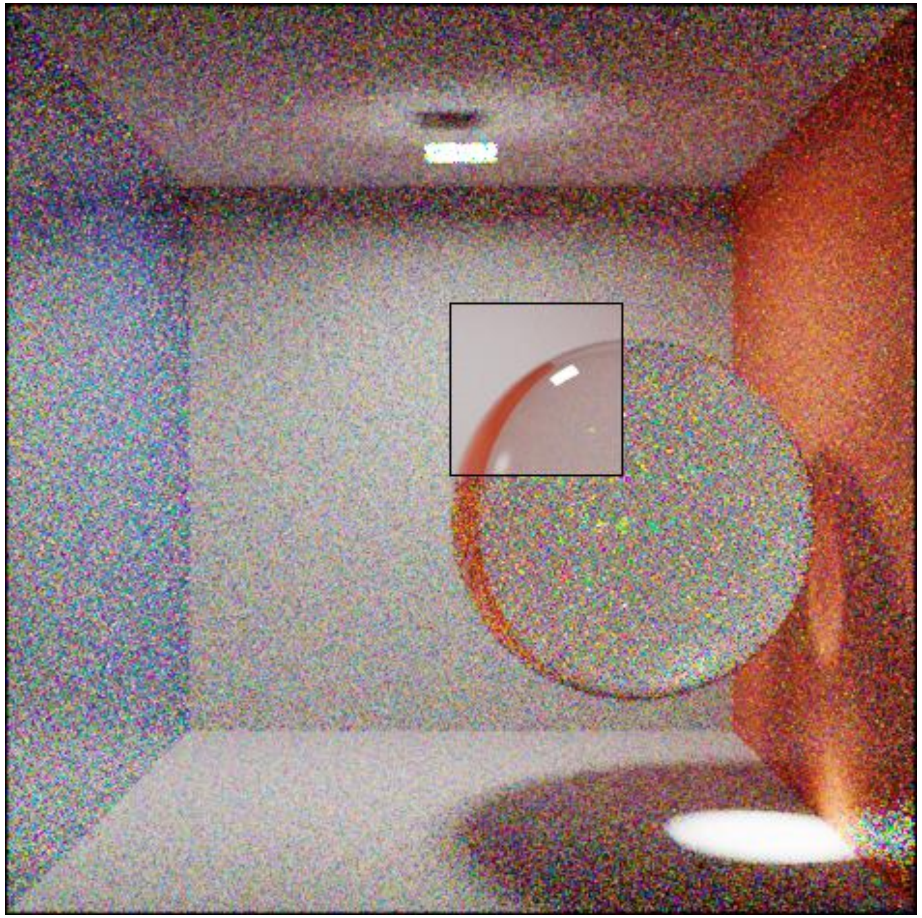
- Each wavelength could have been picked as the *hero* wavelength as well
- $\langle I \rangle = \frac{1}{N} \sum_i^N \sum_j^C w_j(X_i, \lambda_i^j) \frac{L(X_i, \lambda_i^j)}{p(X_i, \lambda_i^j)}$
- Using balance heuristic
  - $w_j\left(X_i, \lambda_i^j\right) = \sum_k^C \frac{p(\lambda_i^H) p(X_i | \lambda_i^H)}{p(\lambda_i^k) p(X_i | \lambda_i^k)}$

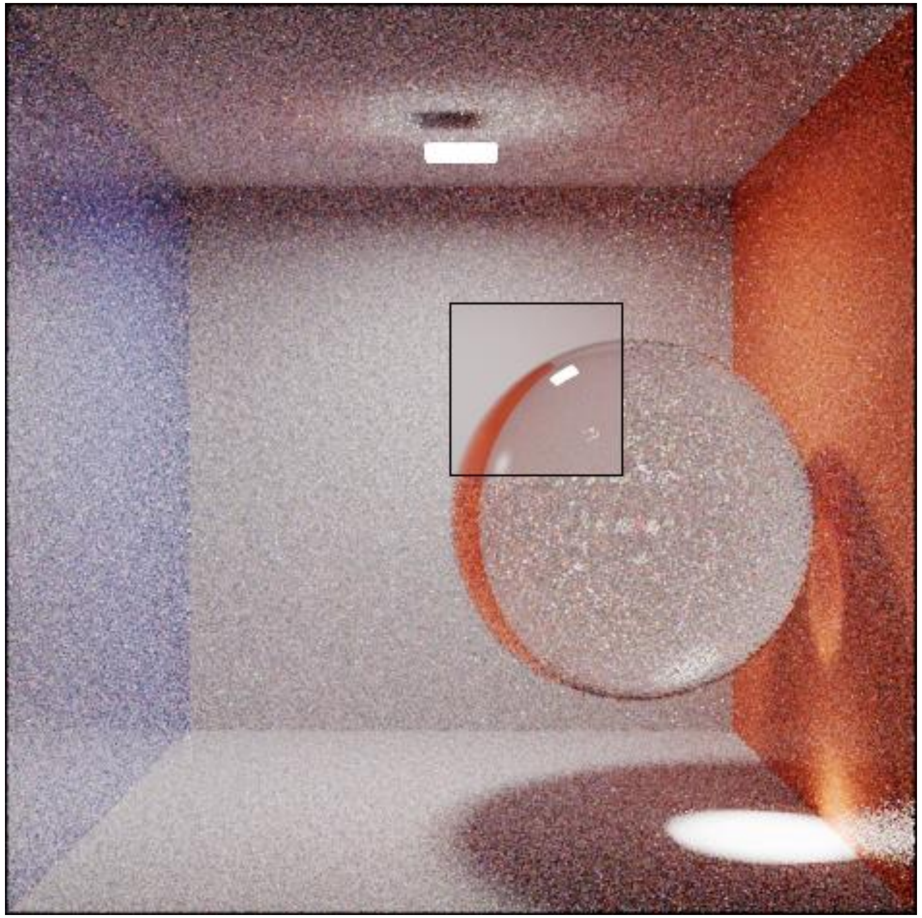
HWS: Delta Response & Singularities

- A ray might hit a material with a delta response, e.g., <u>perfect</u> glass
  - The IOR is wavelength dependent and so is the outgoing direction
  - The pdf for other wavelengths would be zero
  - Solve by setting all weights except the hero wavelength to zero



Results


Single Wavelength Sampling


Hero Wavelength Sampling

HWS Problems

- Shifting is agnostic to wavelength probability
  - Let's pick each wavelength according to $\lambda_i \sim p(\lambda_i)$
  - But still use only one wavelength as the *hero*
  - Continuous Multiple Importance Sampling* allows us to combine everything together
- Fluorescent paths have wavelengths per vertex but have no impact on the actual HWS approach
  - They are integrated in the path pdf
- Bidirectional approaches do get <u>very</u> complicated

** Rex West, Iliyan Georgiev, Adrien Gruson, and Toshiya Hachisuka. 2020. Continuous multiple importance sampling. ACM Trans. Graph. 39, 4, Article 136 (July 2020), 12 pages. DOI:https://doi.org/10.1145/3386569.3392436*

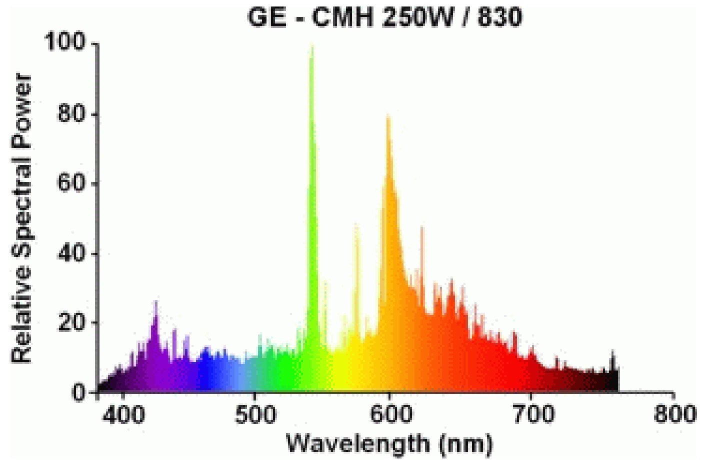
Spectral Upsampling

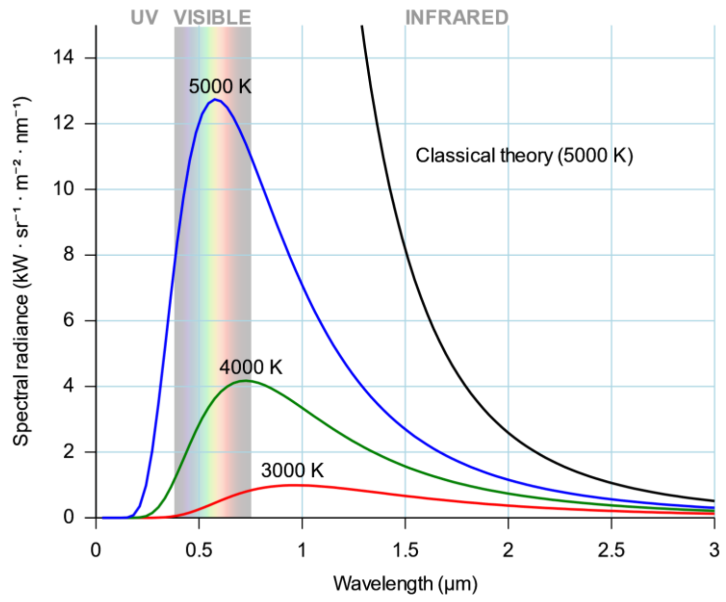
Spectral Upsampling

- RGB -> Spectrum
- Why still use RGB?
  - Many assets are still RGB
  - Artists don’t want to work with spectrums
  - Color more intuitive than curves
- Mapping from Spectral to RGB has the signature $\mathbb{R}^\infty \to \mathbb{R}^3$
  - Not injective
  - Therefore, RGB to Spectral $\mathbb{R}^3 \to \mathbb{R}^\infty$ not unique
- Use cases:
  - Emission
  - Reflection/Albedo
  - IOR ($\to$ Sellmeier Eq.)

Emissive Spectrum

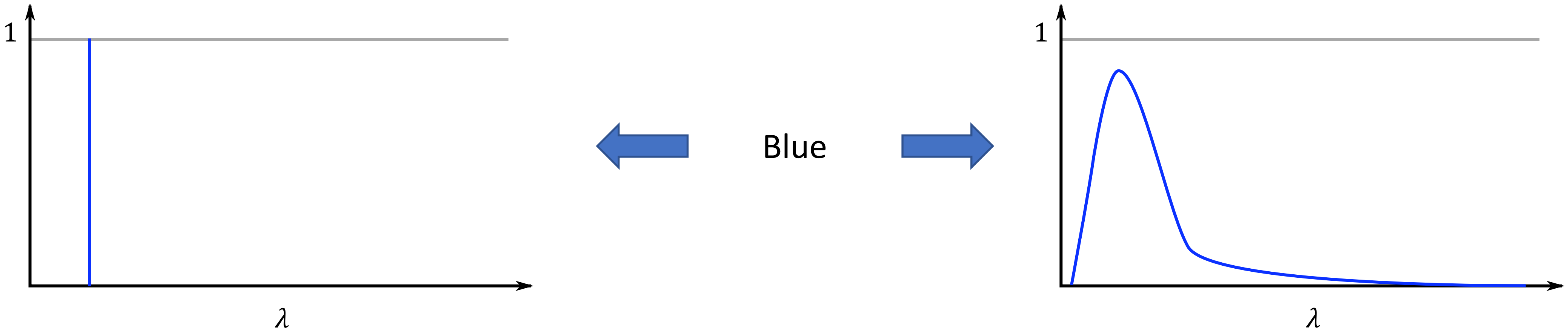
- ~ Power/Radiance at a given wavelength
- Unbounded but positive
- Common to be real measured data
  - ⇒ Noisy
- Blackbody curves
- Standard Illuminants
  - D65, D50, F4, …
- Colored light
  - Multiply reflective/albedo curve with emissive curve





Reflective/Albedo Spectrum

- ~ Reflection/Absorption at a given wavelength
- Usually bound to $[0, 1]$
- White would be just a constant 1
- But no unique way to define - for example - blue?



Problem Definition

- We want to construct a function $f(\lambda)$ given a RGB triplet
- The function shall be smooth
- The function shall be continuous in a given range
- RGB (1,1,1) shall be the standard illuminant of the color space
  - sRGB $\rightarrow$ D65
- Mapping from RGB to spectral and back should be as precise as possible

Optimization

- $\arg\min_{f} \left\| b - T \int_{\lambda} f(\lambda) W(\lambda) xyz(\lambda) d\lambda \right\|$
  - T - Transform matrix to map from CIE XYZ to RGB
  - b - RGB value we optimize for
  - $f(\lambda)$ - Function we optimize for
  - $W(\lambda)$ - SPD of whitepoint (e.g., sRGB → D65)
  - $xyz(\lambda)$ - CIE 1931 color matching functions
- The conditions are given implicitly by fixing the base for $f(\lambda)$

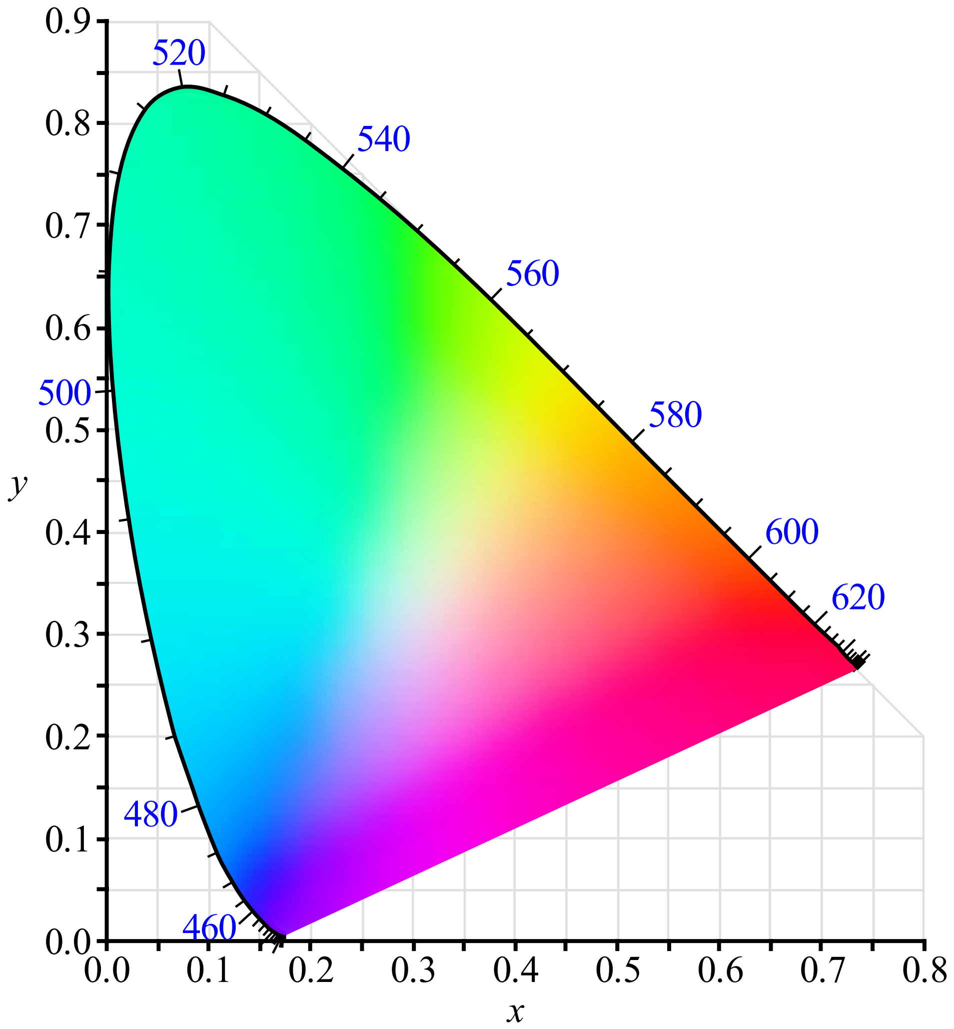
Base function

- Jakob & Hanika* picked:
  - $f(\lambda) = S(c_0\lambda^2 + c_1\lambda + c_2)$
  - $S(x) = \frac{1}{2} + \frac{x}{2\sqrt{1+x^2}}$ being a sigmoid function
- The solver optimizes the three coefficients c_0, c_1 and c_2
- Mapping a RGB triplet returns another triplet!
  - Memory efficient
  - Fast to calculate
  - In-place replacement
- Other possible functions:
  - Gaussian mixture models
  - Polynomials
  - Moment based approach (see EGSR 2021)
  - Etc…

** Wenzel Jakob and Johannes Hanika. 2019. A Low-Dimensional Function Space for Efficient Spectral Upsampling. In Computer Graphics Forum (Proceedings of Eurographics) 38(2).*

Runtime Preprocessing

- Optimization on the fly is not an option
- Pre-calculate some reference points inside the CIE *horseshoe*
  - This is done once for a color space
- Use interpolation for points in-between
- Requires closest neighbor search and interpolation for each RGB triplet
  - This can be done as a scene preprocess step
- Using only $f(\lambda)$ in BSDF evaluation



Problems

- Interpolation between coefficients not as same as interpolating between RGB values
  - Have a look at the Jacobian of $f(\lambda)$
- Black is not easy to represent
  - $f(\lambda) = 0 \ \forall\lambda$ would be black but no possible combination of c_0, c_1, c_2 exists to make it possible
  - Handle this as a special case
- Quality of mapping proportional to the size of precalculated reference points
- Mapping depends on the color space
  - You need an optimized dataset for each color space you use

Conclusion

What did we talk about?

- Inclusion of spectra into the rendering equation
- Using and sampling of wavelengths in a raytracer
- Handling of fluorescence BSDFs
- Mapping a spectrum to RGB
- Mapping RGB to spectrum

What did we NOT talk about?

- History
- Actual fluorescence BSDF models
- HWS + Bidirectional methods
- Spectral differentials
  - Like ray differentials, but spectral
- Stylized rendering
  - Yes, it’s possible
- More recent research

More realism?

- Add phosphorescence
- Add polarization
- Use wave characteristic effects
- Go outside the visible spectrum and *display* it!