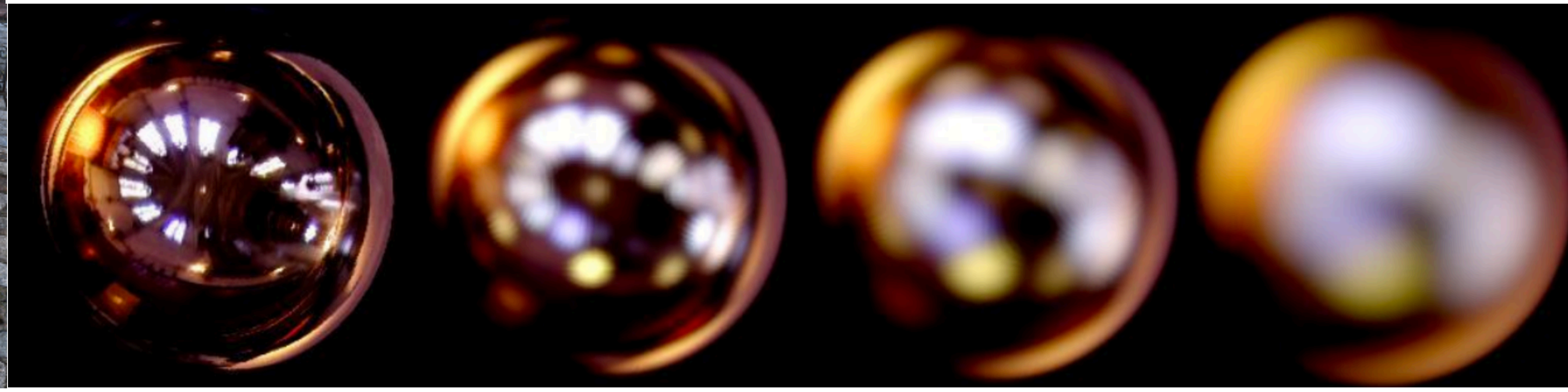


Path Tracing & Microfacet BSDFs

Gurprit Singh


Noise

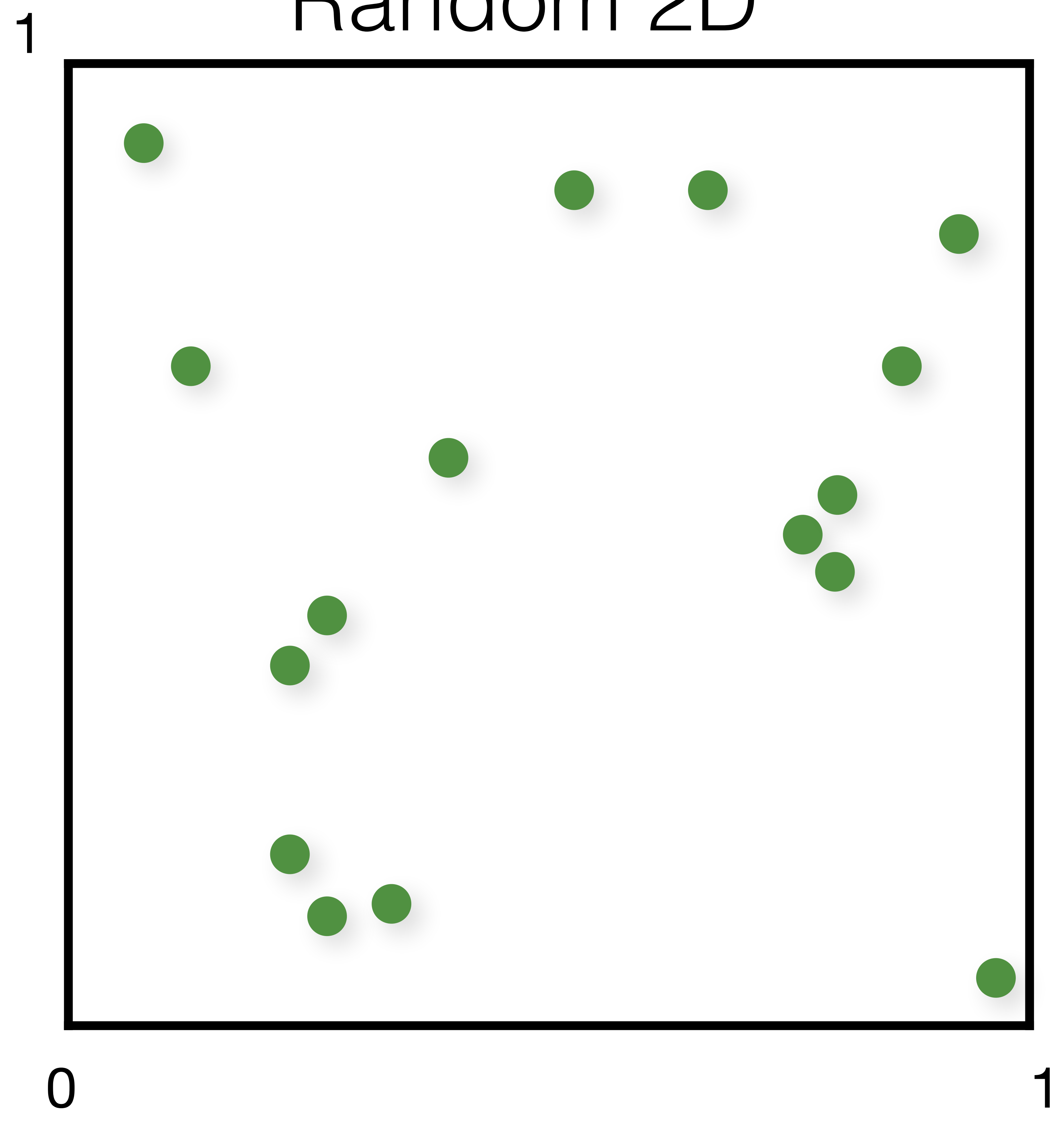

Variance Reduction Techniques

- Correlated Sampling
- Importance Sampling
- Perceptual Error Distribution

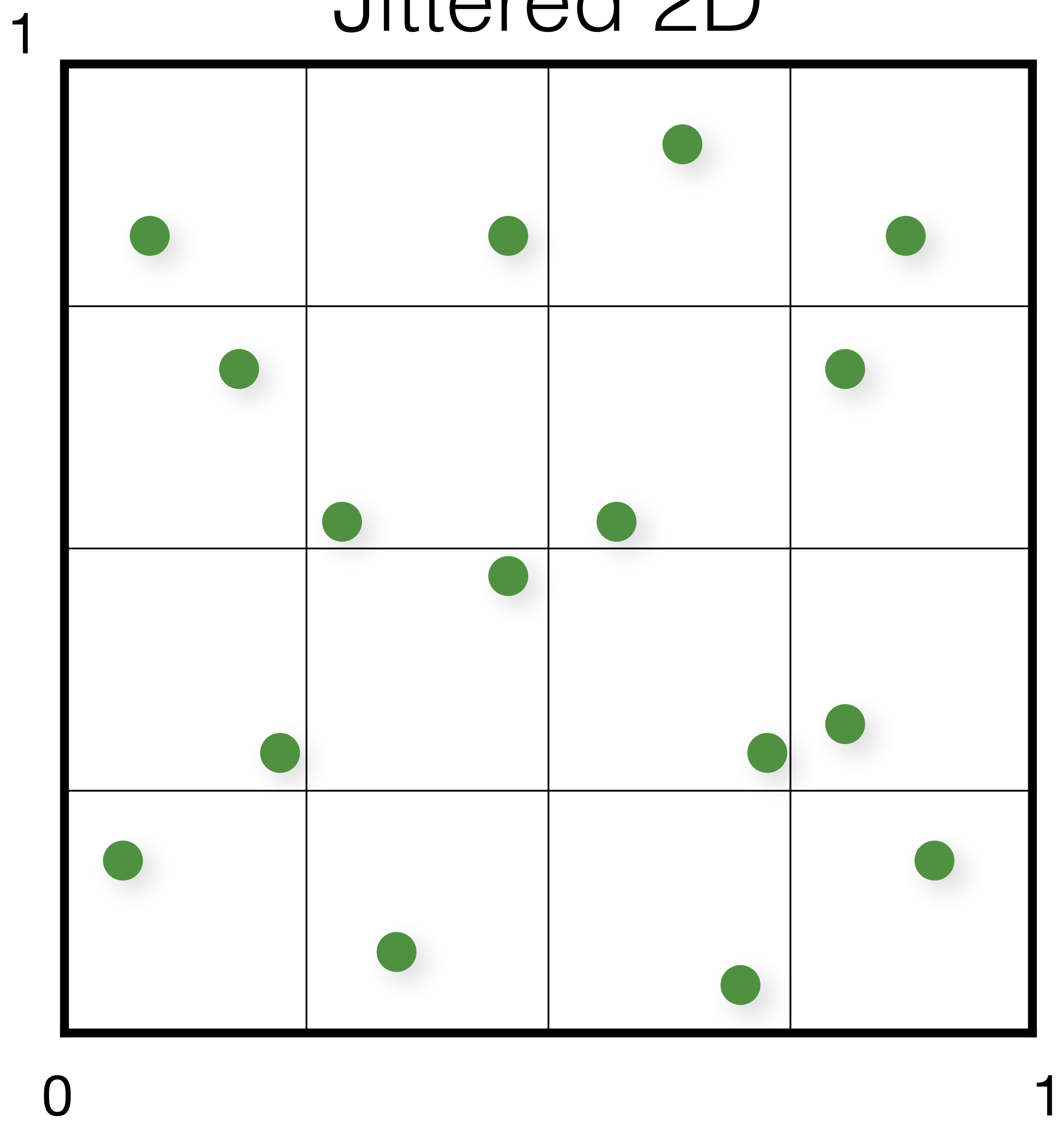
Correlated Sampling: Jittered Sampling

Variance reduction: Stratified Sampling

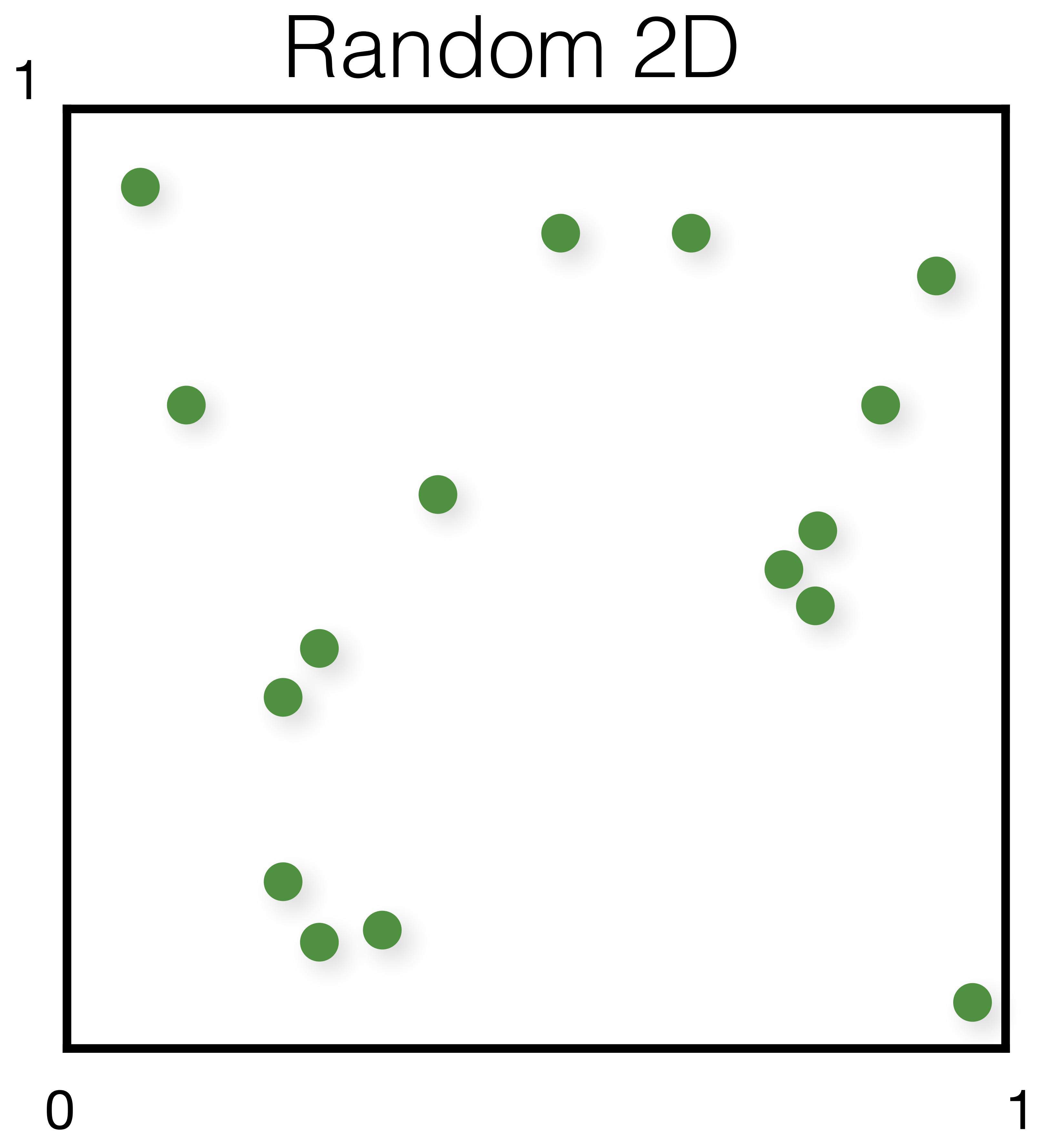
Random 2D

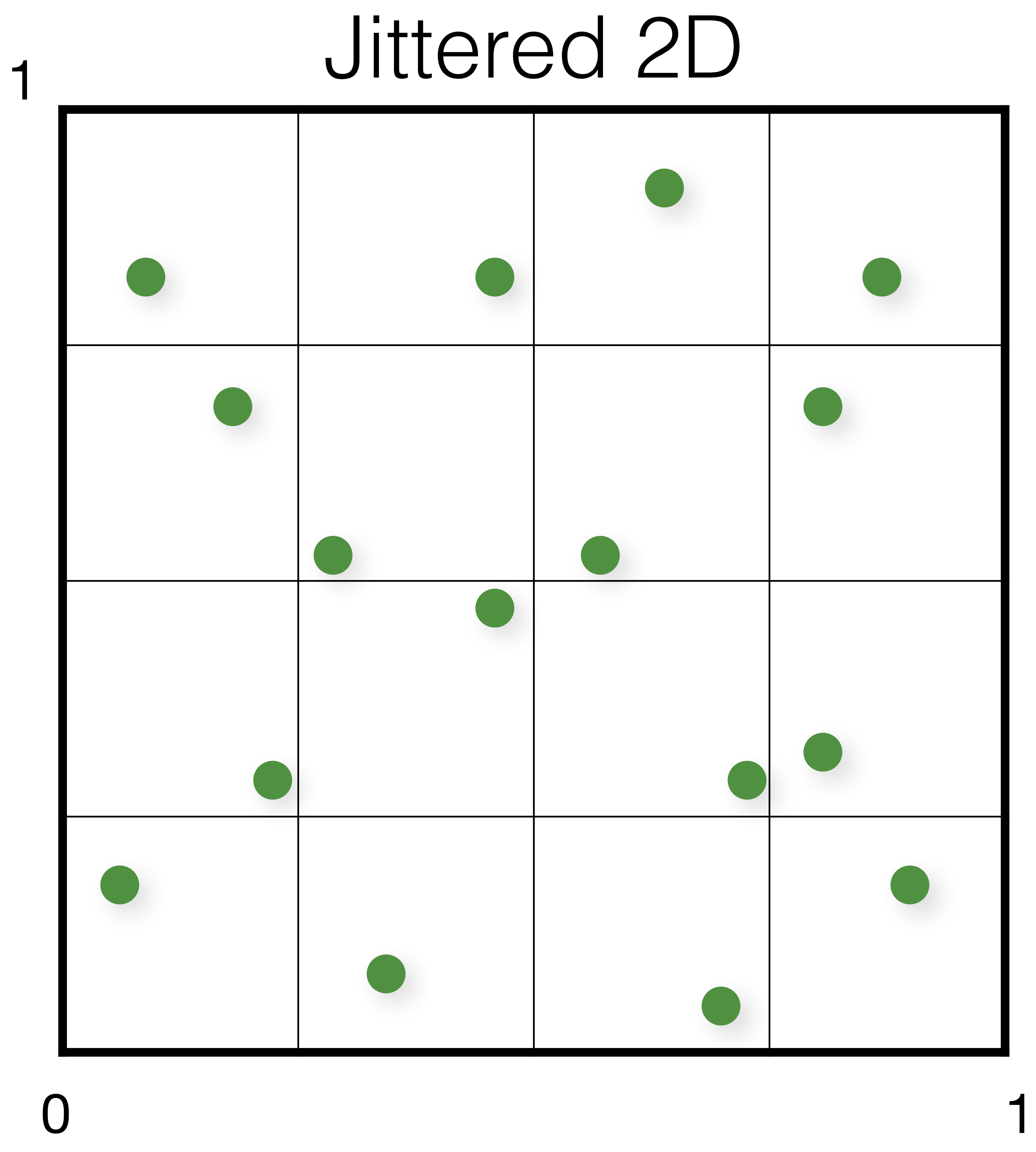


Jittered 2D



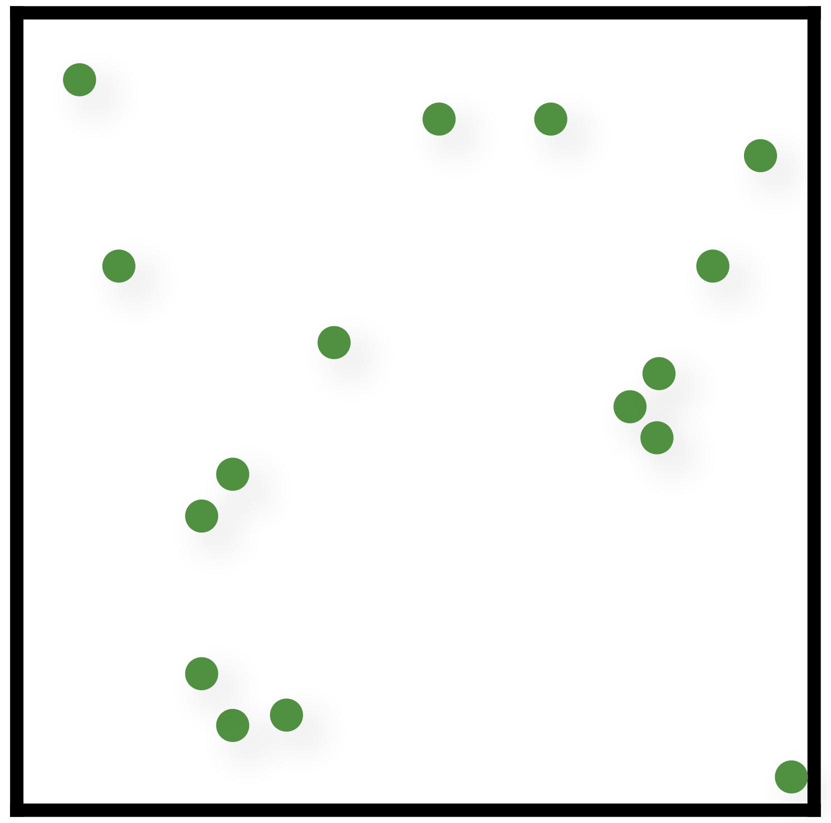
Variance reduction: Stratified Sampling

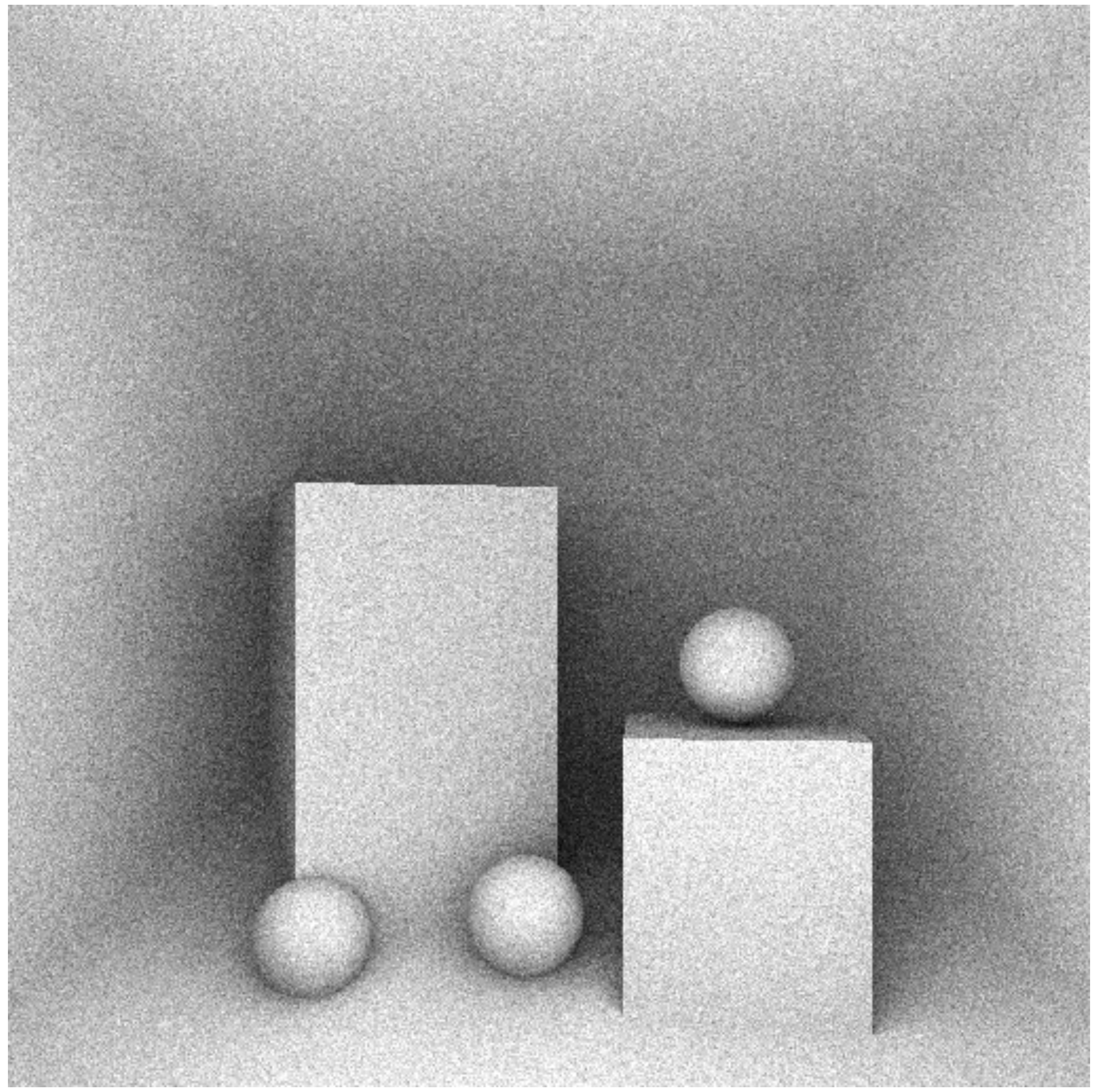




Random vs. Stratified Sampling

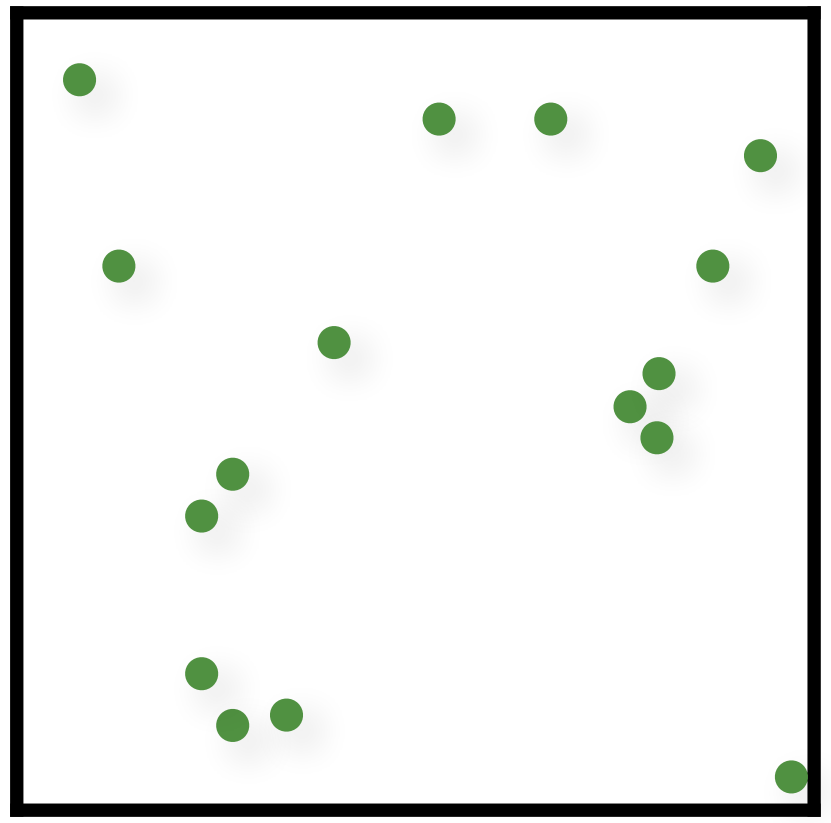
Random Samples

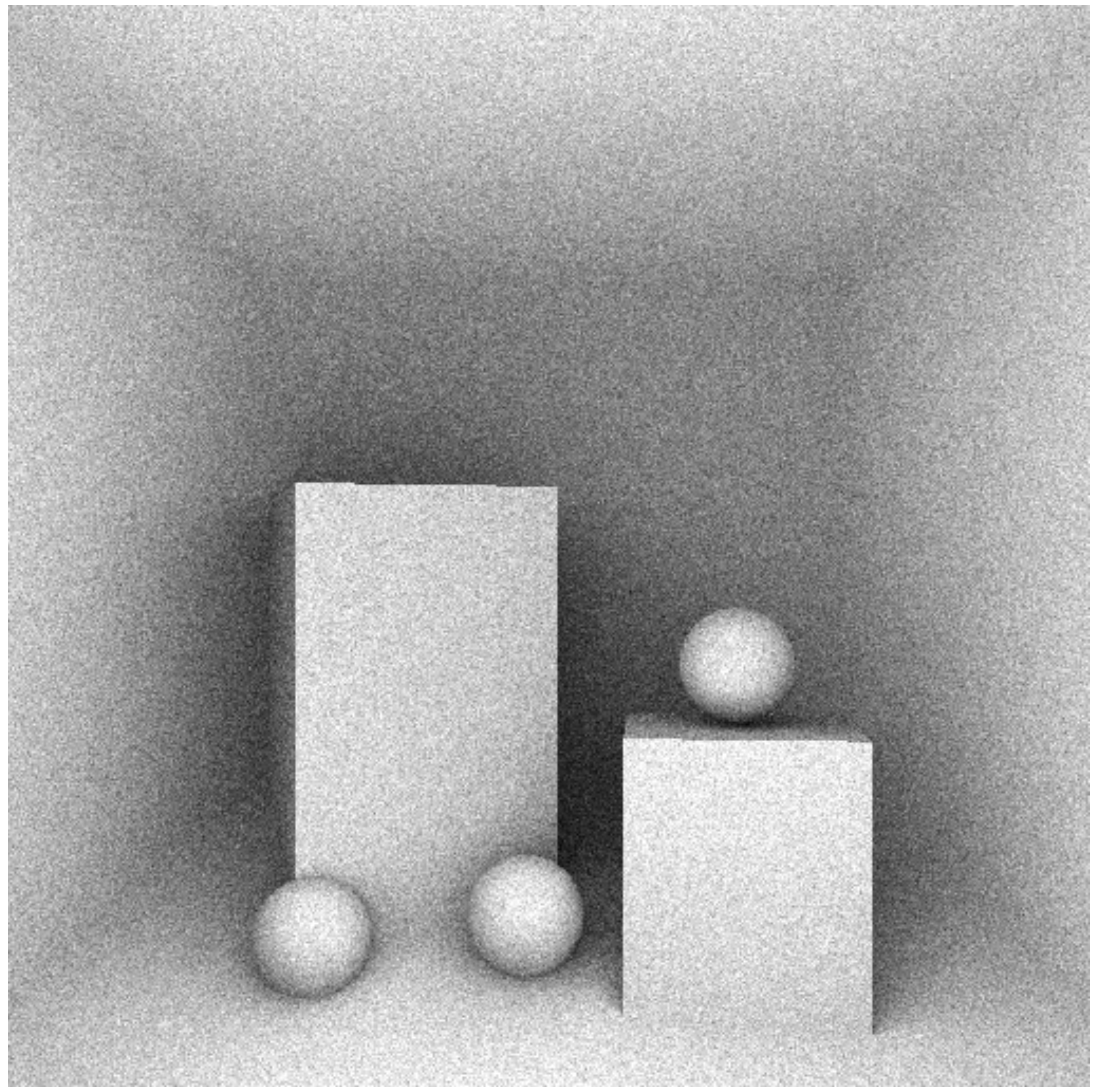




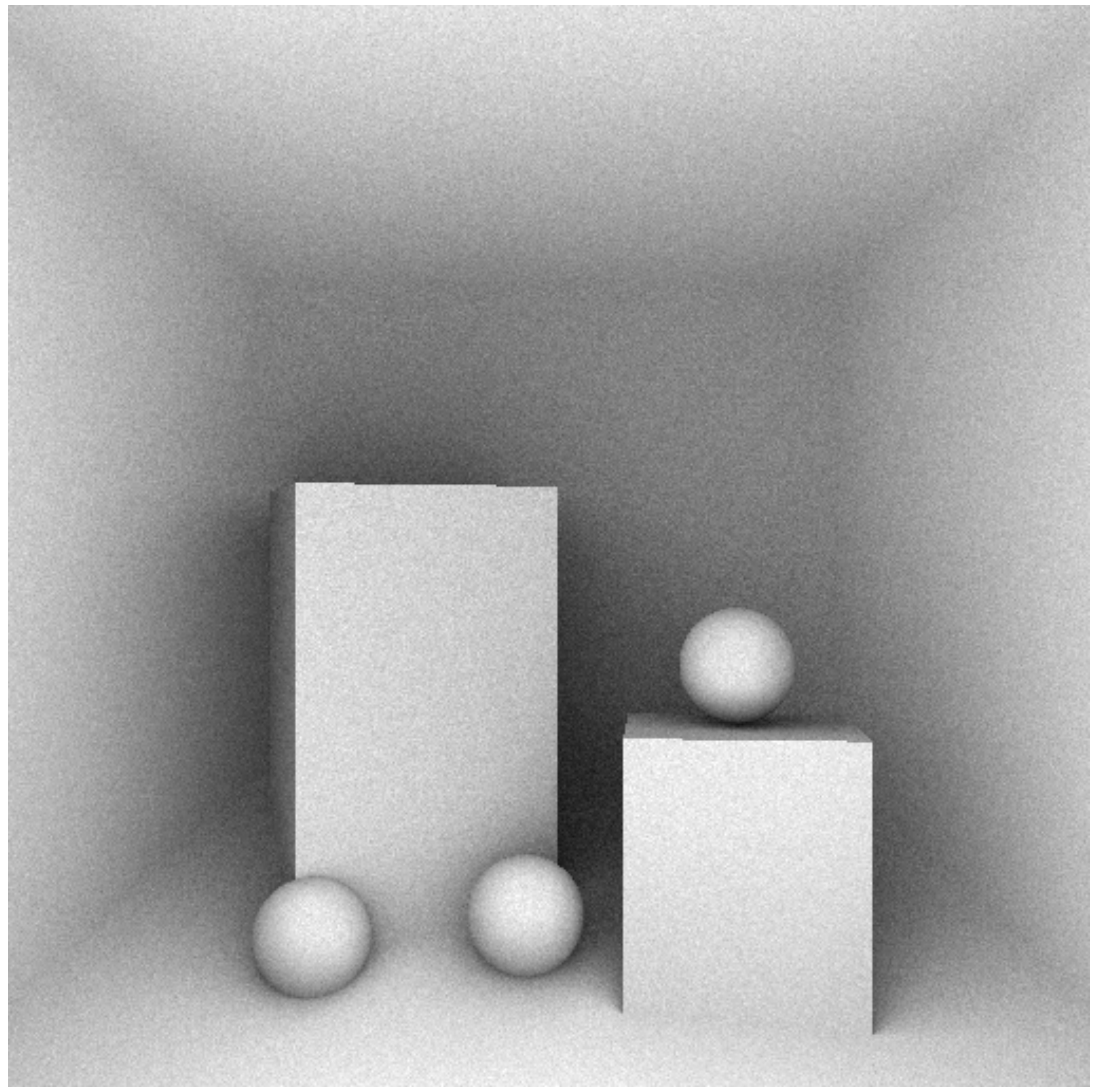
Random vs. Stratified Sampling

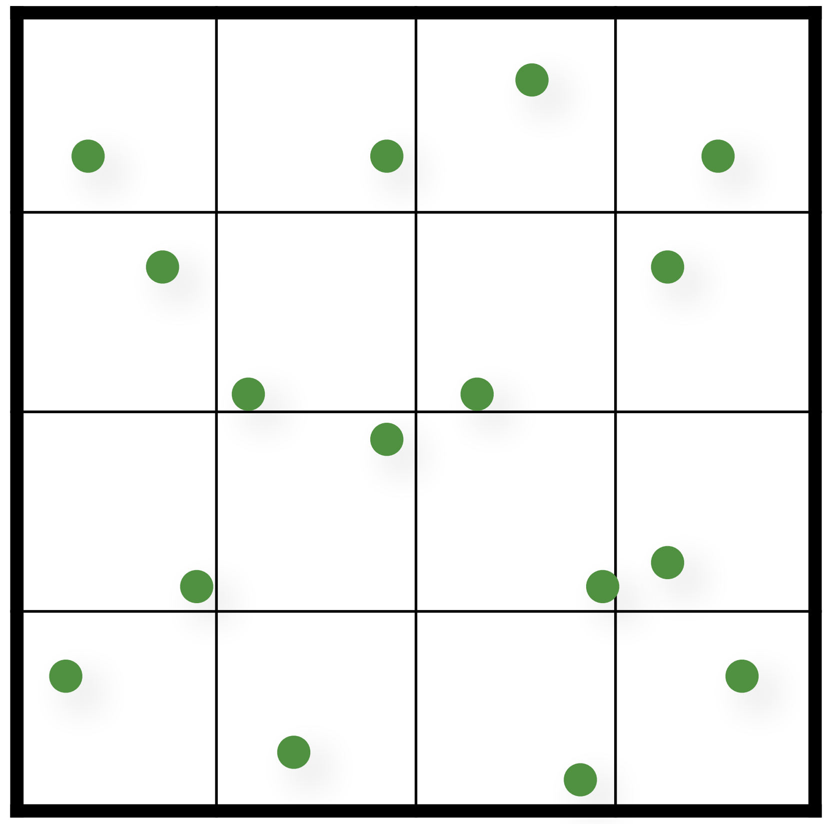
Random Samples





Jittered Samples



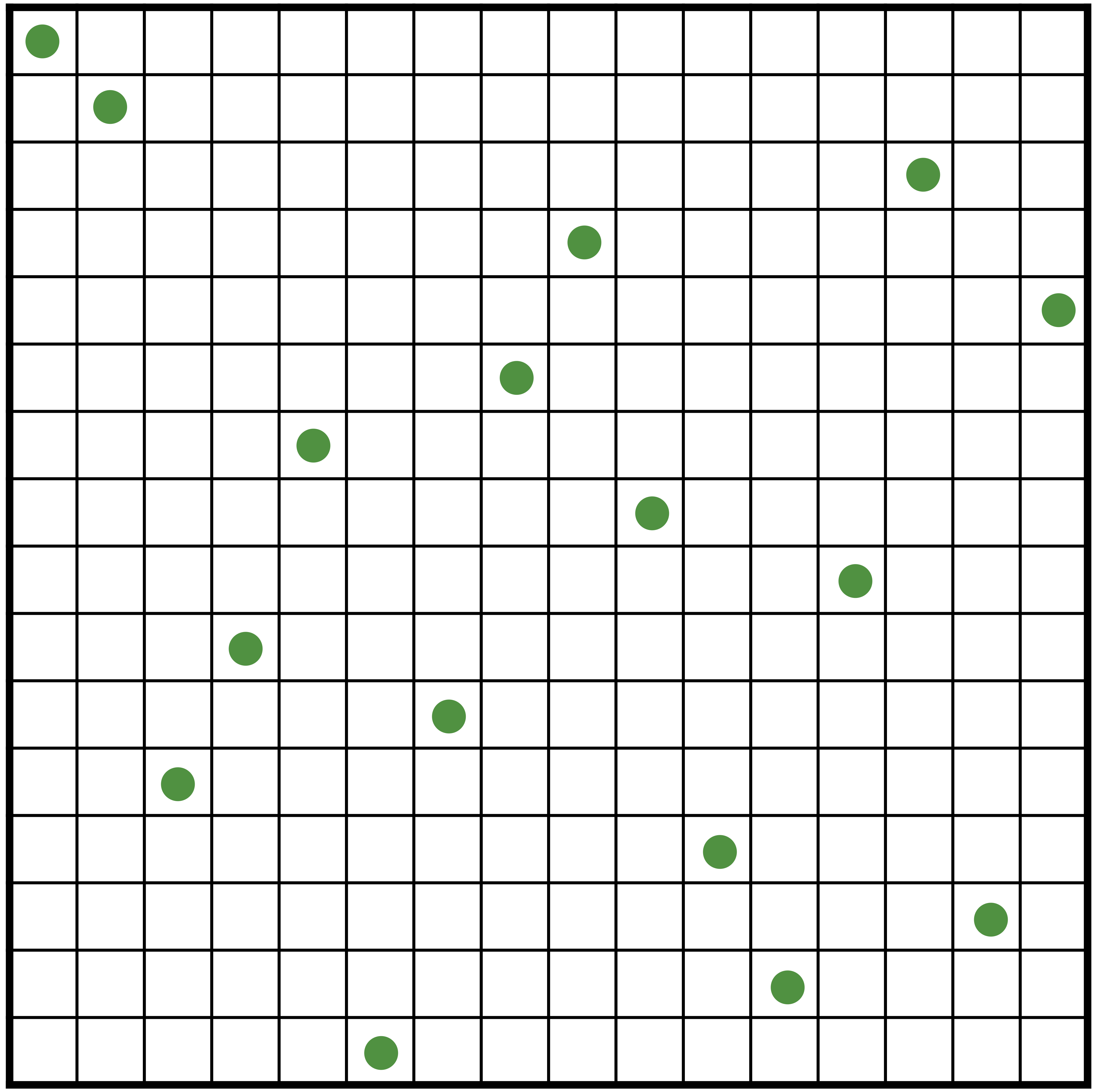


N = 64 spp

Stratified sampling suffers from the curse of dimensionality

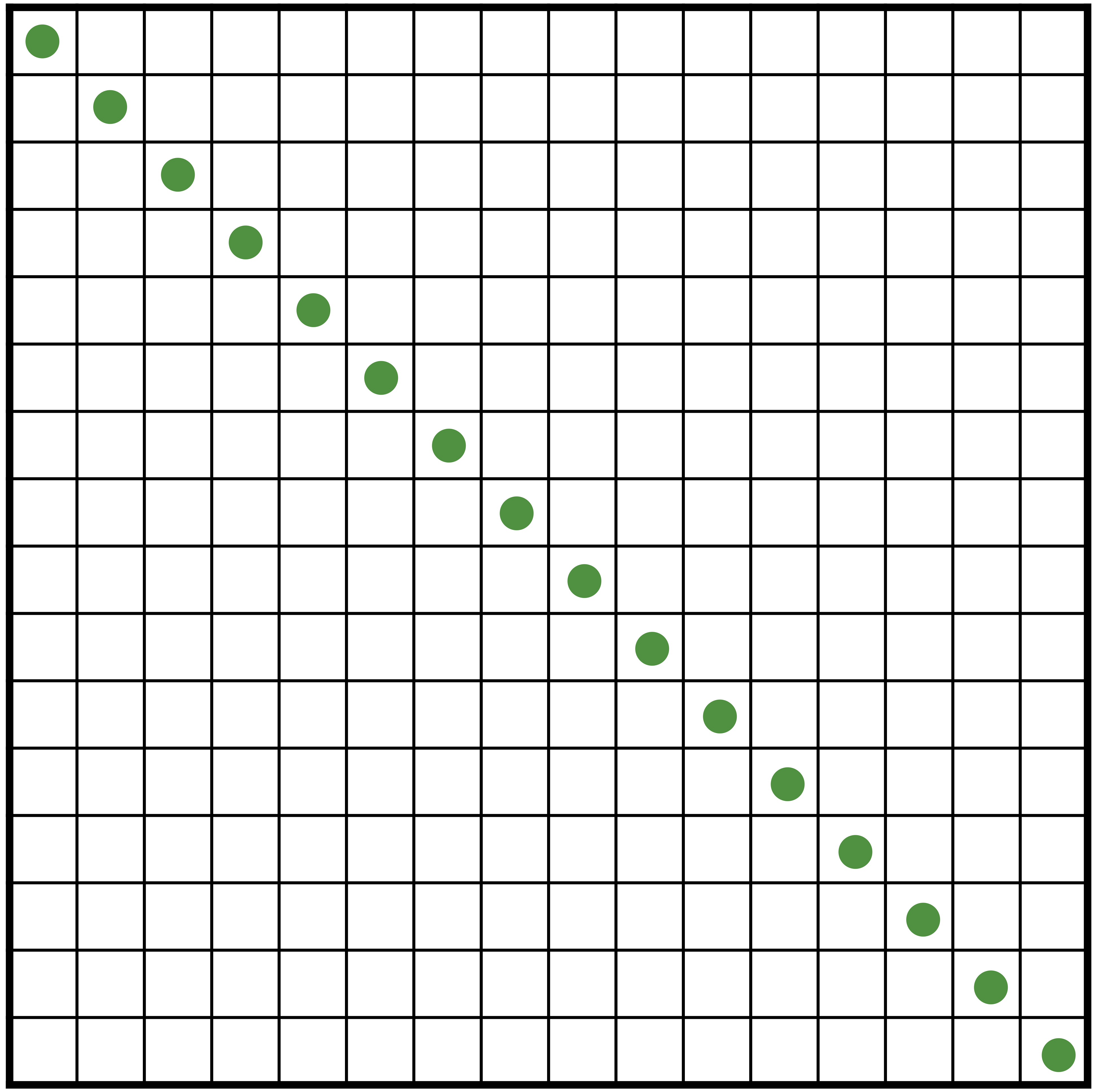
Correlated Sampling: Latin Hypercube Sampling

Latin Hypercube Sampler (N-rooks)



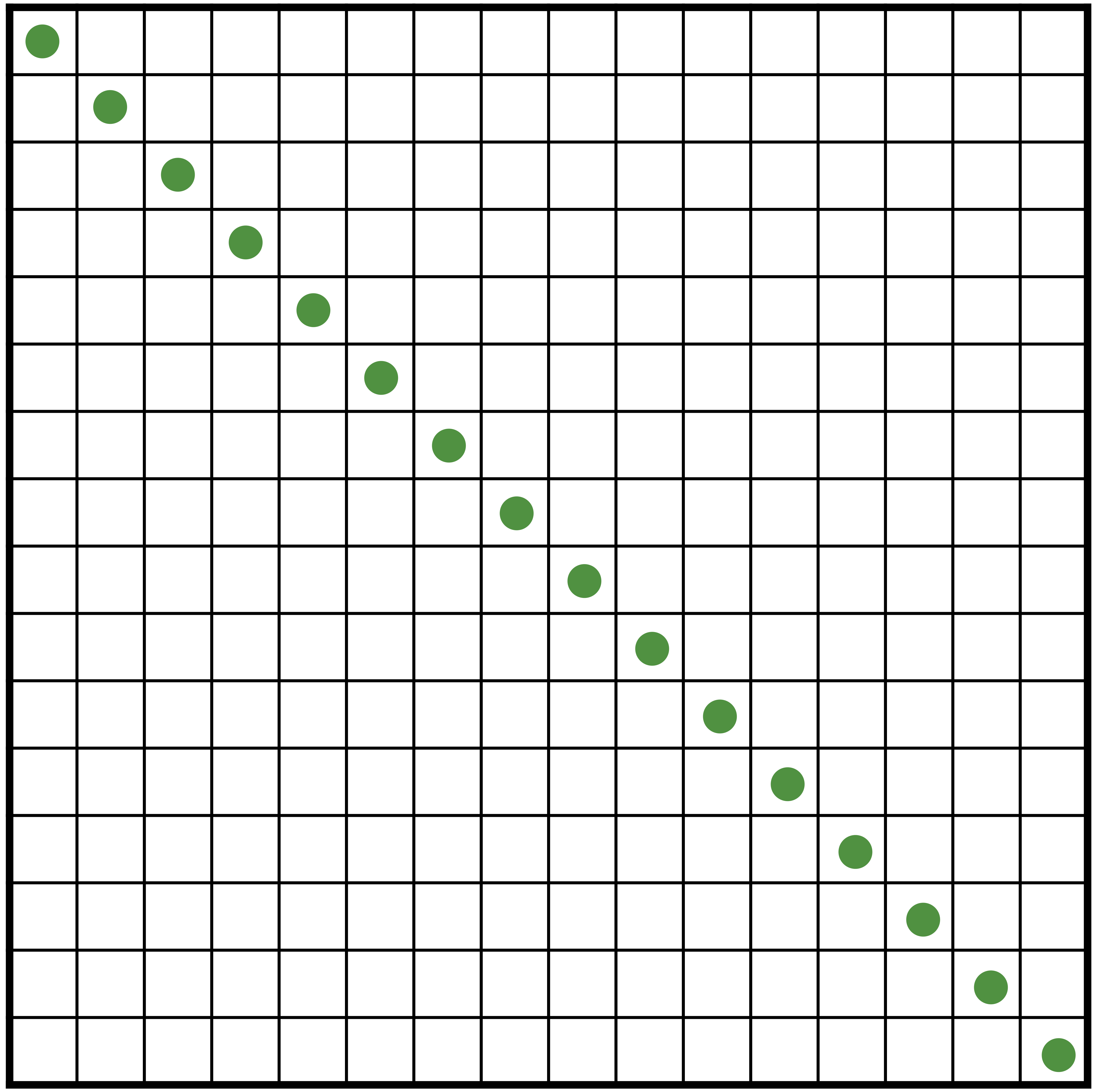
Latin Hypercube Sampler (N-rooks)

Initialize



Latin Hypercube Sampler (N-rooks)

Shuffle rows



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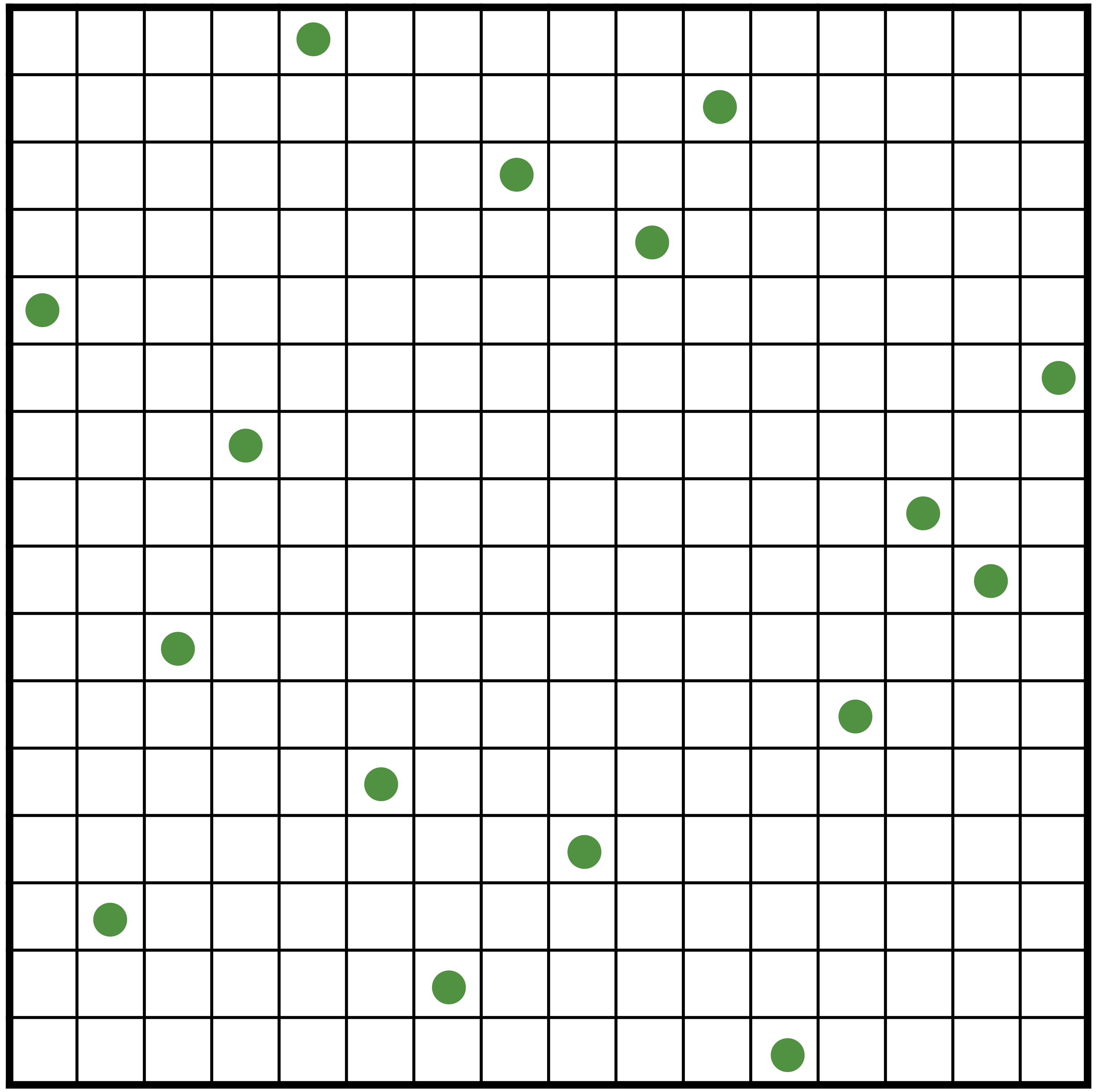
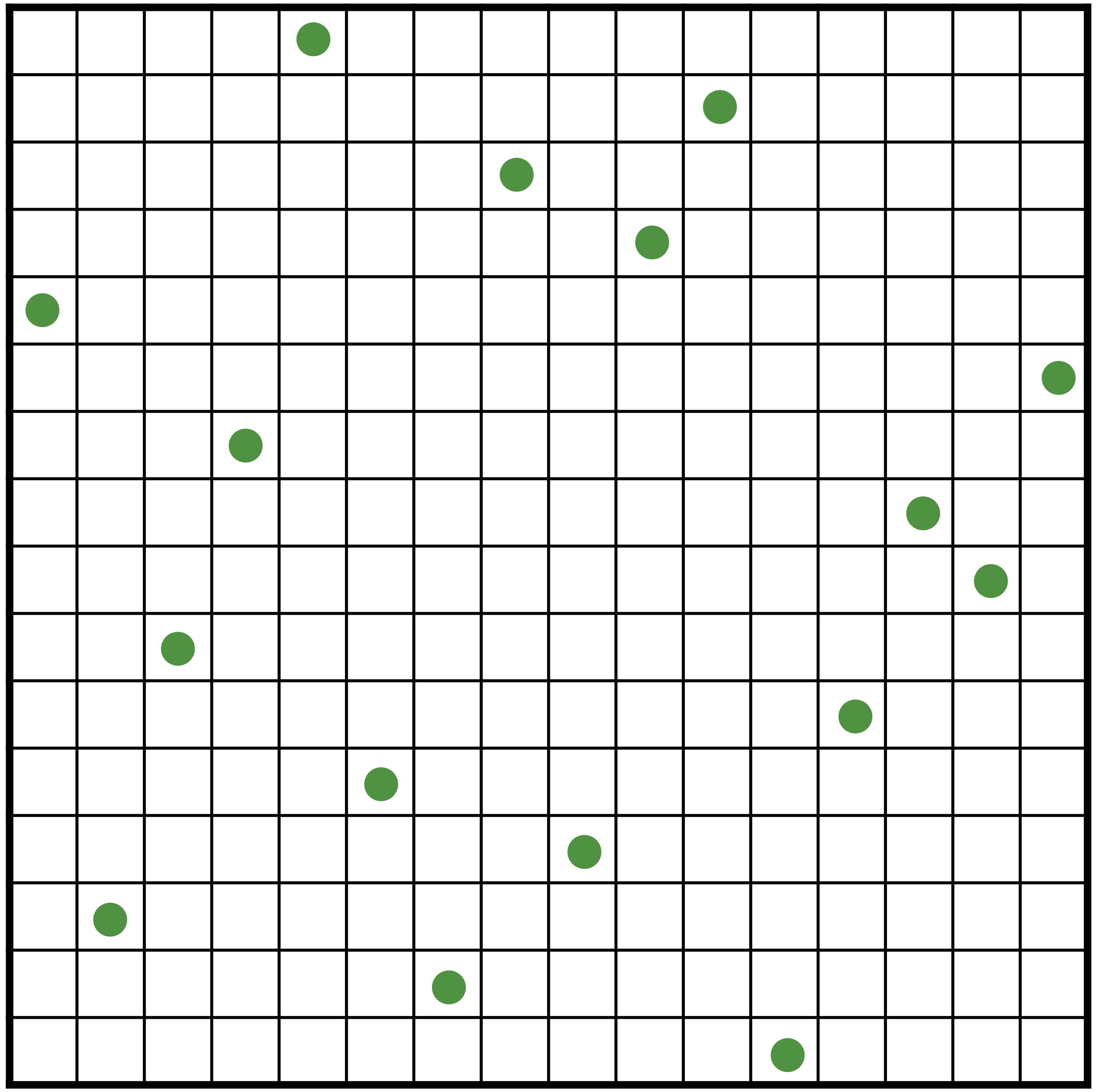
Latin Hypercube Sampler (N-rooks)

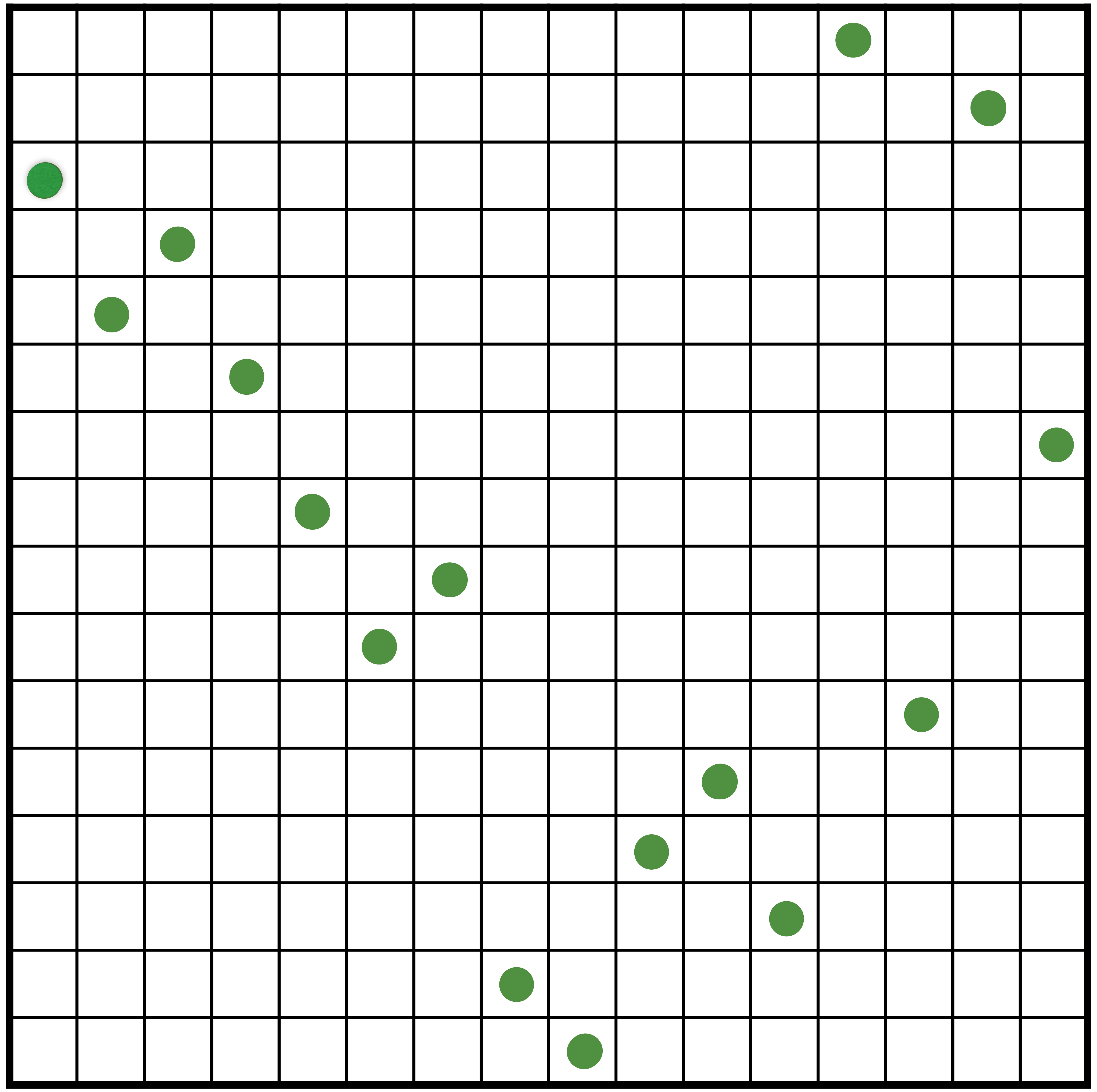


Latin Hypercube Sampler (N-rooks)

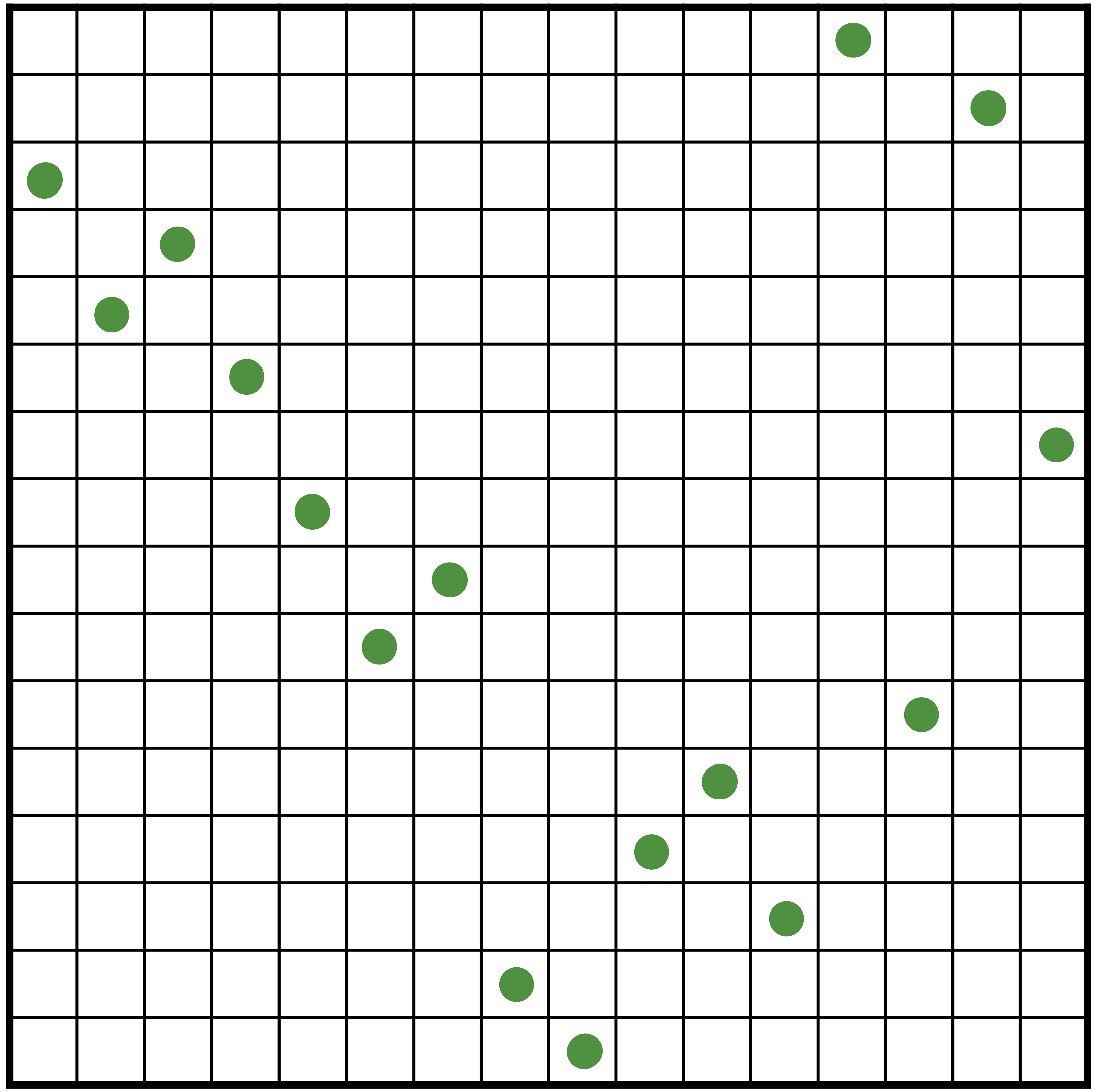
Shuffle columns



Latin Hypercube Sampler (N-rooks)



Latin Hypercube Sampler (N-rooks)



Variants of stratified sampling

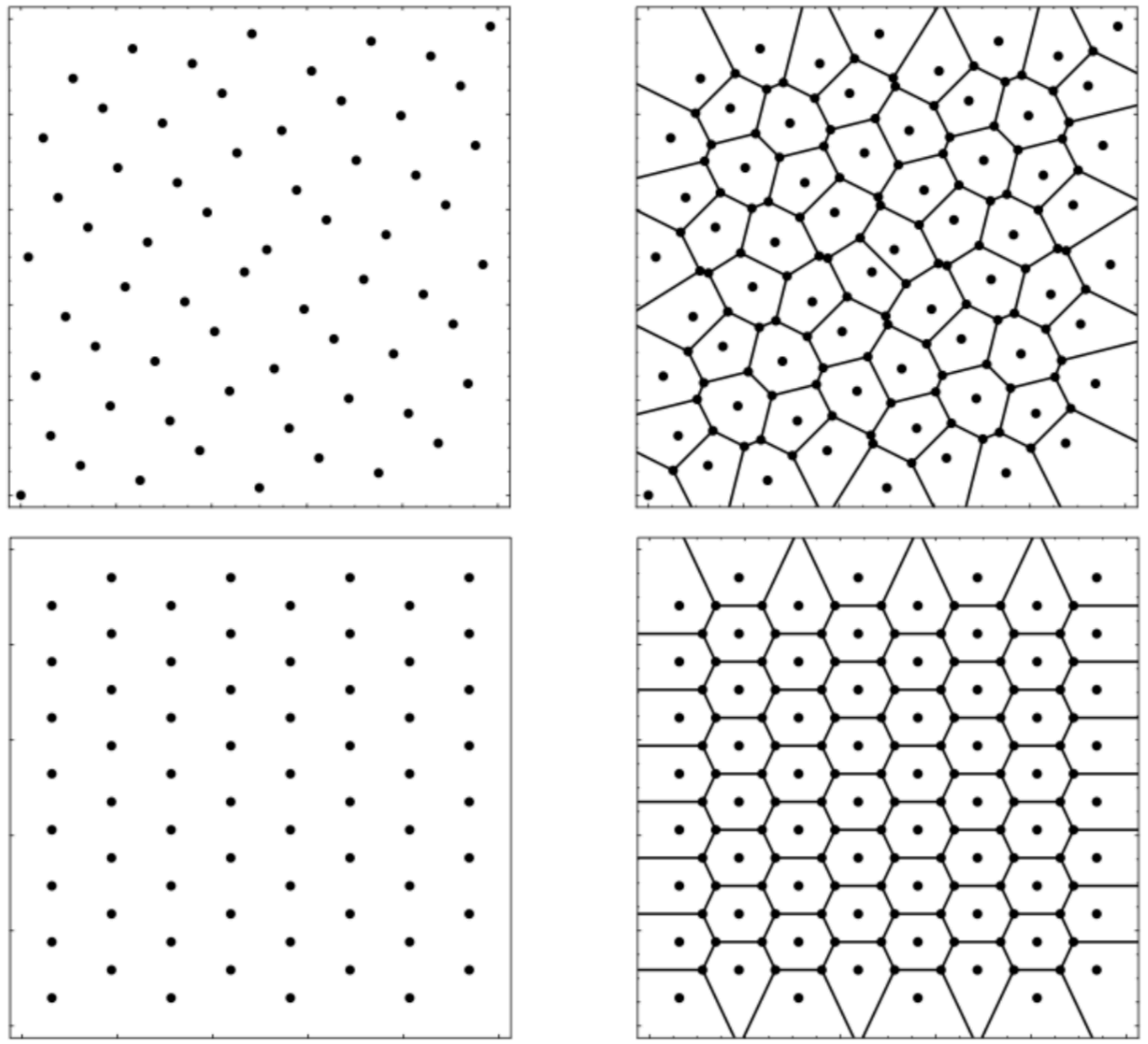


Figure 2.25: Stratification of I^2 with Voronoi diagrams. (a) 64-element Hammersley point set; (b) Voronoi diagram implied through (a); (c) 64-element hexagonal grid; (d) Voronoi diagram implied through (c).

Slide from Philipp Slusallek

Correlated Sampling: Quasi-Monte Carlo Integration

Quasi-Monte Carlo Integration

- Monte Carlo integration suffers, apart from the slow convergence rate, from the disadvantages that only probabilistic statements on convergence and error boundaries are possible

- The success of any Monte Carlo procedure stands or falls with the quality of these random samples

- If the distribution of the sample points is not uniform then there are large regions where there are no samples at all, which can increases the error

- Closely related to this is the fact that a smooth function is evaluated at unnecessary many locations if samples are clumped

Quasi-Monte Carlo Integration

- Deterministic generation of samples, while making sure uniform distributions
- Based on number-theoretic approaches
- Samples with good uniform properties can be generated in very high dimensions.
- Sample generation is pretty fast: (almost) no pre-processing

Quasi-Monte Carlo Integration

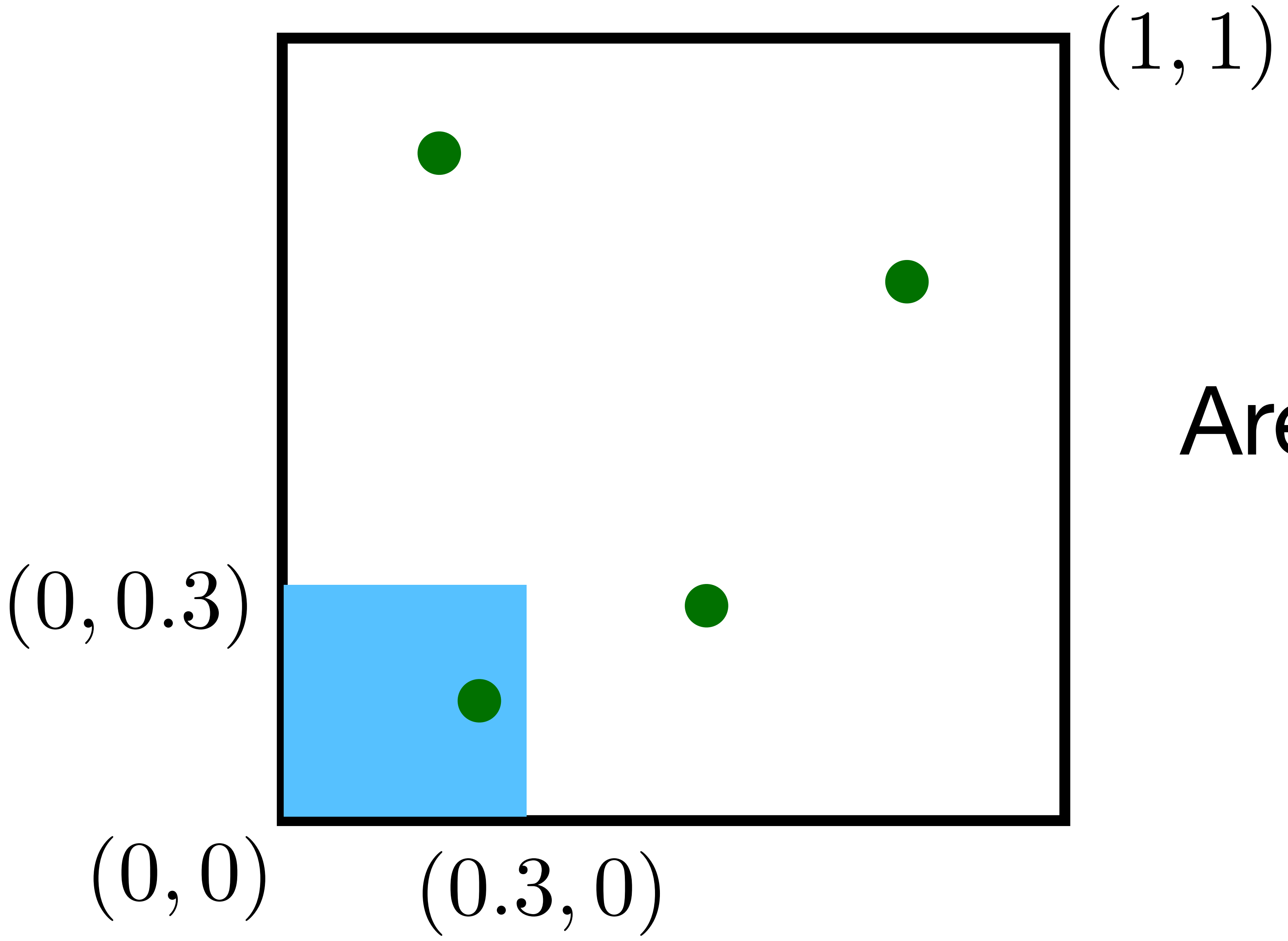
- Low discrepancy sequences
  - Halton and Hammerslay sequences
  - Scrambled sequences
- Discrepancy

Discrepancy: Basic idea

- The concept of discrepancy can be viewed as a quantitative measure for the deviation of a given point set from a uniform distribution

Discrepancy: Basic idea

- The concept of discrepancy can be viewed as a quantitative measure for the deviation of a given point set from a uniform distribution

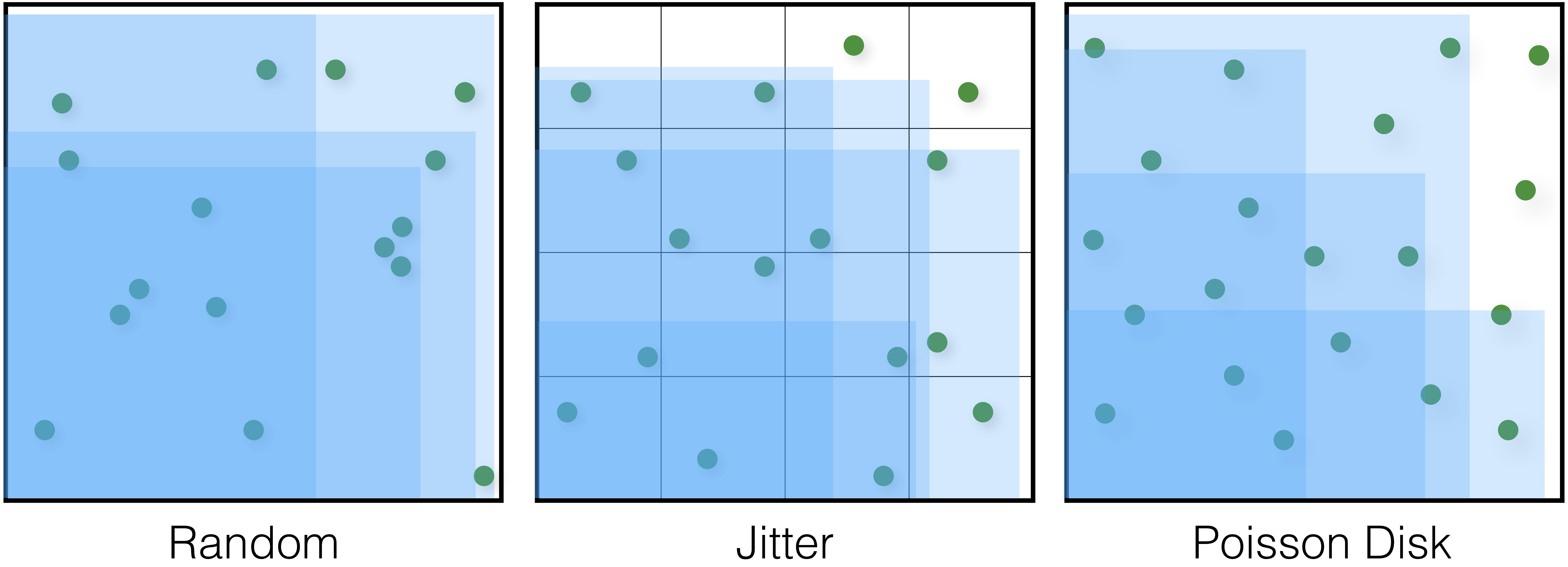


Area of the blue box: 0.09

Area associated to each sample: 0.25

Discrepancy: 0.25 - 0.09 = 0.16

Spatial Statistics: Discrepancy



Discrepancy = BoxArea - FractionSamples

Star Discrepancy

Radical Inverse

Techniques based on a construction called as **radical inverse**

Any integer can be represented in the form:

$$n = \sum_{i=1}^{\infty} d_i b^{i-1}$$

n	Binary	$\Phi_b(n)$
1	1	
2	01	
3	11	
4	001	
5	101	

Radical Inverse

Techniques based on a construction called as **radical inverse**

Any integer can be represented in the form:

$$n = \sum_{i=1}^{\infty} d_i b^{i-1}$$

Radical inverse:

$$\Phi_b(n) = 0.d_1 d_2 ... d_m$$

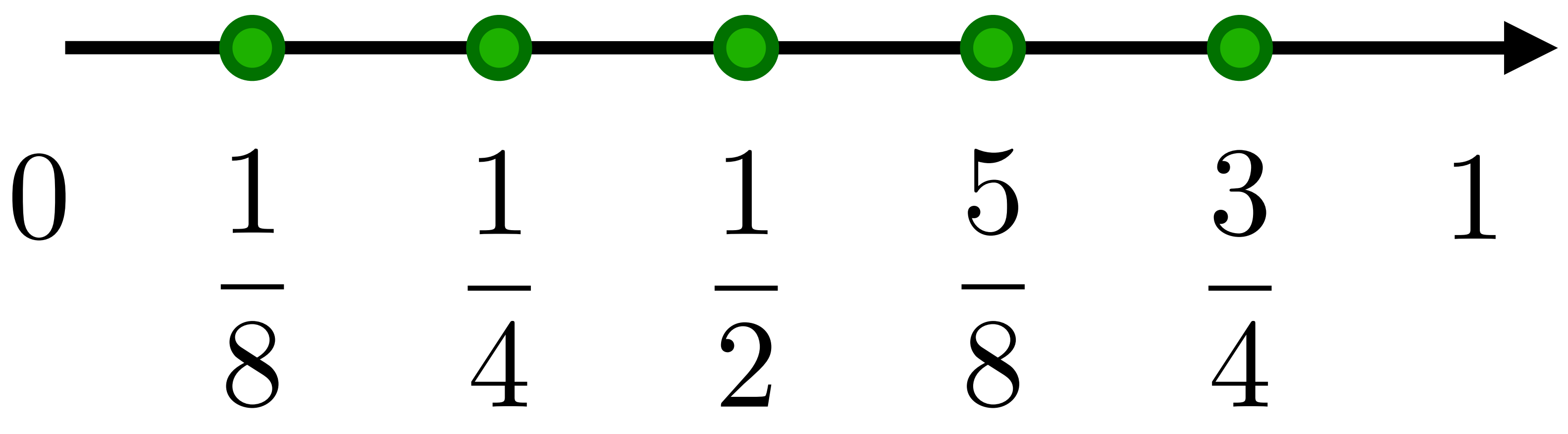
n	Binary	$\Phi_b(n)$
1	1	0.1
2	01	0.01
3	11	0.11
4	001	0.001
5	101	0.101

Radical Inverse

Techniques based on a construction called as **radical inverse**

Radical inverse:

$$\Phi_b(n) = 0.d_1 d_2 \ldots d_m$$



n	Binary	$\Phi_b(n)$
1	1	0.1 = 1/2
2	01	0.01 = 1/4
3	11	0.11 = 3/4
4	001	0.001 = 1/8
5	101	0.101 = 5/8

Halton and Hammerslay Sequence

Techniques based on a construction called as **radical inverse**

Radical inverse: $\Phi_b(n) = 0.d_1 d_2 ... d_m$

Halton Sequence: For n-dimensional sequence, we use different base b for each dimension

$$x_i = (\Phi_2(i), \Phi_3(i), \Phi_5(i), \ldots, \Phi_{p_n}(i))$$

Halton and Hammerslay Sequence

Techniques based on a construction called as **radical inverse**

Radical inverse: $\Phi_b(n) = 0.d_1 d_2 ... d_m$

Halton Sequence: For n-dimensional sequence, we use different base b for each dimension

$$x_i = (\Phi_2(i), \Phi_3(i), \Phi_5(i), \ldots, \Phi_{p_n}(i))$$

Hammerslay Sequence: All except the first dimension has co-prime bases

$$x_i = \left(\frac{i}{N}, \Phi_{b_1}(i), \Phi_{b_2}(i), \ldots, \Phi_{b_n}(i) \right)$$

Halton and Hammerslay Sequence

Techniques based on a construction called as **radical inverse**

Radical inverse: $\Phi_b(n) = 0.d_1 d_2 ... d_m$

Halton Sequence:

$$x_i = (\Phi_2(i), \Phi_3(i), \Phi_5(i), \ldots, \Phi_{p_n}(i))$$

Hammerslay Sequence:

$$x_i = \left(\frac{i}{N}, \Phi_{b_1}(i), \Phi_{b_2}(i), \ldots, \Phi_{b_n}(i)\right)$$

Hammerslay has slightly **lower** discrepancy than Halton


Halton 4spp


Halton 8spp


Sobol 4spp

Sobol 8spp


Random 8spp

Visualizing samples

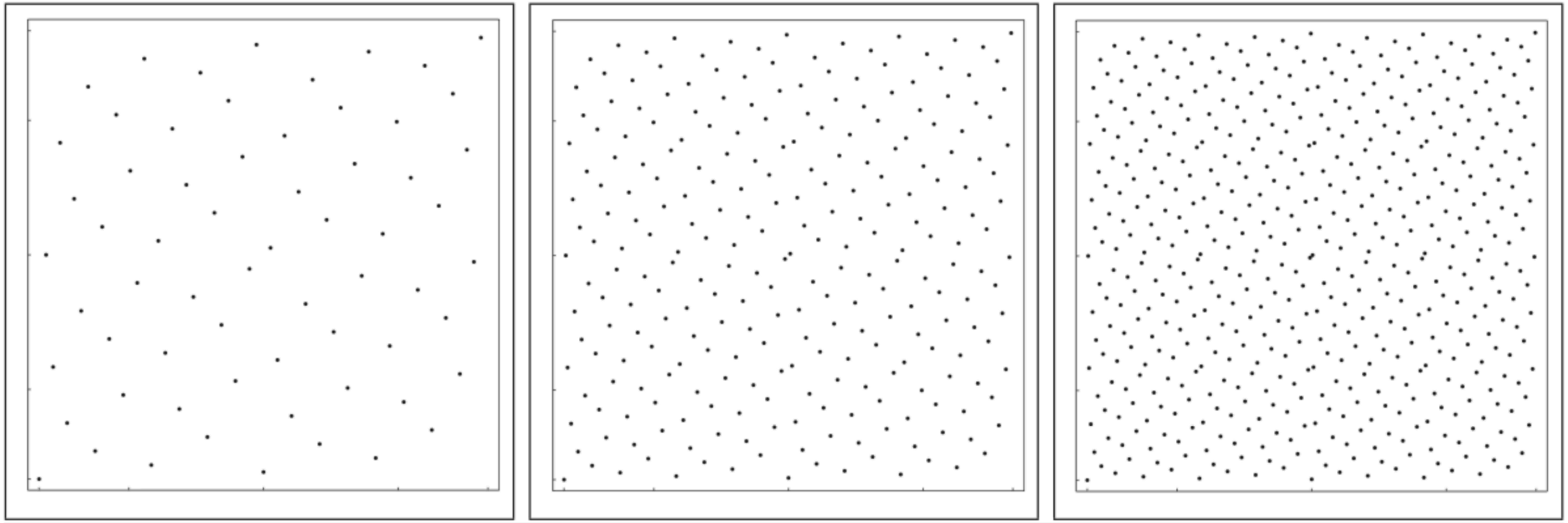


Figure 2.7: Hammersley Point Set on the 2D Plane. Three 2-dimensional Hammersley point sets $\mathbf{P}^2_{\mathrm{HAM}} = \left(\frac{i}{N}, \Phi_2(i)\right)_{i \in (0,\ldots,N-1)}$ of sizes $N = 64$-element, $N = 256$-element and $N = 512$-element.

Slide from Philipp Slusallek

Visualizing samples

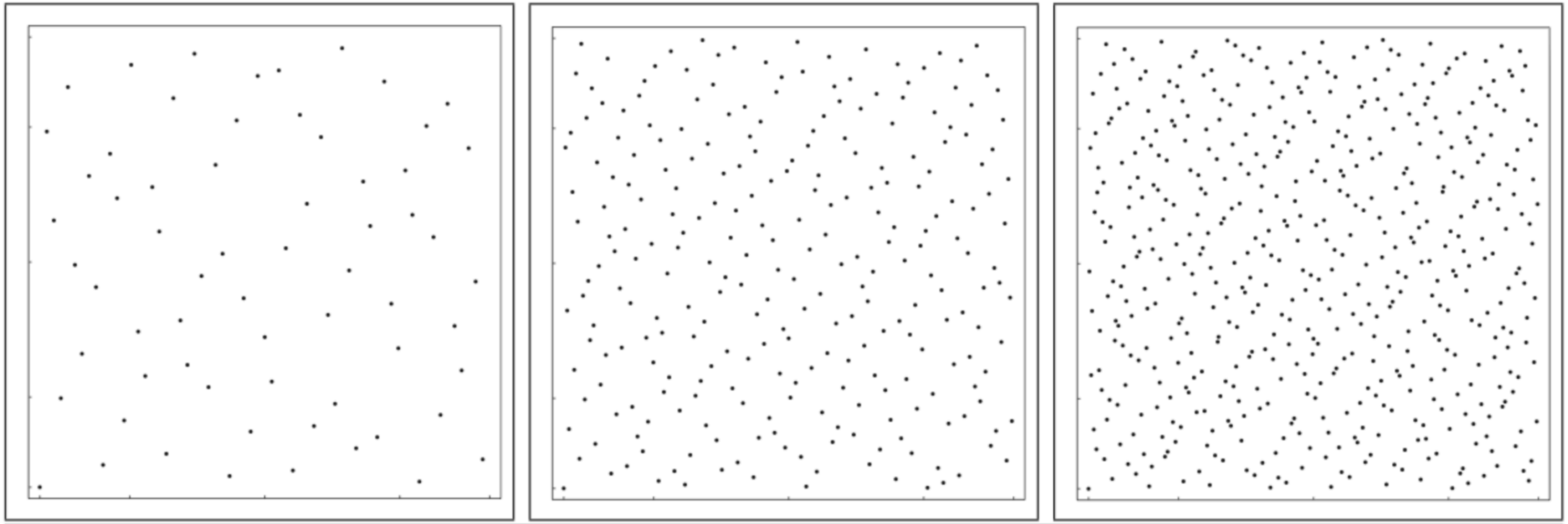
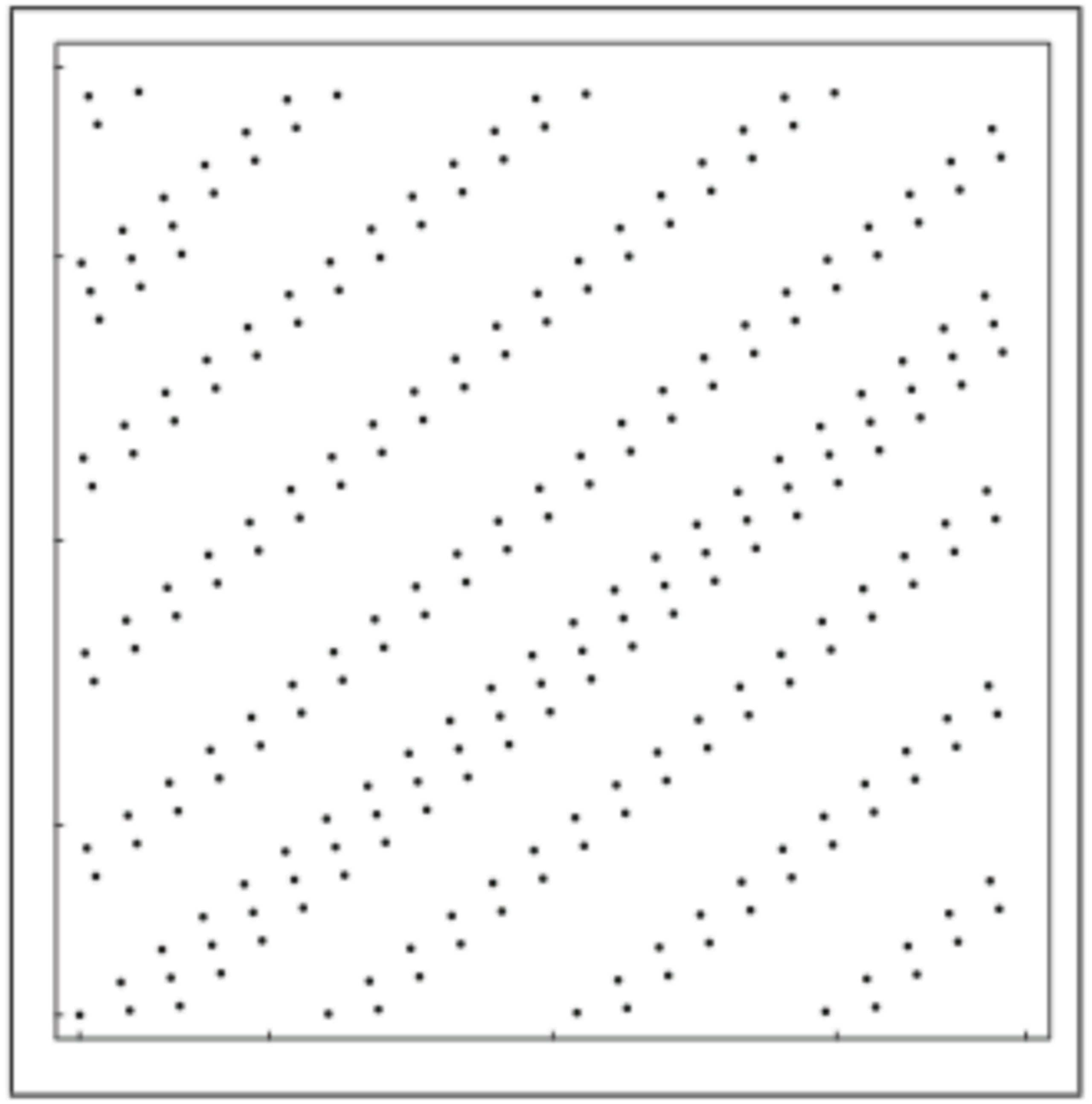


Figure 2.5: Halton sequence. The first 64, 256, and 512 points of the 2-dimensional Halton Sequence $\mathbf{P}^2_{\mathrm{HAL}} = (\Phi_2(i), \Phi_3(i))_{i \in \mathbb{N}_0}$.

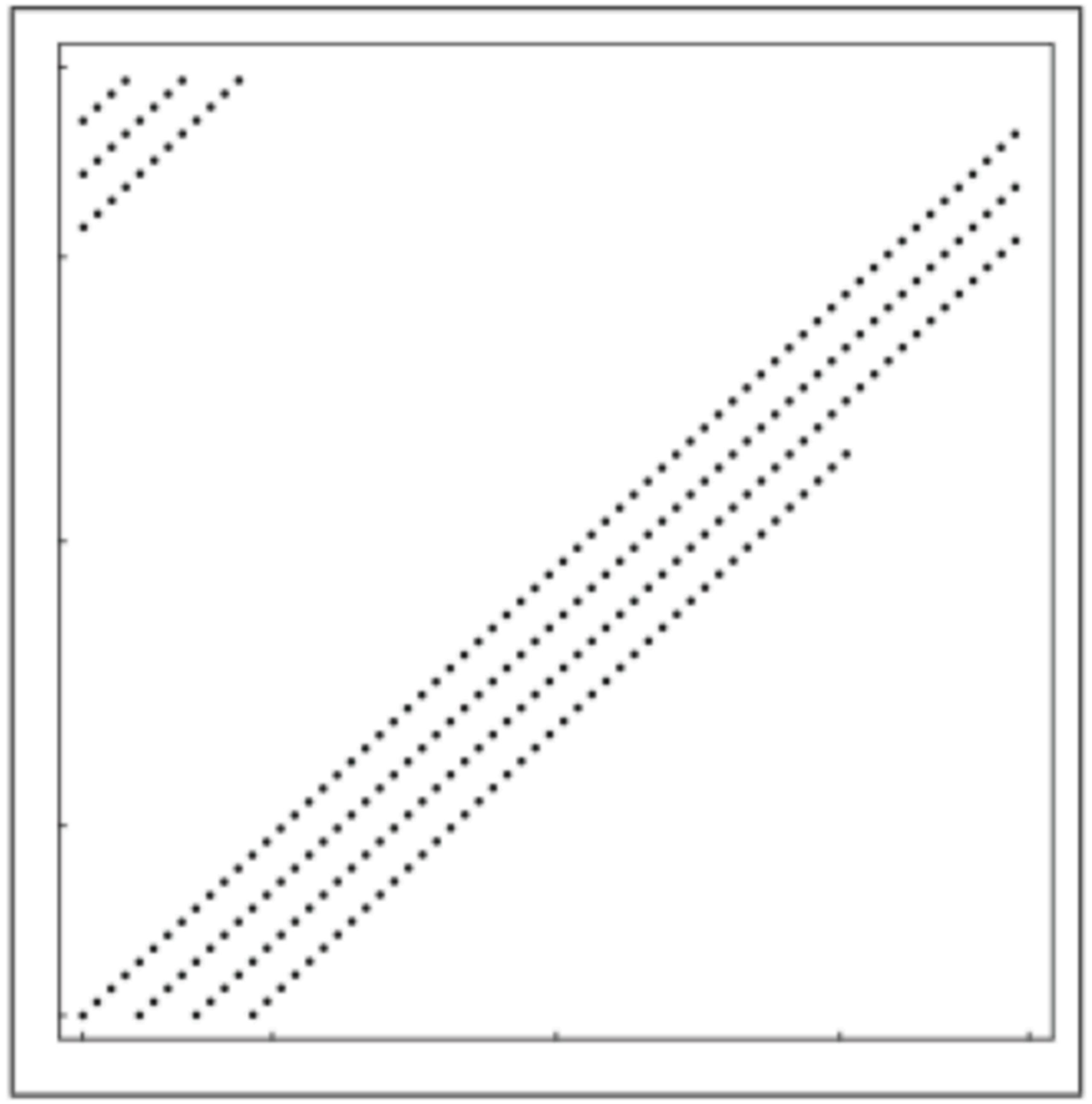
Slide from Philipp Slusallek

Visualizing samples

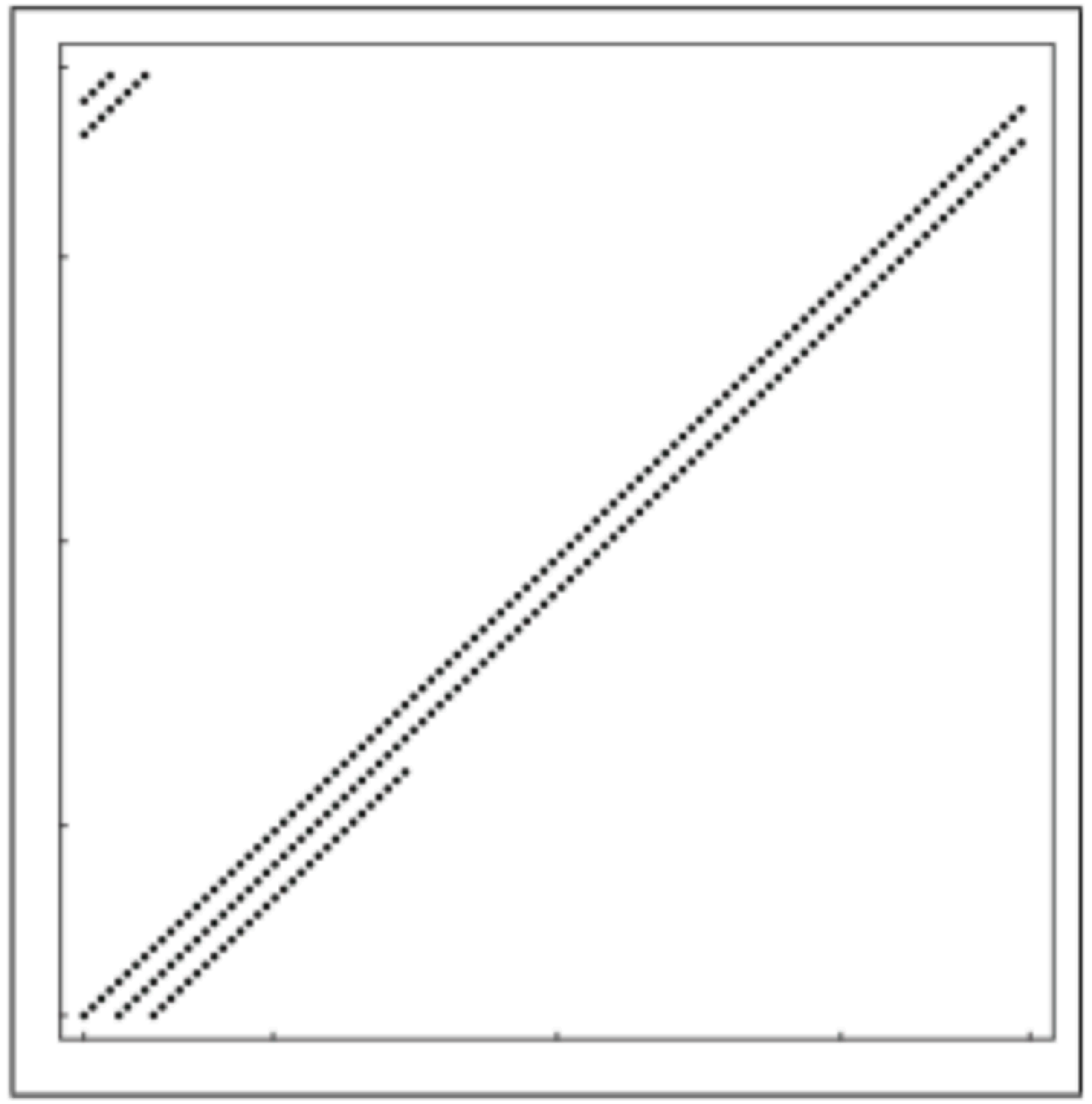
Projection: (9,10)



Projection: (19,20)



Projection: (29,30)



Halton Sequence

Slide from Philipp Slusallek

Faure's permutation

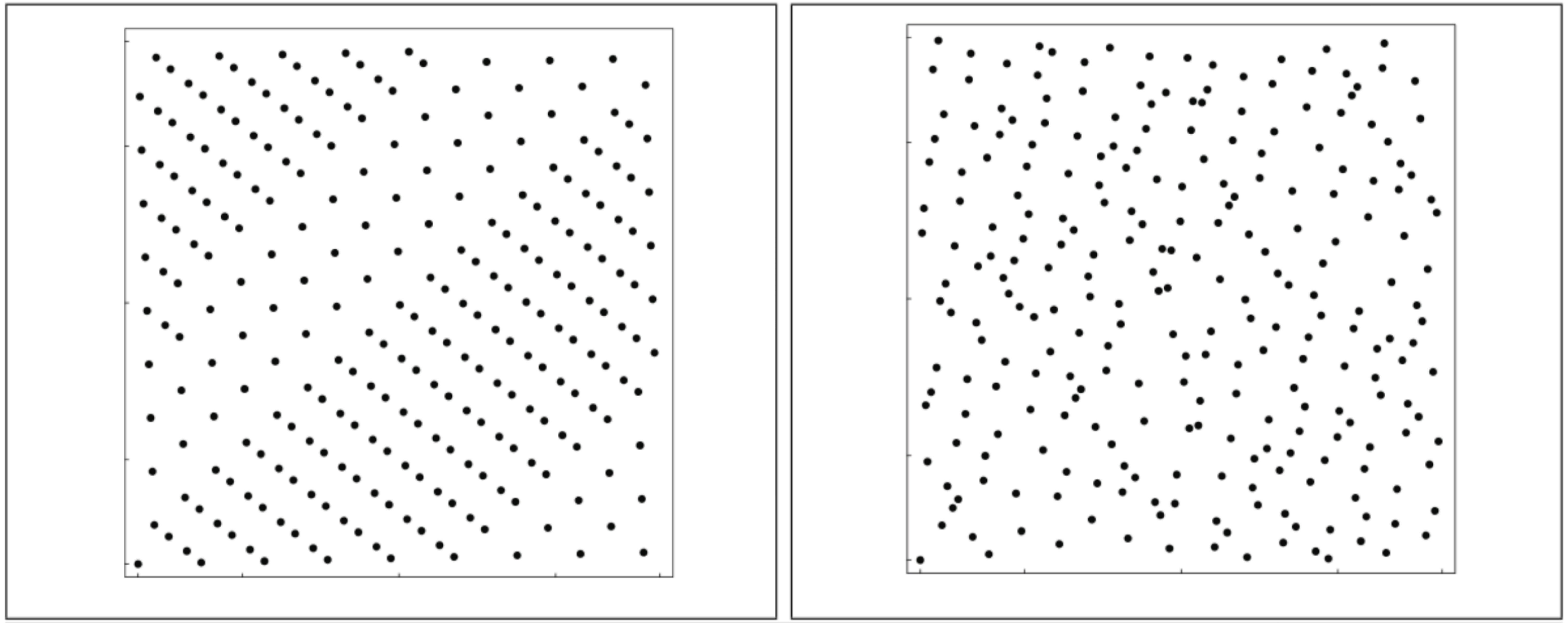


Figure 2.12: Halton Sequence and Scrambled Halton Sequence, Dimensions 7 and 8. **(a)** The first 256 elements of the 2-dimensional Halton sequence $\mathbf{P}^2_{\text{HAL}} = (\Phi_7(i), \Phi_8(i))$ and the scrambled versions of dimension 7 and 8 generated according to procedure of Faure.

Slide from Philipp Slusallek

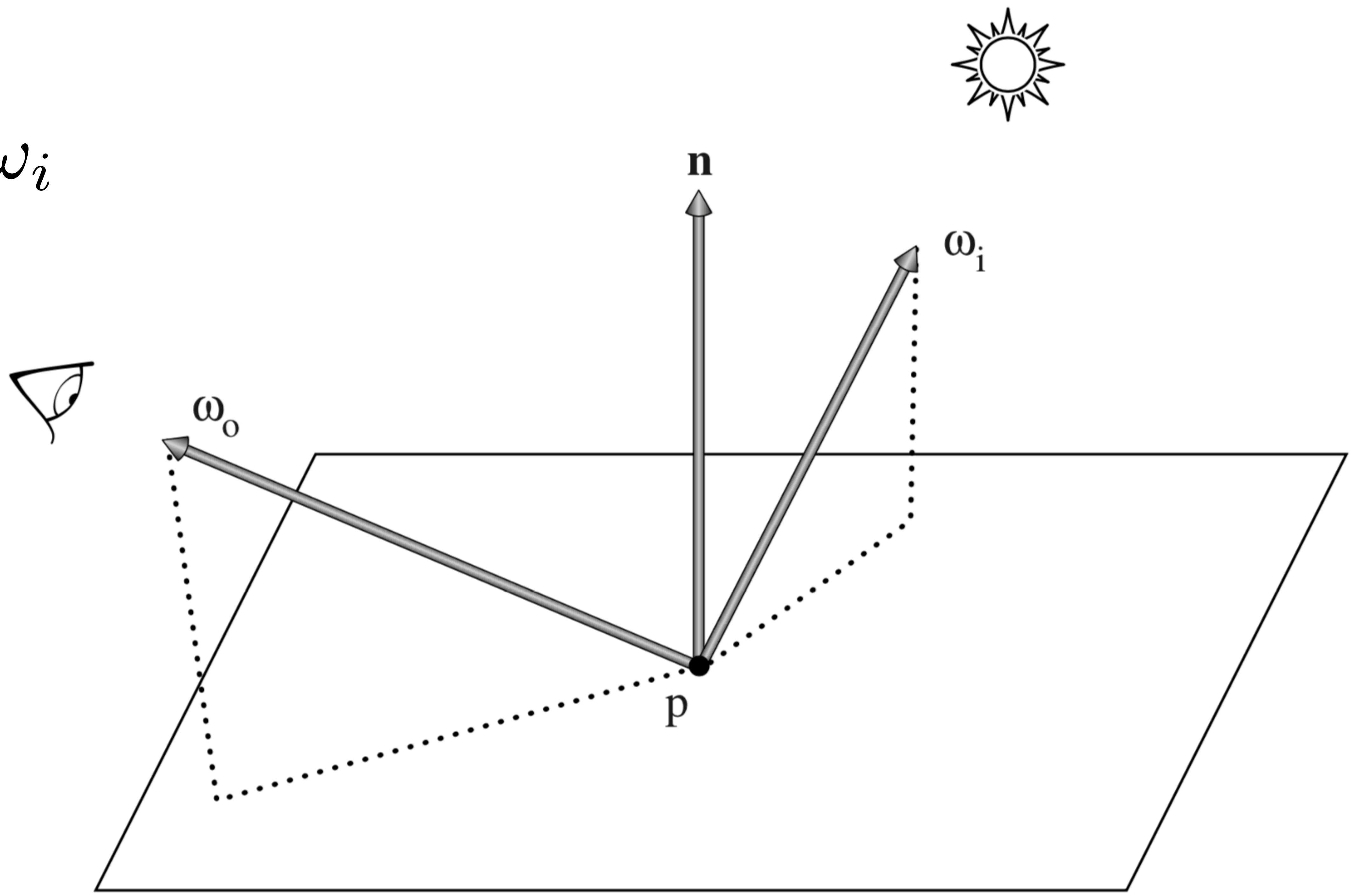
Questions?

Gaussian Material Synthesis by Zsolnai-Feher, Wonka, Wimmer [SIGGRAPH 2018]

Importance Sampling

Importance Sampling

$$L_o(p,\omega) = \int_{\mathcal{H}^2} f_r(x,\omega_0,\omega_i) L_i(x,\omega_i) |\cos\theta_i| d\omega_i$$

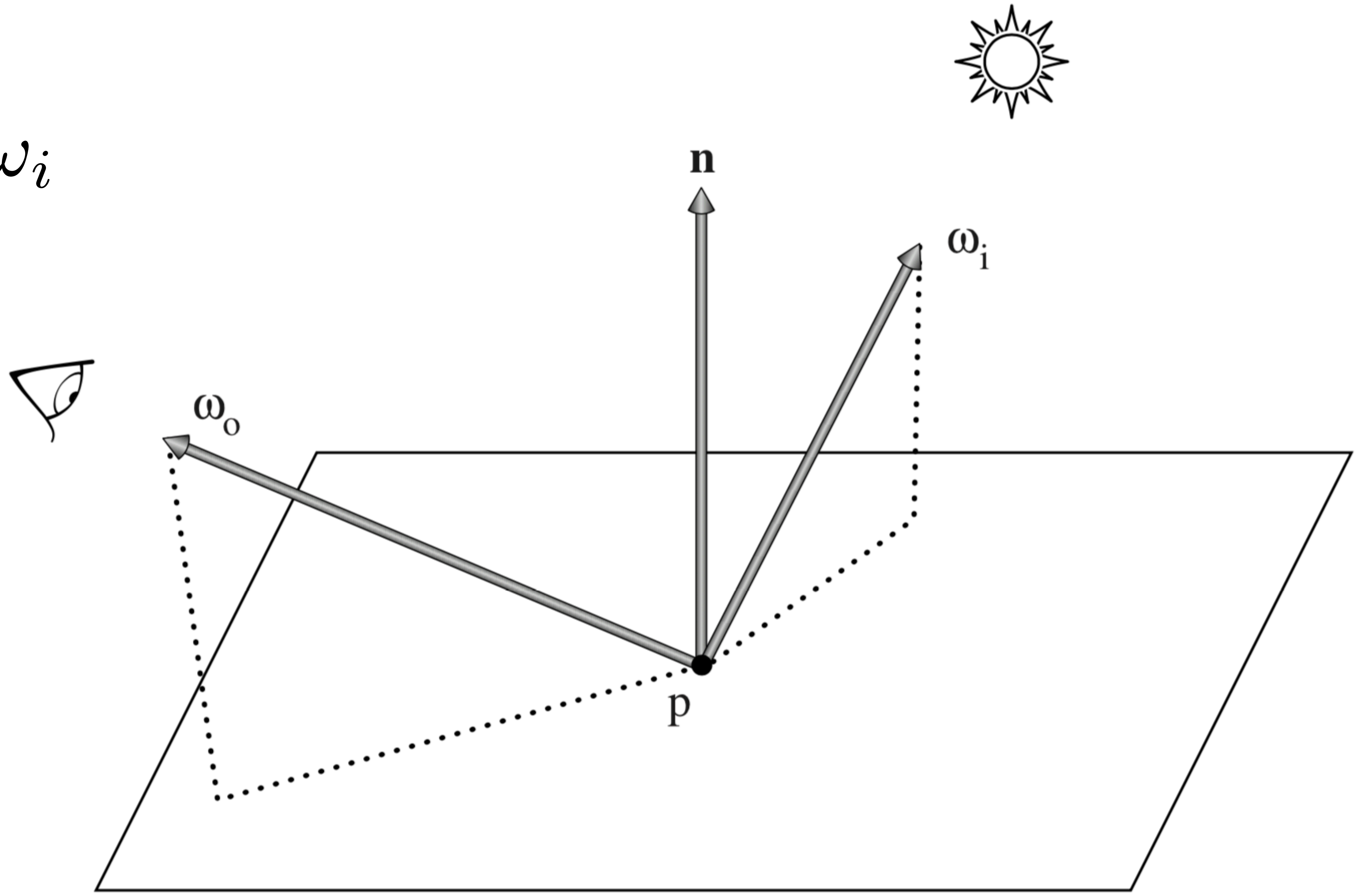


What terms can we importance sample?

- BSDF
- Incident radiance
- cosine term

Importance Sampling: Cosine term

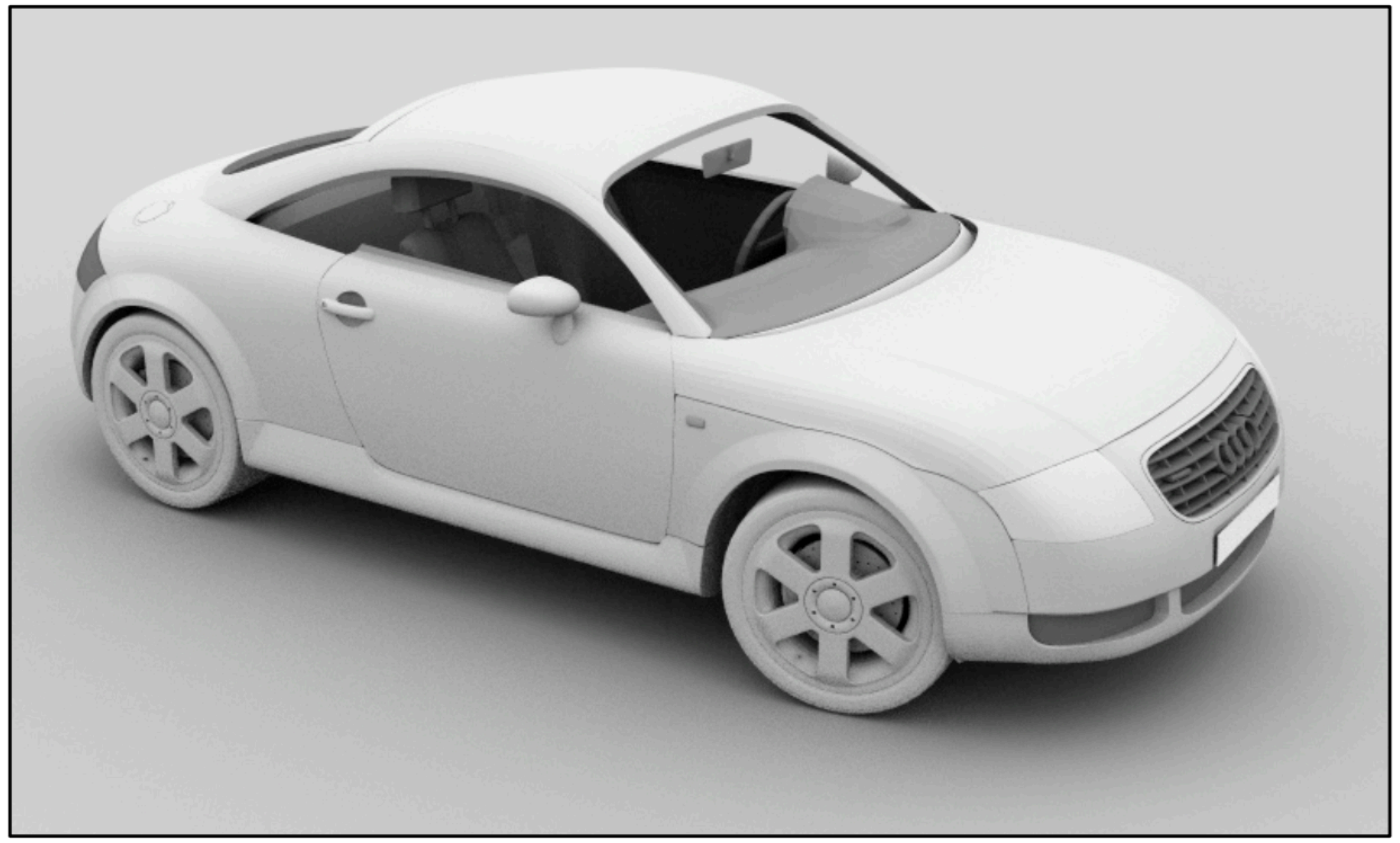
$$L_o(p,\omega) = \int_{\mathcal{H}^2} f_r(x,\omega_0,\omega_i) L_i(x,\omega_i) |\cos\theta_i| d\omega_i$$

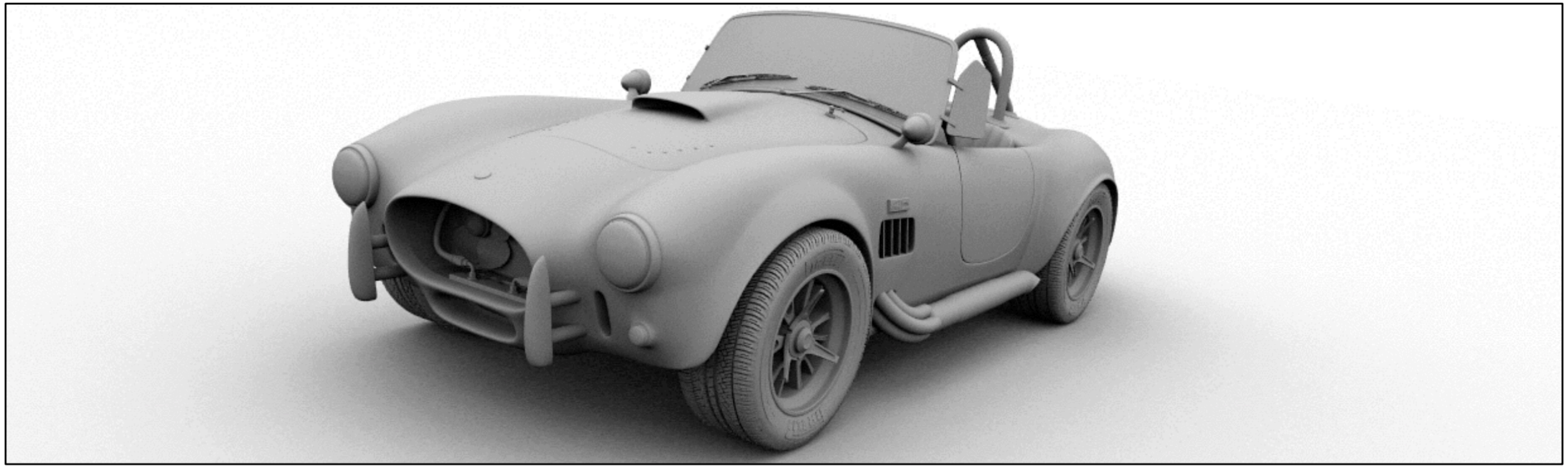


What terms can we importance sample?

- BSDF
- Incident radiance
- cosine term

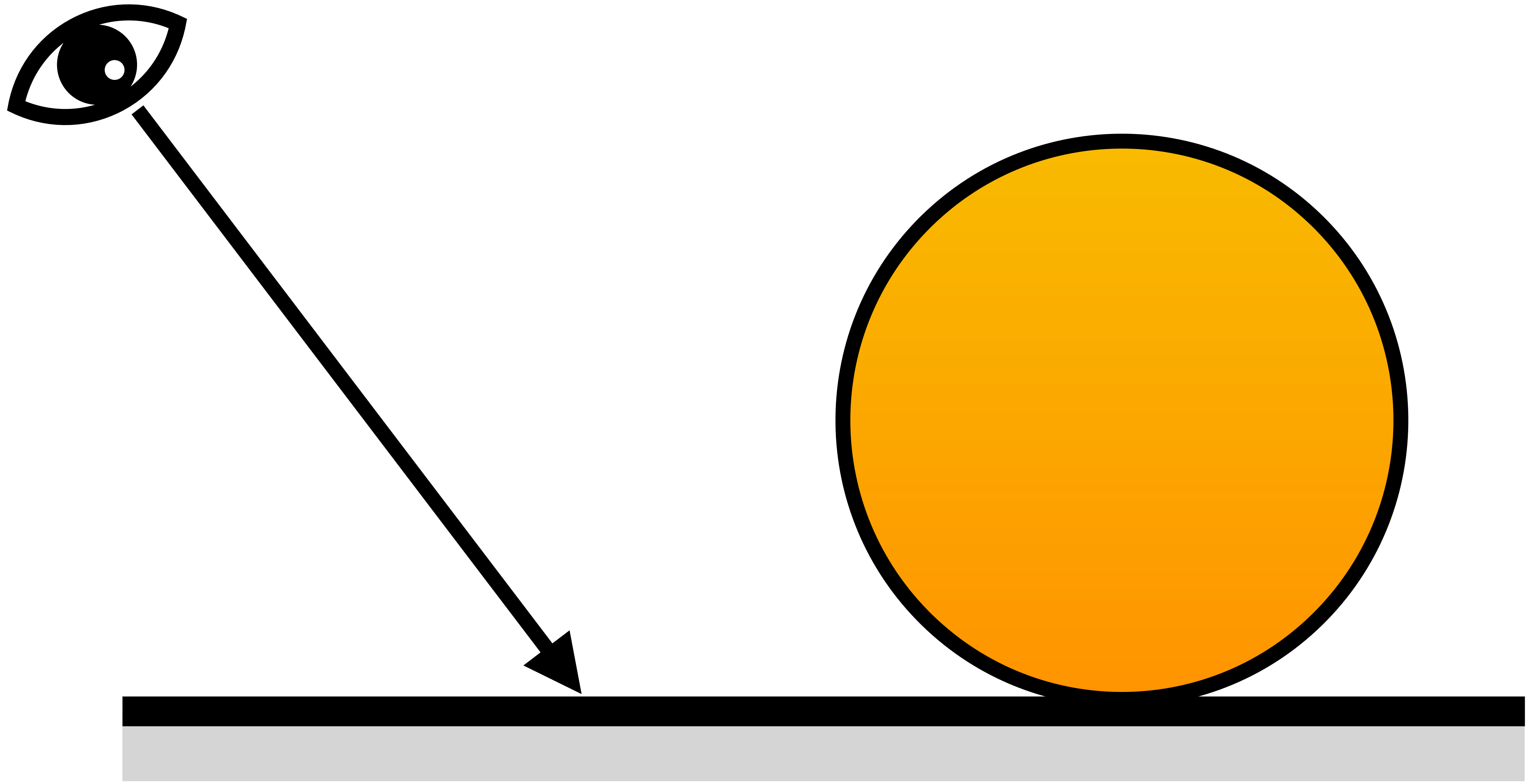
Example: Ambient Occlusion





Example: Ambient Occlusion

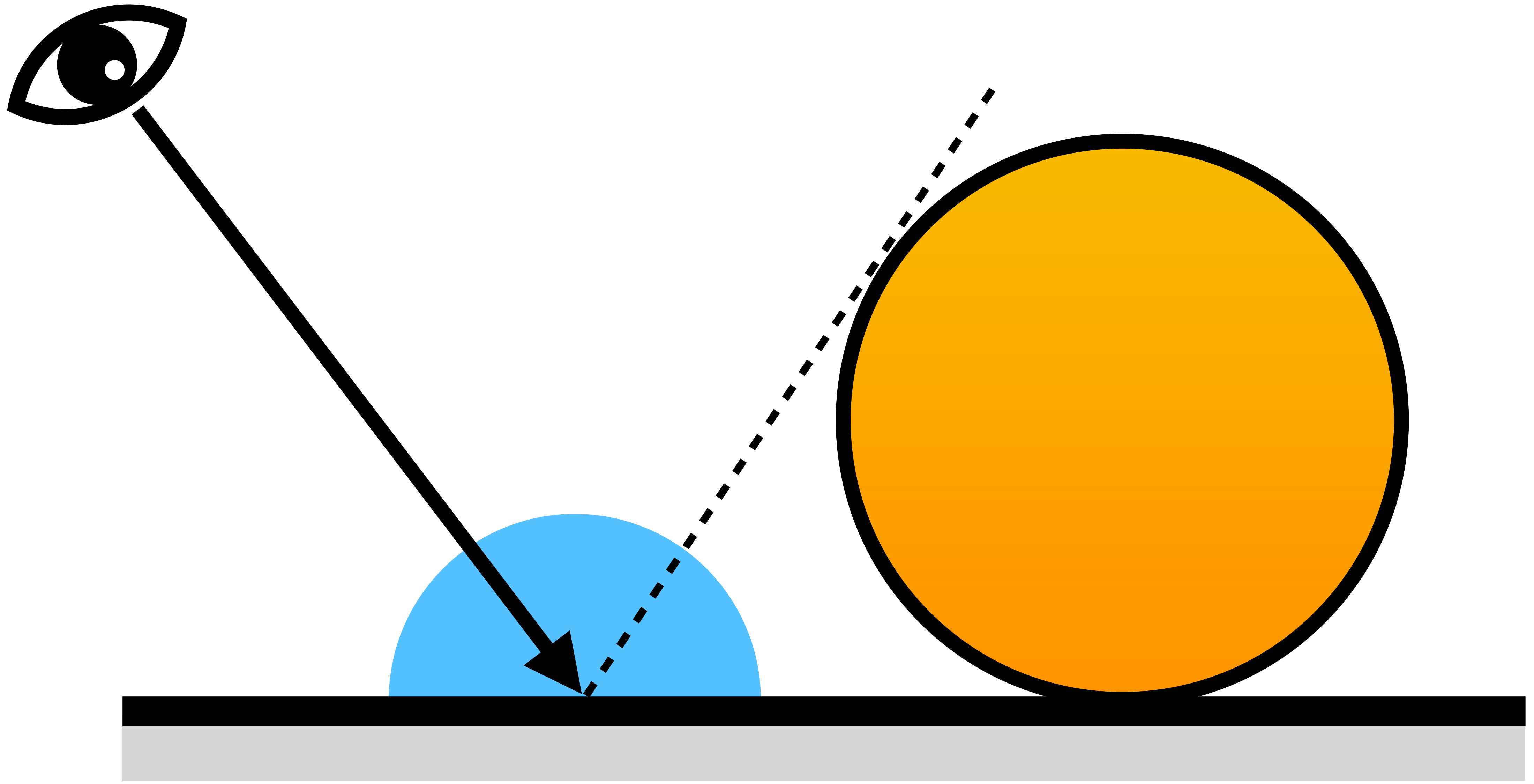
$$L_o(p,\omega) = \int_{\mathcal{H}^2} f_r(x,\omega_0,\omega_i) L_i(x,\omega_i) |\cos\theta_i| d\omega_i$$



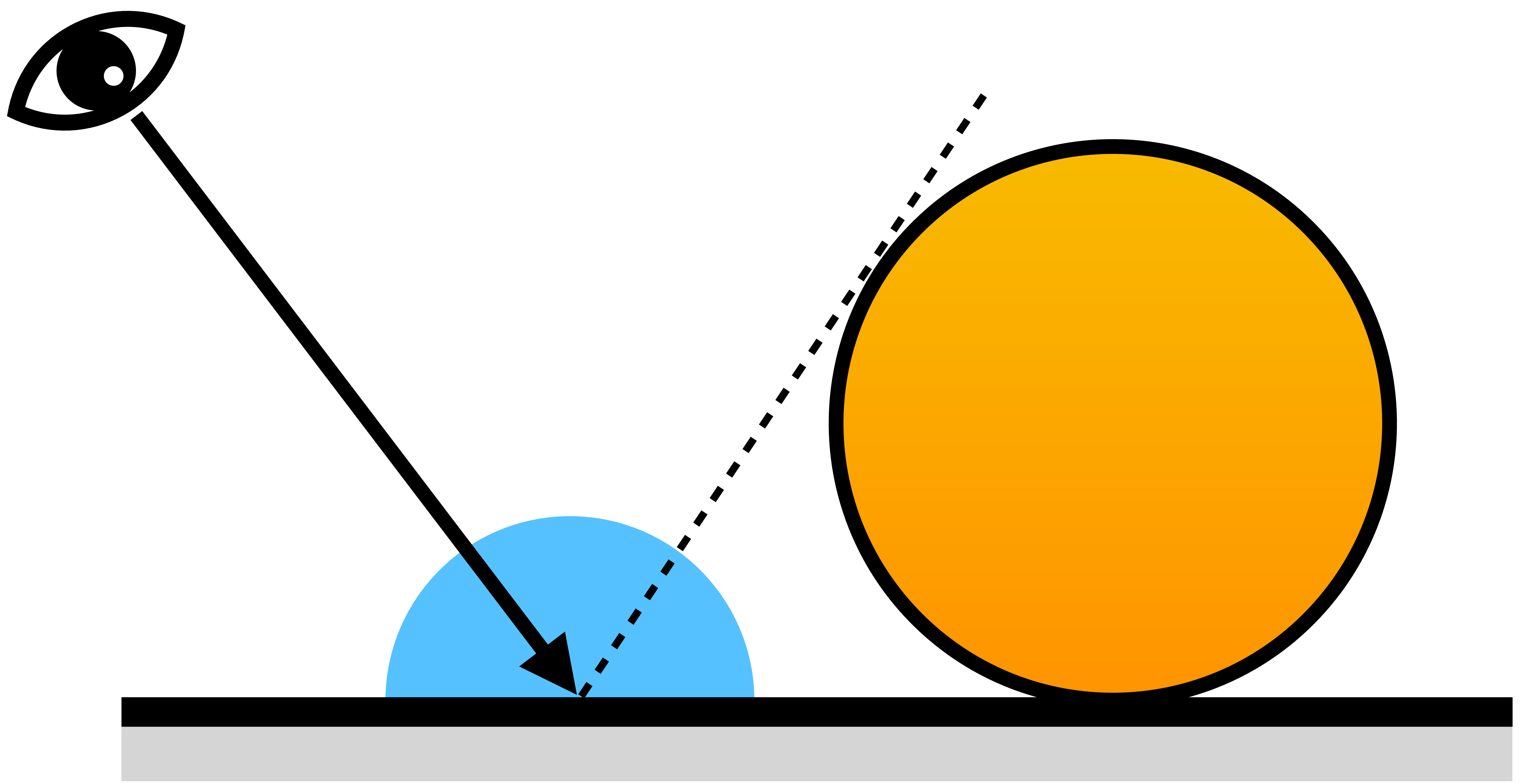
$$L_o(p,\omega) = \frac{\rho}{\pi} \int_{\mathcal{H}^2} V(x,\omega_i) |\cos\theta_i| d\omega_i$$

Example: Ambient Occlusion

$$L_o(p,\omega) = \frac{\rho}{\pi}\int_{\mathcal{H}^2} V(x,\omega_i)|\cos\theta_i|d\omega_i$$



Example: Ambient Occlusion

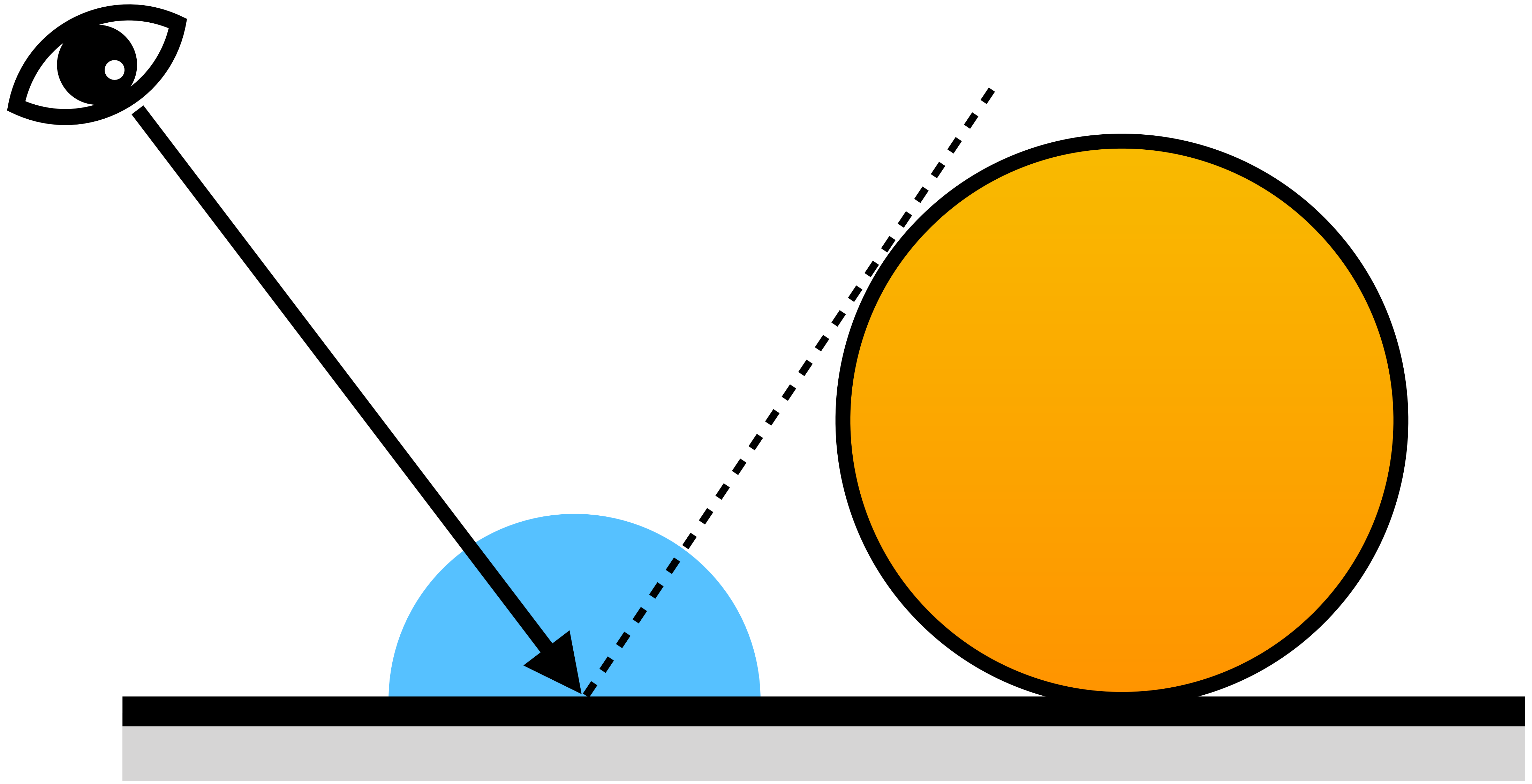


$$L_o(p, \omega) = \frac{\rho}{\pi} \int_{\mathcal{H}^2} V(x, \omega_i) |\cos\theta_i| d\omega_i$$

$$L_o(p, \omega) = \frac{\rho}{\pi} \frac{1}{N} \sum_{k=1}^{N} \frac{V(x, \omega_{i,k}) |\cos\theta_{i,k}|}{p(x, \omega_{i,k})}$$

Example: Ambient Occlusion

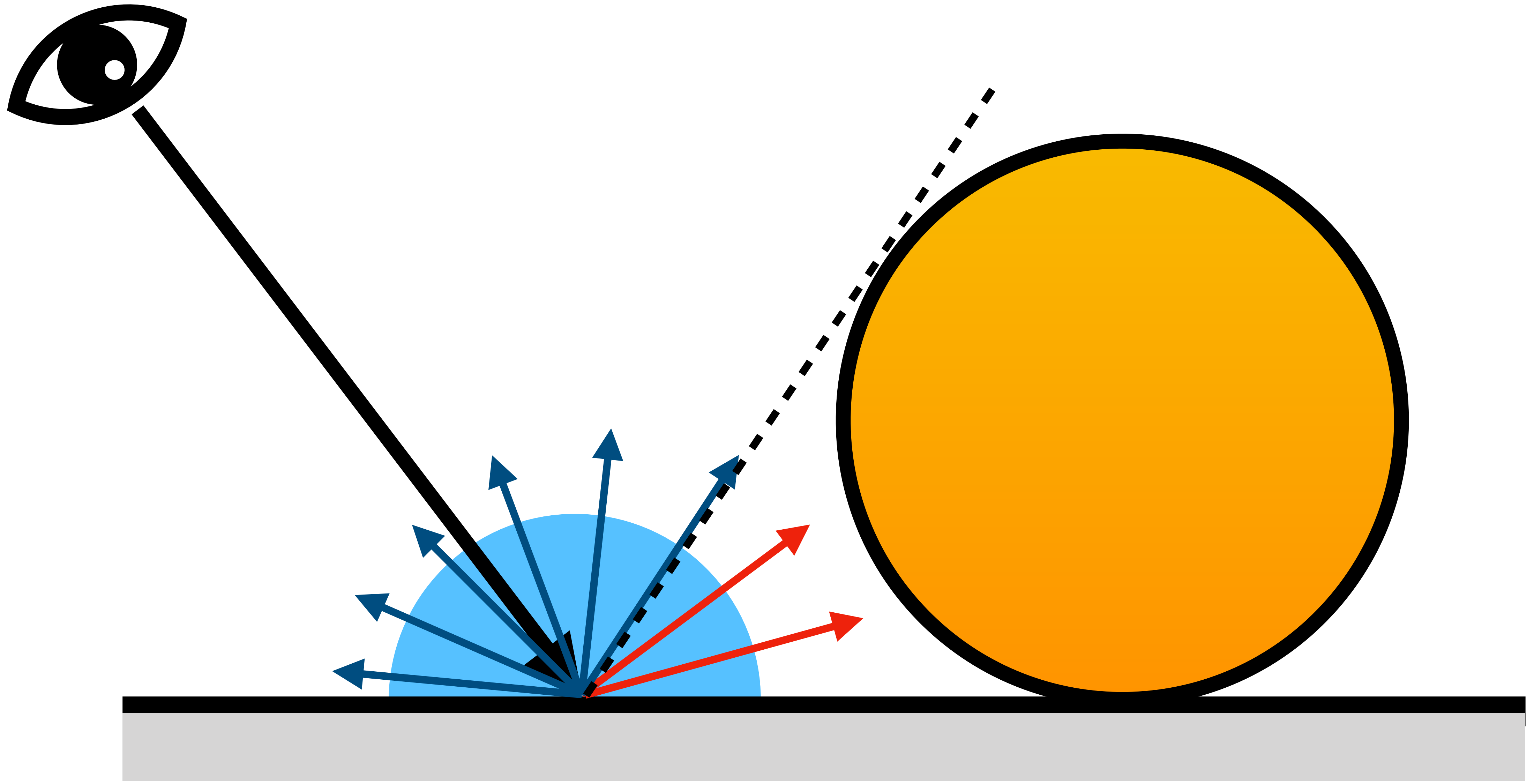
$$L_o(p,\omega) = \frac{\rho}{\pi}\frac{1}{N}\sum_{k=1}^{N}\frac{V(x,\omega_{i,k})|\cos\theta_{i,k}|}{p(x,\omega_{i,k})}$$



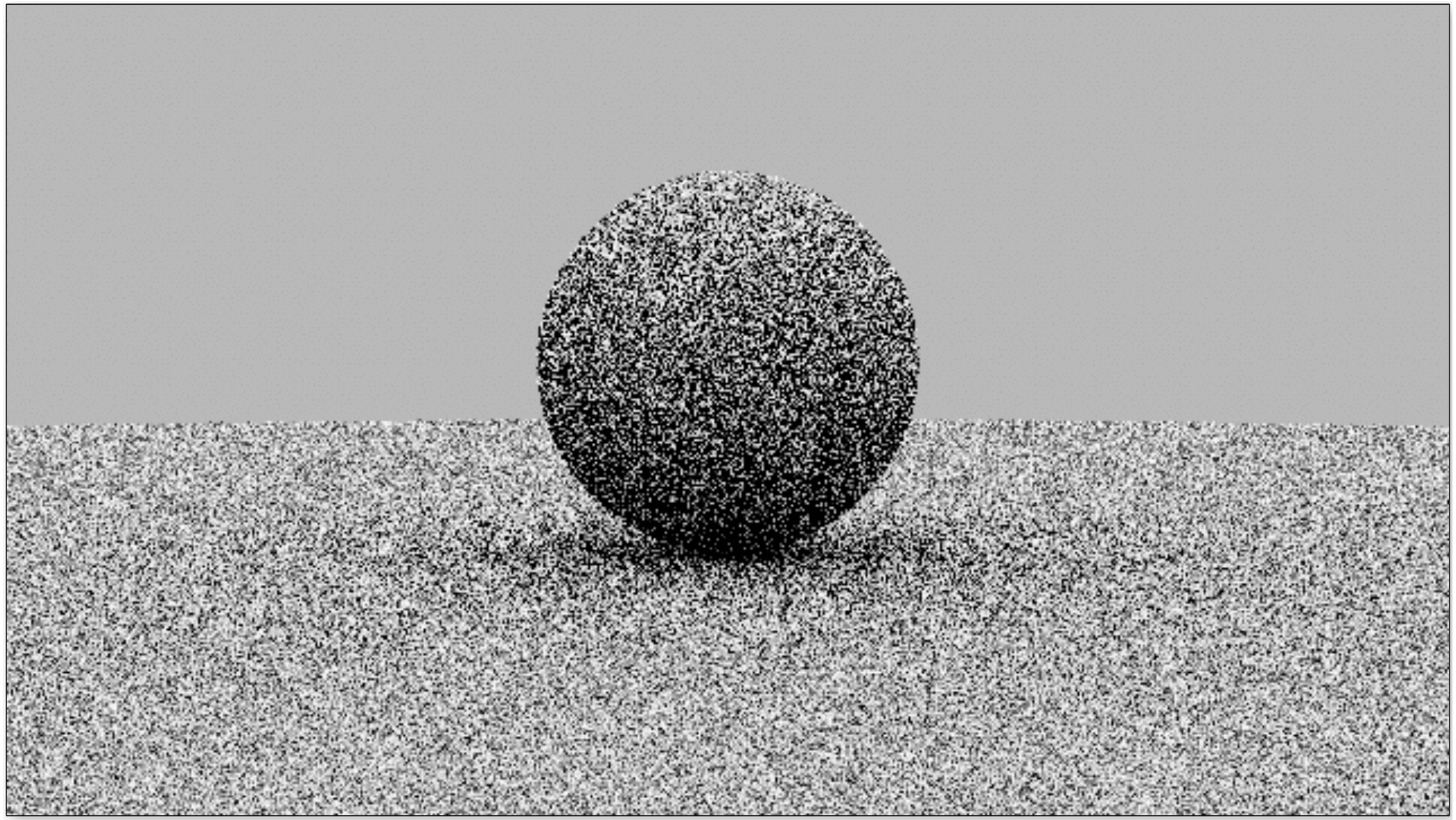
$$p(x,\omega_{i,k}) \propto \;???$$

Hemispherical Sampling: Constant PDF

Uniform Hemispherical Sampling

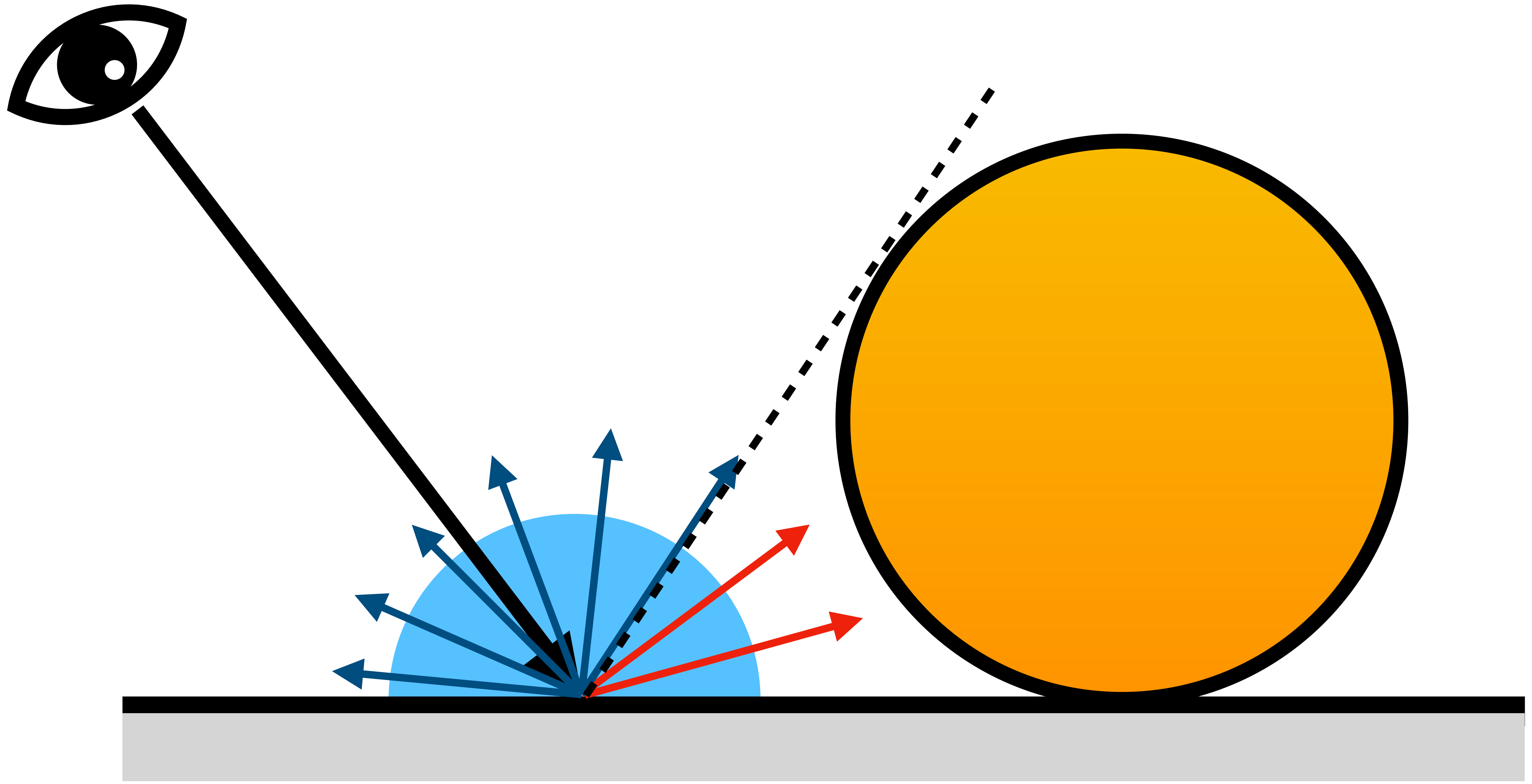


(1 Sample)



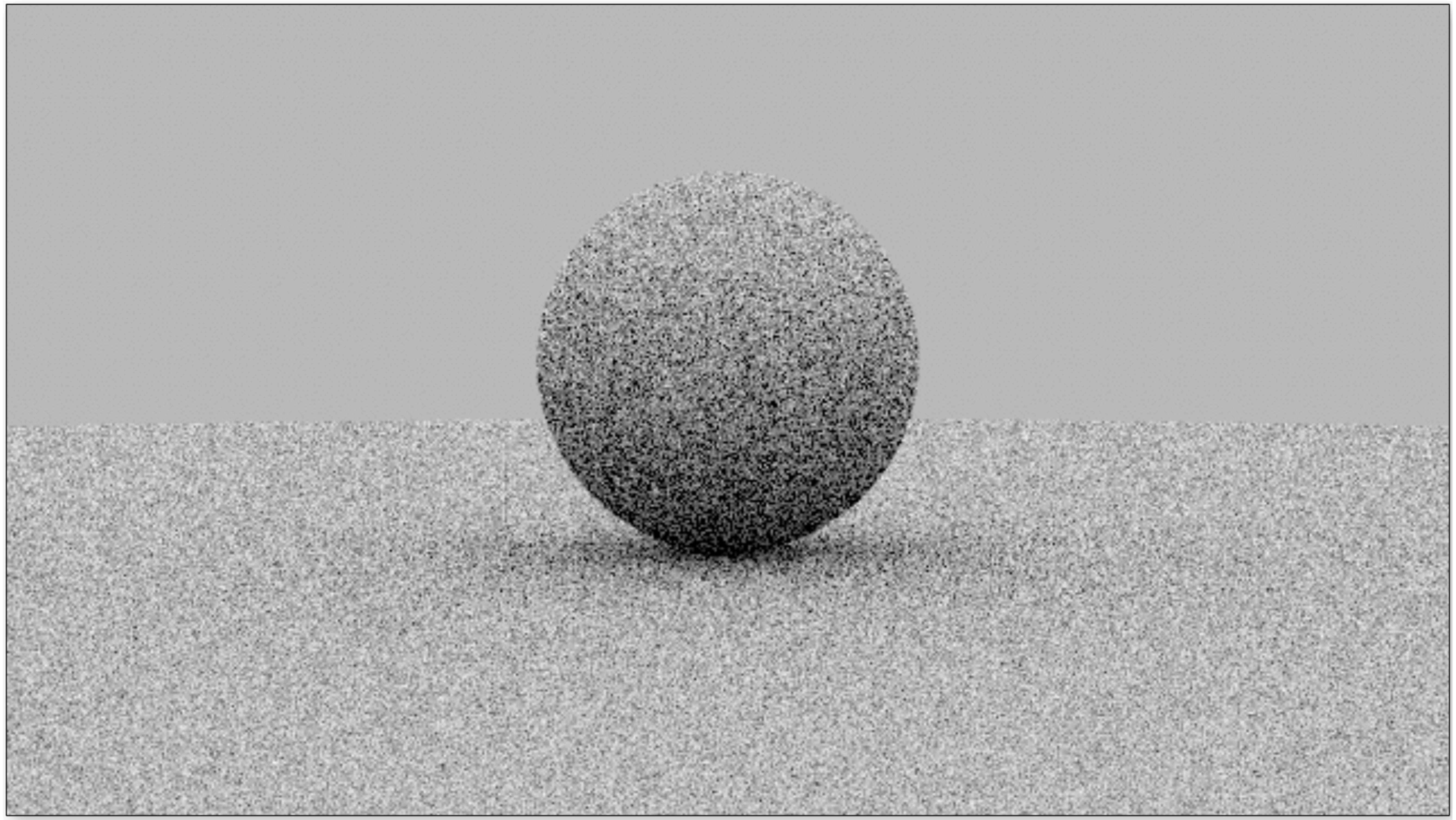
Hemispherical Sampling: Constant PDF

Uniform Hemispherical Sampling



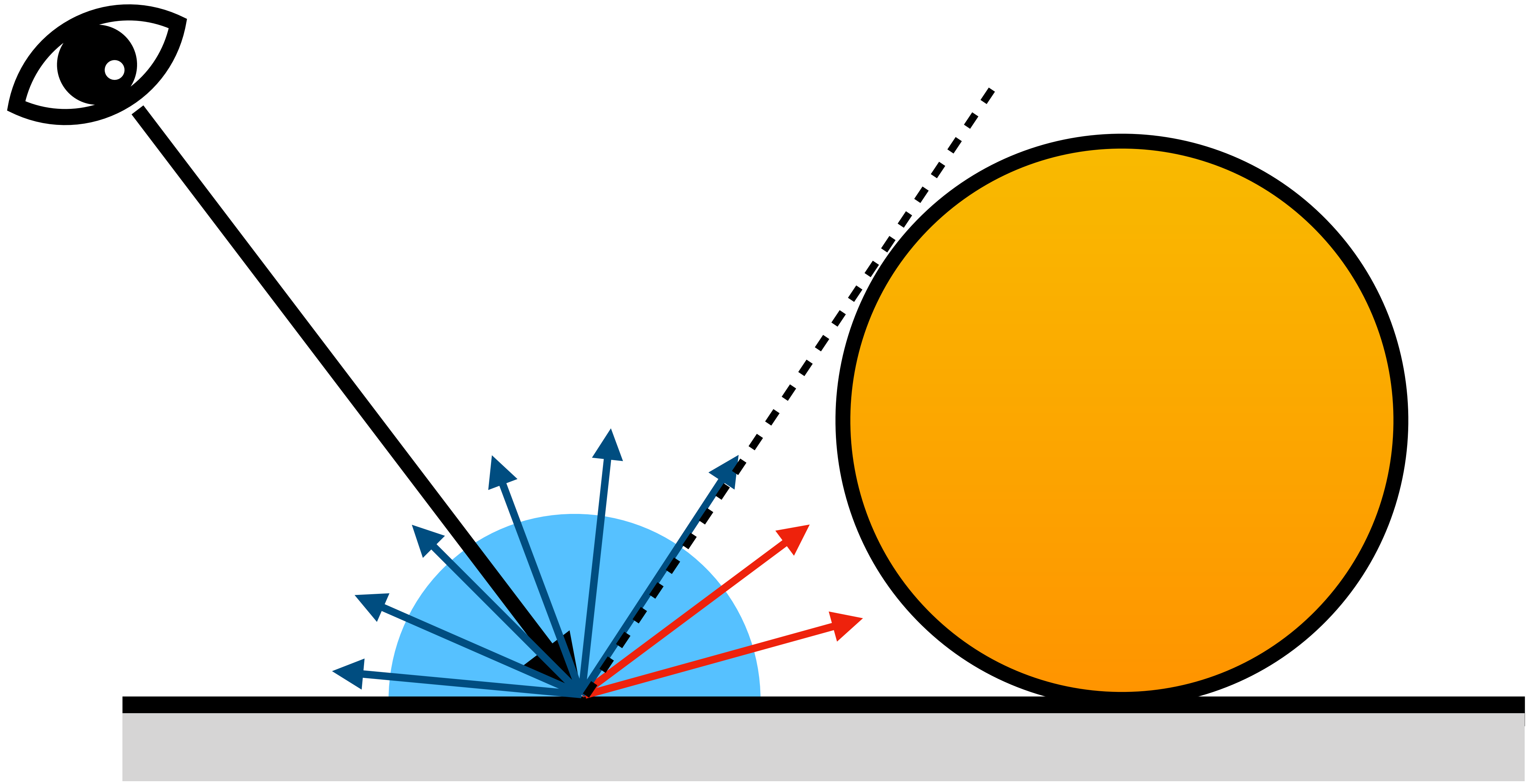
$$p(x, \omega_i) = \frac{1}{2\pi}$$

(4 Samples)



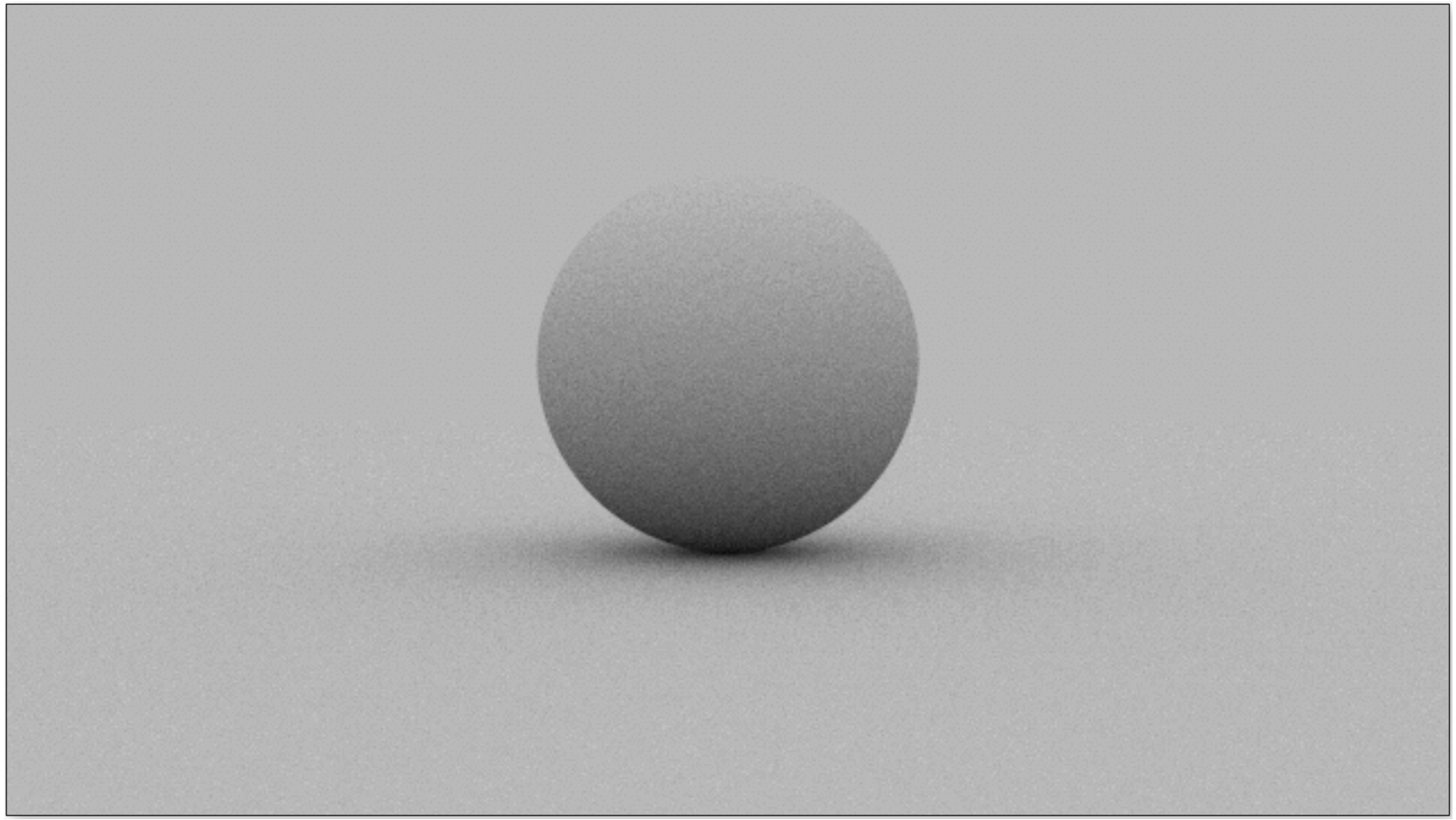
Hemispherical Sampling: Constant PDF

Uniform Hemispherical Sampling



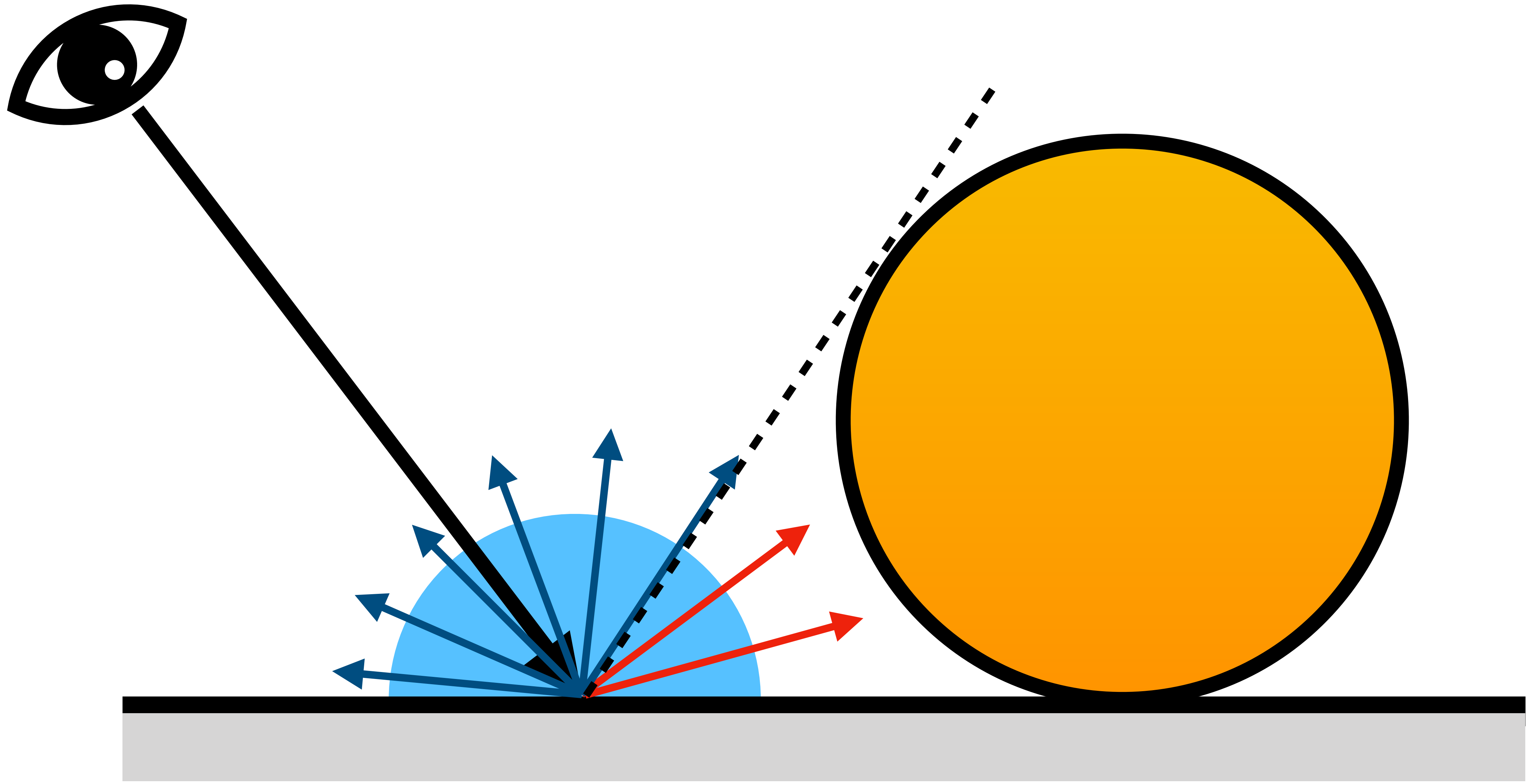
$$p(x, \omega_i) = \frac{1}{2\pi}$$

(256 Samples)



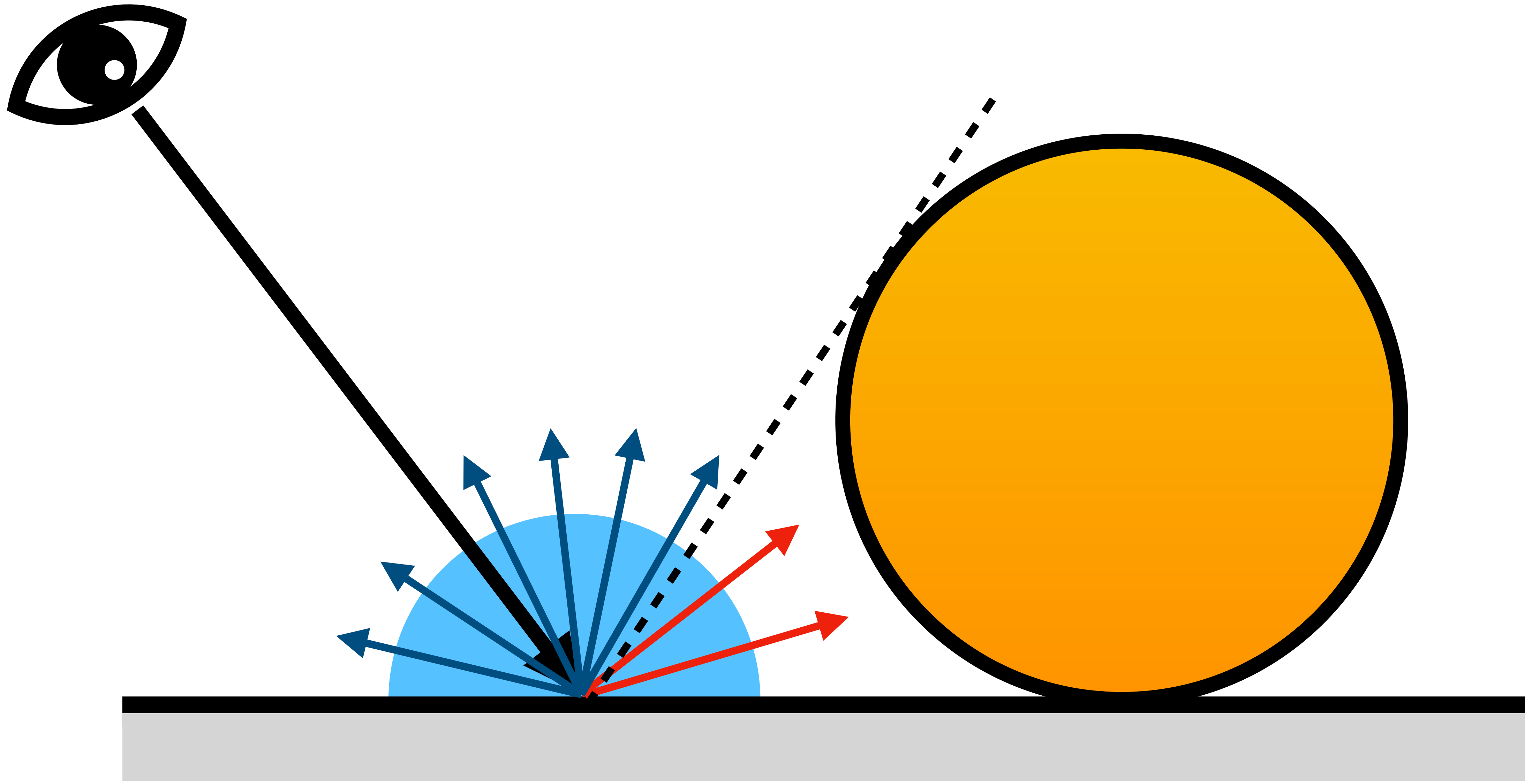
Importance Sampling: Cosine term

Uniform Hemispherical Sampling



$$p(x, \omega_i) = \frac{1}{2\pi}$$

Cosine-weighted Importance Sampling



$$p(x, \omega_i) = \cos\theta_i$$

Uniform hemispherical sampling 1 sample/pixel **Cosine-weighted importance sampling**

Slide from Wojciech Jarosz

Uniform hemispherical sampling

4 sample/pixel

Cosine-weighted importance sampling

Slide from Wojciech Jarosz

Uniform hemispherical sampling

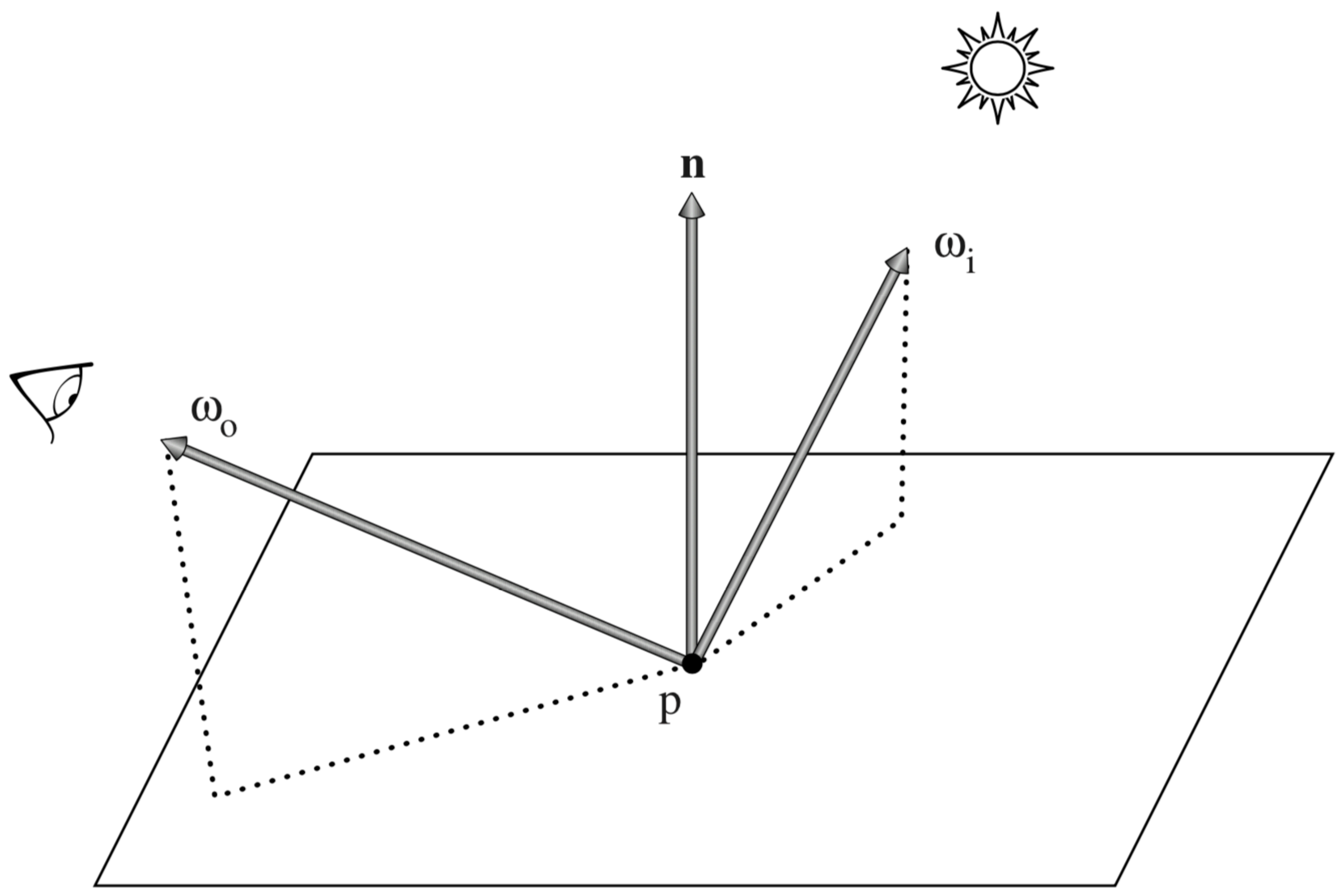
16 sample/pixel

Cosine-weighted importance sampling

Slide from Wojciech Jarosz

Importance Sampling: Incident Radiance

$$L_o(p, \omega) = \int_{\mathcal{H}^2} f(p, \omega_0, \omega_i) L_i(x, \omega_i) |\cos\theta_i| d\omega_i$$



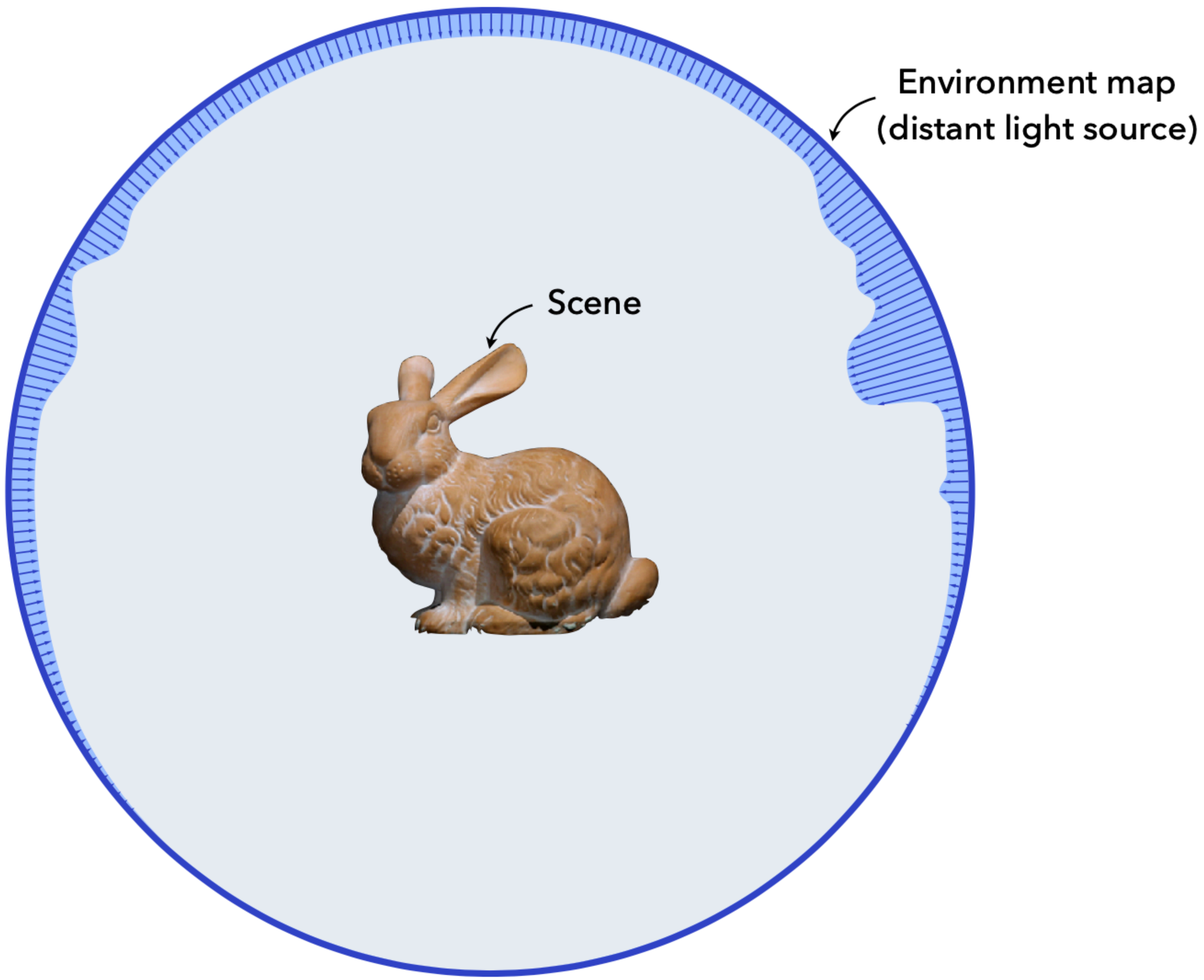
What terms can we importance sample?

- BSDF
- Incident radiance
- cosine term

Example: Environment Lighting

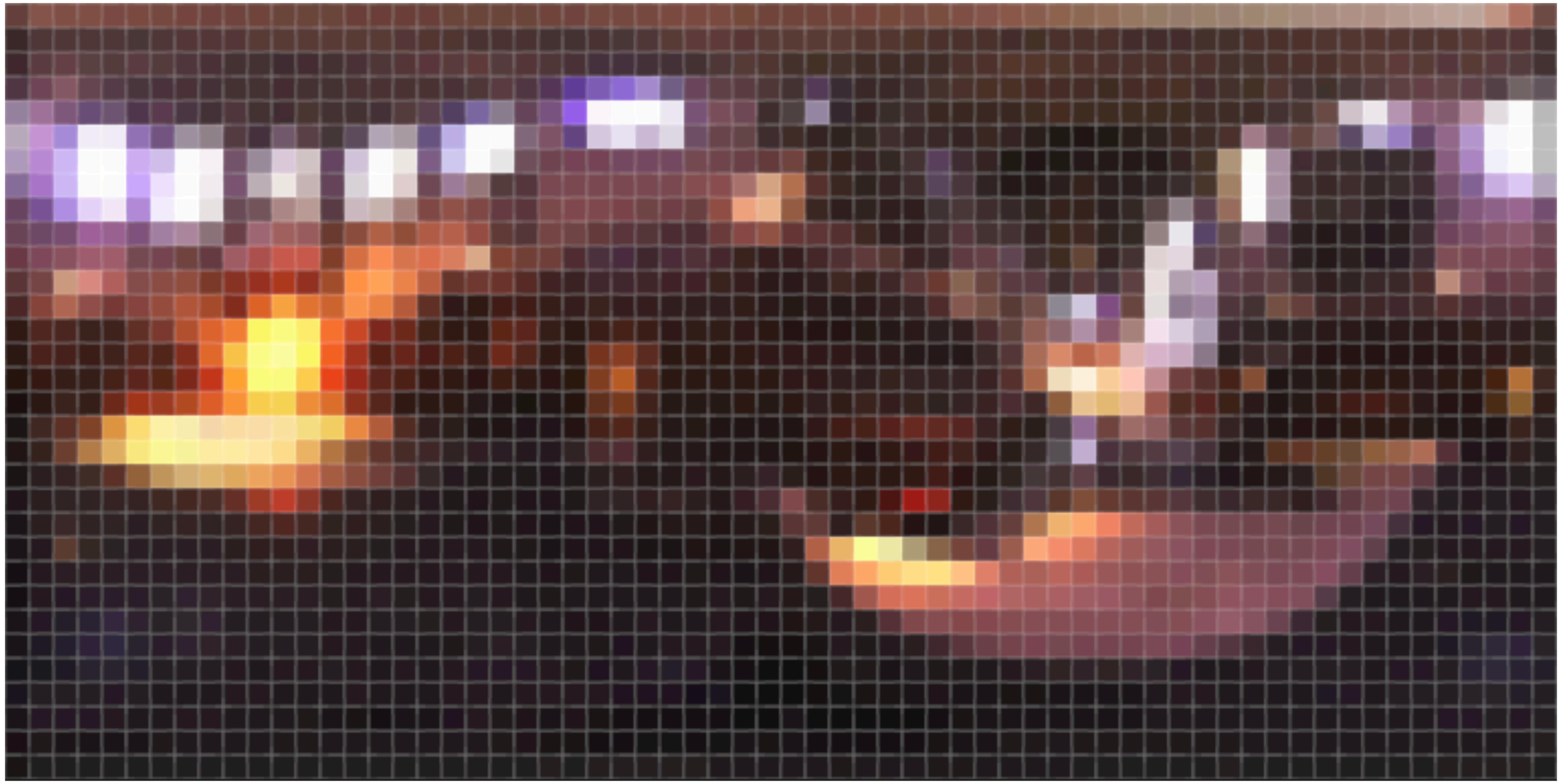
Example: Environment Lighting

Environment Lighting



Slide after Wojciech Jarosz

Importance function



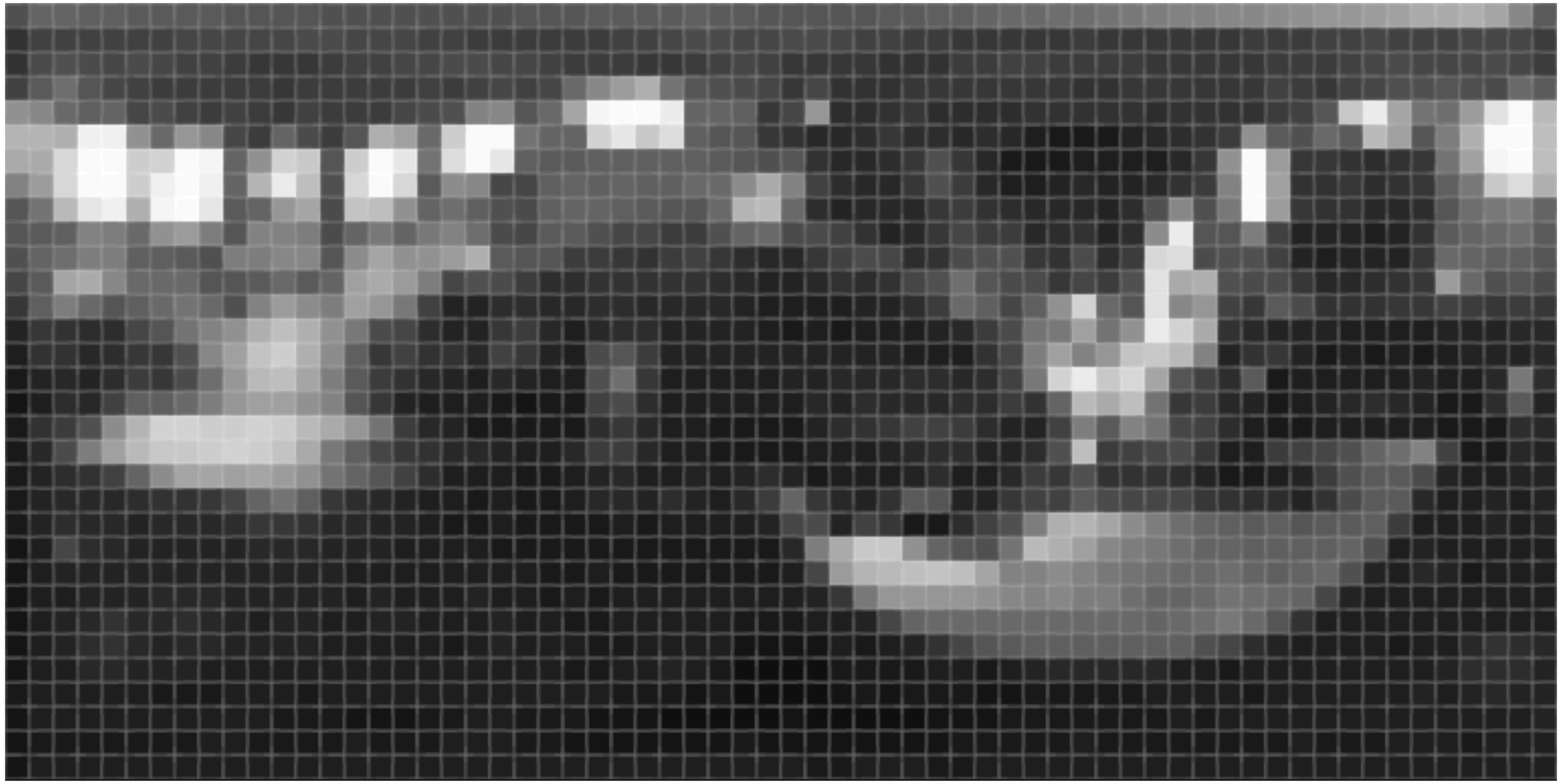
θ

ϕ

Slide after Wojciech Jarosz

Importance function

Scalar version e.g., luminance channel only



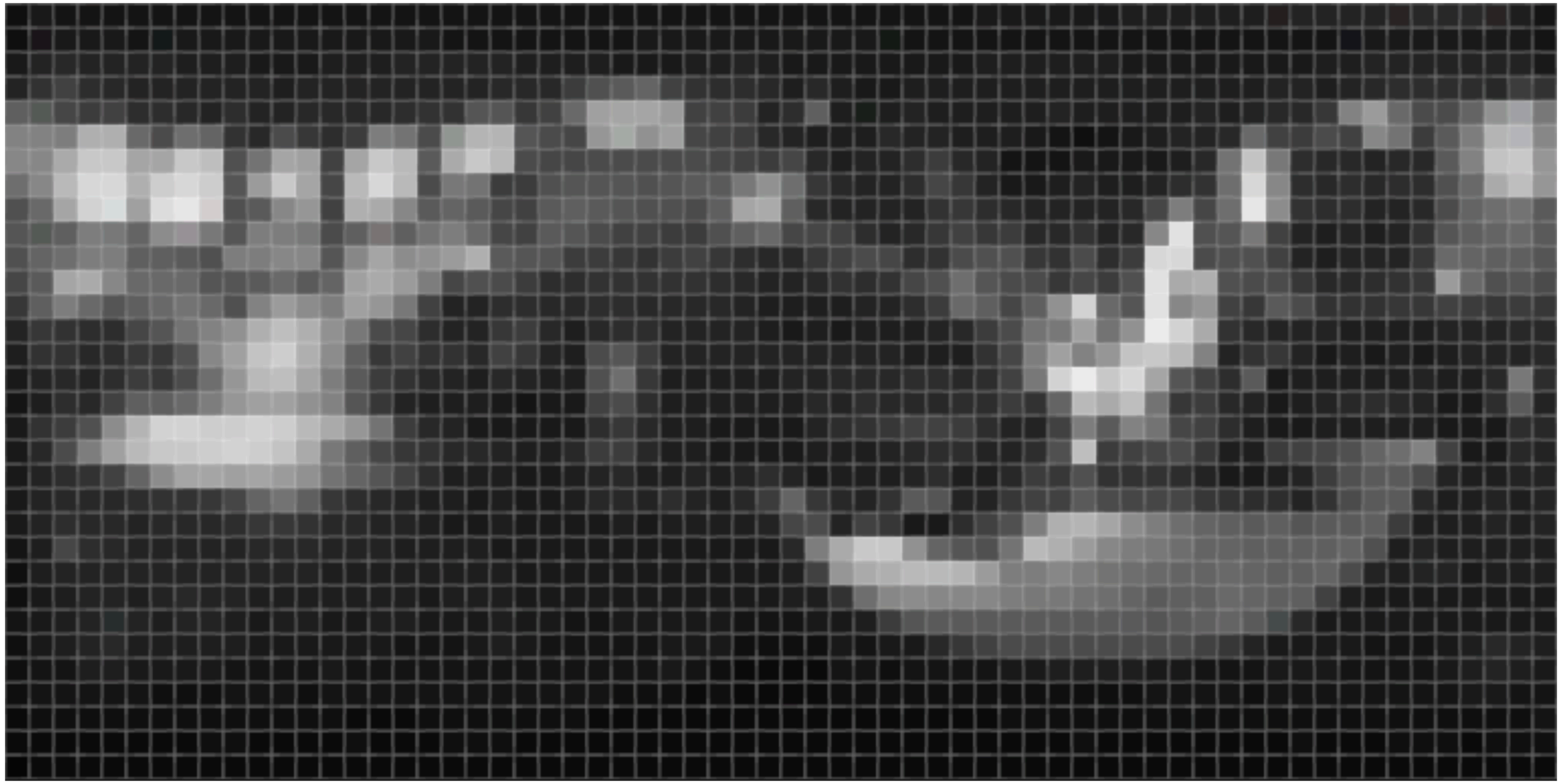
θ

ϕ

Slide after Wojciech Jarosz

Importance function: Scalar function

Multiplication with $\sin\theta$

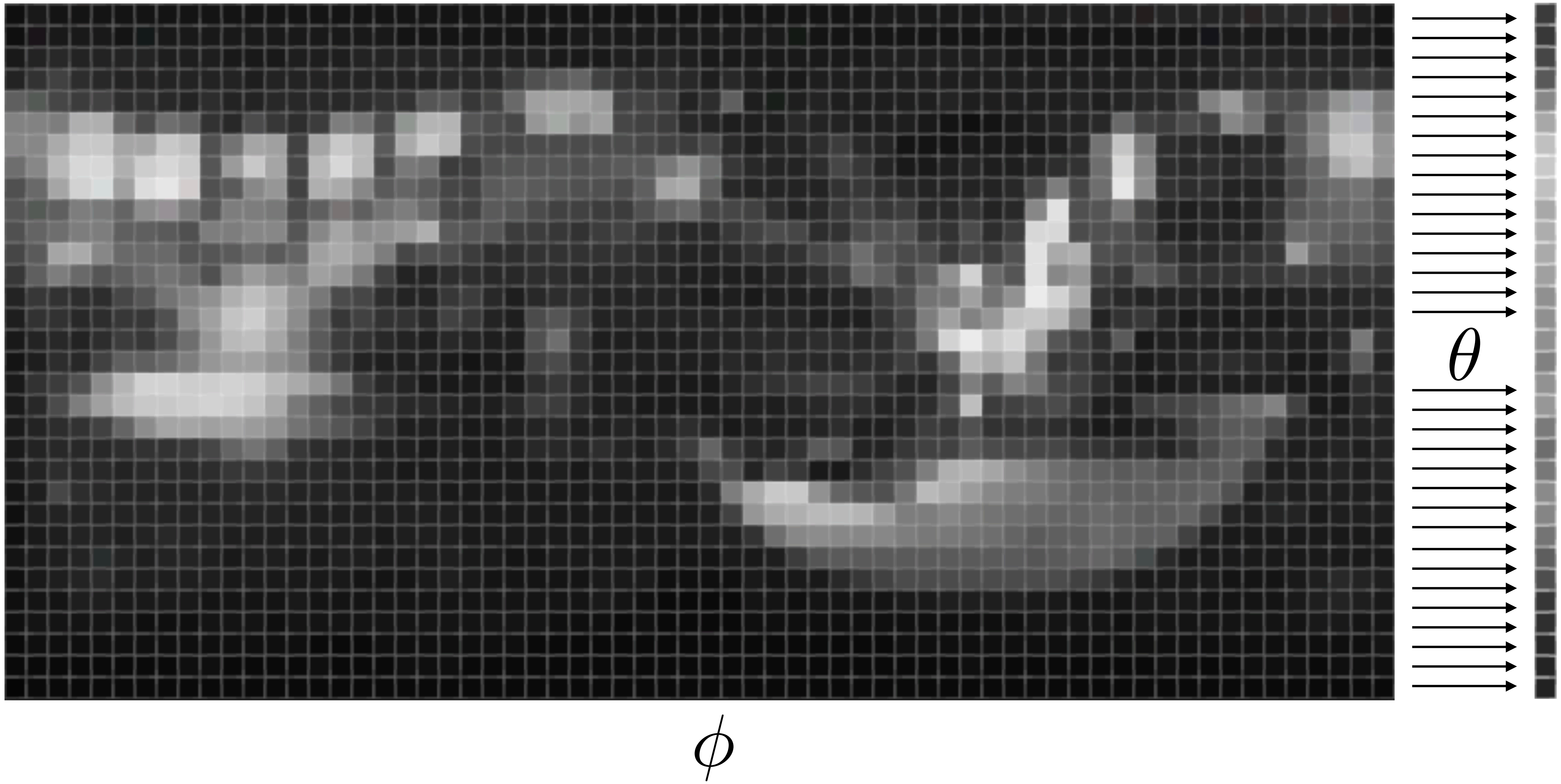


θ

ϕ

Slide after Wojciech Jarosz

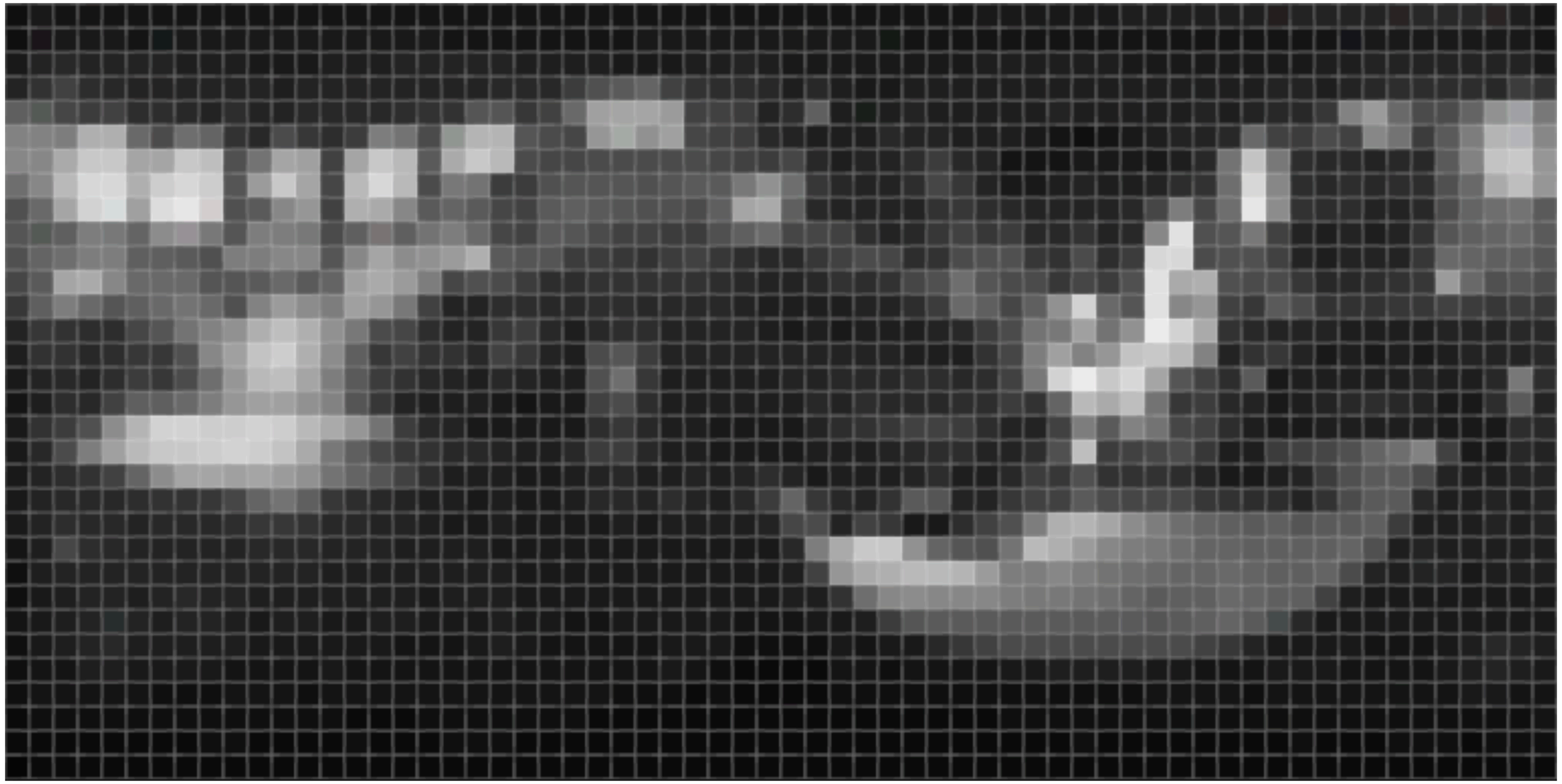
Importance function: Marginalization



Slide after Wojciech Jarosz

Importance function: Conditional PDFs

Once normalized, each row can serve as the conditional PDF

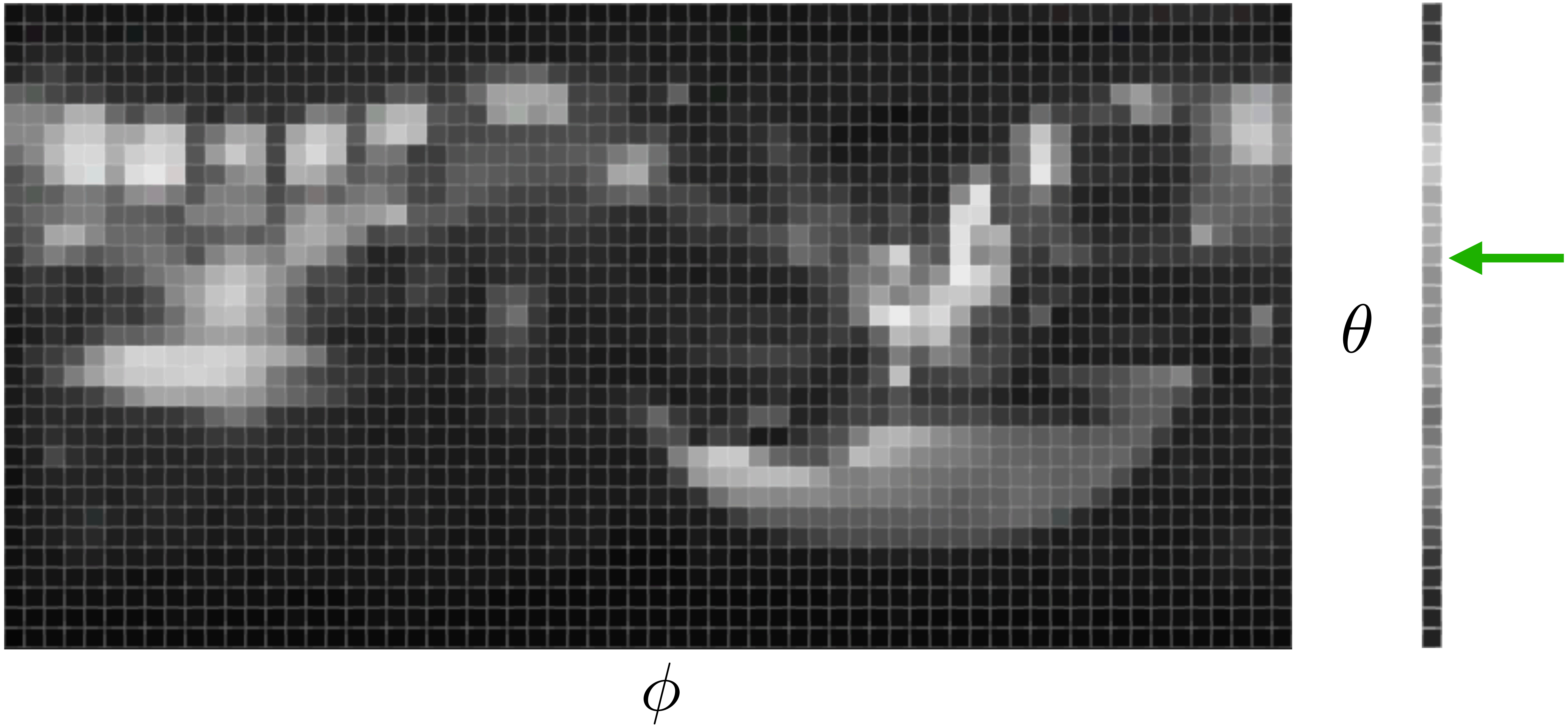


θ

ϕ

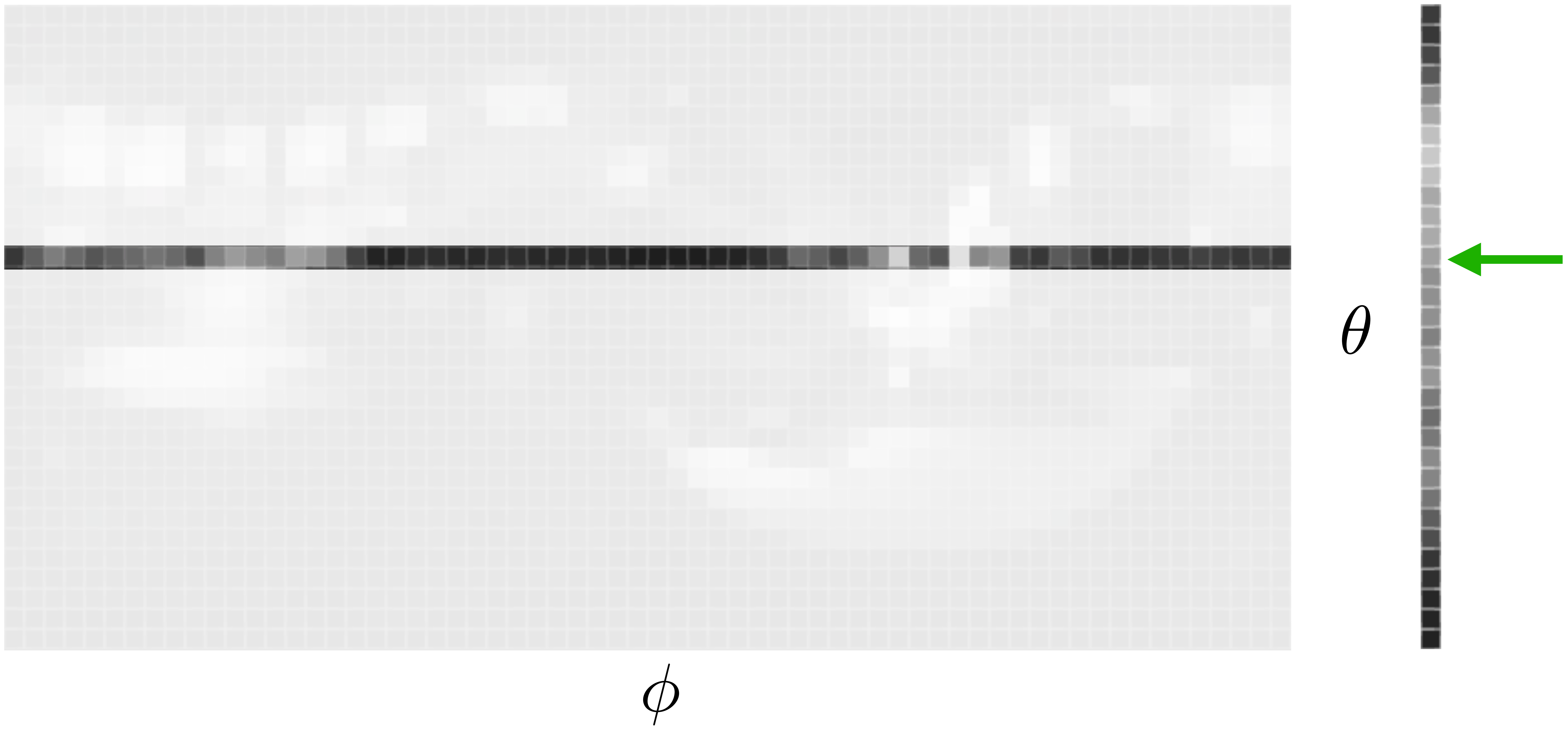
Slide after Wojciech Jarosz

Importance function: Sampling



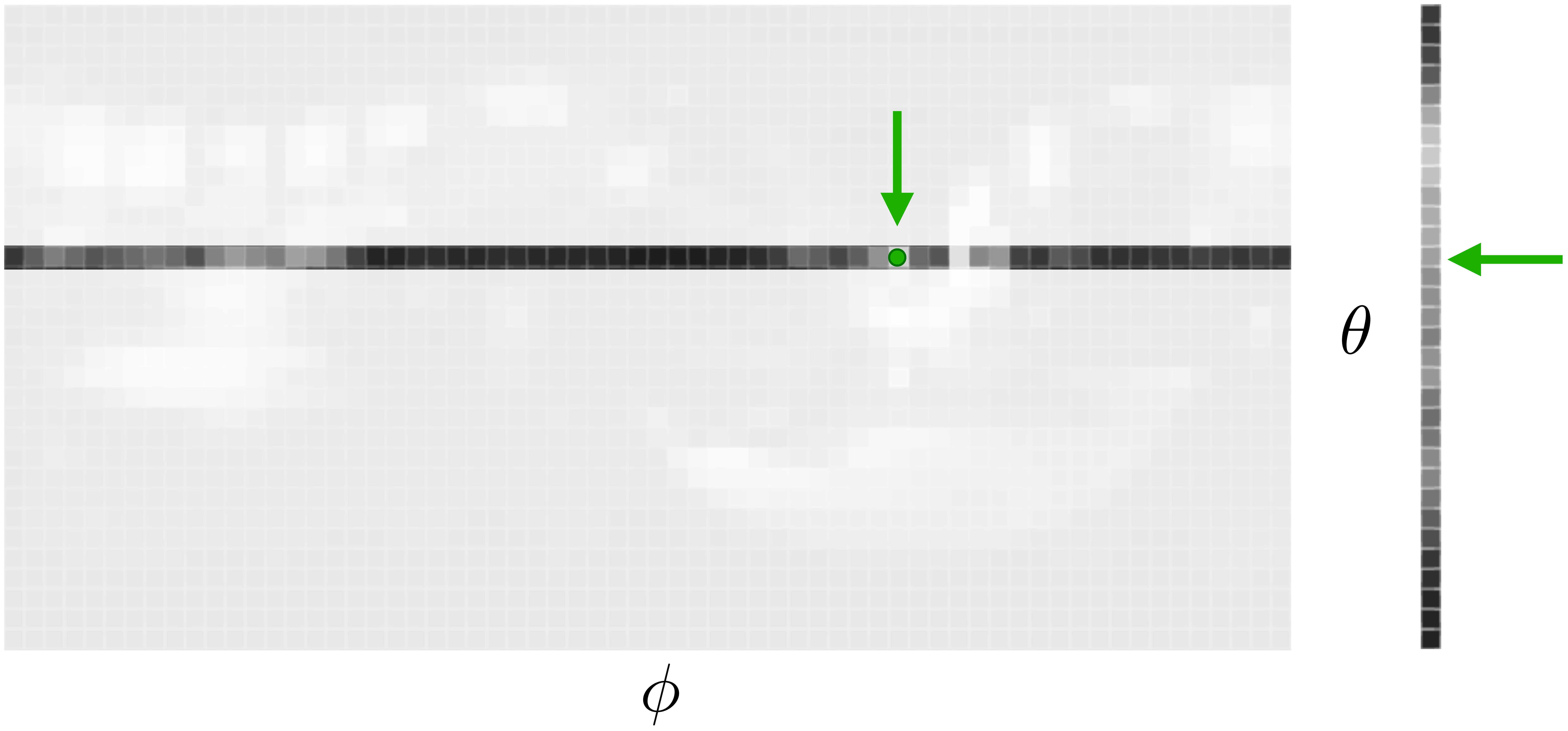
Slide after Wojciech Jarosz

Importance function: Sampling



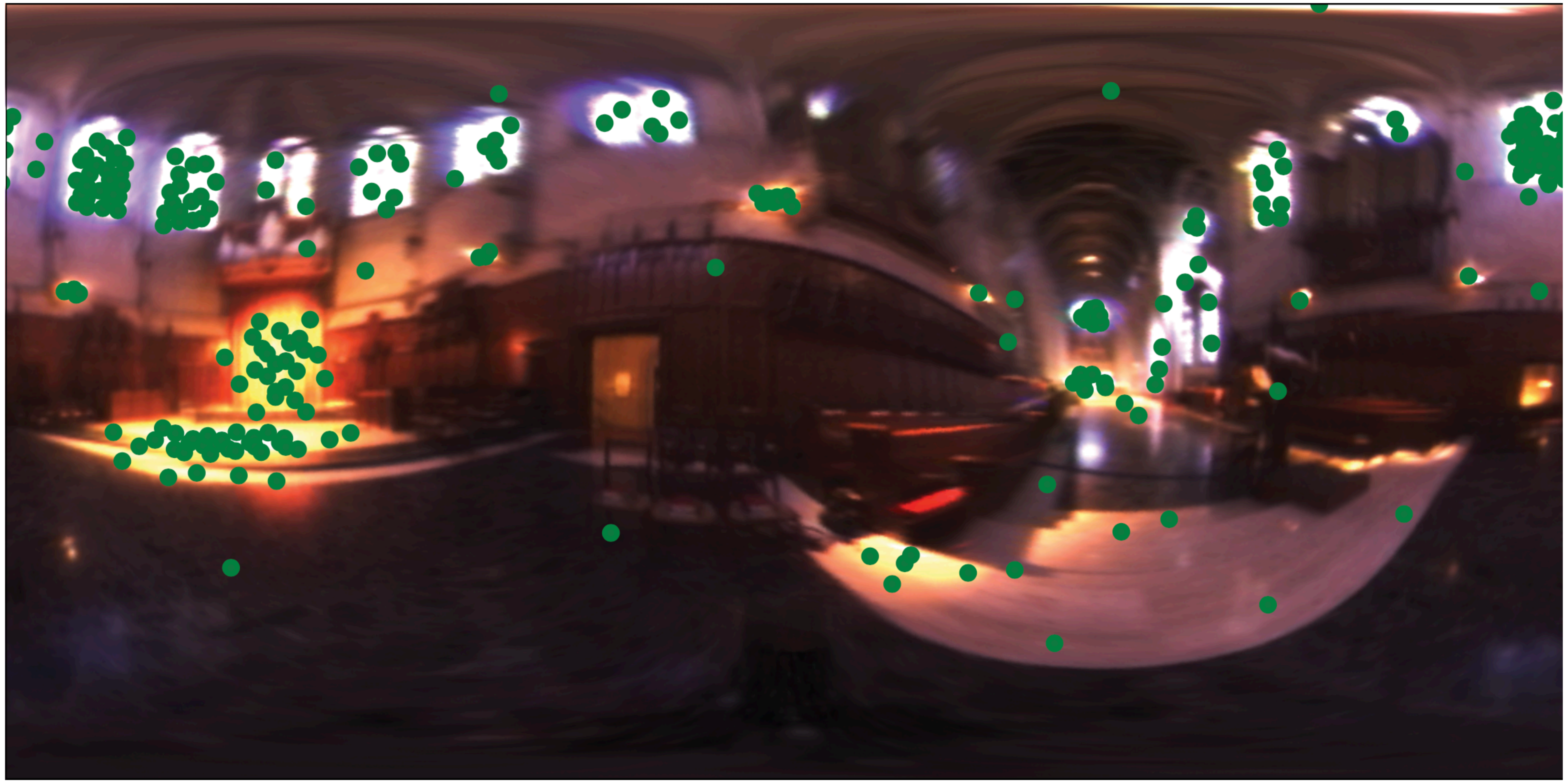
Slide after Wojciech Jarosz

Importance function: Sampling



Slide after Wojciech Jarosz

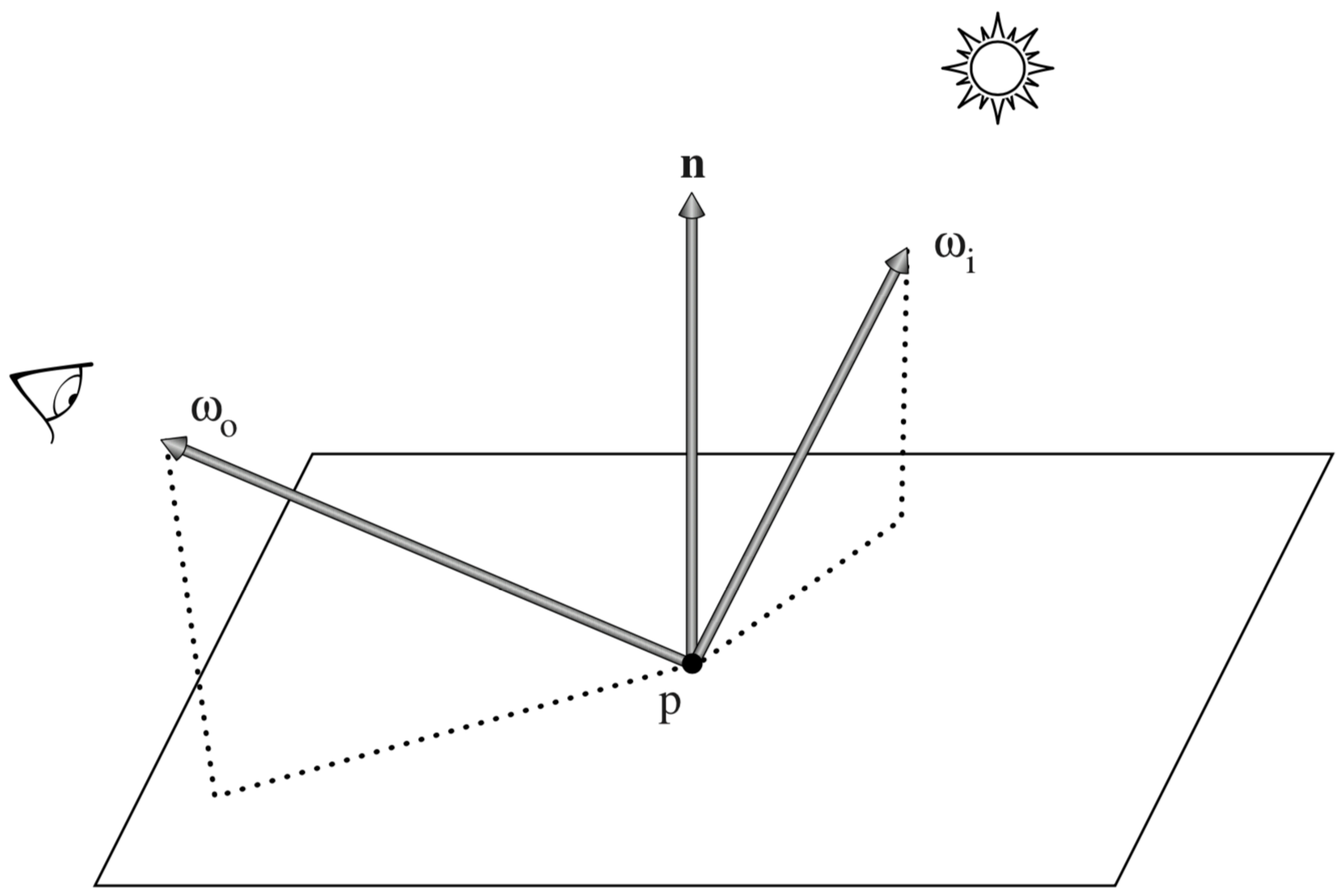
Importance Sampling



For more details, see PBRTv3: 13.2 and 13.6.7

Importance Sampling

$$L_o(p,\omega) = \int_{\mathcal{H}^2} f(p,\omega_0,\omega_i) L_i(x,\omega_i) |\cos\theta_i| d\omega_i$$



What terms can we importance sample?

- BSDF
- Incident radiance
- cosine term

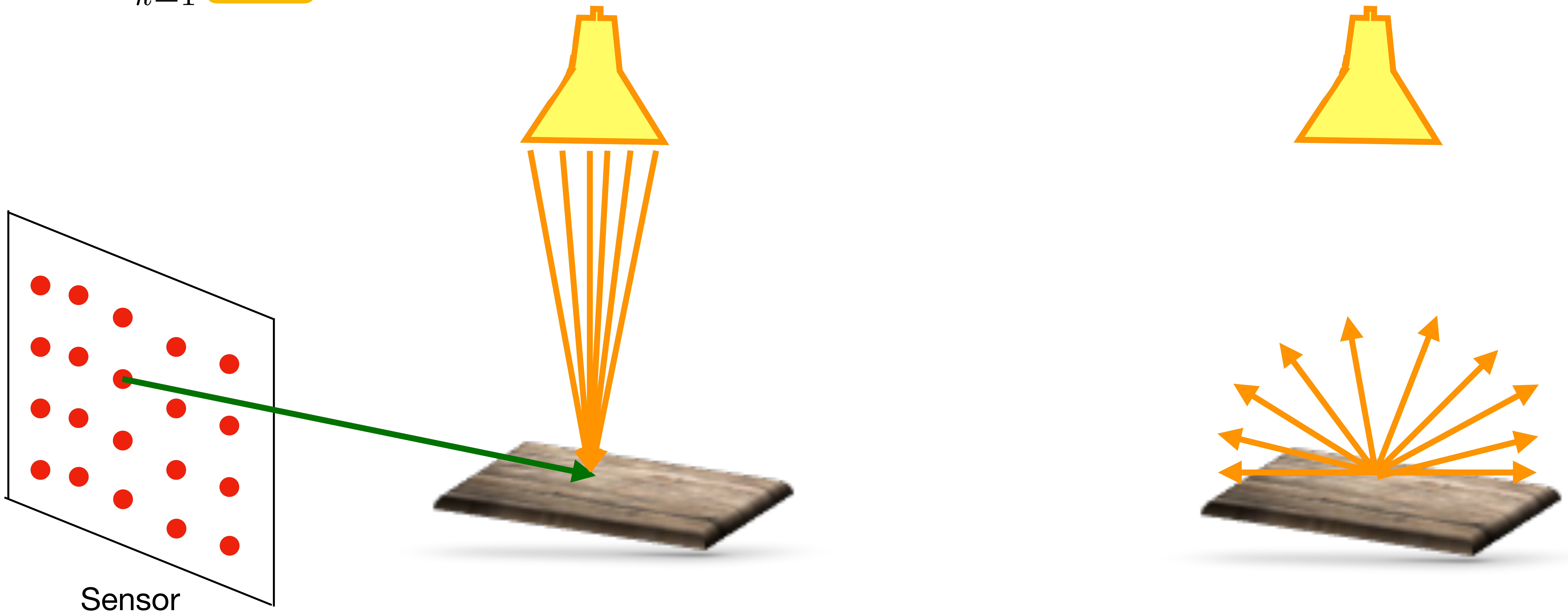
To handle this, we will introduce Microfacet BSDF theory in the later part of the lecture.

Light vs. BSDF Importance Sampling

$$I_N = \frac{1}{N} \sum_{k=1}^{N} \frac{f(\vec{x}_k)}{p(\vec{x}_k)}$$

Light PDF Sampling

BSDF PDF Sampling

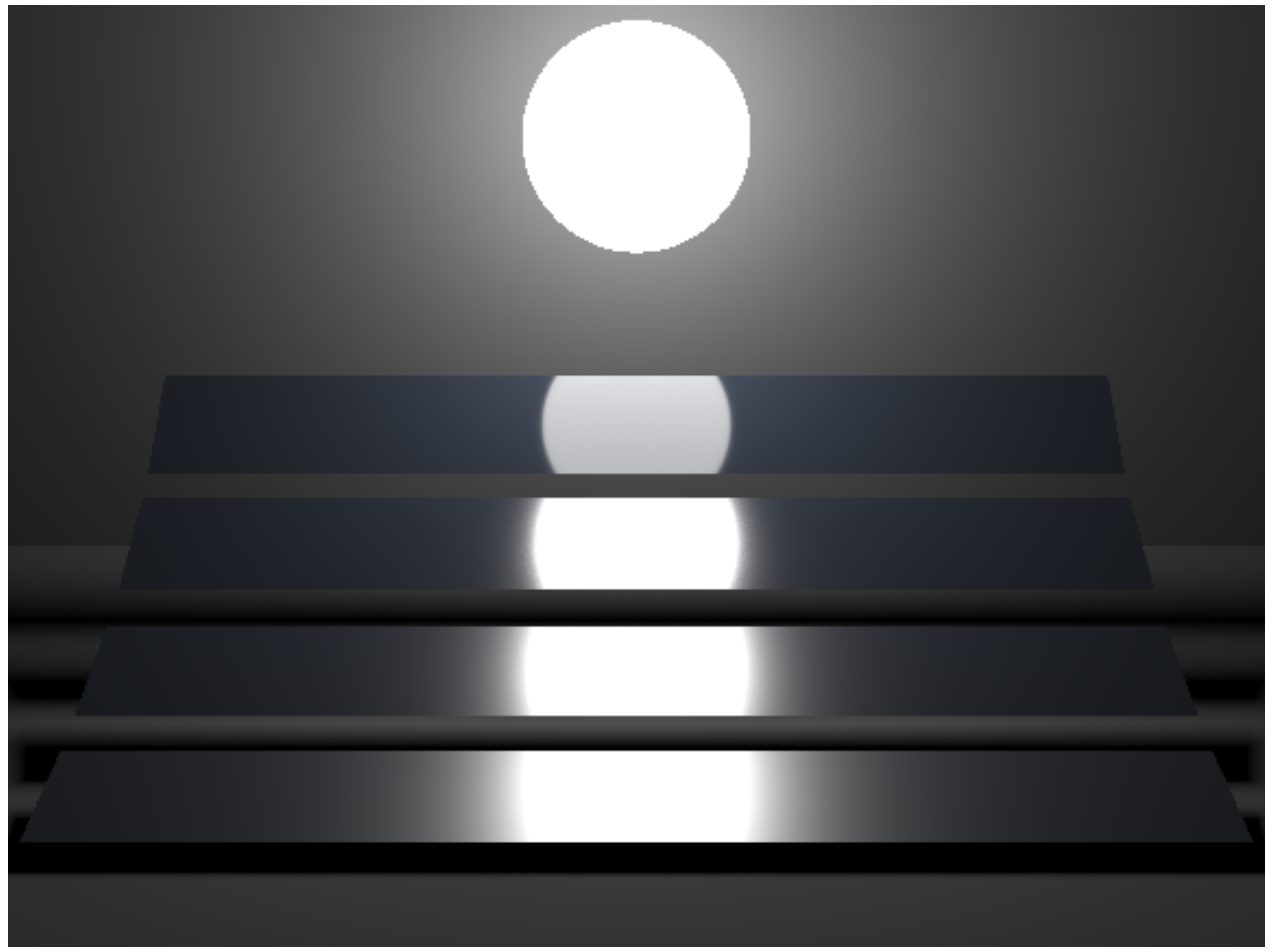


Sensor

Light IS

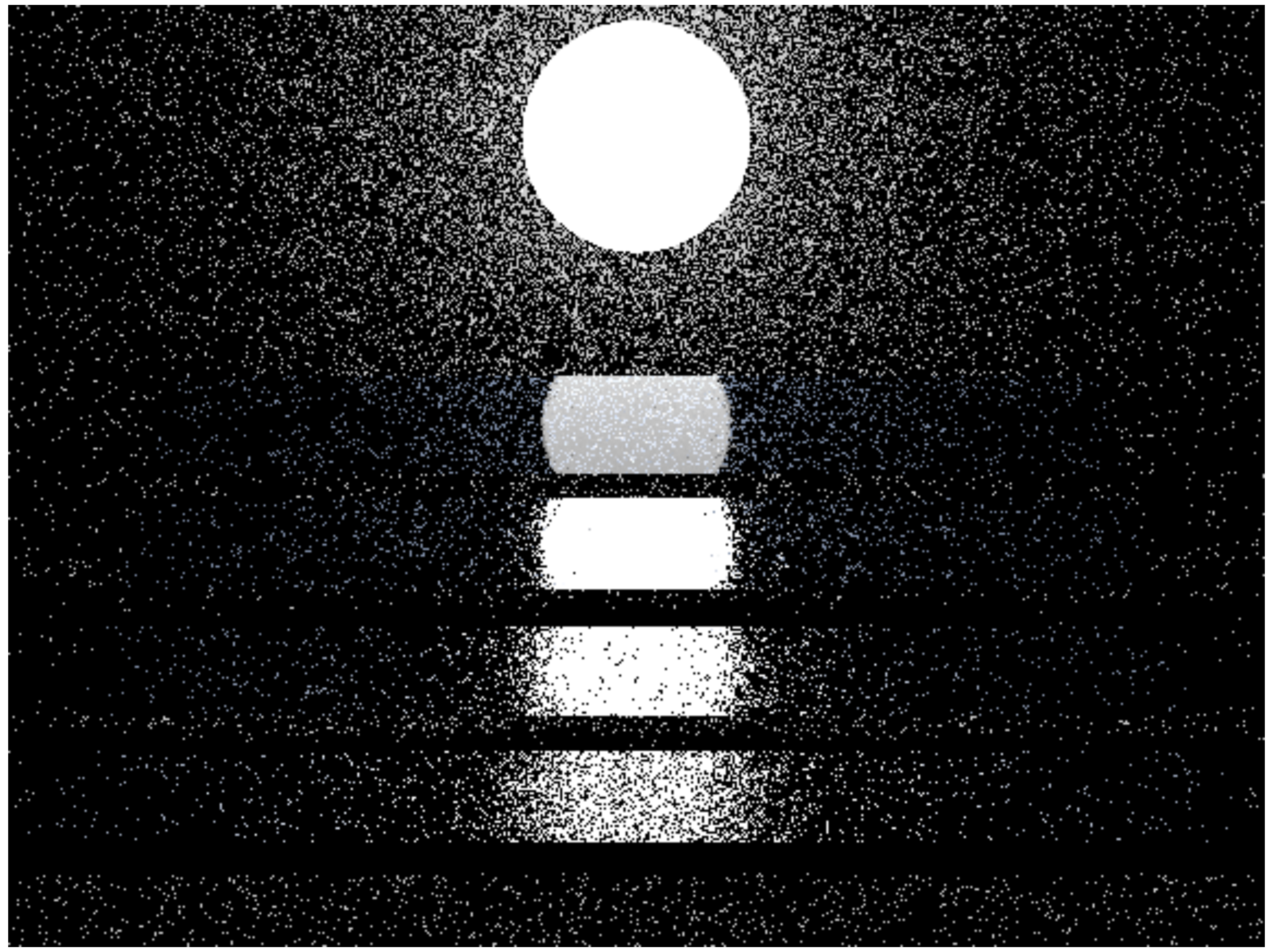
BSDF IS

Variance reduction: Importance sampling



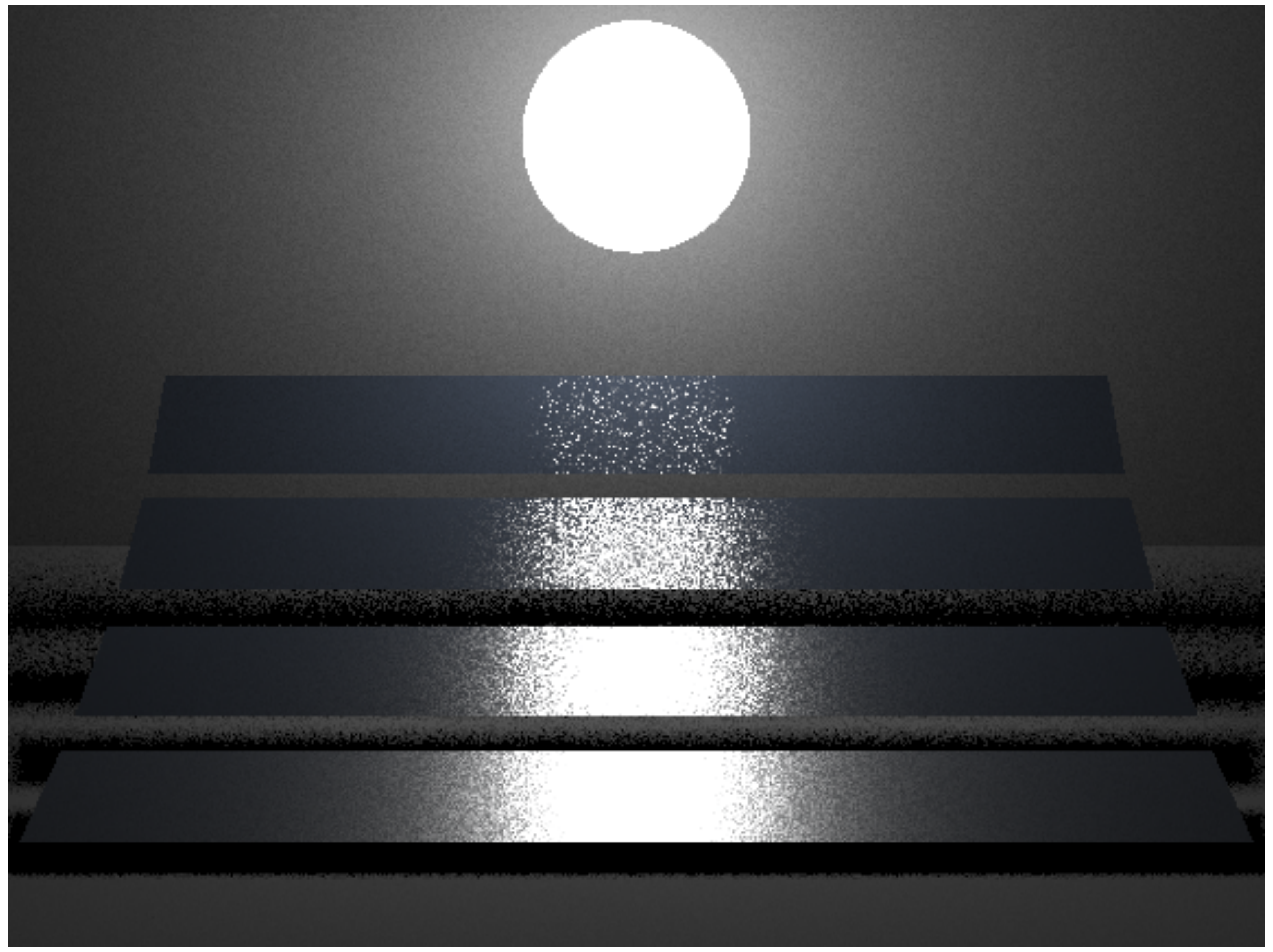
Reference image

N = 1024 spp



BSDF importance sampling

N = 4 spp



Light importance sampling

N = 4 spp

Variance reduction: Importance sampling

Reference image

N = 1024 spp

BSDF importance sampling

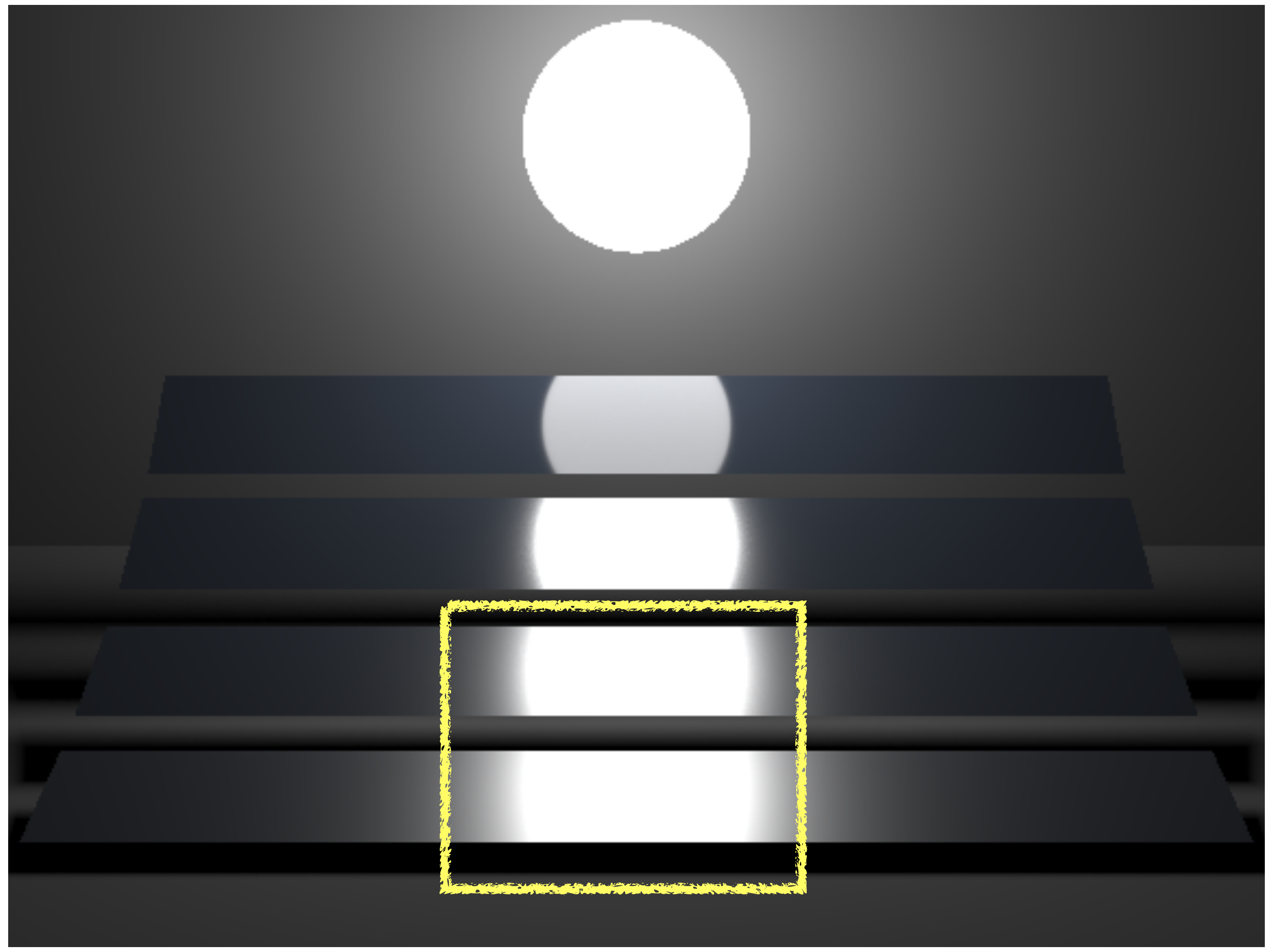
N = 4 spp

Light importance sampling

N = 4 spp

BSDF sampling is better in some regions

Variance reduction: Importance sampling



Reference image

N = 1024 spp

BSDF importance sampling

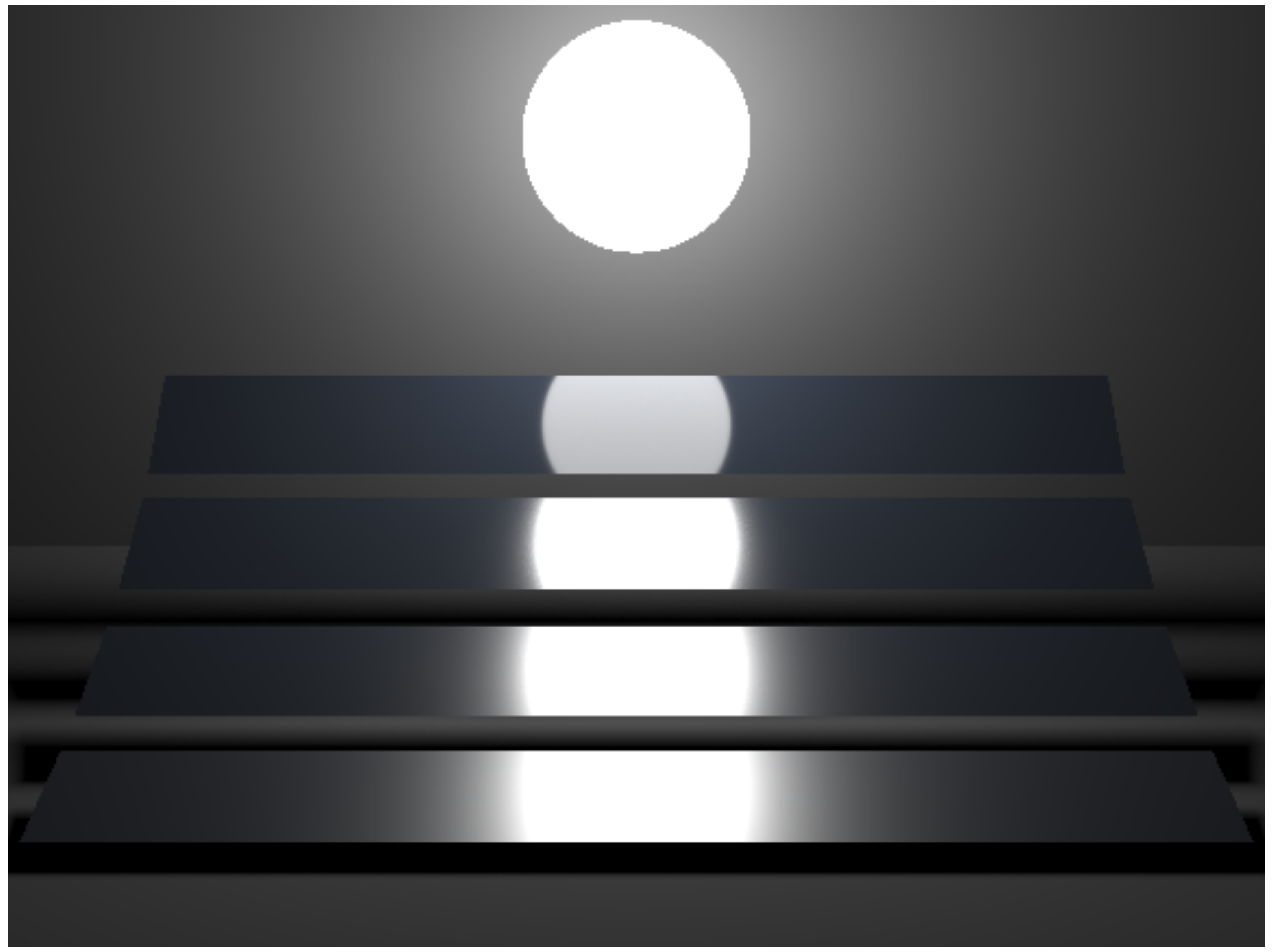
N = 4 spp

Light importance sampling

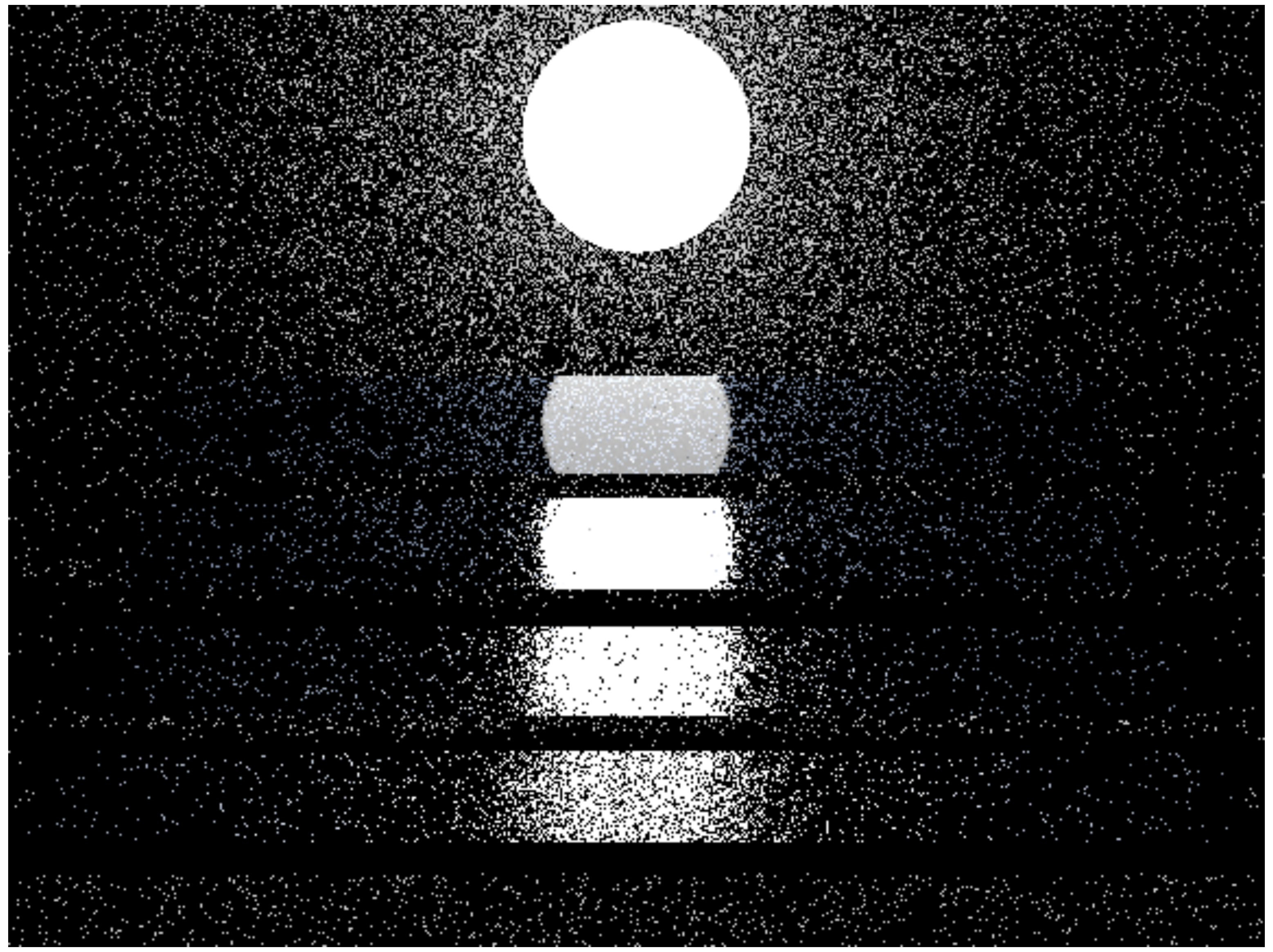
N = 4 spp

Light sampling is better in other regions

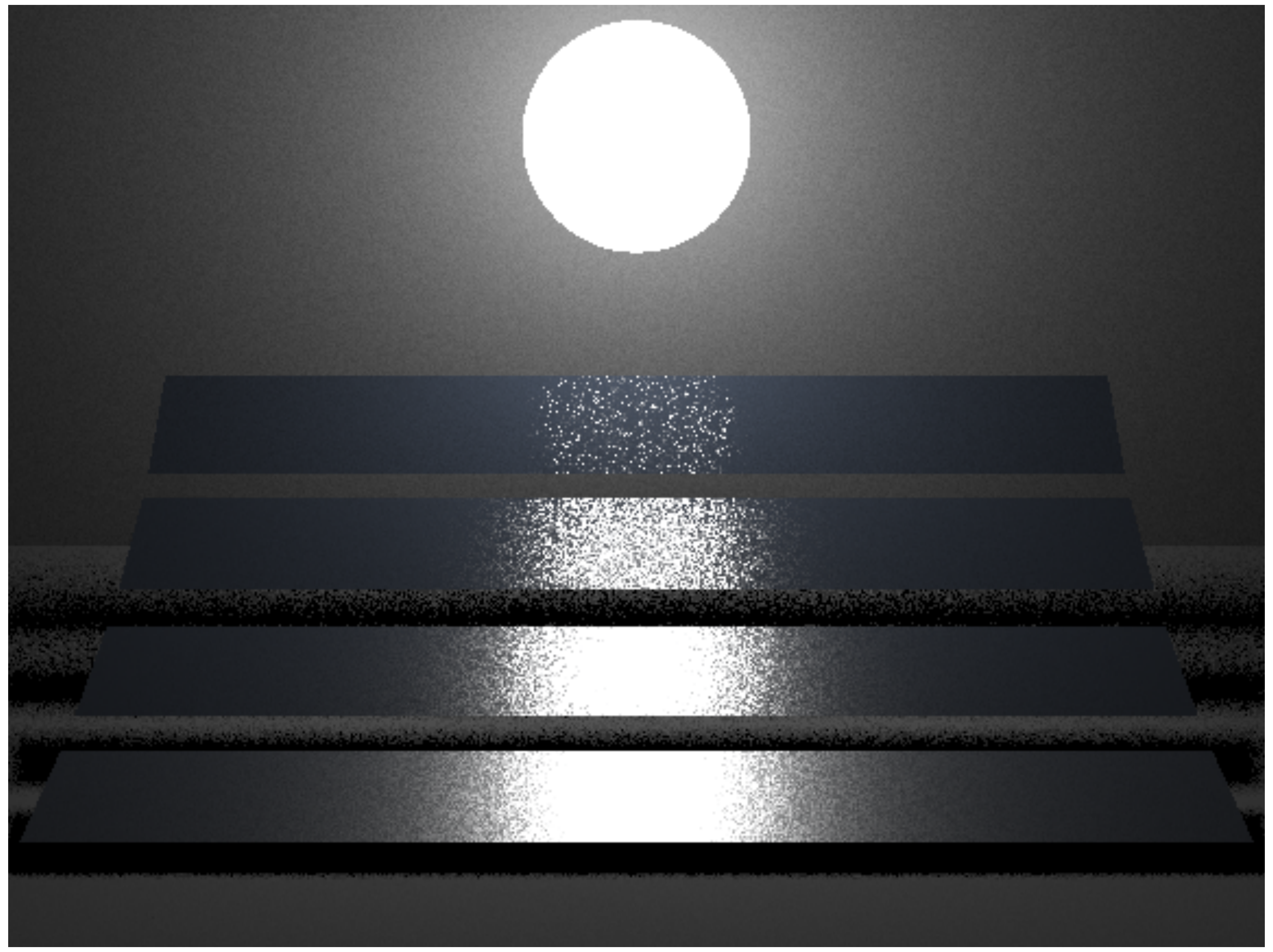
Variance reduction: Importance sampling



Reference image



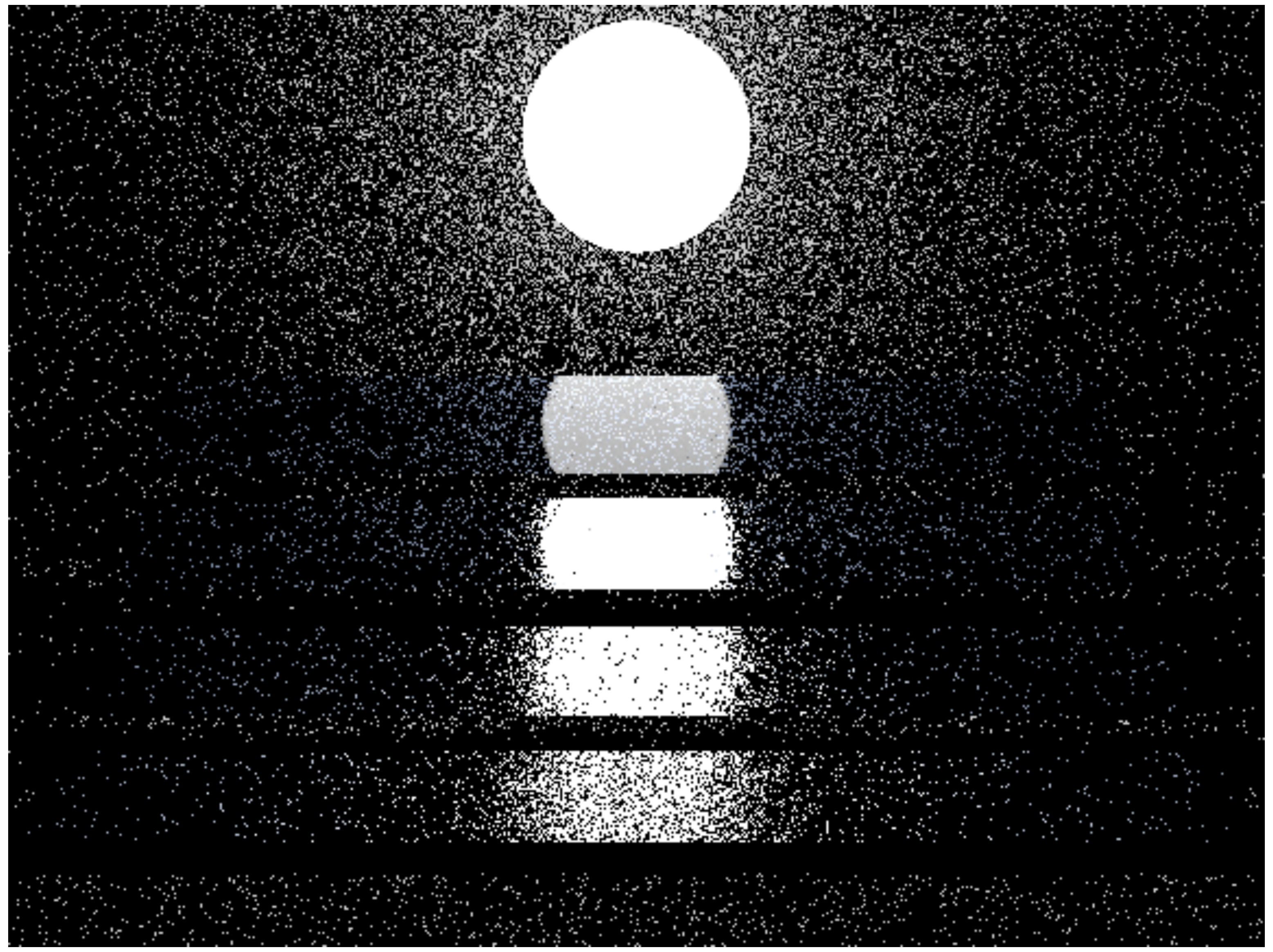
BSDF importance sampling



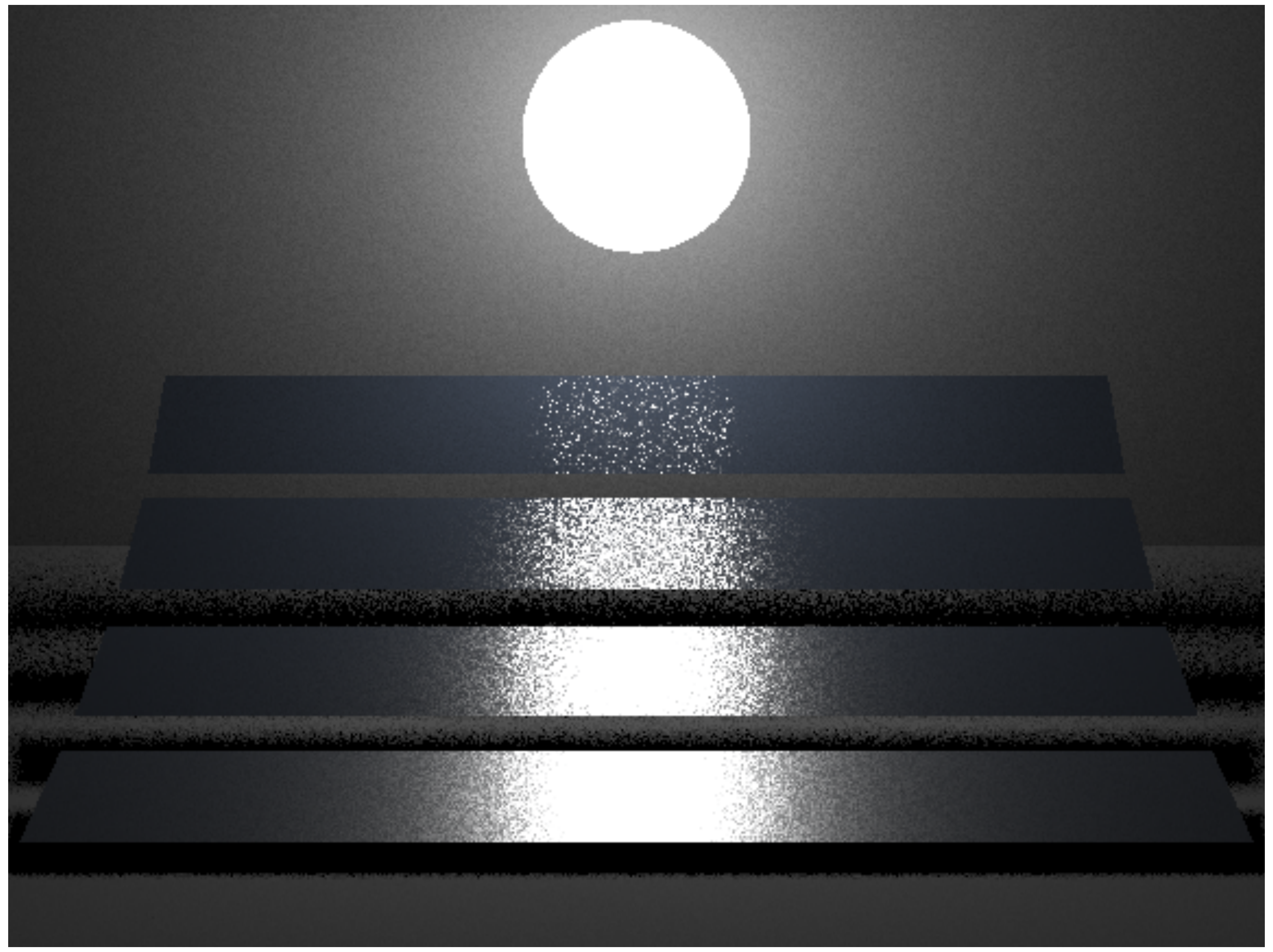
Light importance sampling

Can we combine the benefits of different PDFs ? **Yes!**

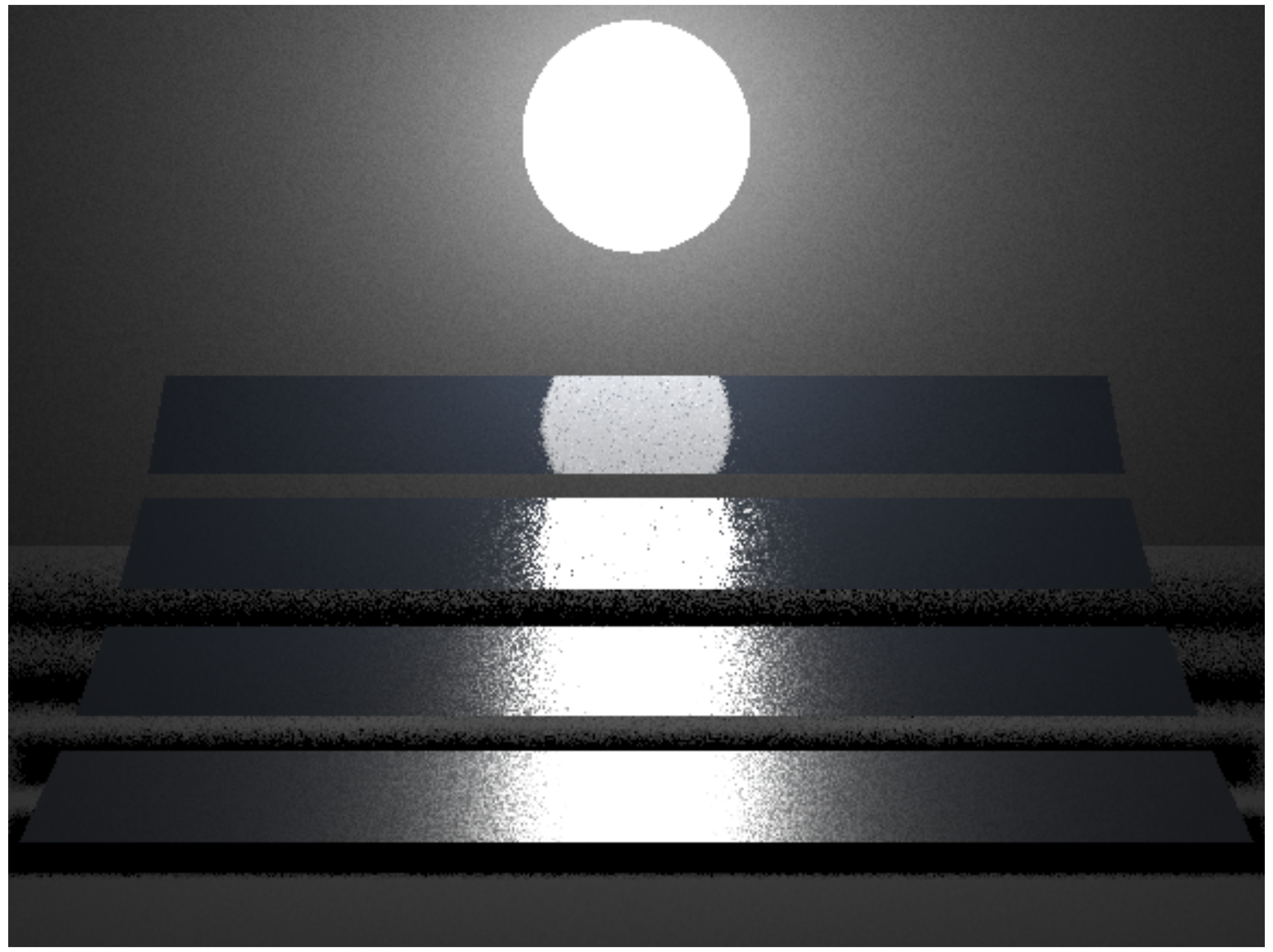
Variance reduction: Importance sampling



BSDF importance sampling



Light importance sampling



Multiple Importance Sampling

Can we combine the benefits of different PDFs ? **Yes!**

Variance reduction: Multiple Importance sampling

Multiple Importance Sampling

$$I_N = \frac{1}{N} \sum_{i=1}^{N} \frac{f(x)g(x)}{p(x)}$$

$$p(x) \propto \text{???}$$

$$\mathbf{I}_N = \frac{1}{n_f} \sum_{i=1}^{n_f} \frac{f(X_i)g(X_i)w_f(X_i)}{p_f(X_i)} + \frac{1}{n_g} \sum_{j=1}^{n_g} \frac{f(Y_j)g(Y_j)w_g(Y_j)}{p_g(Y_j)}$$

Variance reduction: Multiple Importance sampling

Multiple Importance Sampling

$$\mathbf{I}_N = \frac{1}{n_f} \sum_{i=1}^{n_f} \frac{f(X_i) g(X_i) w_f(X_i)}{p_f(X_i)} + \frac{1}{n_g} \sum_{j=1}^{n_g} \frac{f(Y_j) g(Y_j) w_g(Y_j)}{p_g(Y_j)}$$

Balance heuristic: $$w_s(x) = \frac{n_s p_s(x)}{\sum_i n_i p_i(x)}$$

Power heuristic: $$w_s(x) = \frac{(n_s p_s(x))^\beta}{\sum_i (n_i p_i(x))^\beta}$$ $\beta = 2$

Rendering Equation

Rendering Equation

$$L_o(x, \omega_o) = L_e(x, \omega_o) + L_r(x, \omega_o)$$

Outgoing emitted reflected

James Kajiya, The Rendering Equation, SIGGRAPH 1986

Rendering Equation

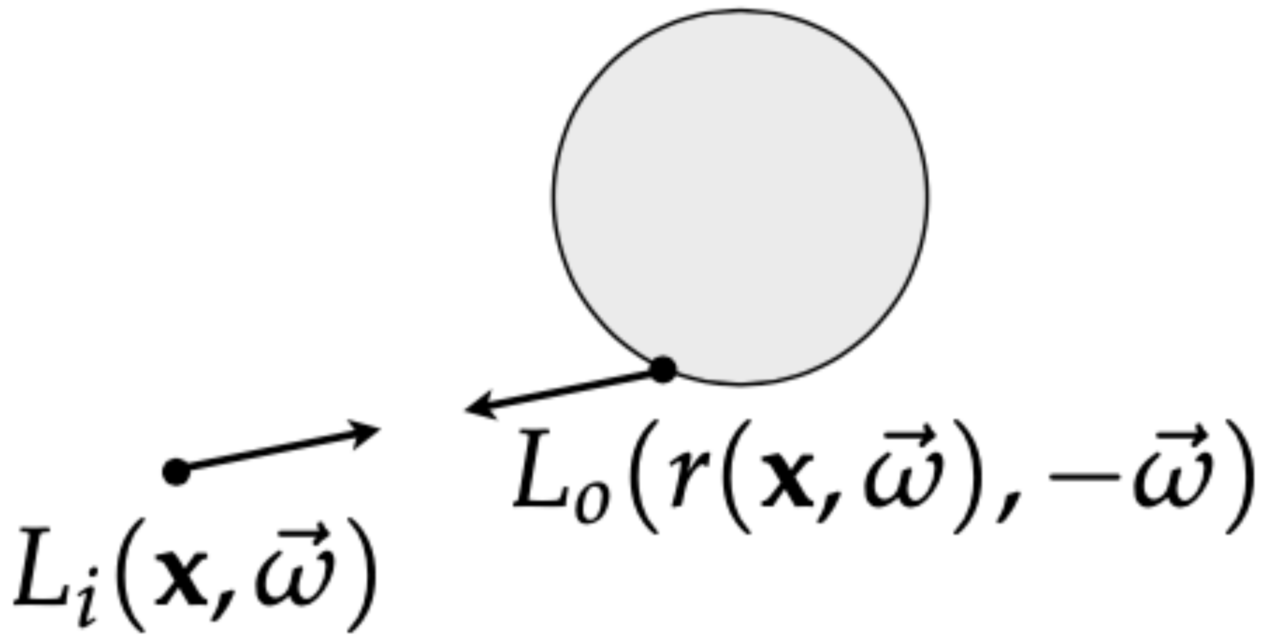
$$L_o(x,\omega_o) = L_e(x,\omega_o) + \int_{\mathcal{H}^2} f_r(x,\omega_0,\omega_i) L_i(x,\omega_i) |\cos\theta_i| d\omega_i$$

Outgoing emitted reflected

Rendering Equation: Light Transport

In vaccum, radiance is constant along rays

We can relate out-going radiance to the incoming radiance



Rendering Equation: Light Transport

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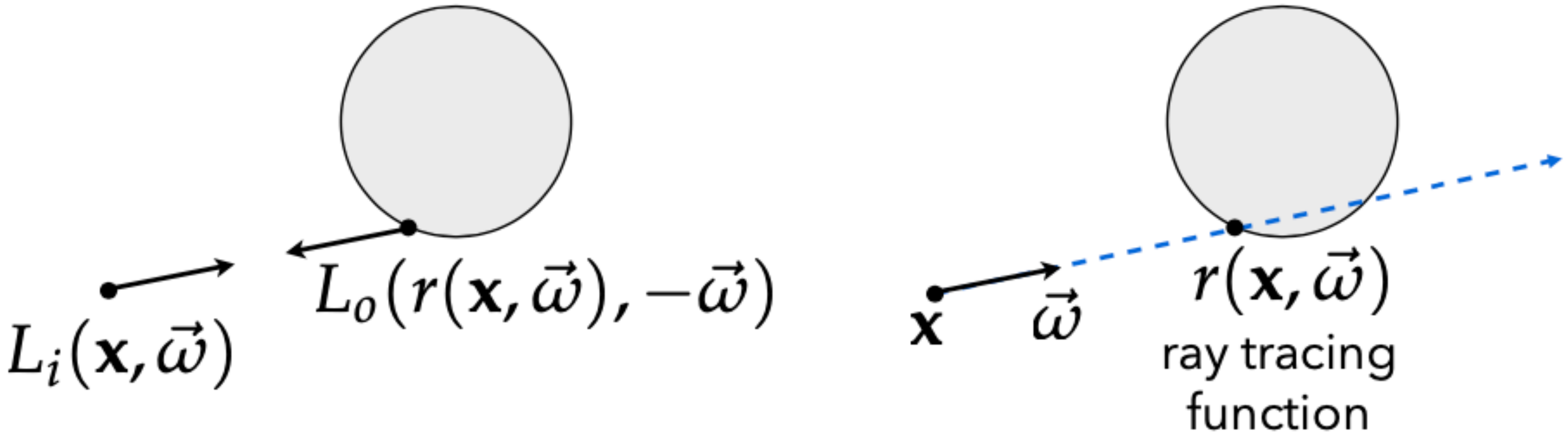


Rendering Equation

$$L_o(x,\omega) = L_e(x,\omega) + \int_{\mathcal{H}^2} f(p,\omega',\omega) L_i(x,\omega) |\cos\theta'| d\omega'$$

Rendering Equation

ray tracing function

$$L(x,\omega) = L_e(x,\omega) + \int_{\mathcal{H}^2} f(p,\omega',\omega) L(r(x,\omega), -\omega') |\cos\theta'| d\omega'$$

Only outgoing radiance on both sides

- we drop the "o" subscript

- Becomes Fredholm equation of the second kind (recursive)

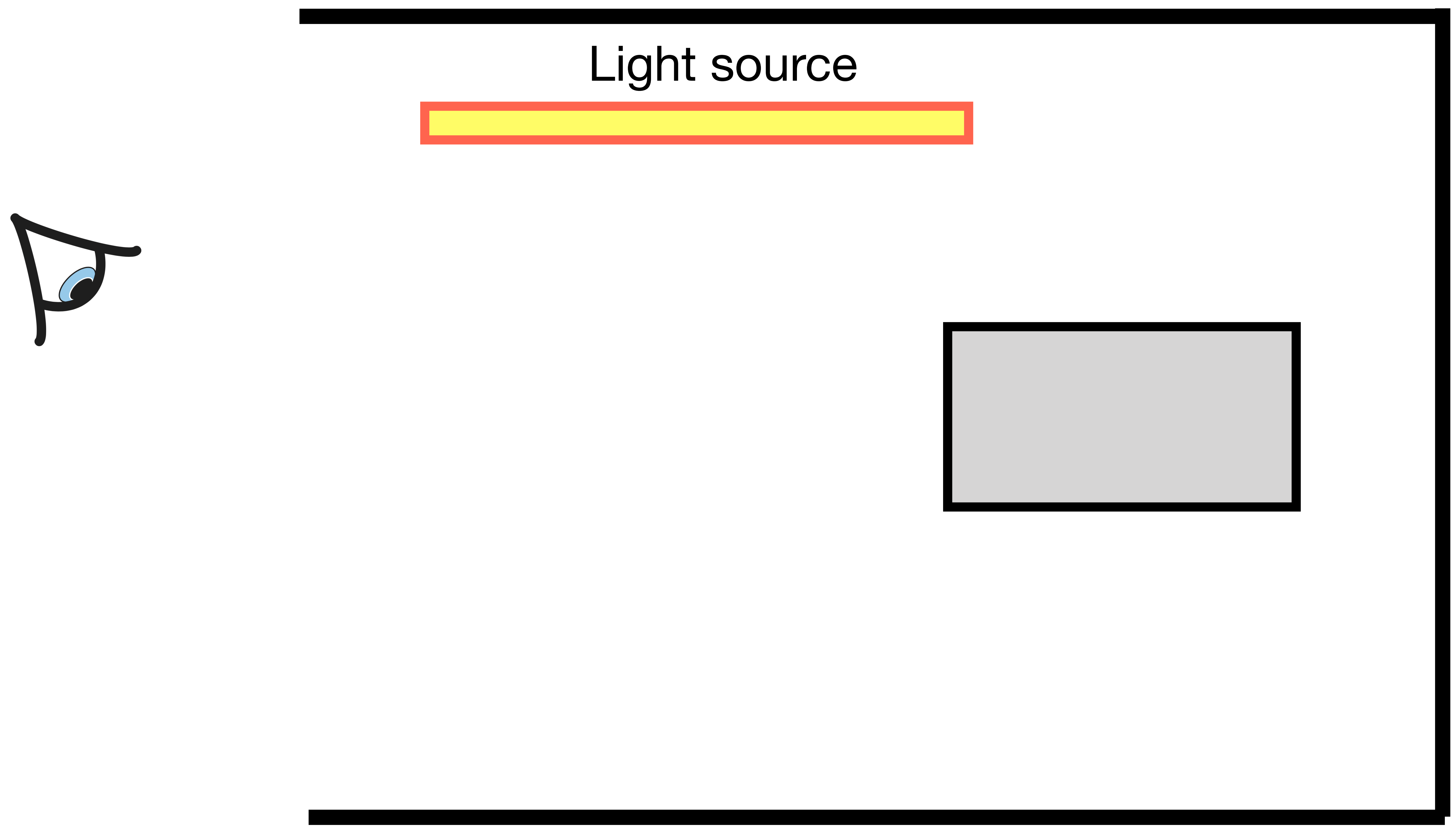
Rendering Equation

$$L(x, \vec{\omega}) = L_e(x, \vec{\omega}) + \int_{\mathcal{H}^2} f(x, \vec{\omega}', \vec{\omega}) L(r(x, \vec{\omega}'), -\vec{\omega}') |\cos\theta'| d\vec{\omega}'$$

Rendering Equation

ray tracing function

$$L(x,\vec{\omega}) = L_e(x,\vec{\omega}) + \int_{\mathcal{H}^2} f(x,\vec{\omega}',\vec{\omega})L(r(x,\vec{\omega}'),-\vec{\omega}')|\cos\theta'|d\vec{\omega}'$$



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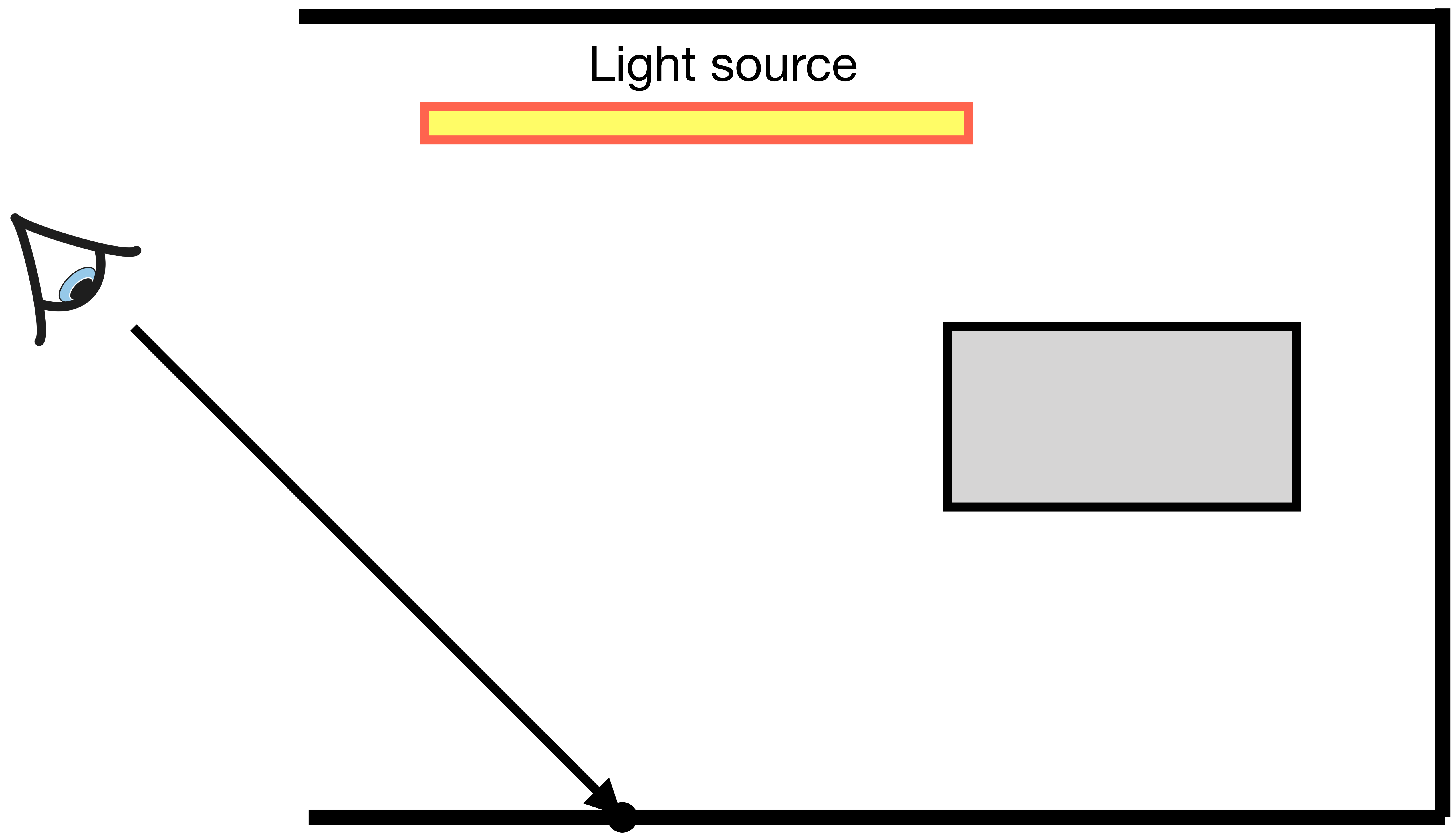
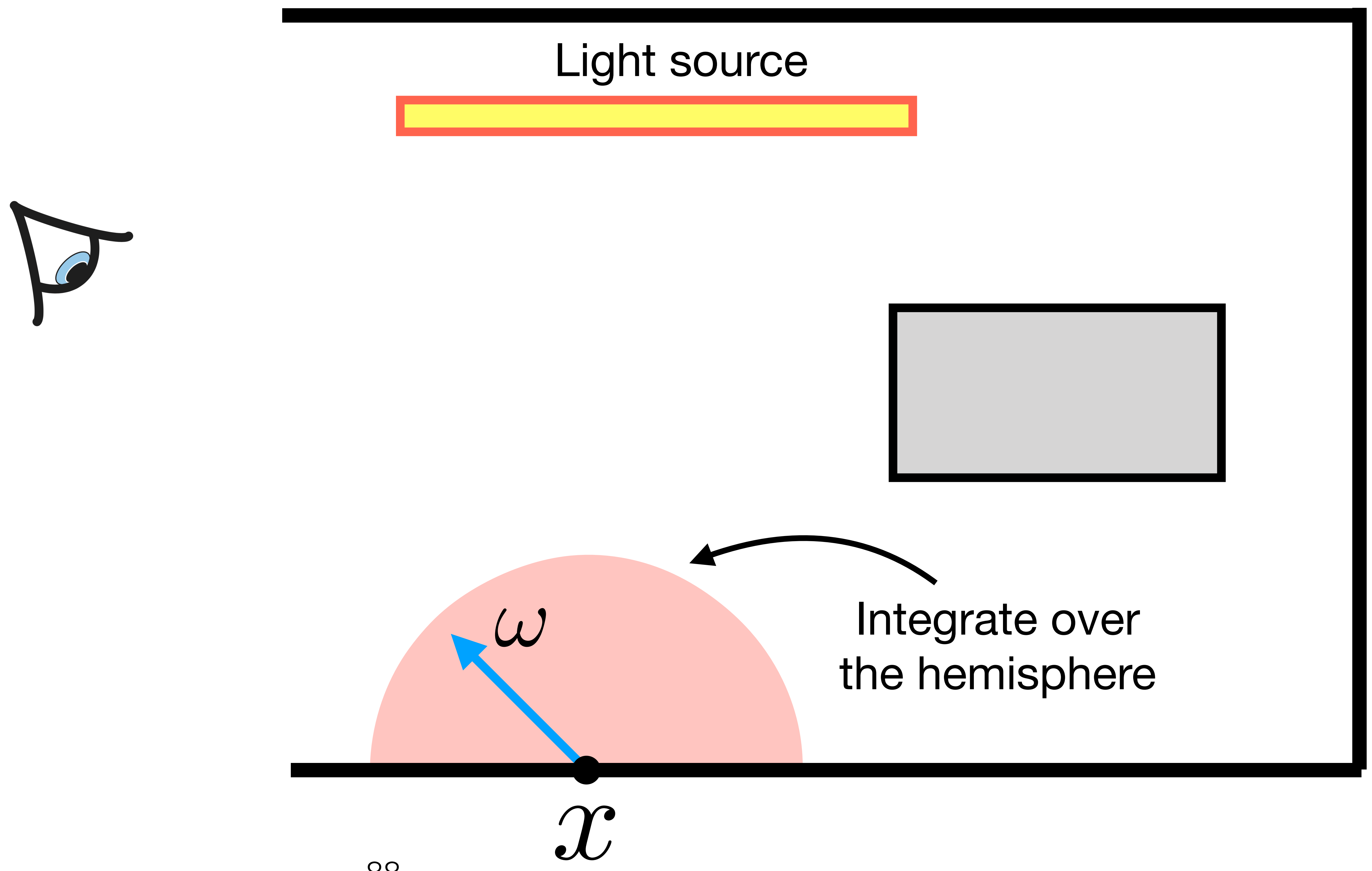


Rendering Equation

$$L(x, \vec{\omega}) = L_e(x, \vec{\omega}) + \int_{\mathcal{H}^2} f(x, \vec{\omega}', \vec{\omega}) L(r(x, \vec{\omega}'), -\vec{\omega}') |\cos\theta'| d\vec{\omega}'$$



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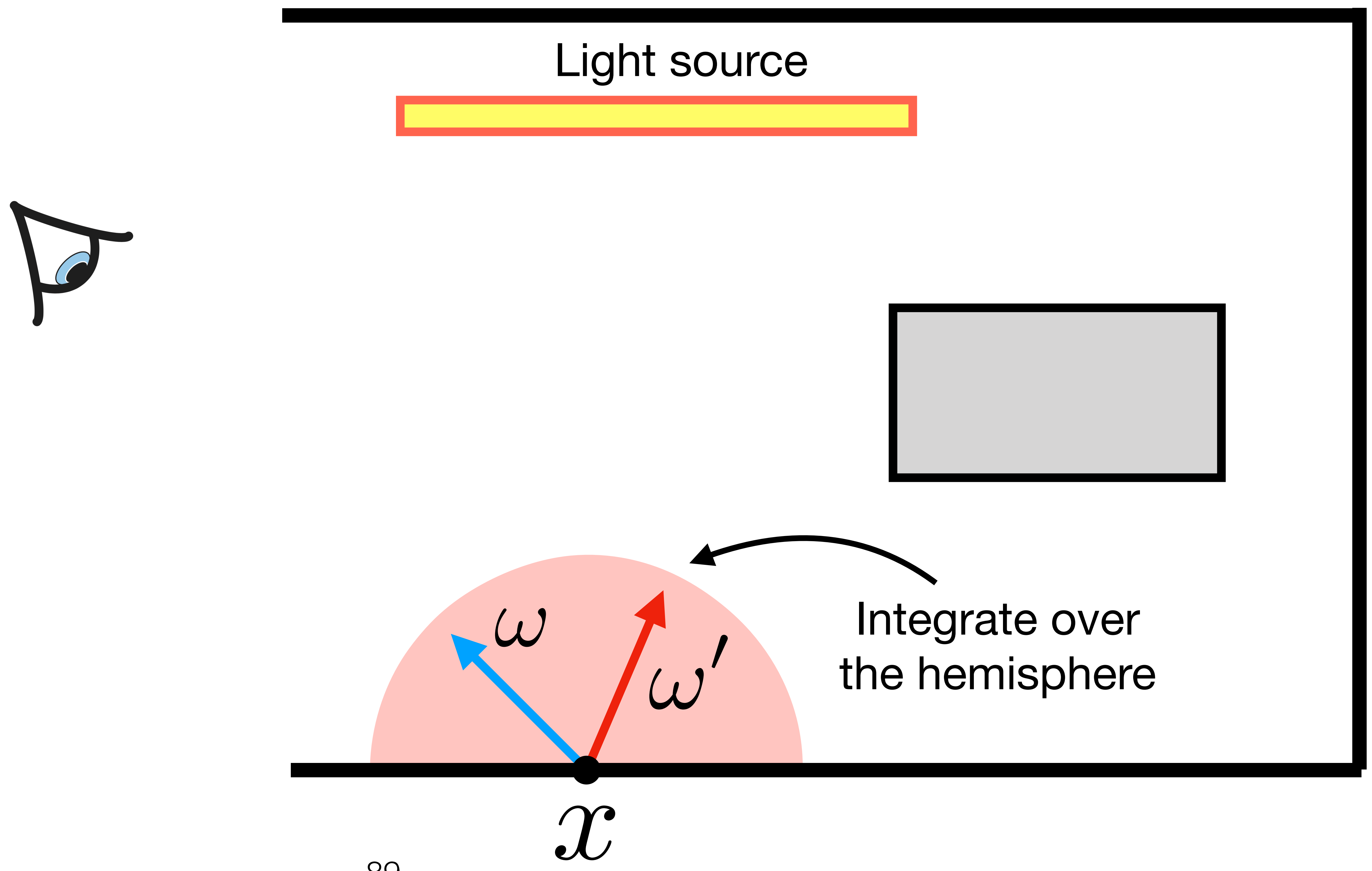

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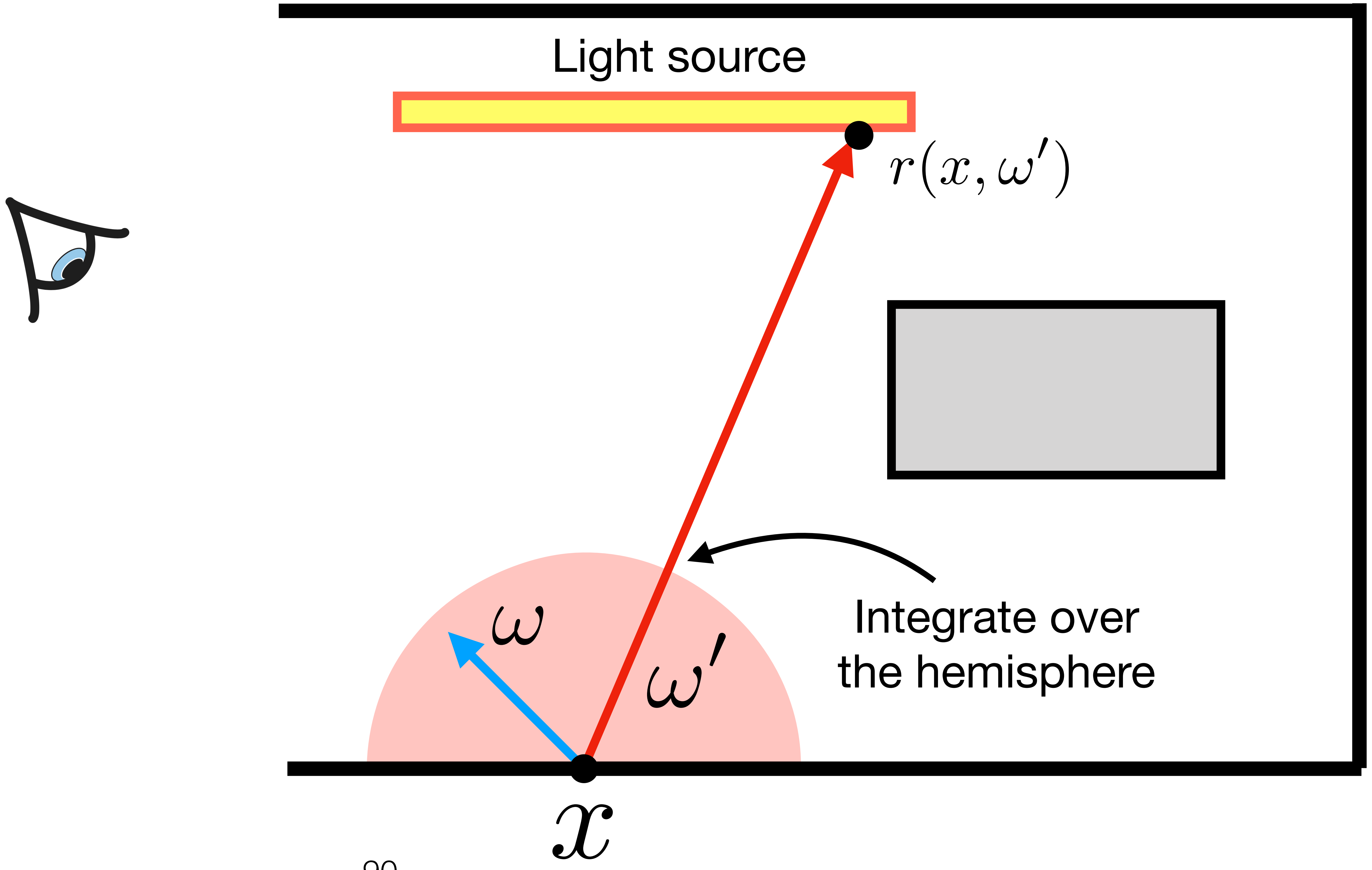

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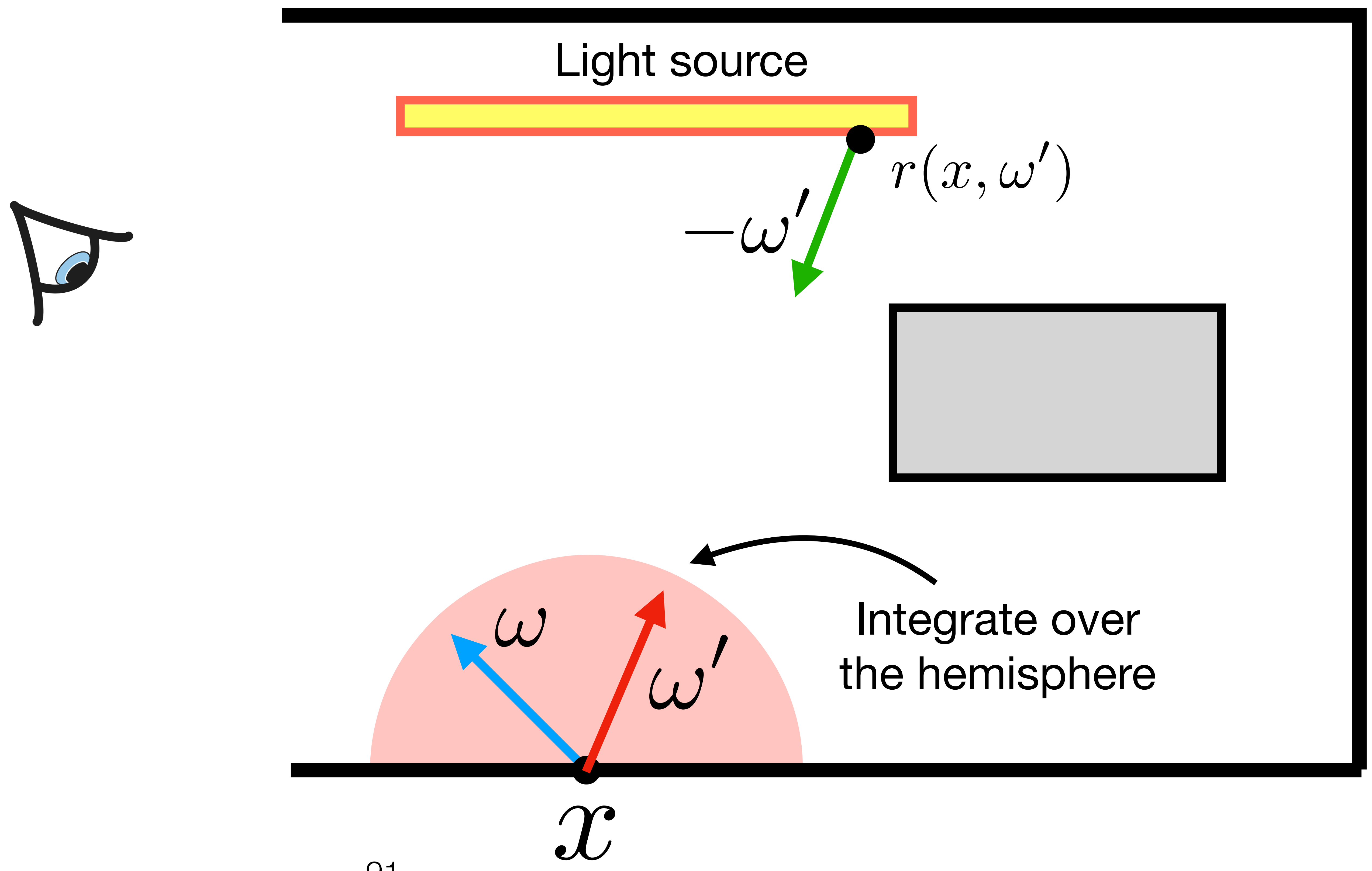

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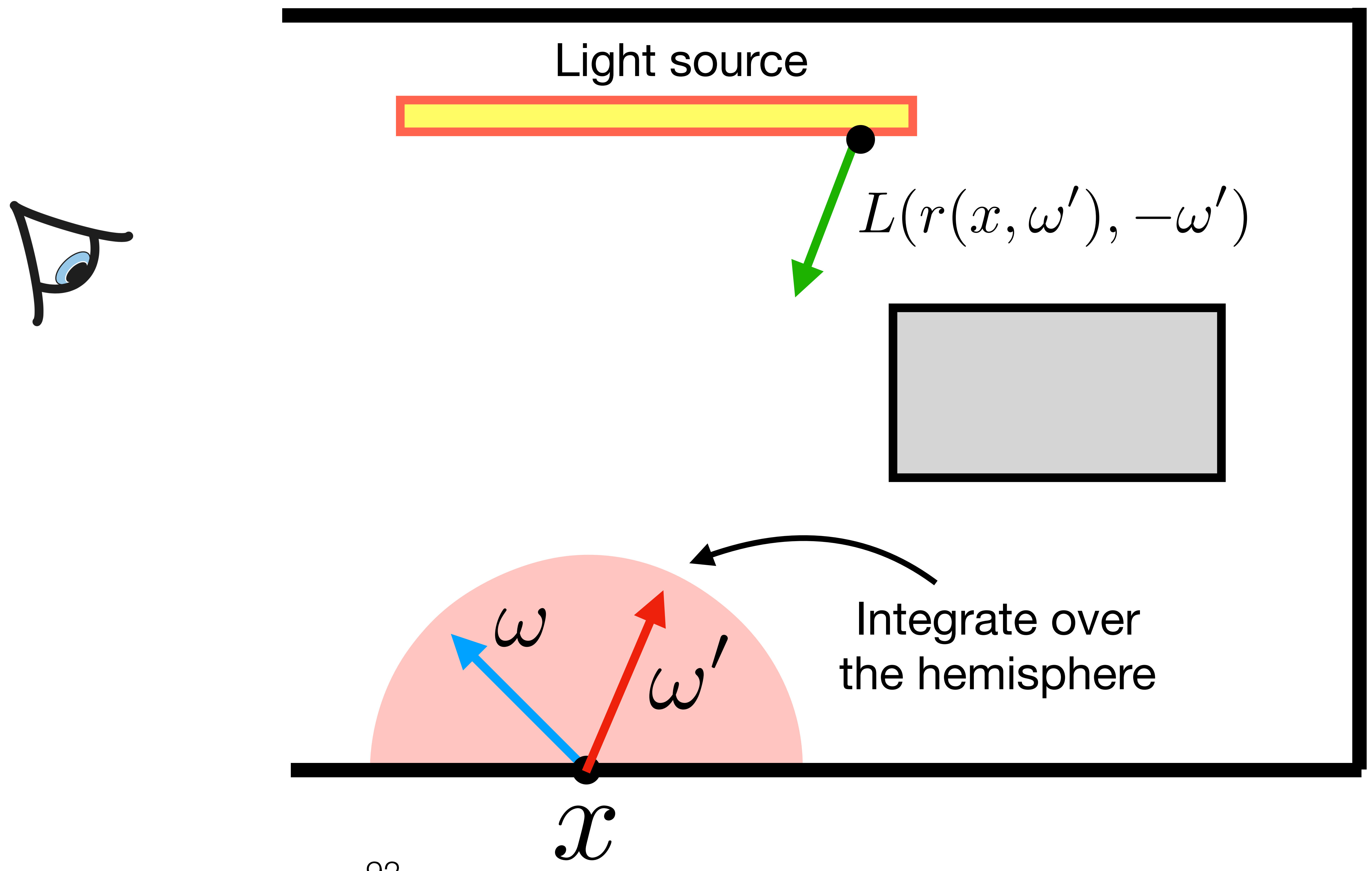

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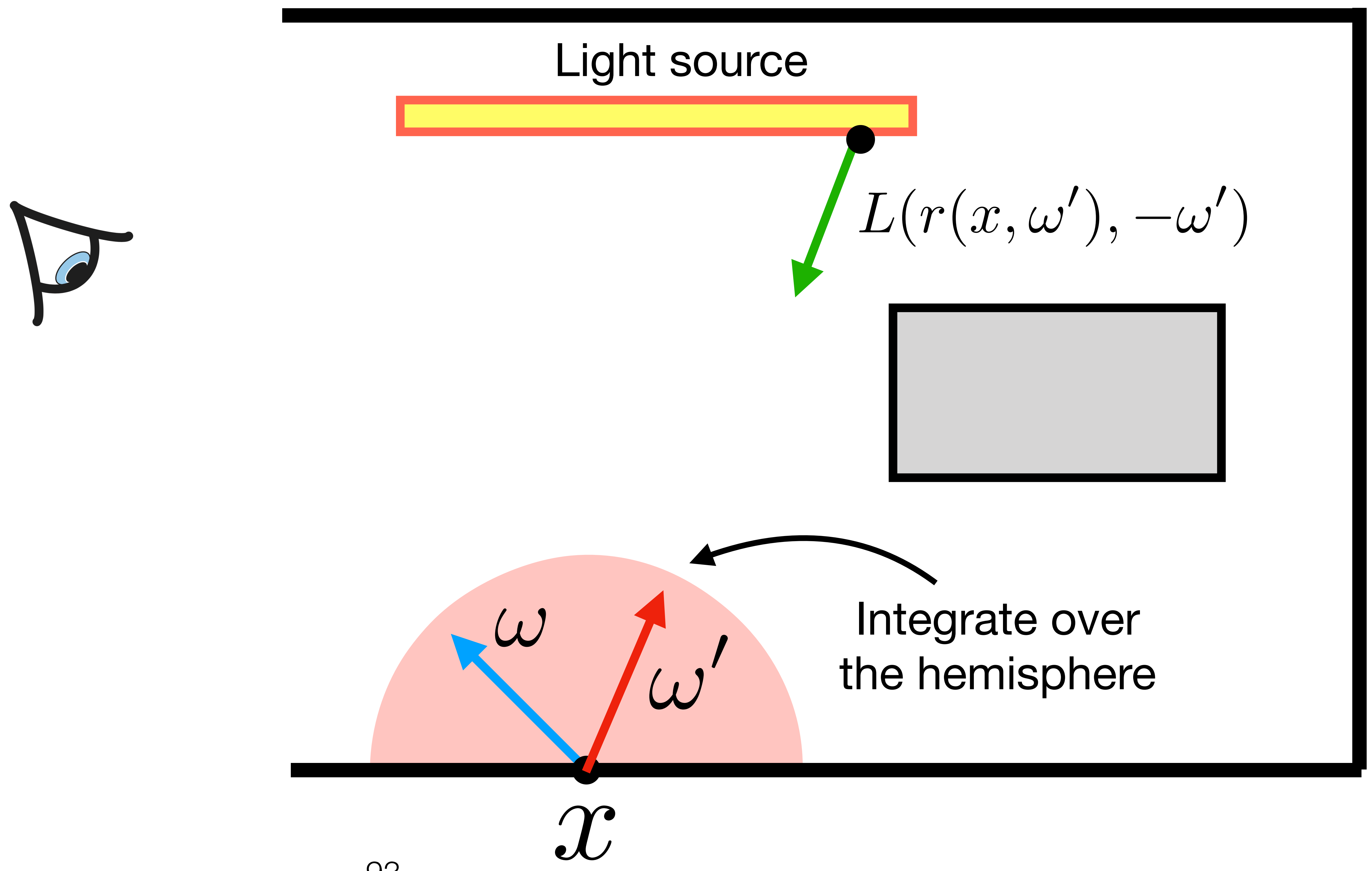

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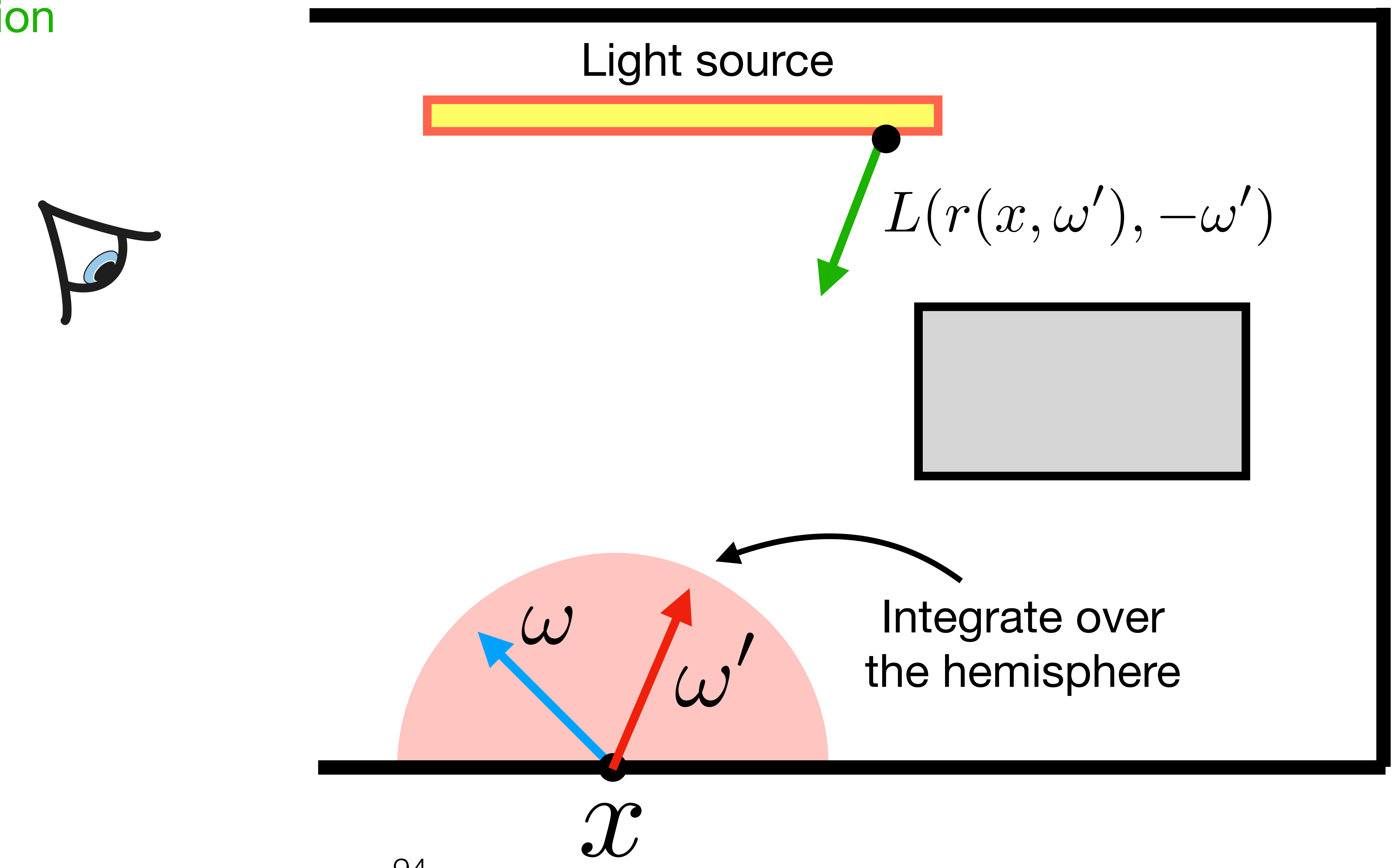
recursion



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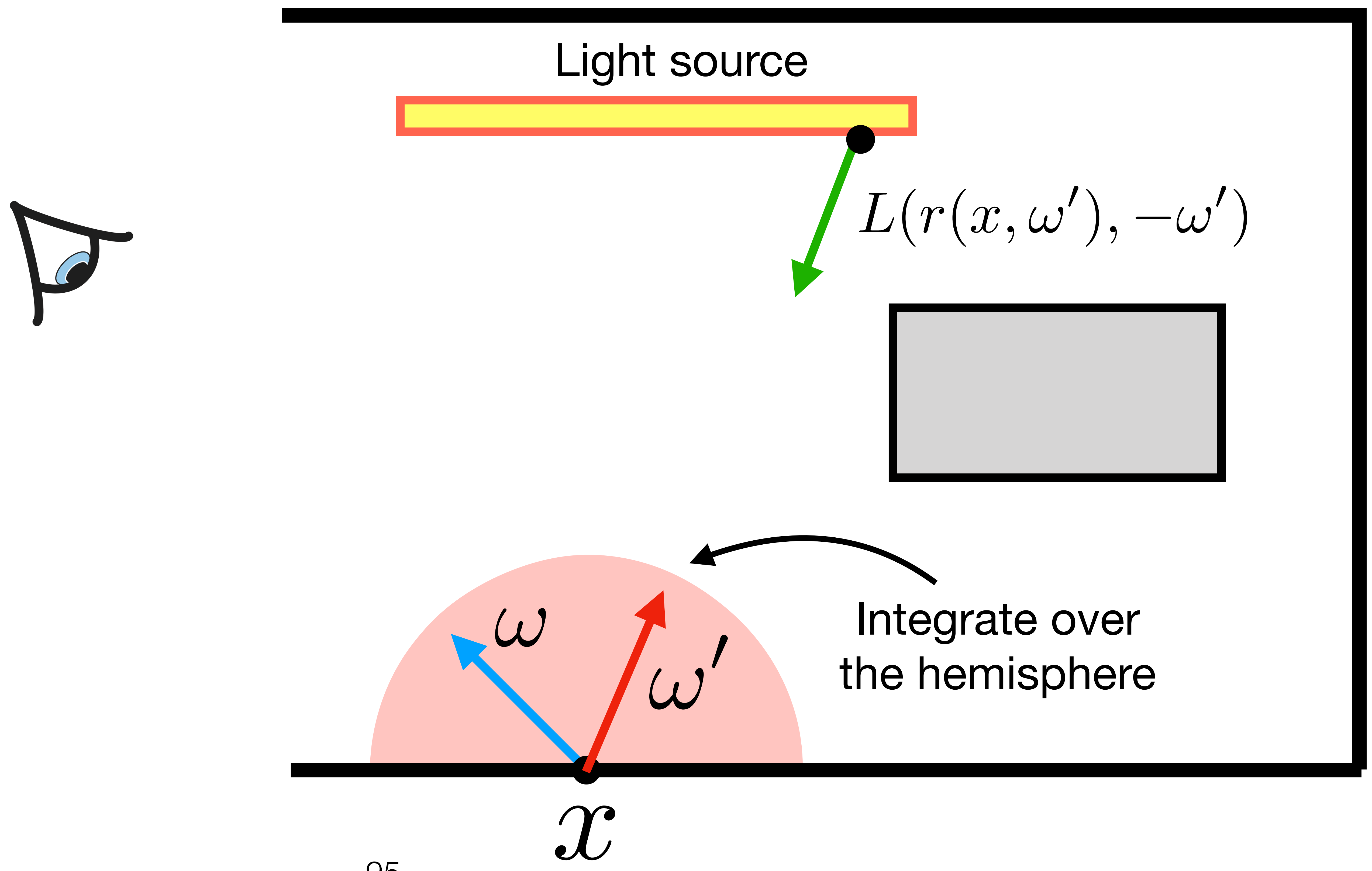

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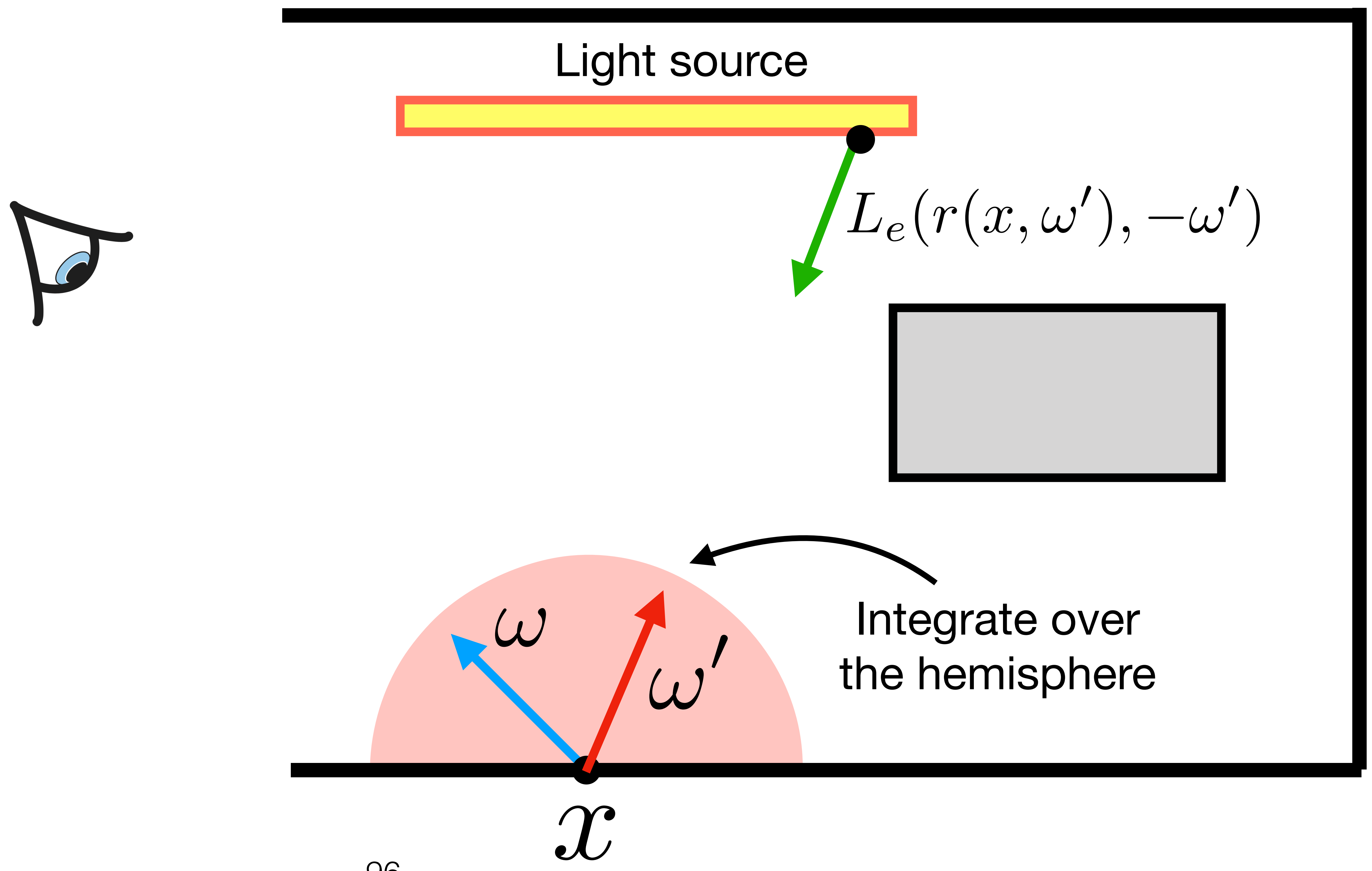
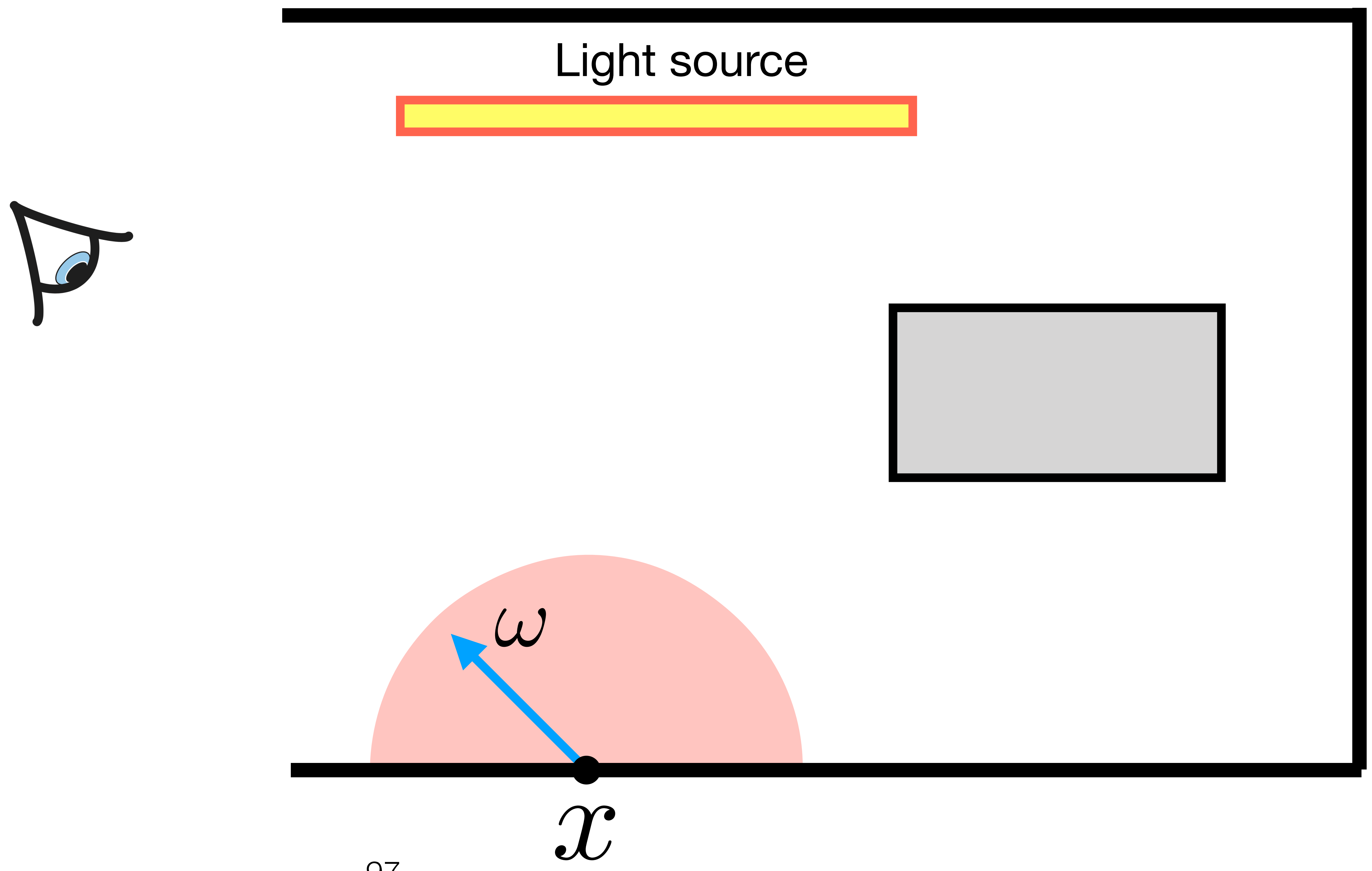


Rendering Equation

$$L(x, \vec{\omega}) = L_e(x, \vec{\omega}) + \int_{\mathcal{H}^2} f(x, \vec{\omega}', \vec{\omega}) L(r(x, \vec{\omega}'), -\vec{\omega}') |\cos\theta'| d\vec{\omega}'$$



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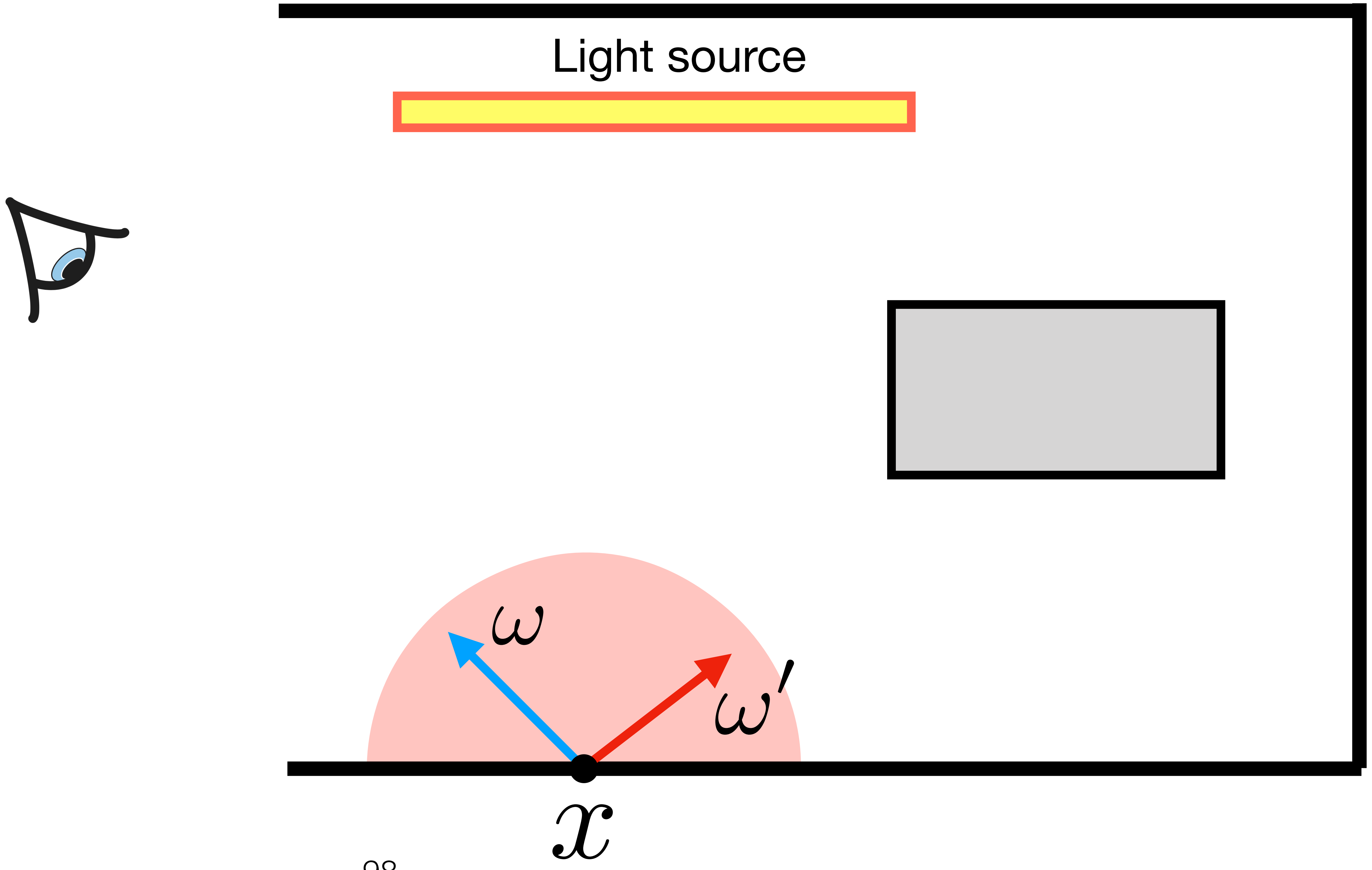

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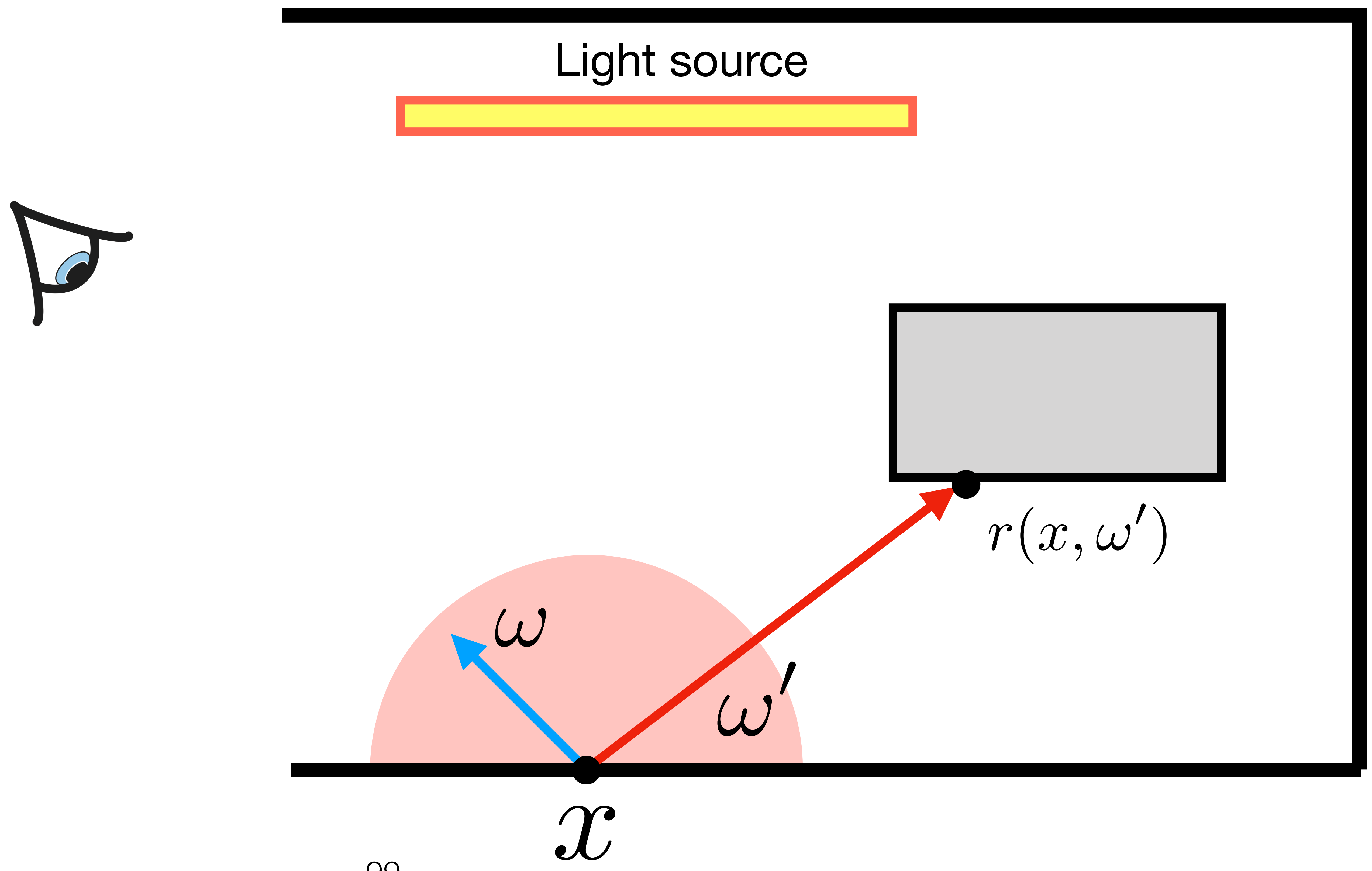

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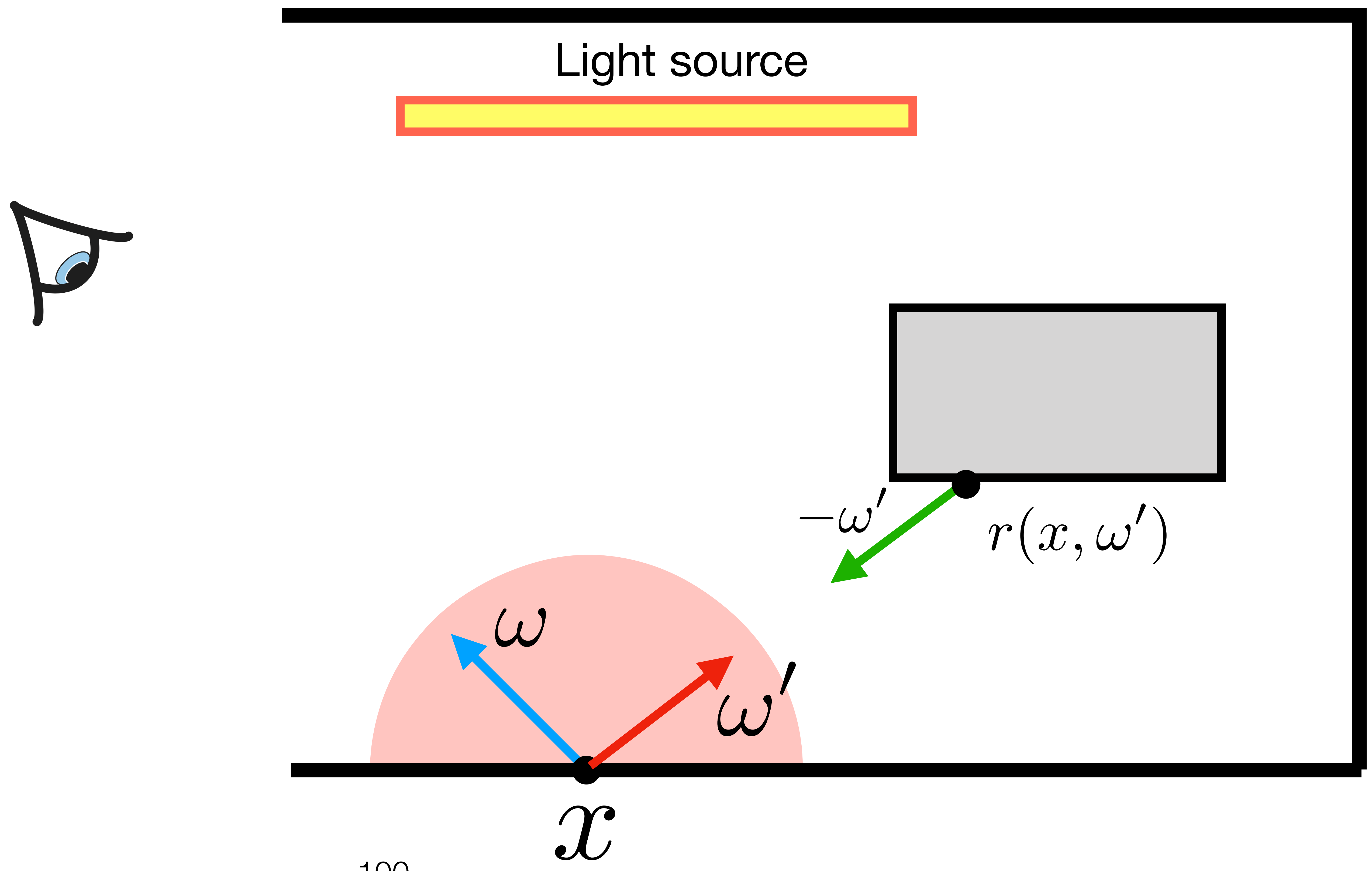

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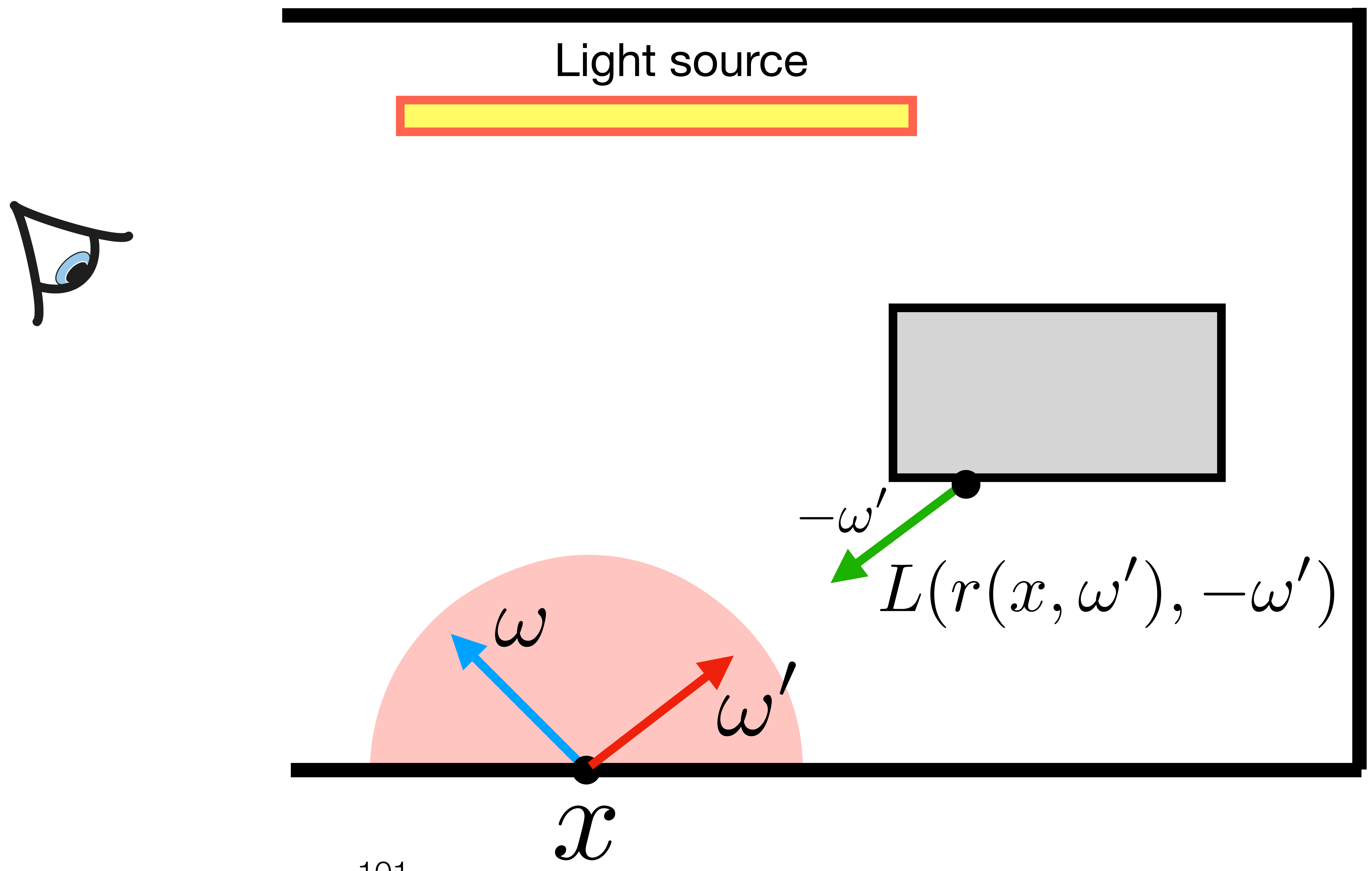

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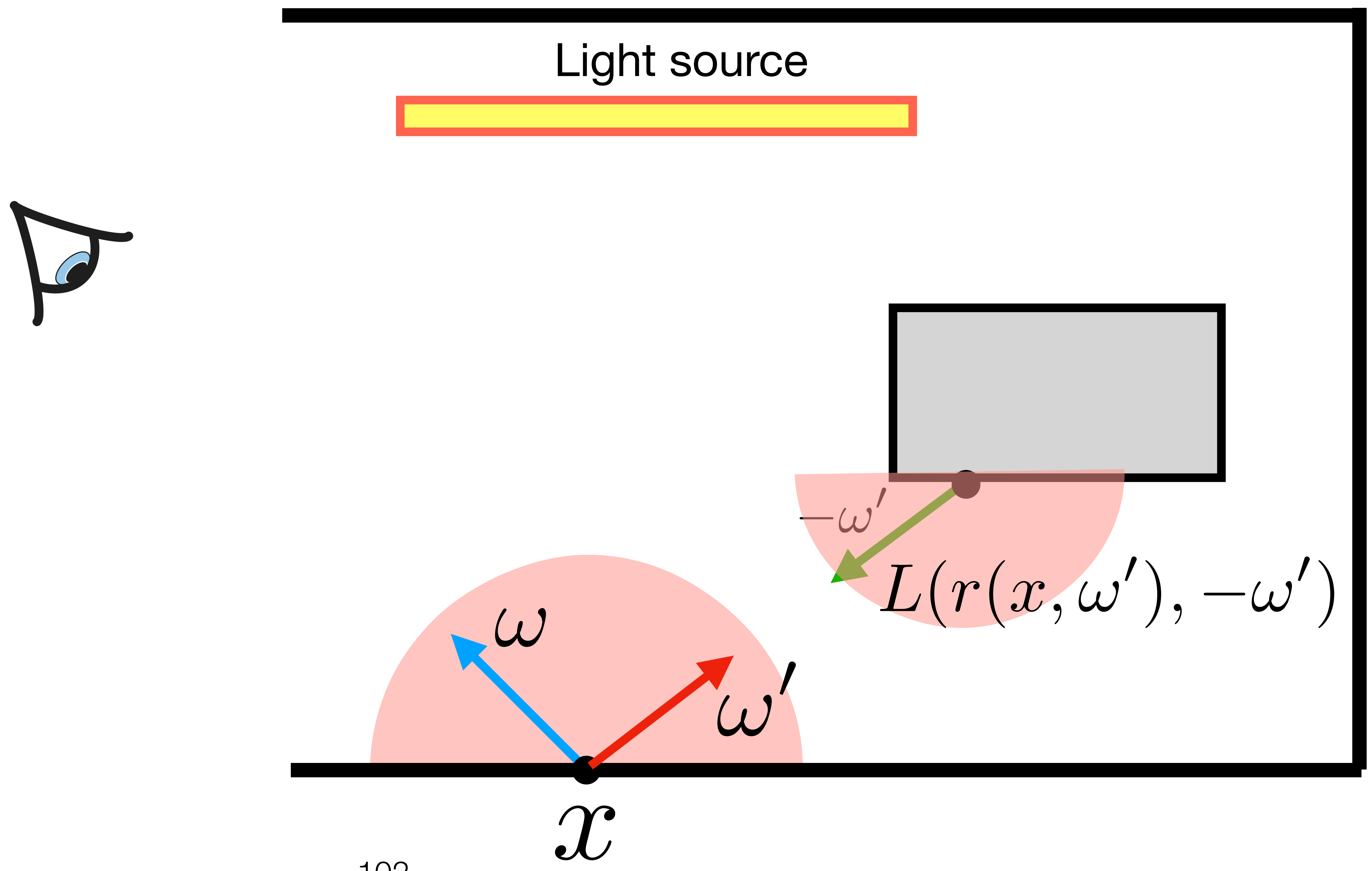
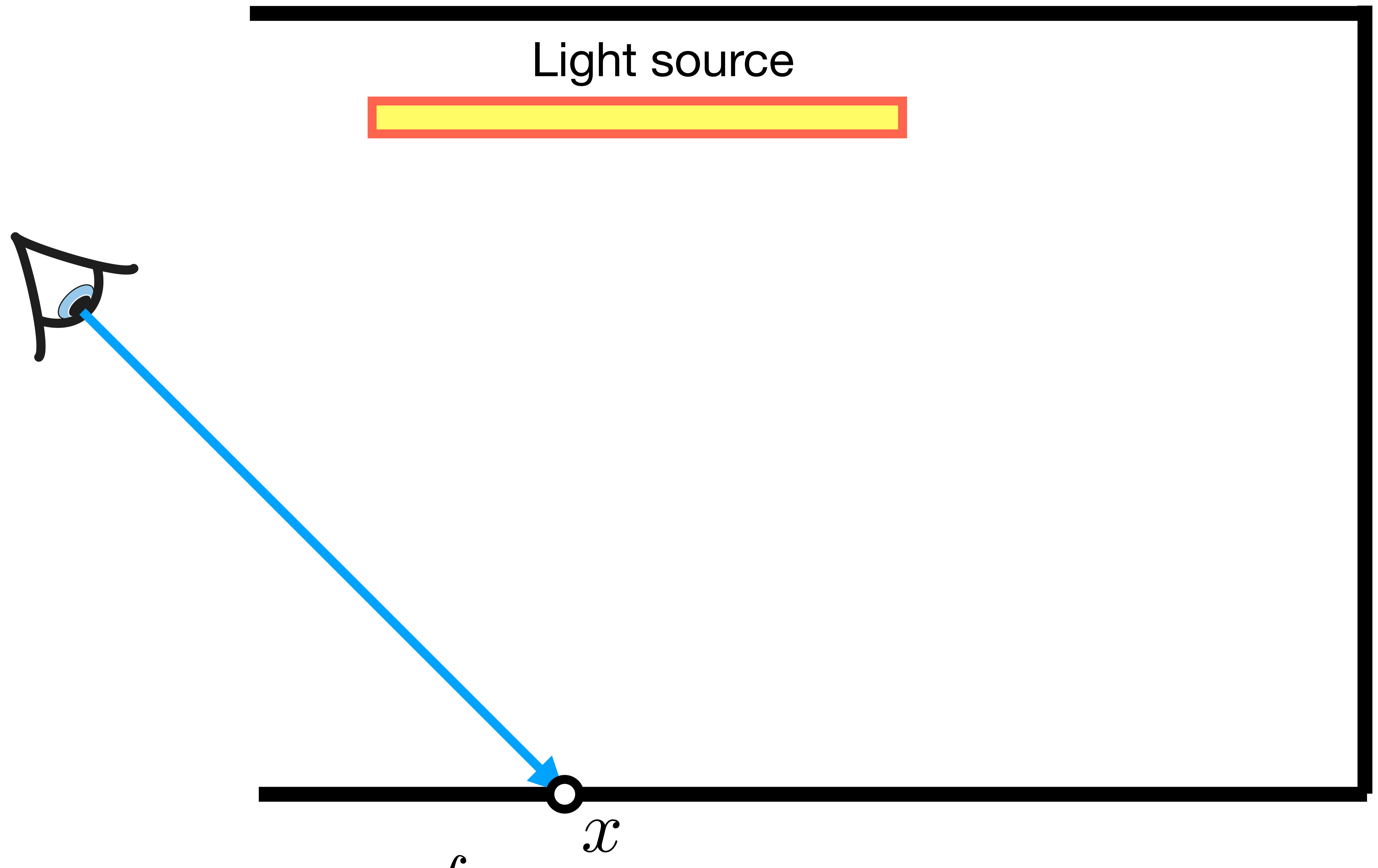


Questions?

Gaussian Material Synthesis by Zsolnai-Feher, Wonka, Wimmer [SIGGRAPH 2018]

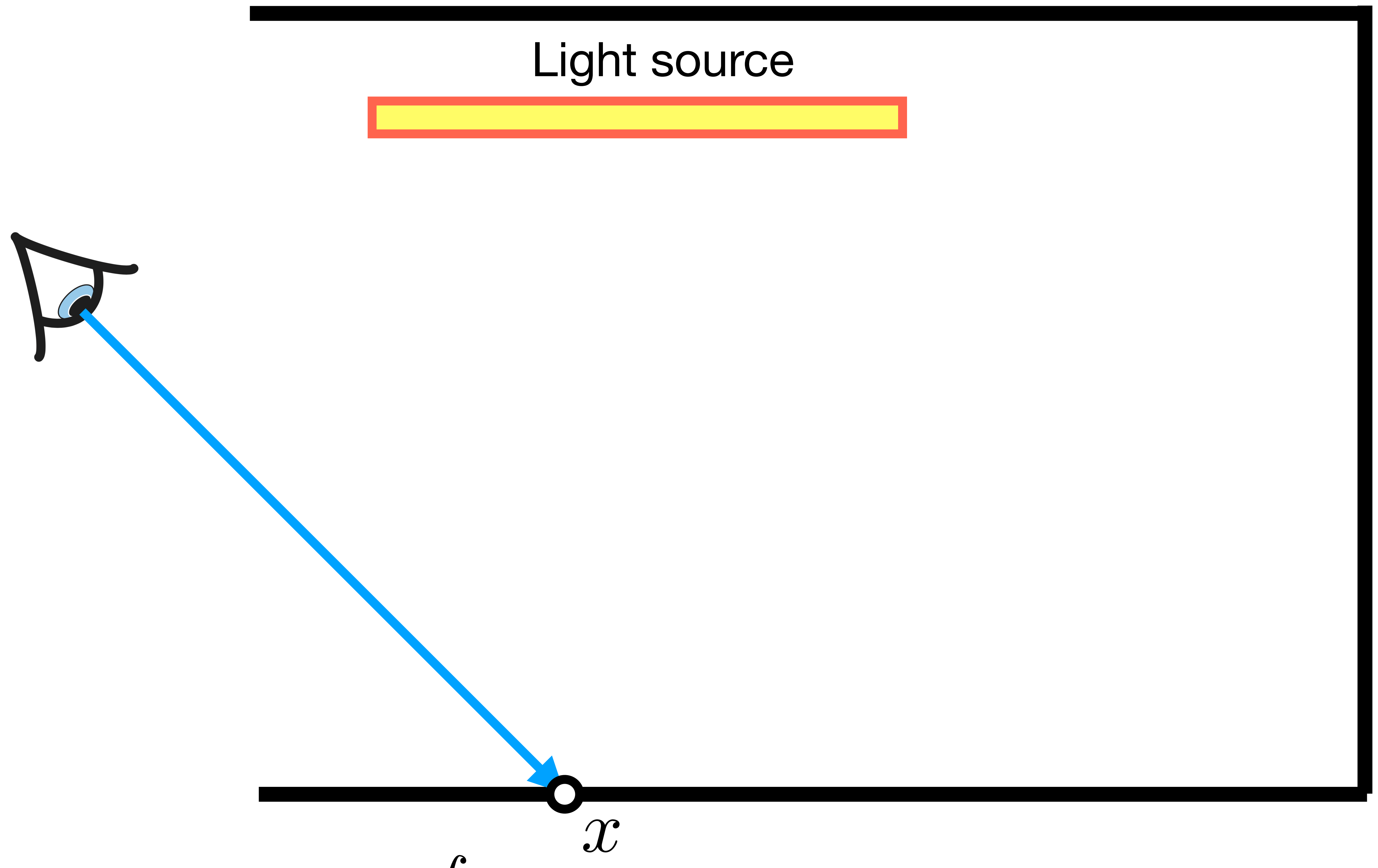
Path Tracing

Path Tracing



$$L(x,\omega) = L_e(x,\omega) + \int_{\mathcal{H}^2} f(x,\omega',\omega) L(r(x,\omega'),-\omega') \cos\theta' d\omega'$$

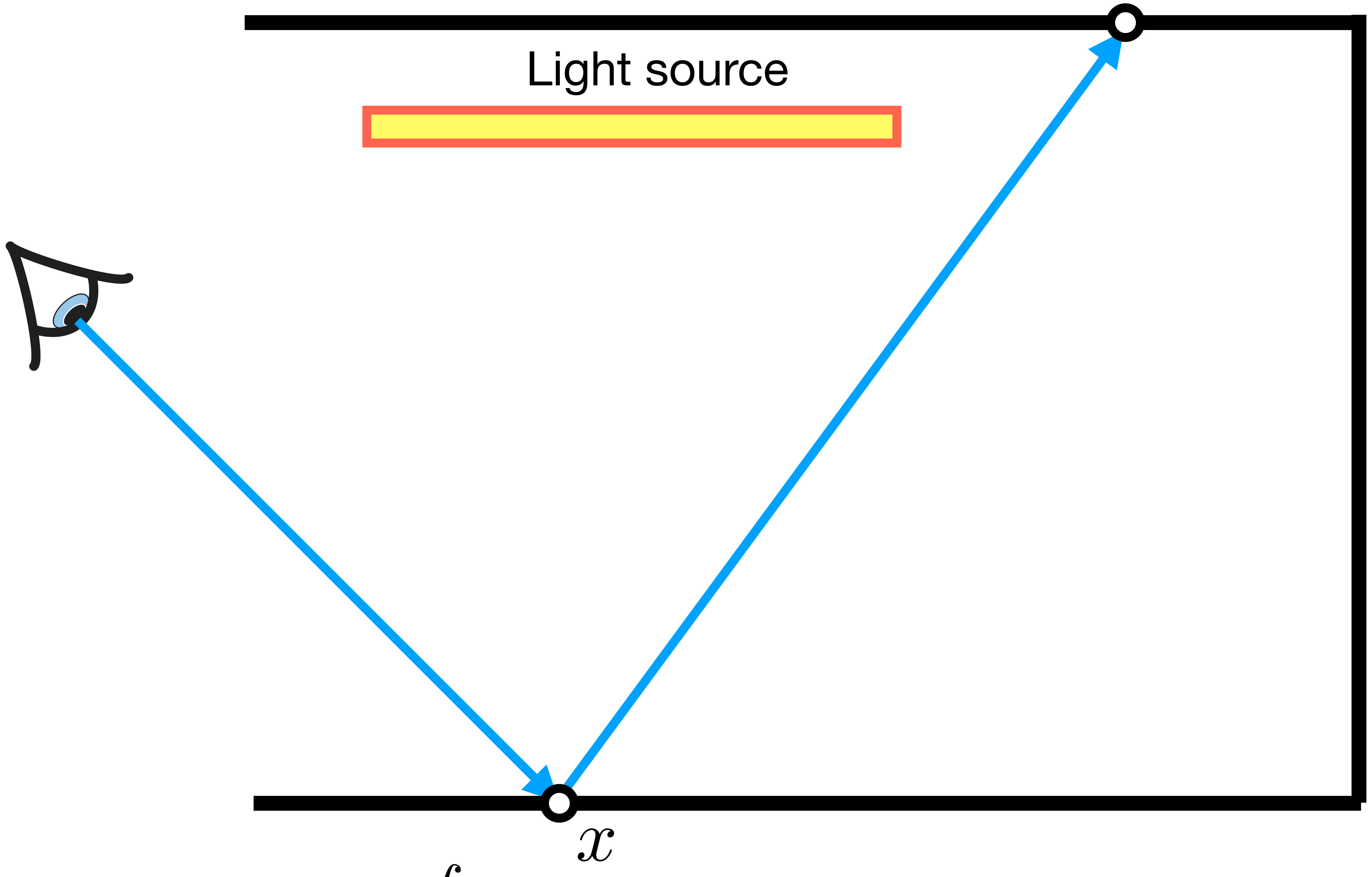
Path Tracing



$$L(x,\omega) = L_e(x,\omega) + \int_{\mathcal{H}^2} f(x,\omega',\omega)L(r(x,\omega'),-\omega')\cos\theta' d\omega'$$

$$\approx L_e(x,\omega) + \frac{f(x,\omega',\omega)L(r(x,\omega'),-\omega')\cos\theta'}{p(\omega')}$$

Path Tracing



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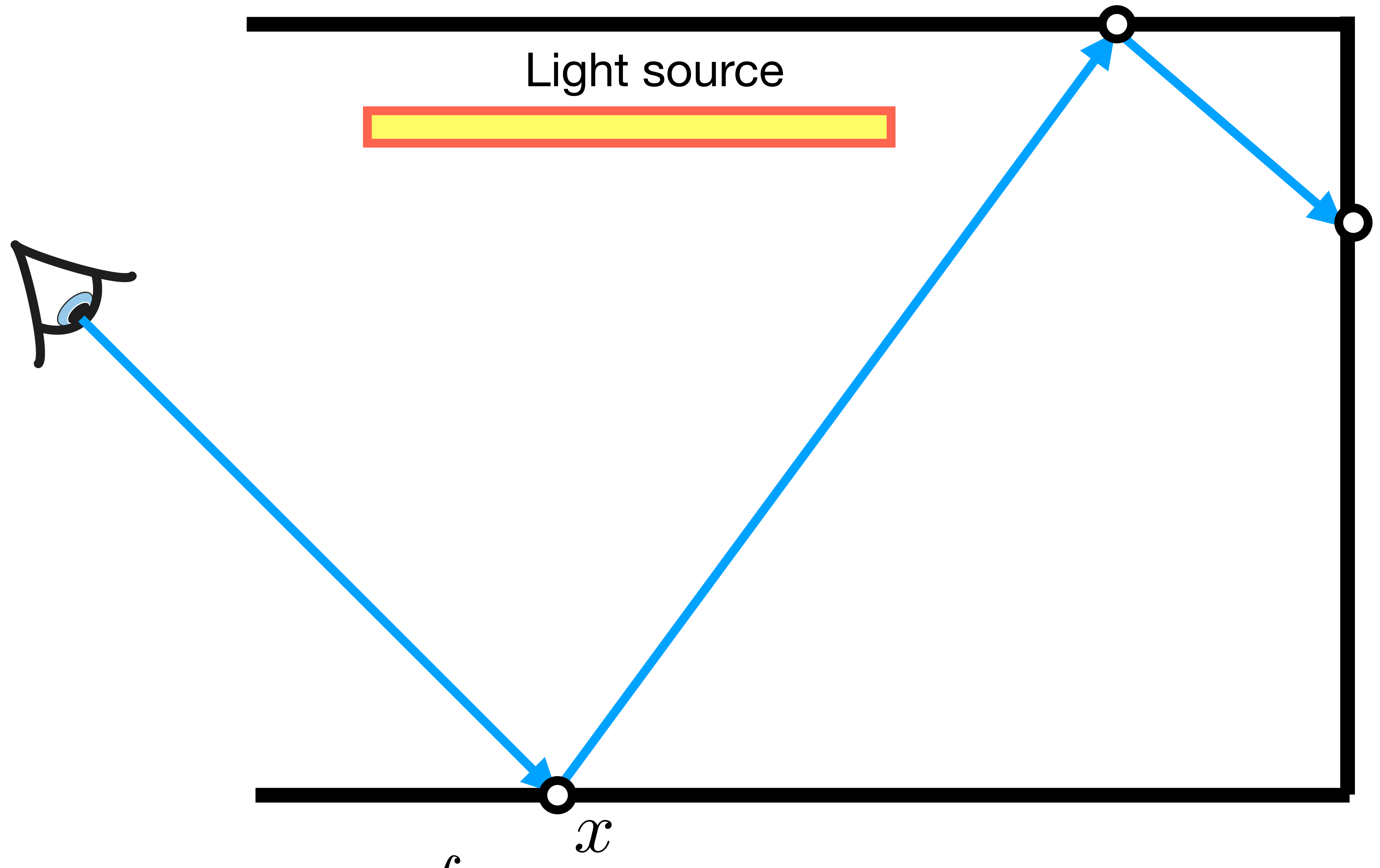
Path Tracing



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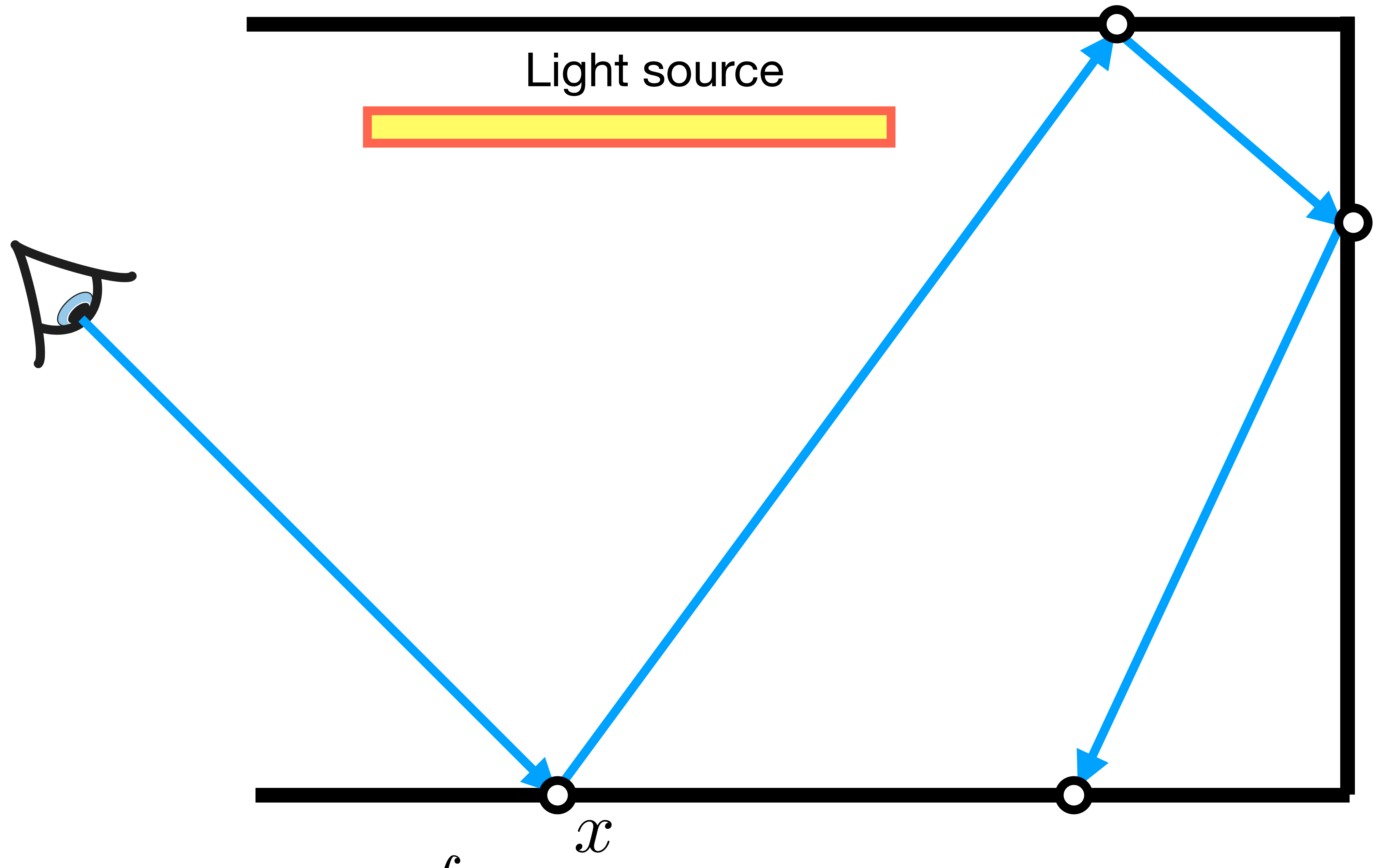
Path Tracing



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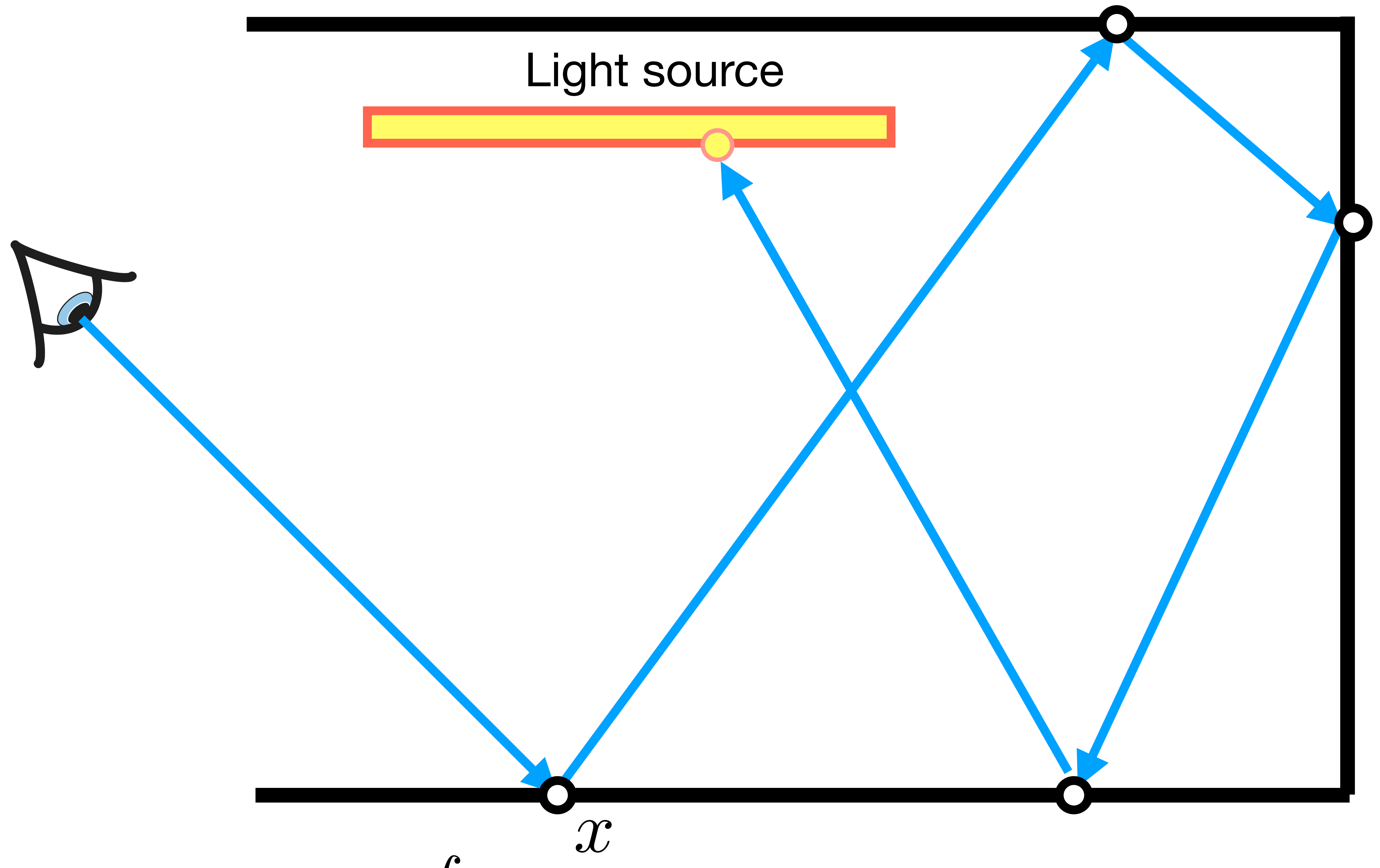
Path Tracing



$$L(x,\omega) = L_e(x,\omega) + \int_{\mathcal{H}^2} f(x,\omega',\omega) L(r(x,\omega'),-\omega') \cos\theta' d\omega'$$

$$\approx L_e(x,\omega) + \frac{f(x,\omega',\omega) L(r(x,\omega'),-\omega') \cos\theta'}{p(\omega')}$$

Path Tracing Algorithm

$$L_o(x, \omega_o) = L_e(x, \omega_o) + L_r(x, \omega_o)$$

```
Color color(Point x, Direction ω, int moreBounces):

    if not moreBounces:
        return Le(x,-ω)

    // sample recursive integral
    ω' = sample from BRDF
    return Le(x,-ω) + BRDF * color(trace(x, ω'), moreBounces-1) * dot(n, ω') / pdf(ω')
```

Partitioning the Integrand

Direct Illumination: sometimes better estimated by sampling the emissive surfaces

Partitioning the Integrand

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Let's estimate direct illumination separately from indirect illumination, then add the two

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- i.e., shoot shadow rays (direct) and gather rays (indirect)

- be careful not to double count!

Partitioning the Integrand

Direct Illumination: sometimes better estimated by sampling the emissive surfaces

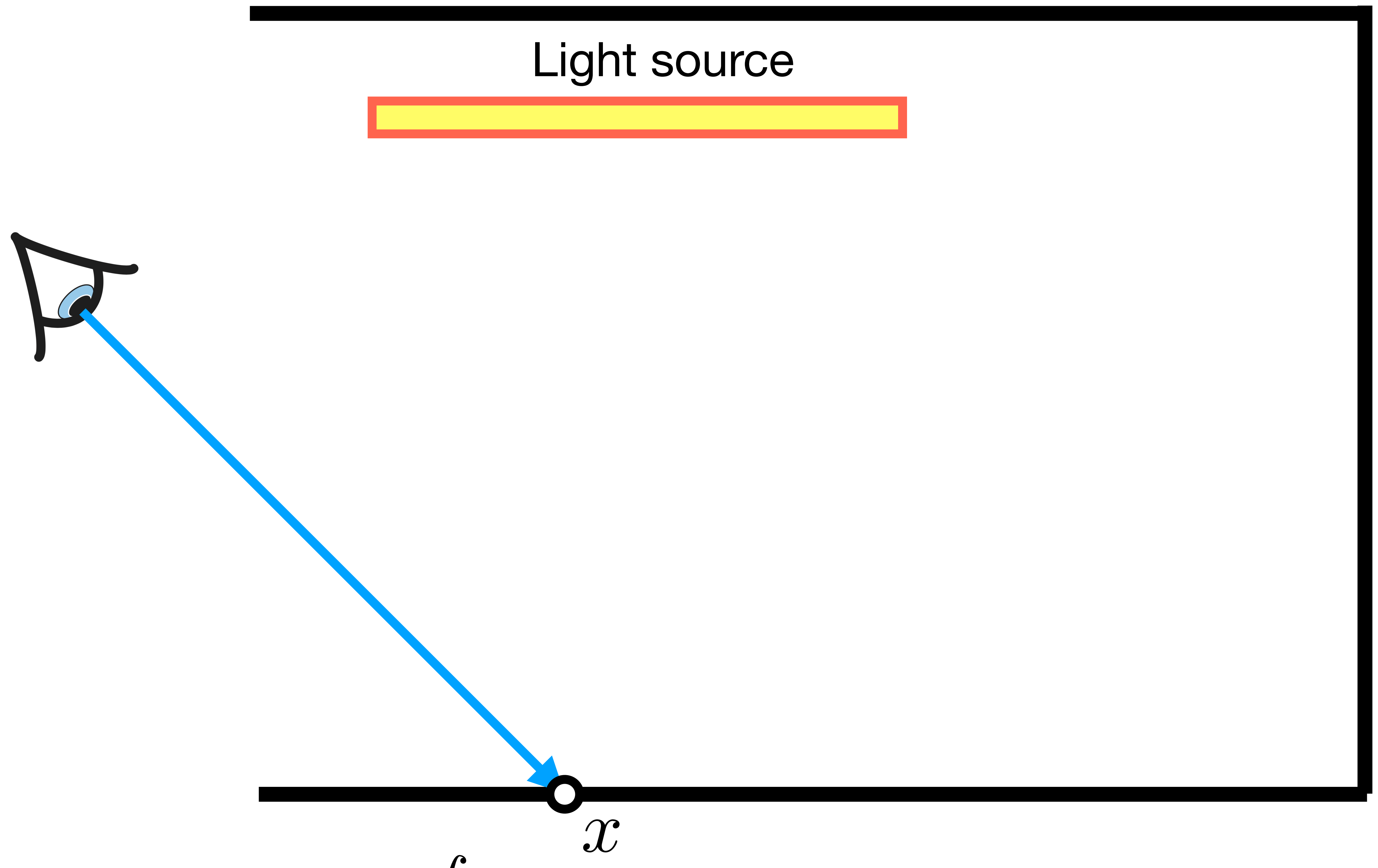
Let's estimate direct illumination separately from indirect illumination, then add the two

- i.e., shoot shadow rays (direct) and gather rays (indirect)

- be careful not to double count!

Also known as Next Event Estimation (NEE)

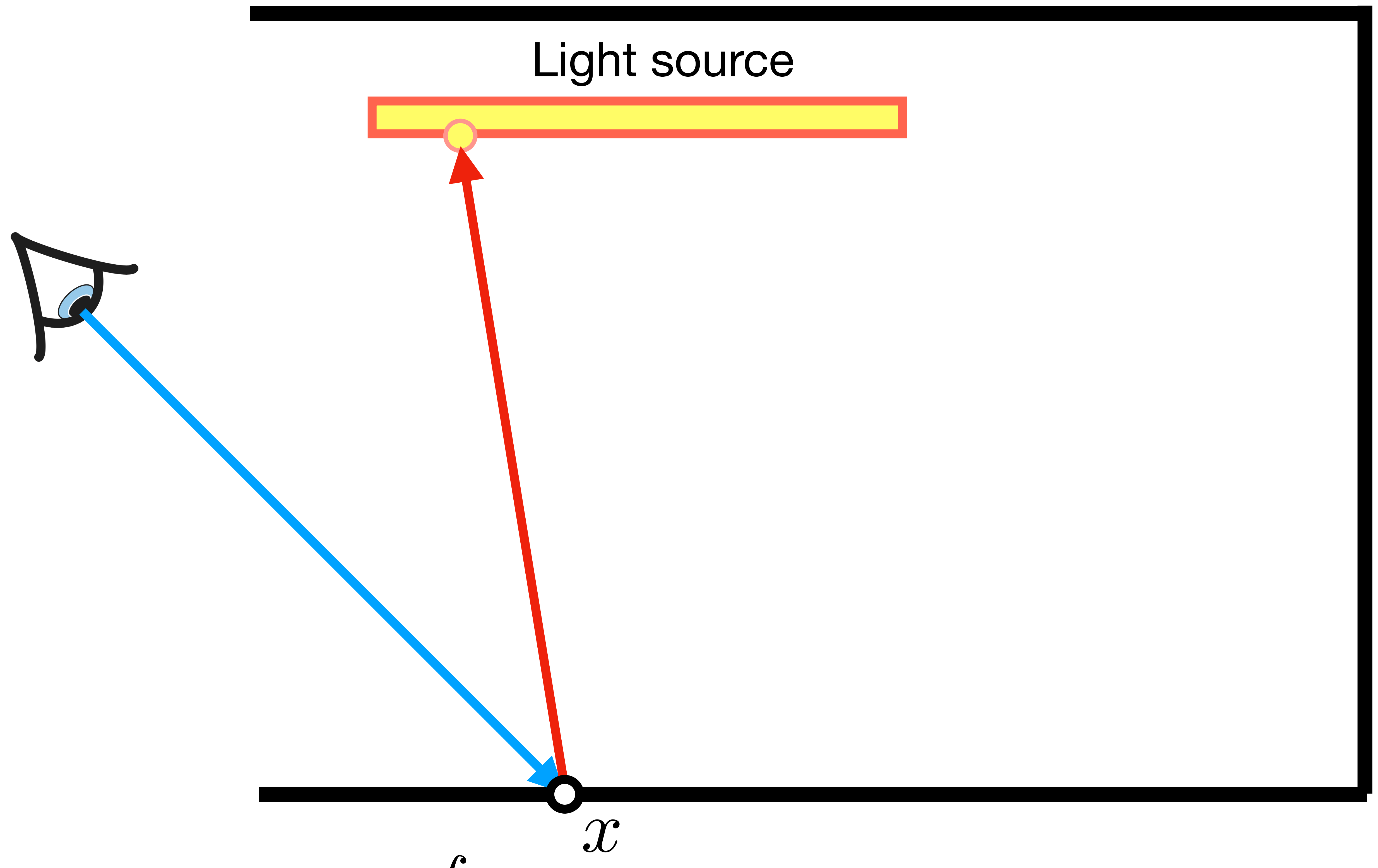
Path Tracing Algorithm with NEE



$$L(x,\omega) = L_e(x,\omega) + \int_{\mathcal{H}^2} f(x,\omega',\omega) L(r(x,\omega'), -\omega') \cos\theta' d\omega'$$

$$\approx L_e(x,\omega) + \frac{f(x,\omega',\omega) L(r(x,\omega'), -\omega') \cos\theta'}{p(\omega')}$$

Path Tracing Algorithm with NEE



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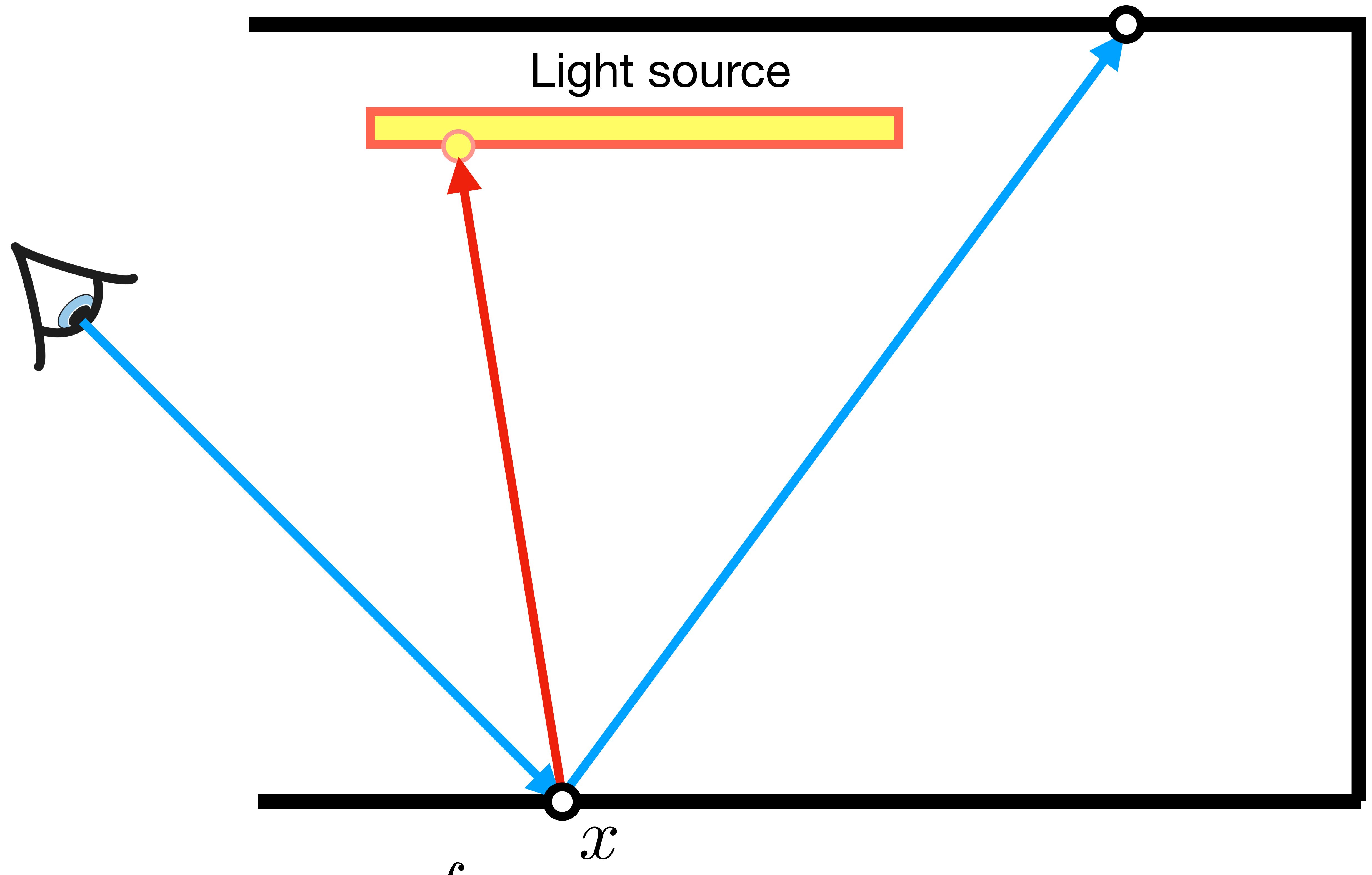

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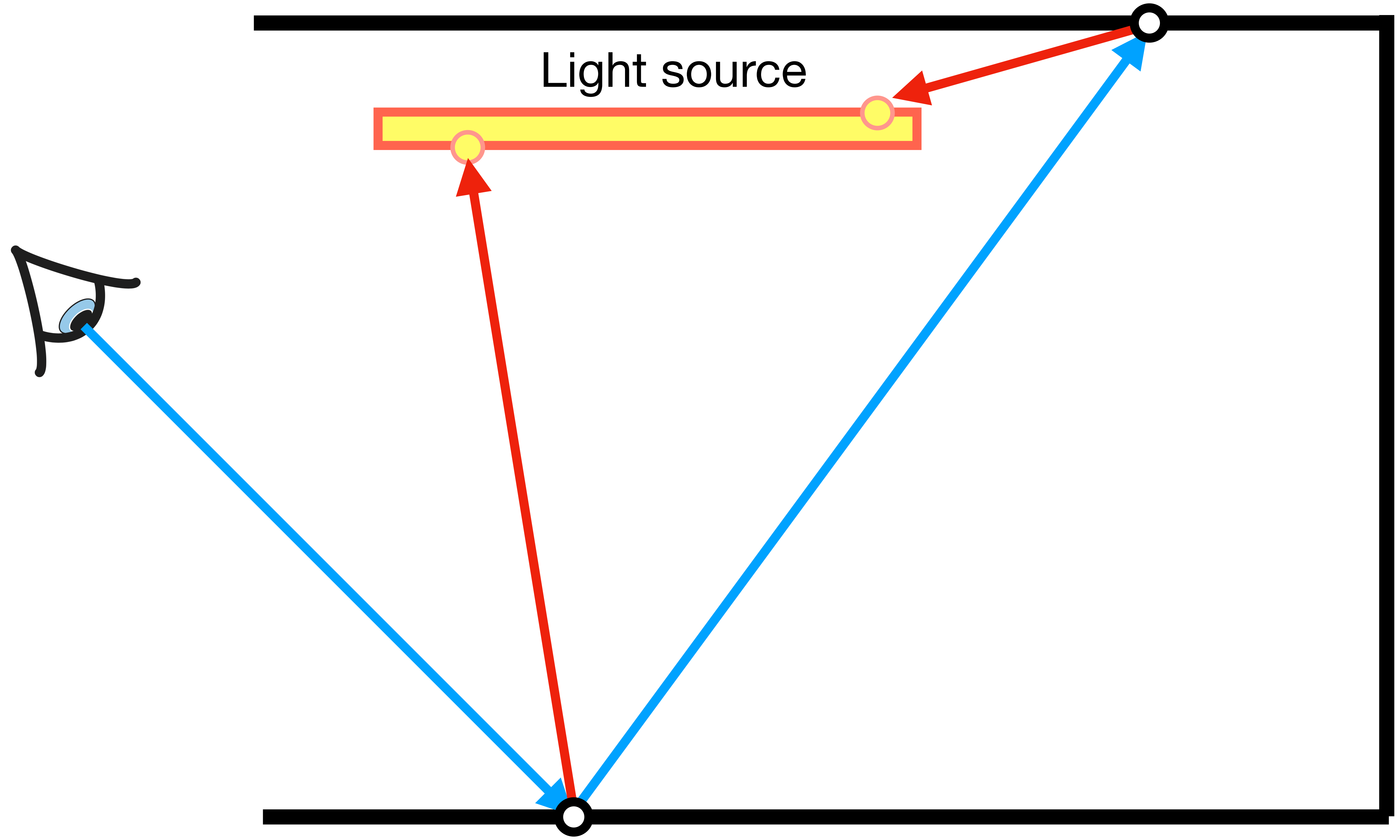

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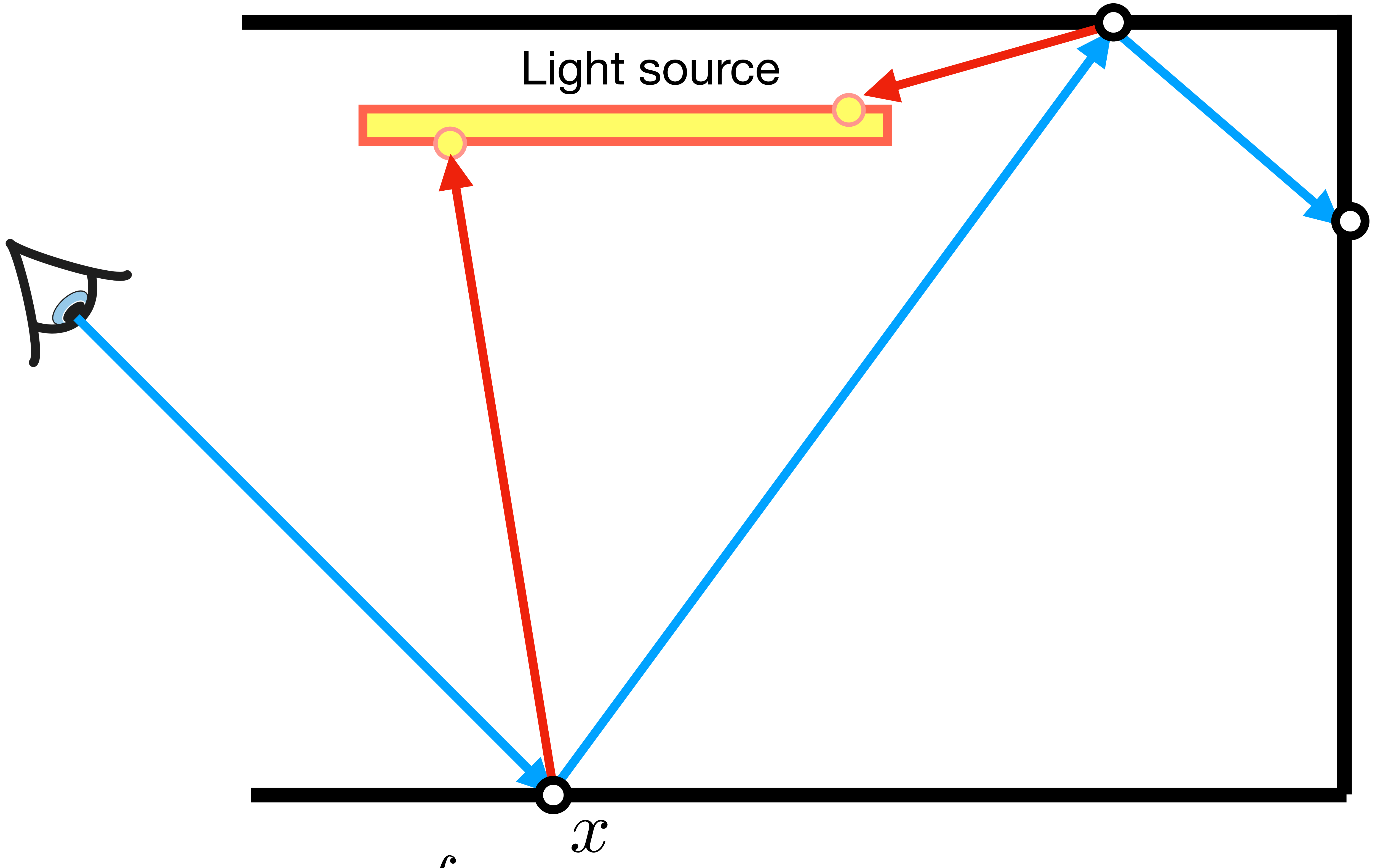

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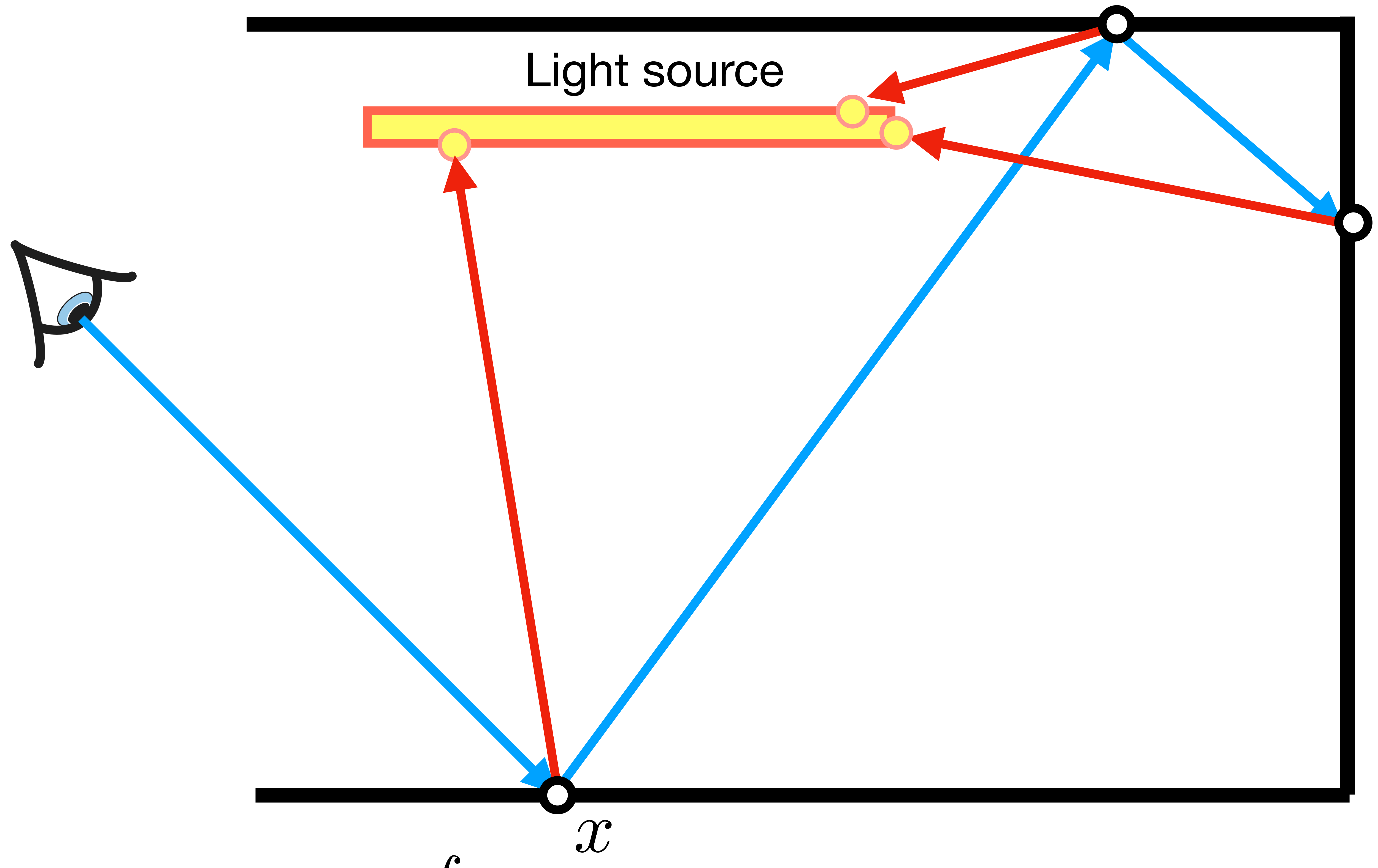

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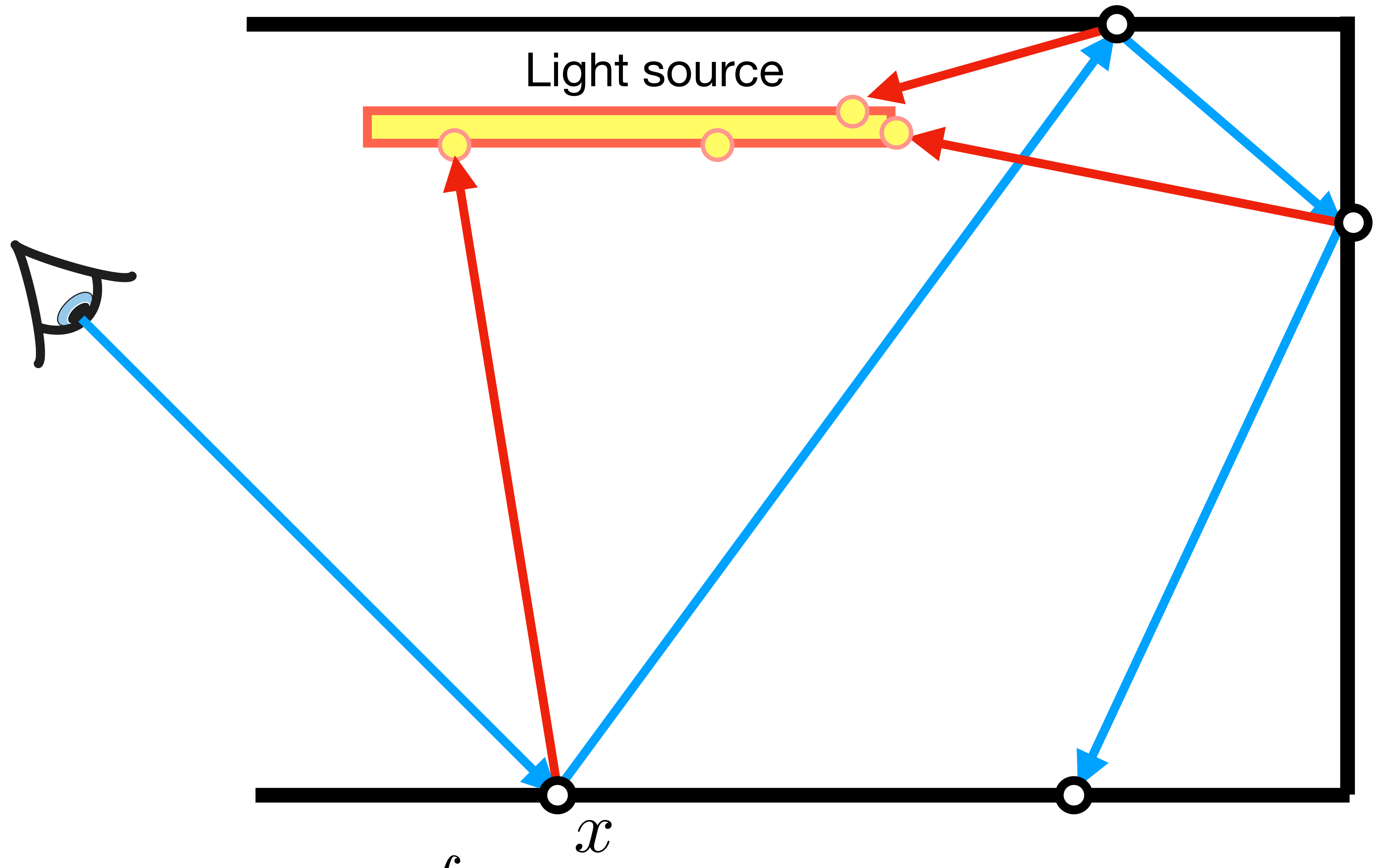

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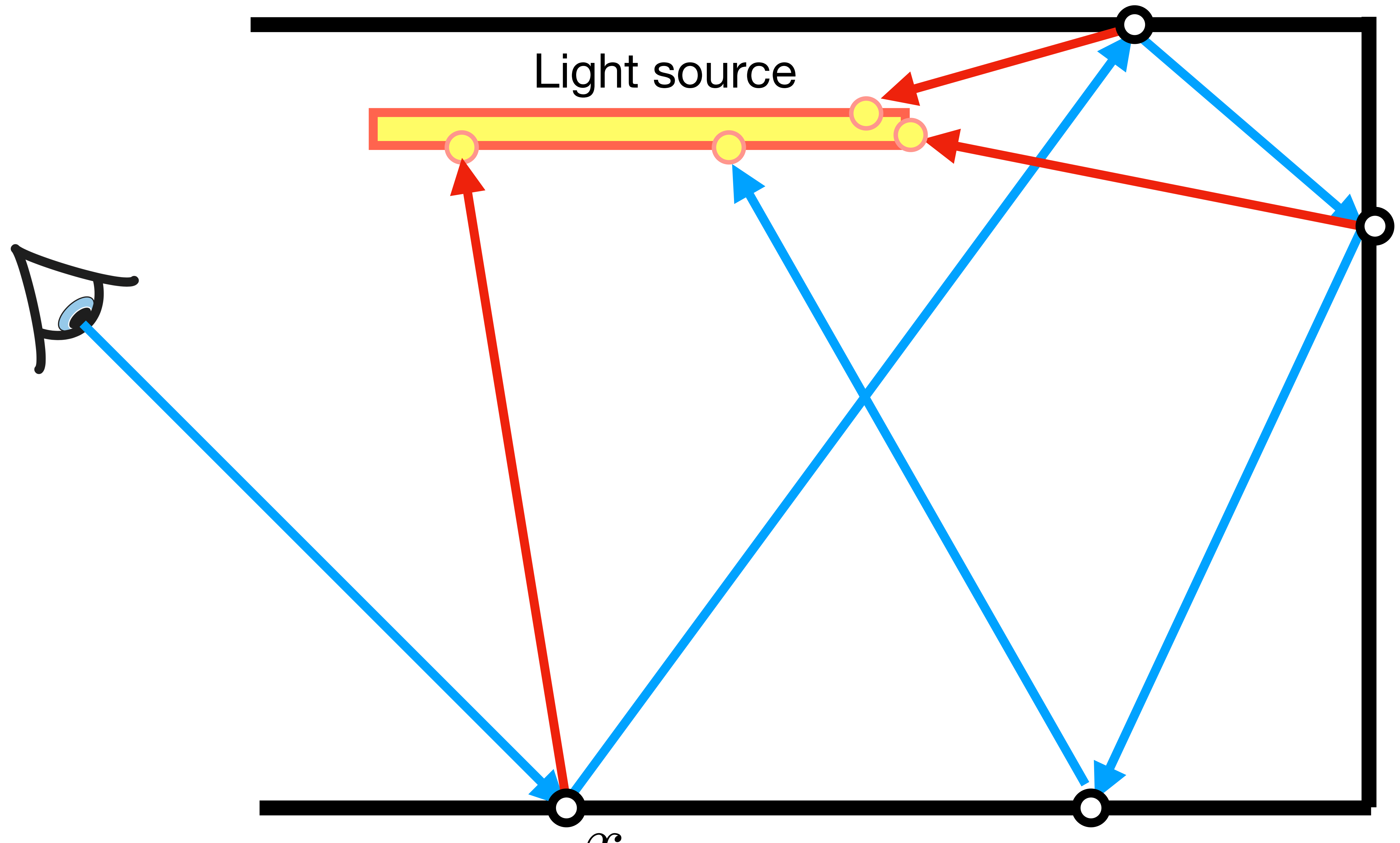
Path Tracing Algorithm with NEE



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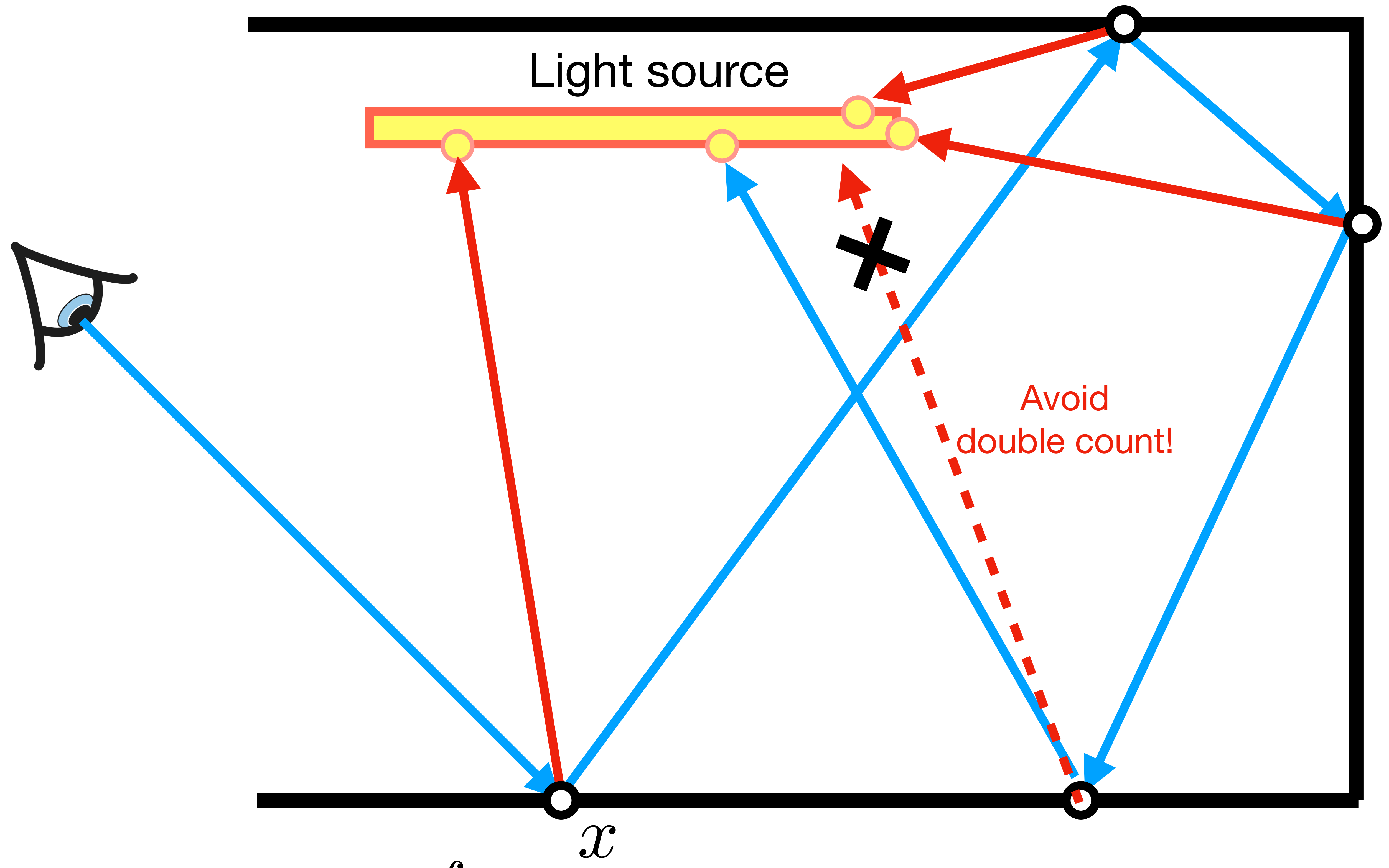

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Path Tracing Algorithm with NEE

$$L(x,\omega) = L_e(x,\omega) + L_{dir}(x,\omega) + L_{ind}(x,\omega)$$

```
Color color(Point x, Direction ω, int moreBounces):

    if not moreBounces:
        return Lₑ;

    // next-event estimation: compute L_dir by sampling the light
    ω1 = sample from light
    L_dir = BRDF * color(trace(x, ω1), 0) * dot(n, ω1) / pdf(ω1)

    // compute L_ind by sampling the BSDF
    ω2 = sample from BSDF;
    L_ind = BSDF * color(trace(x, ω2), moreBounces-1) * dot(n, ω2) / pdf(ω2)

    return Le + L_dir + L_ind
```

Path Tracing Algorithm with NEE

$$L(x,\omega) = L_e(x,\omega) + L_{dir}(x,\omega) + L_{ind}(x,\omega)$$

```
Color color(Point x, Direction ω, int moreBounces):

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    ω1 = sample from light
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    // compute Lind by sampling the BSDF
    ω2 = sample from BSDF;
    Lind = BSDF * color(trace(x, ω2), moreBounces-1) * dot(n, ω2) / pdf(ω2)

    return Le + Ldir + Lind
```

double counting!

Path Tracing Algorithm with NEE

$$L(x, \omega) = L_e(x, \omega) + L_{dir}(x, \omega) + L_{ind}(x, \omega)$$

```
Color color(Point x, Direction ω, int moreBounces):

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    // next-event estimation: compute Ldir by sampling the light
    ω1 = sample from light
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    ω2 = sample from BSDF;
    Lind = BSDF * color(trace(x, ω2), moreBounces-1) * dot(n, ω2) / pdf(ω2)

    return Le + Ldir + Lind
```
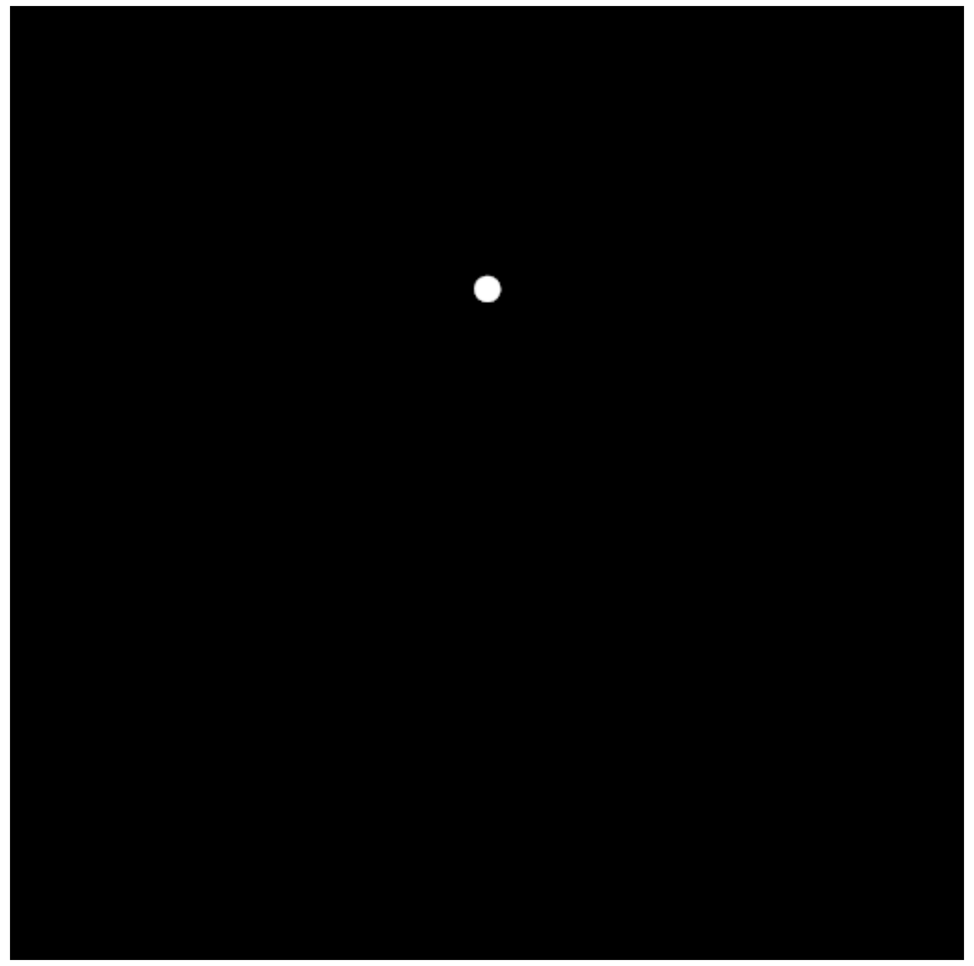

Path Tracing Algorithm with NEE

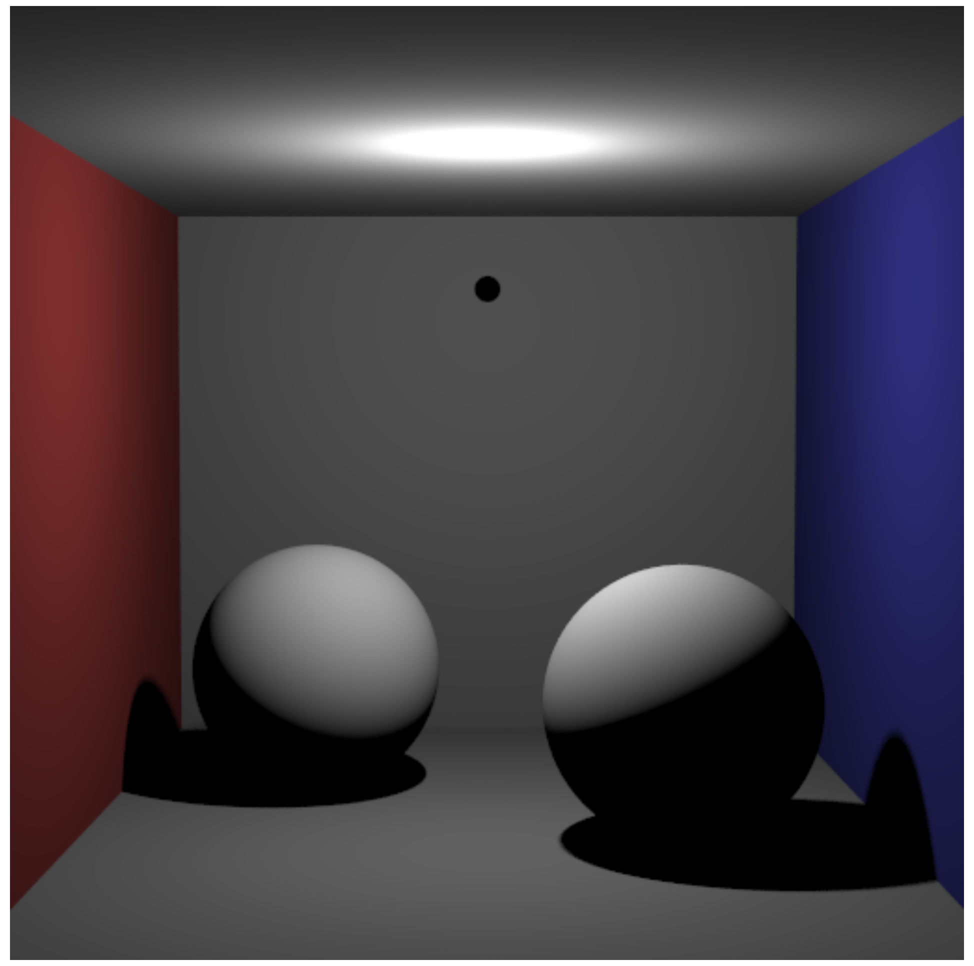
$$L(x,\omega) = L_e(x,\omega) + L_{dir}(x,\omega) + L_{ind}(x,\omega)$$

```
Color color(Point x, Direction ω, int moreBounces, bool includeLe):

    Le = includeLe ? Le(x,-ω) : black

    if not moreBounces:
        return Le

    // next-event estimation: compute Ldir by sampling the light
    ω1 = sample from light
    Ldir = BRDF * color(trace(x, ω1), 0, true) * dot(n, ω1) / pdf(ω1)

    // compute Lind by sampling the BSDF
    ω2 = sample from BSDF
    Lind = BSDF * color(trace(x, ω2), moreBounces-1, false) * dot(n, ω2) / pdf(ω2)

    return Le + Ldir + Lind
```

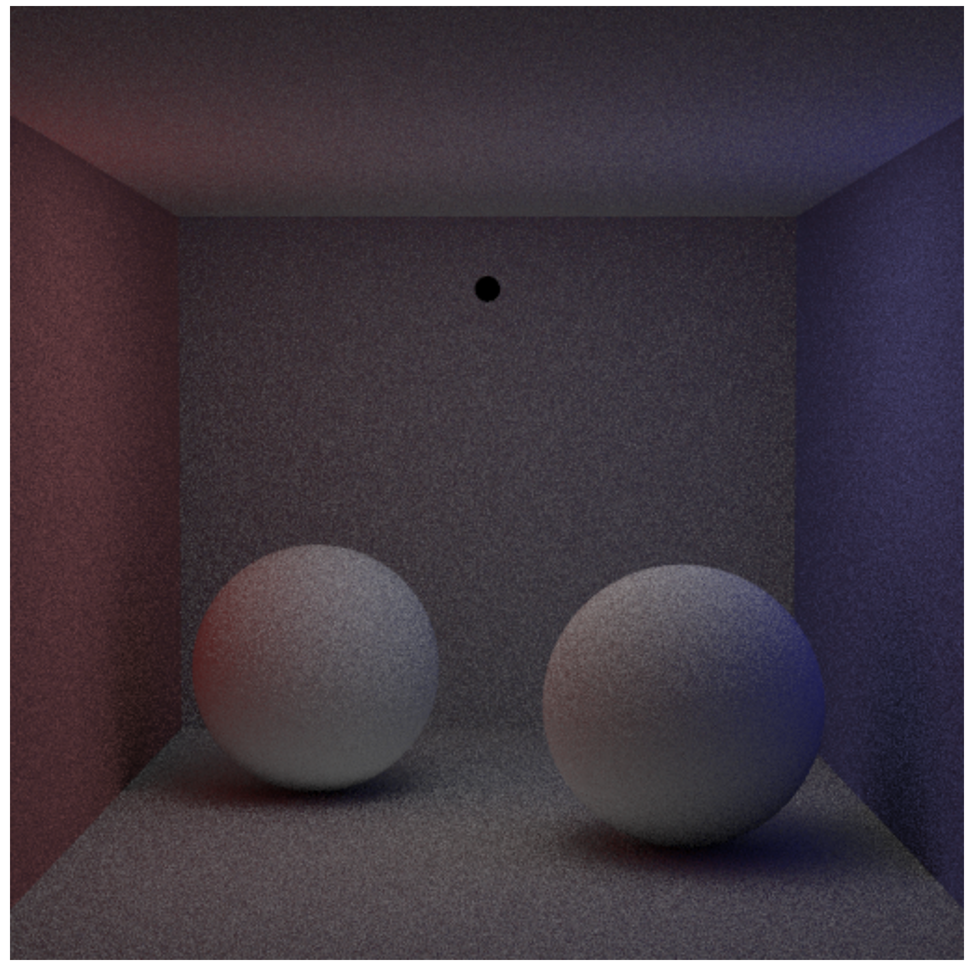
Path-wise Visualization



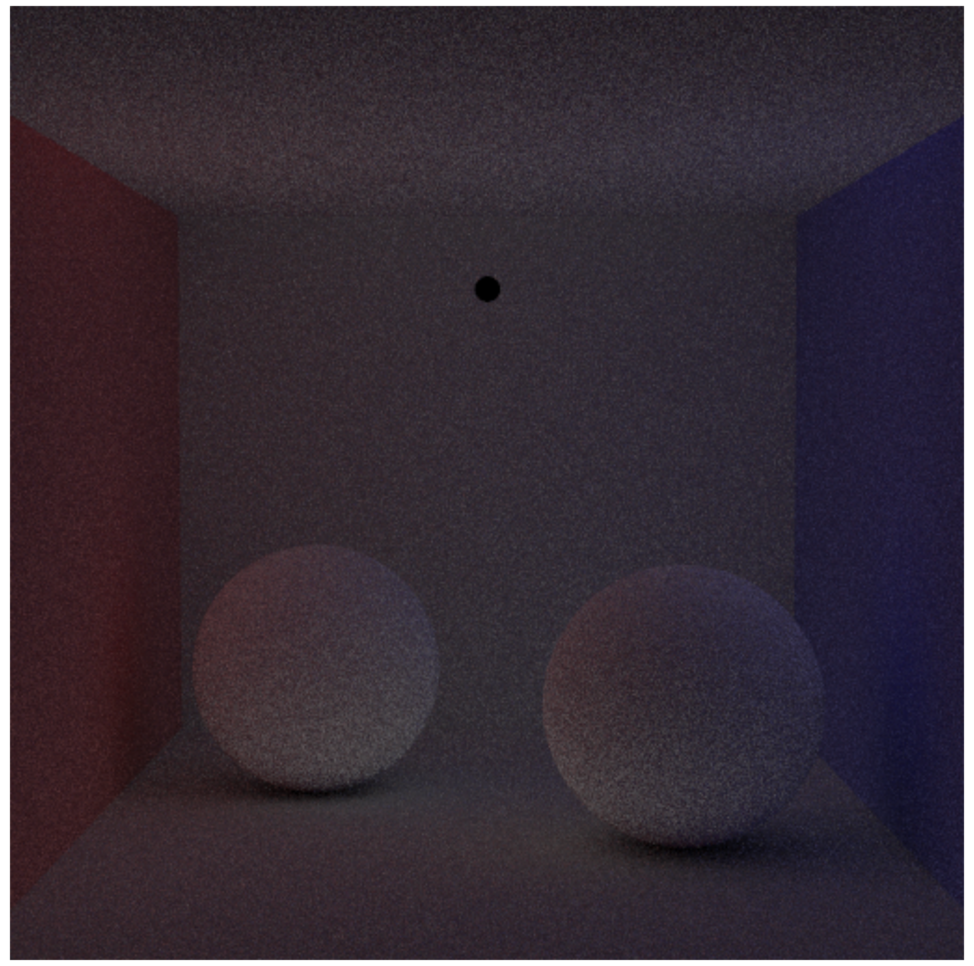
Path: 0



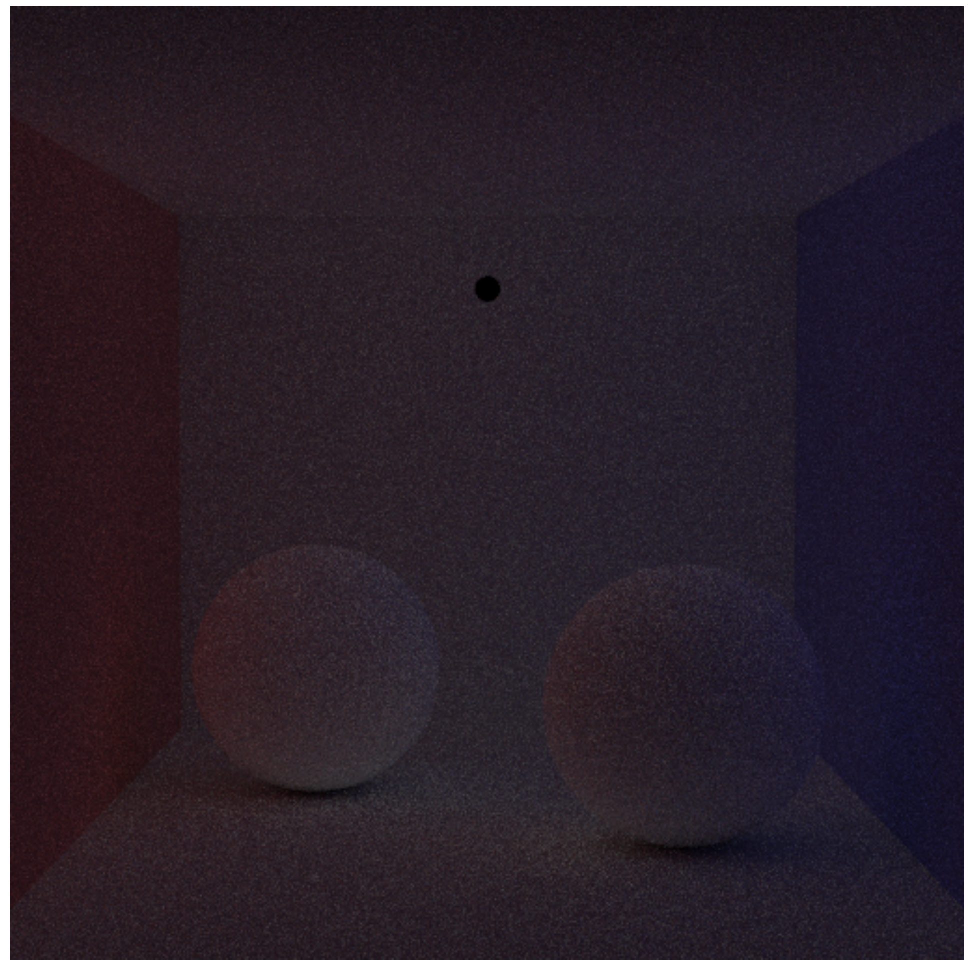
Path: 1



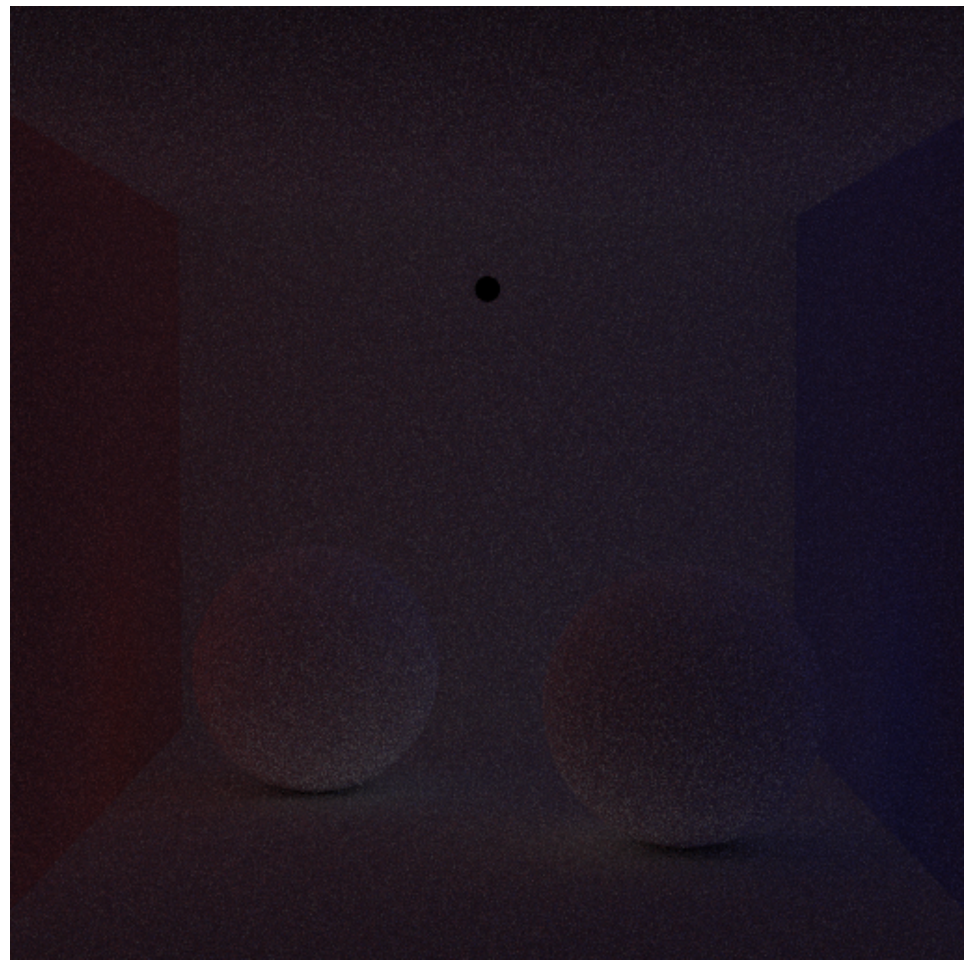
Path: 2



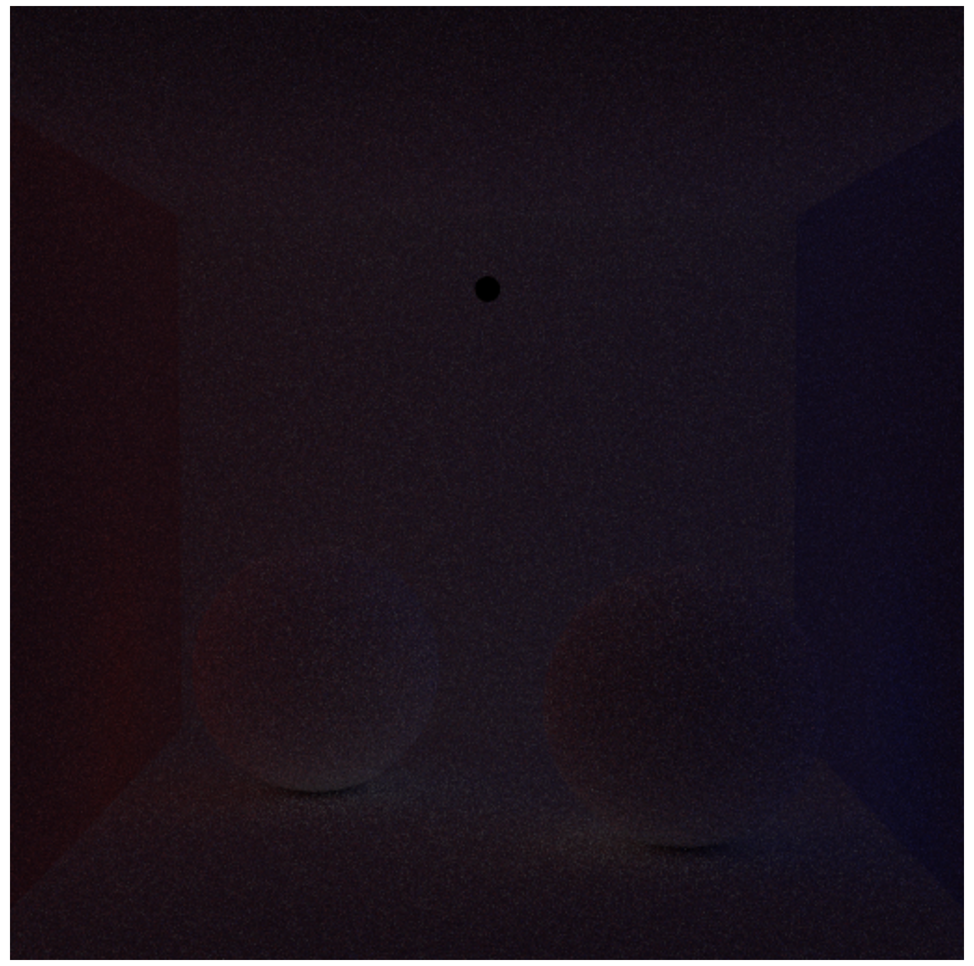
Path: 3



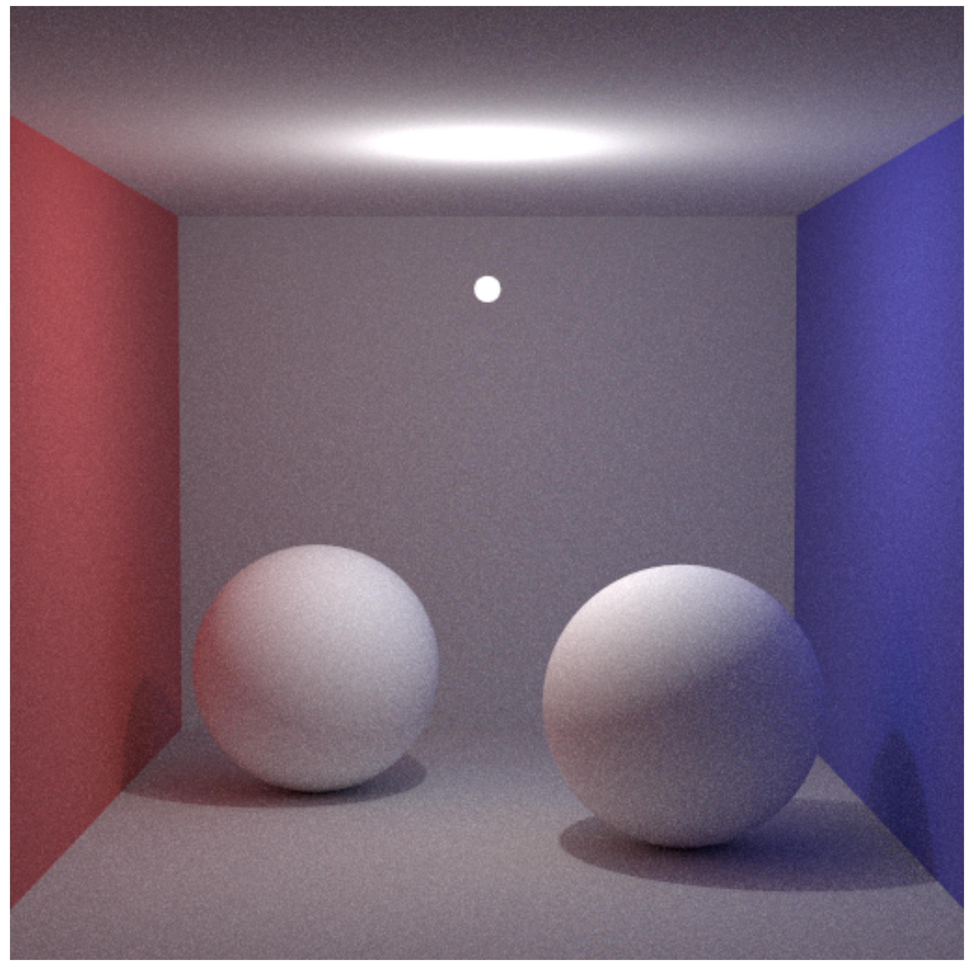
Path: 4



Path: 5



Path: 6



All Paths added

When we do stop recursion?

Truncating at some fixed depth introducing **bias**

Solution: Russian roulette

Russian Roulette

Probabilisticaly terminate the recursion

New estimator: evaluate original estimator X with probability P (but reweighted), otherwise return zero:

$$X_{rr} = \begin{cases} \frac{X}{P} & \xi < P \\ 0 & \text{otherwise} \end{cases}$$

Russian Roulette

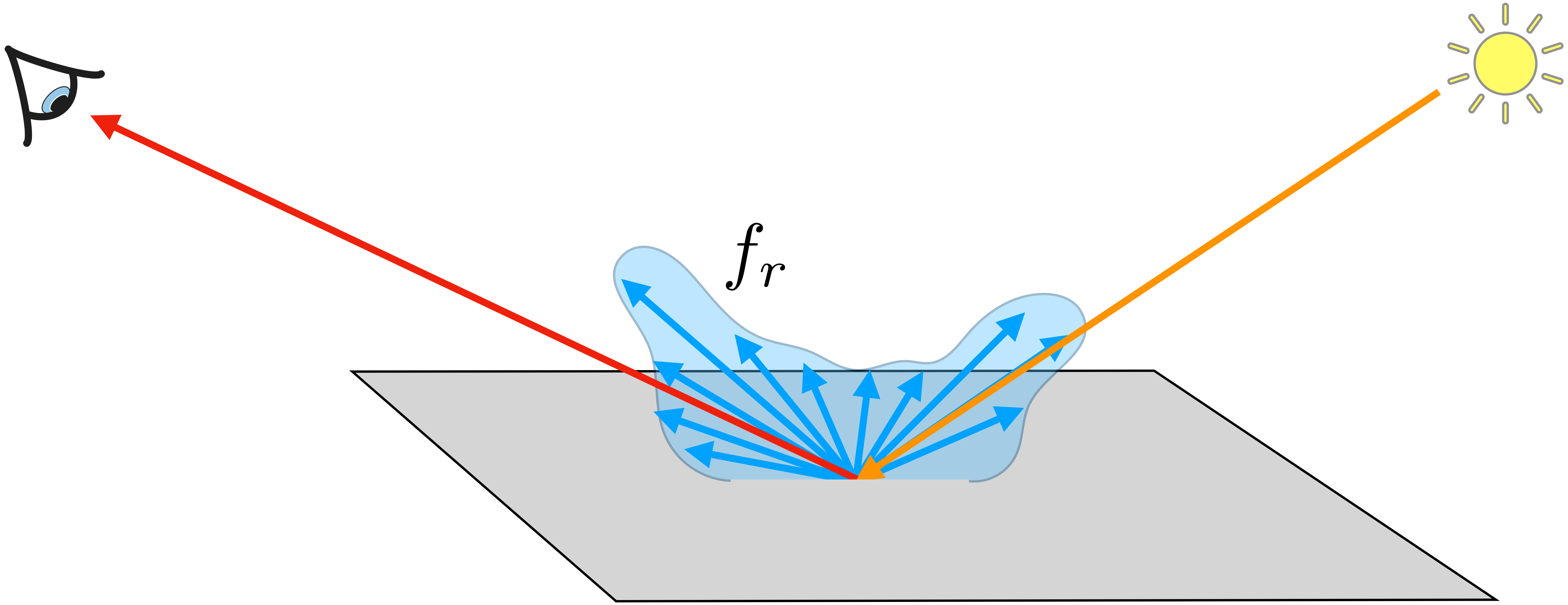
This will increase variance!

- but it will improve efficiency if P is chosen so that the samples that are expensive, but are likely to make small contribution, are skipped

Microfacet BSDFs

BRDF

Bidirectional Reflectance Distribution Function



BRDF Properties

Real/Physically plausible BRDFs obey:

- Energy conservation:

$$\int_{\mathcal{H}^2} f_r(\mathrm{x}, \vec{\omega}_i, \vec{\omega}_r) \cos\theta_i d\vec{\omega}_i \leq 1, \quad \forall\, \vec{\omega}_r$$

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- Helmholtz reciprocity:

$$f_r(\mathrm{x}, \vec{\omega}_i, \vec{\omega}_r) = f_r(\mathrm{x}, \vec{\omega}_r, \vec{\omega}_i)$$

$$f_r(\mathrm{x}, \vec{\omega}_i \leftrightarrow \vec{\omega}_r)$$

Conductors vs. Dielectrics

Copper

Iron

Gold

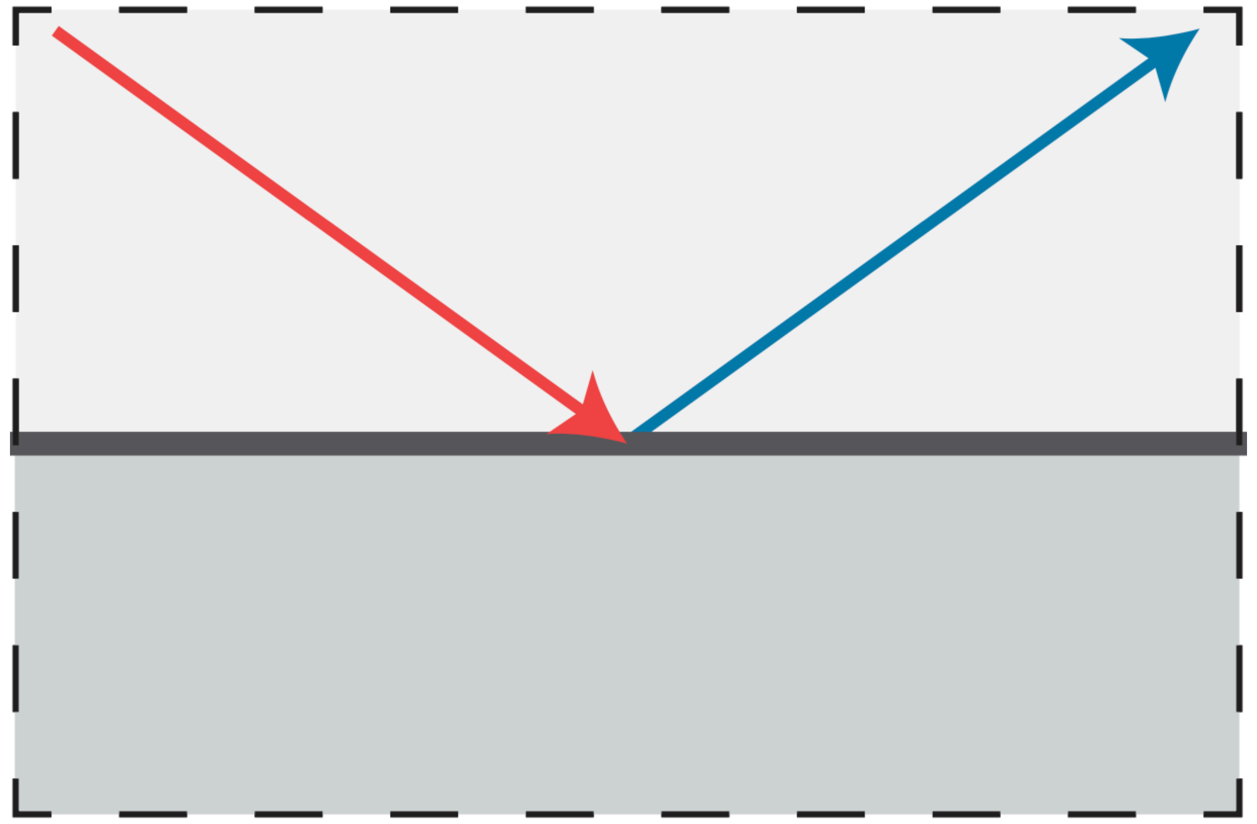
Glass

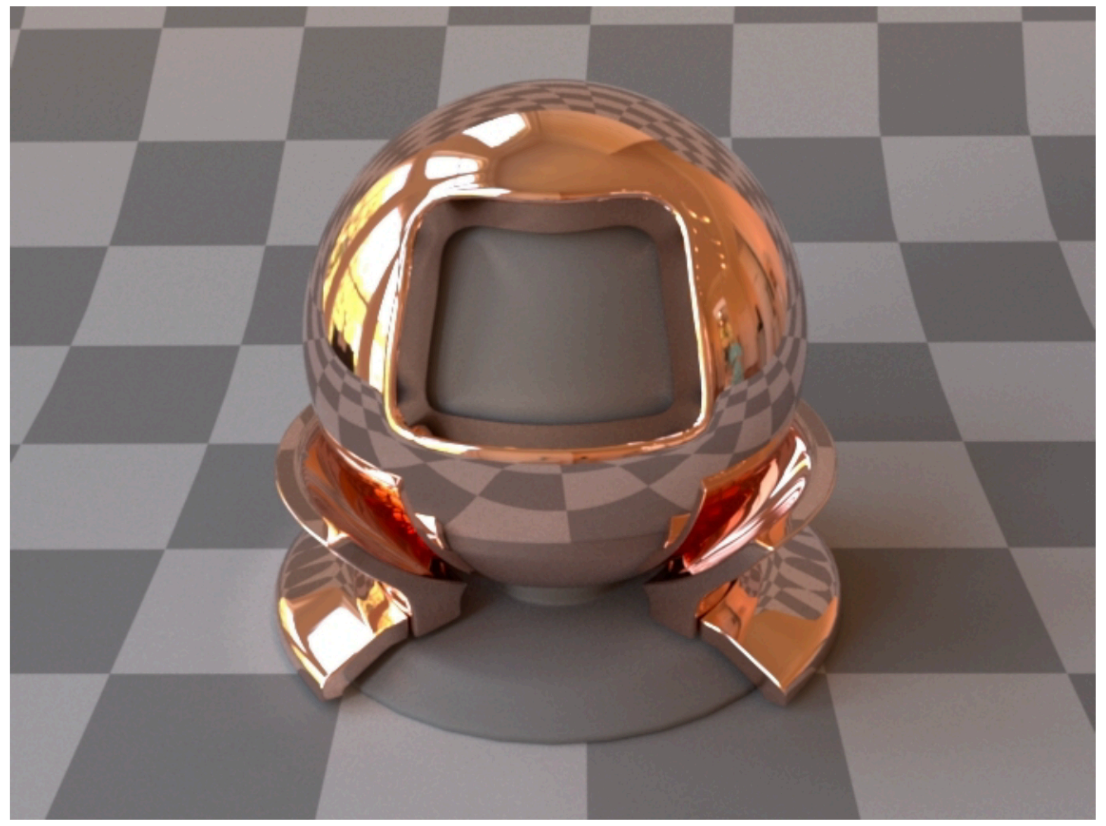
Crystal rocks

Mercury

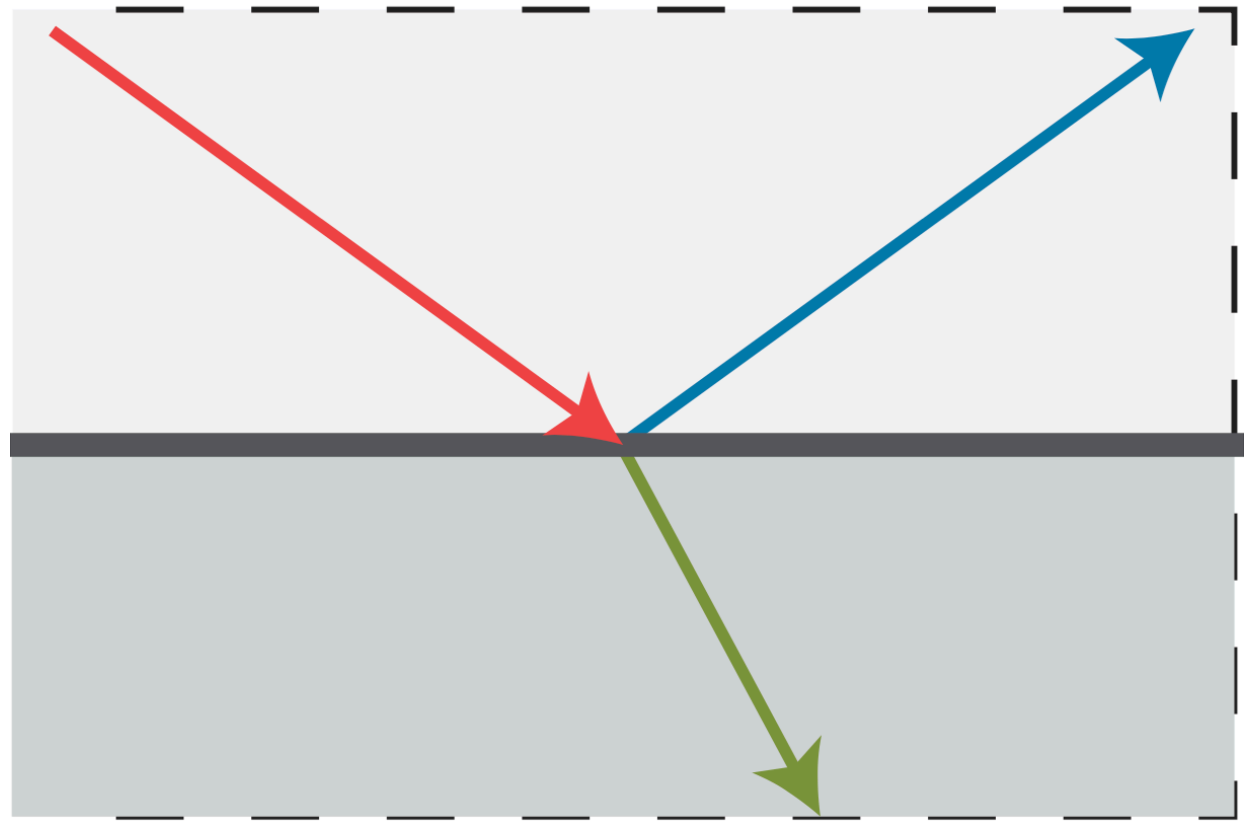
Clouds

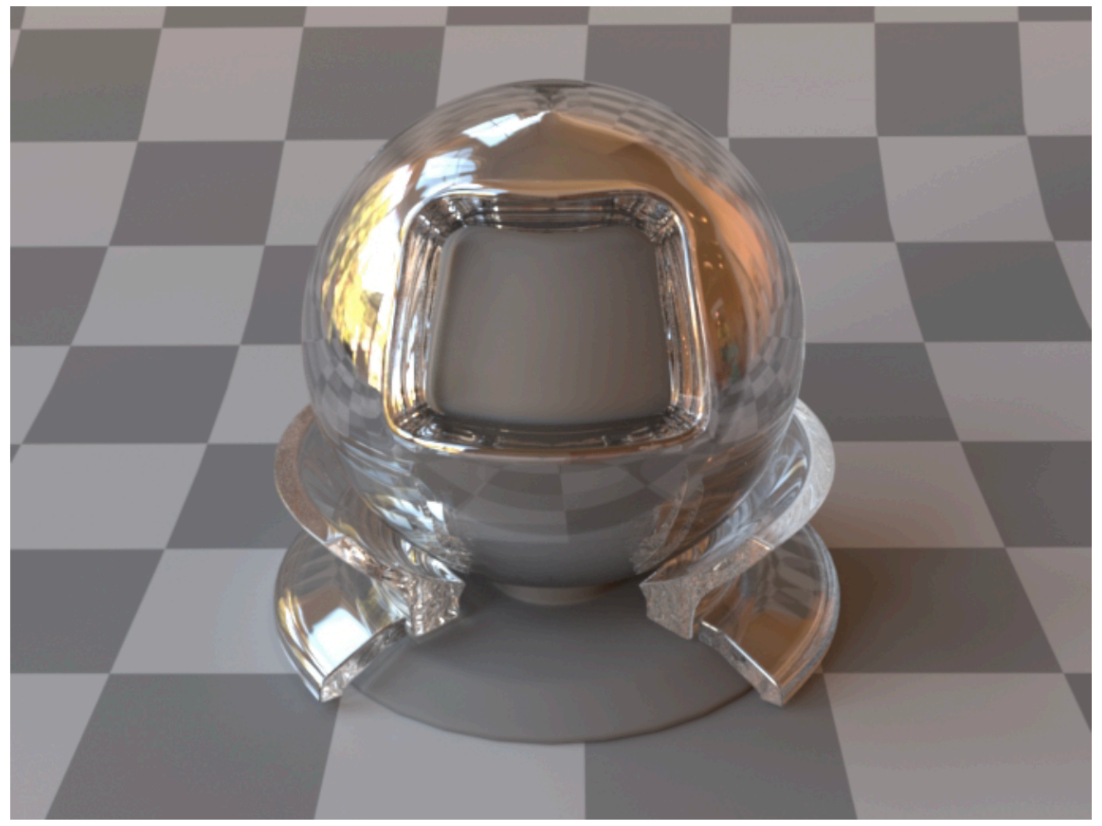
Conductors vs. Dielectrics





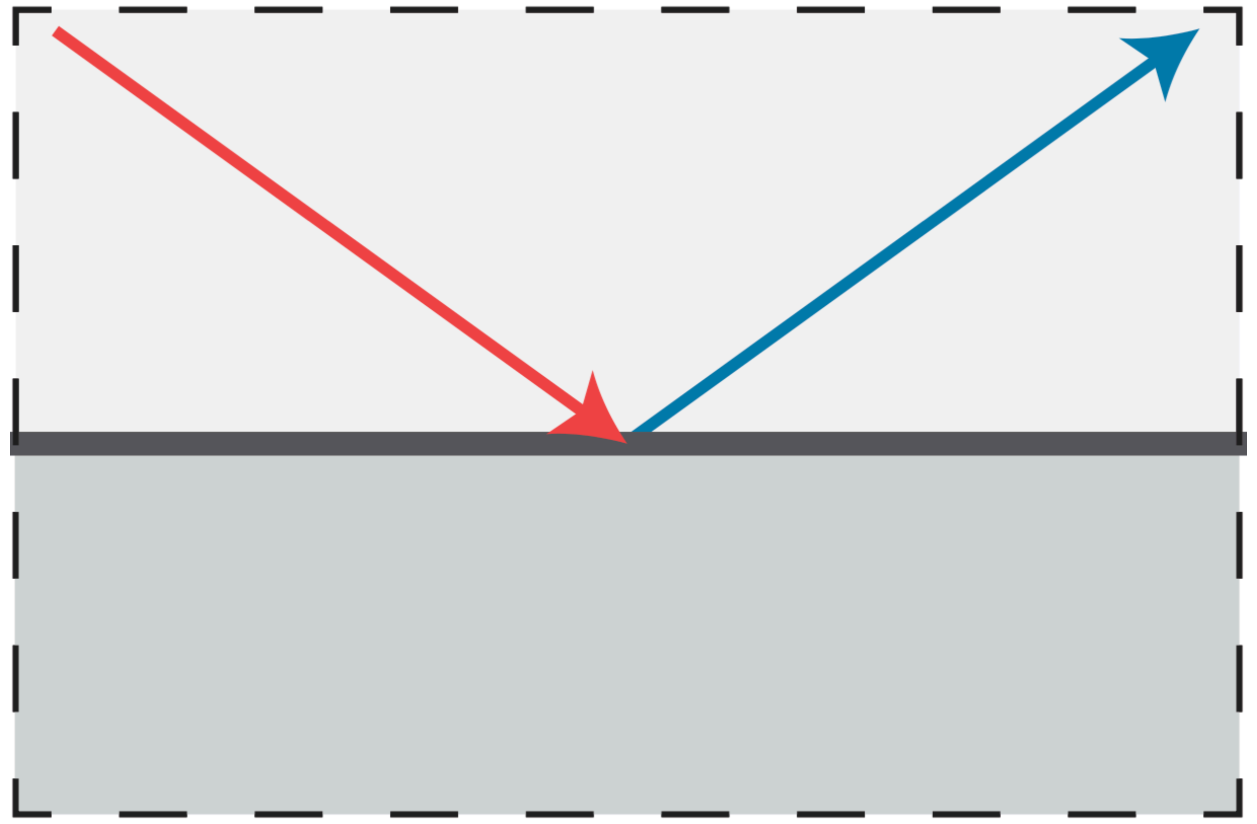
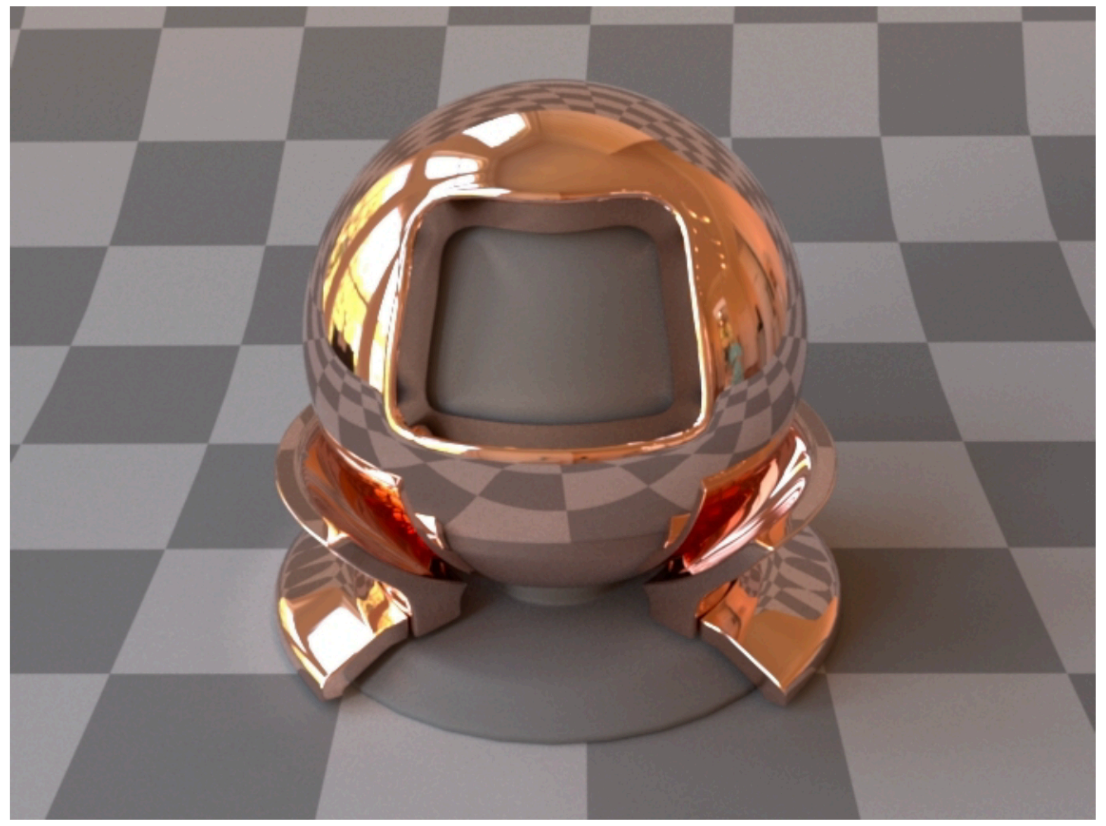
Smooth conducting material





Smooth dielectric material

Conductors vs. Dielectrics

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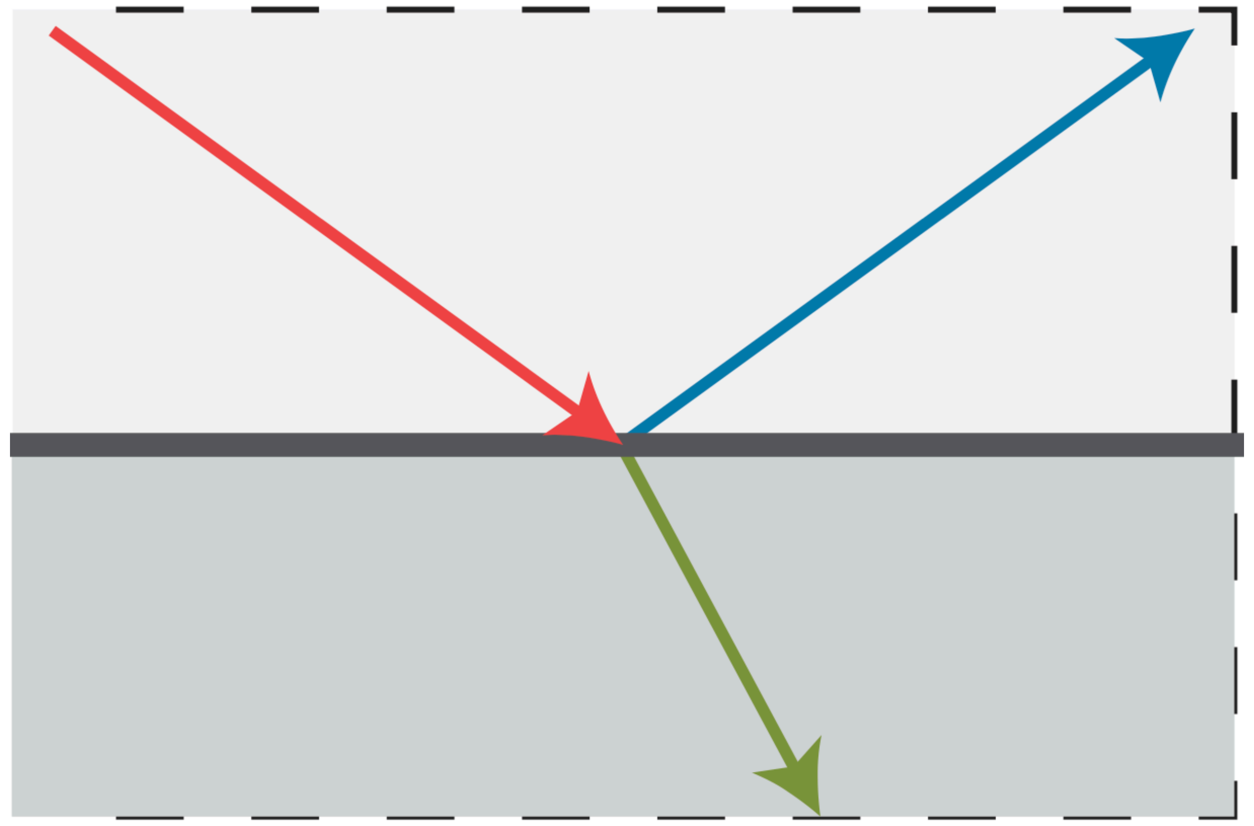
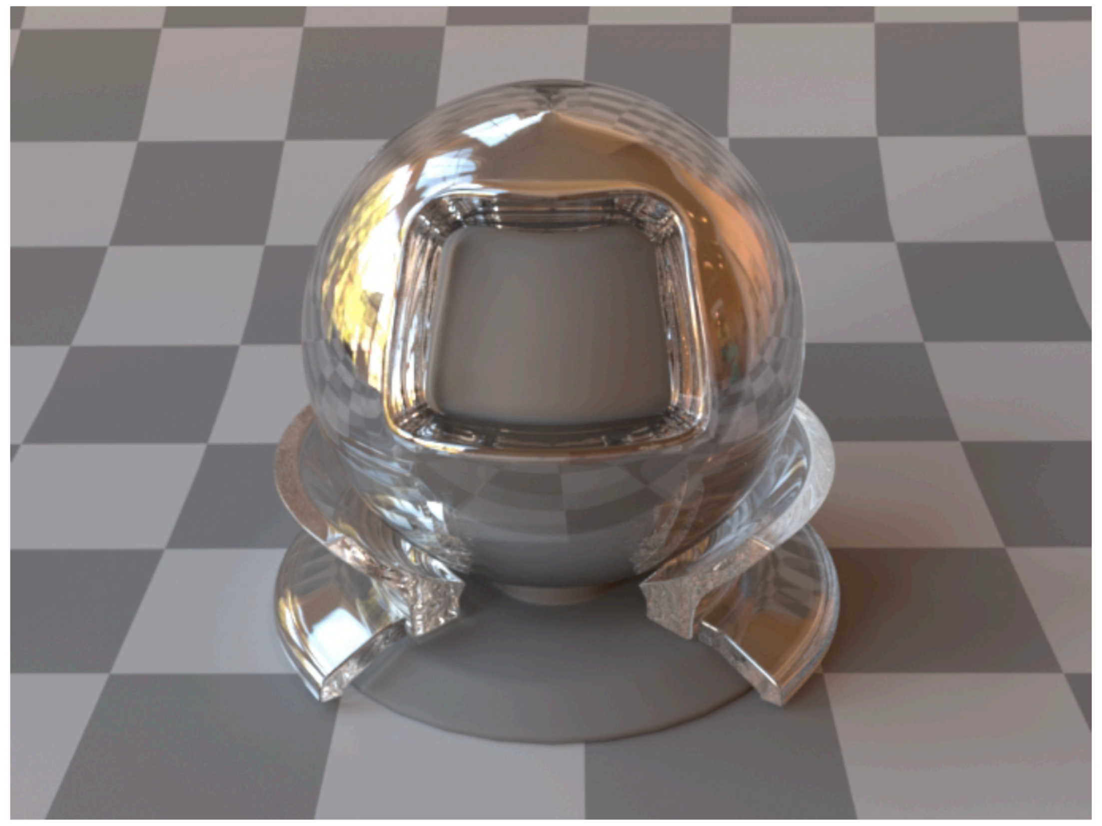

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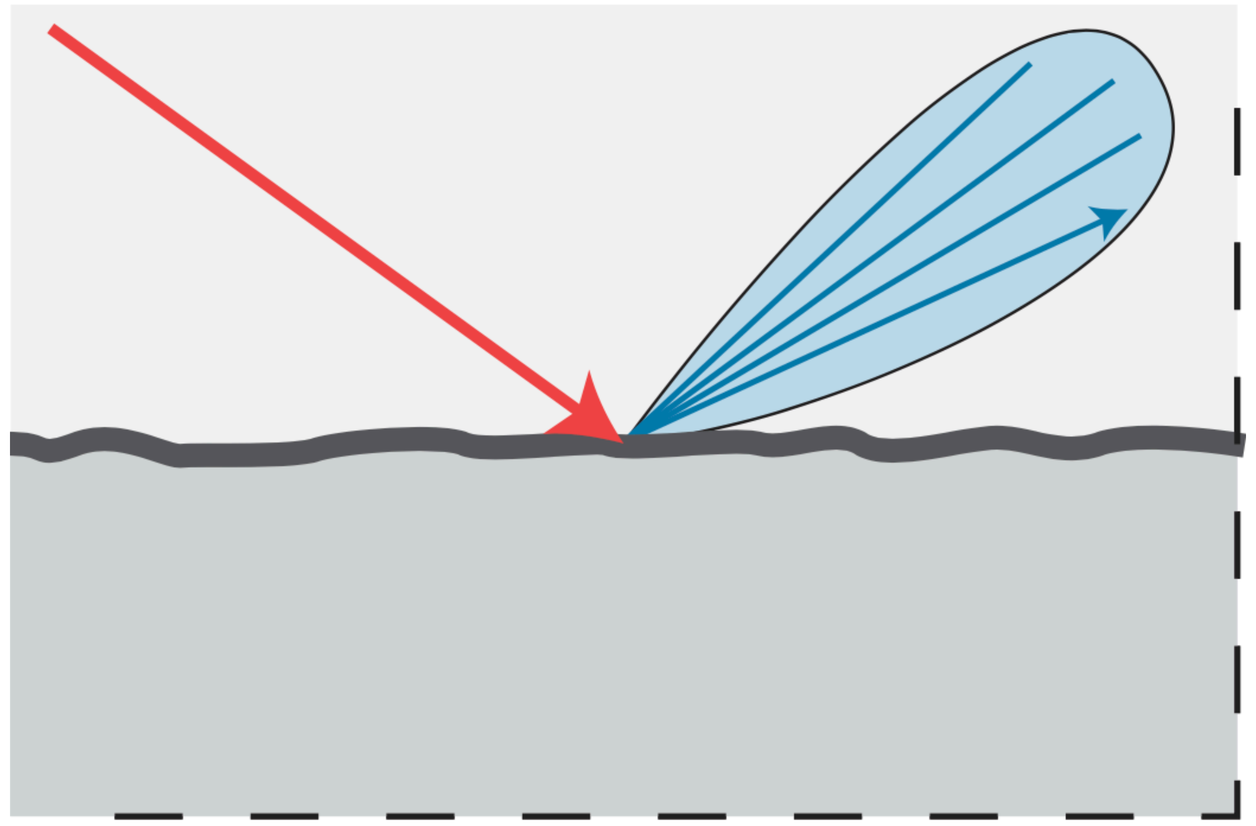
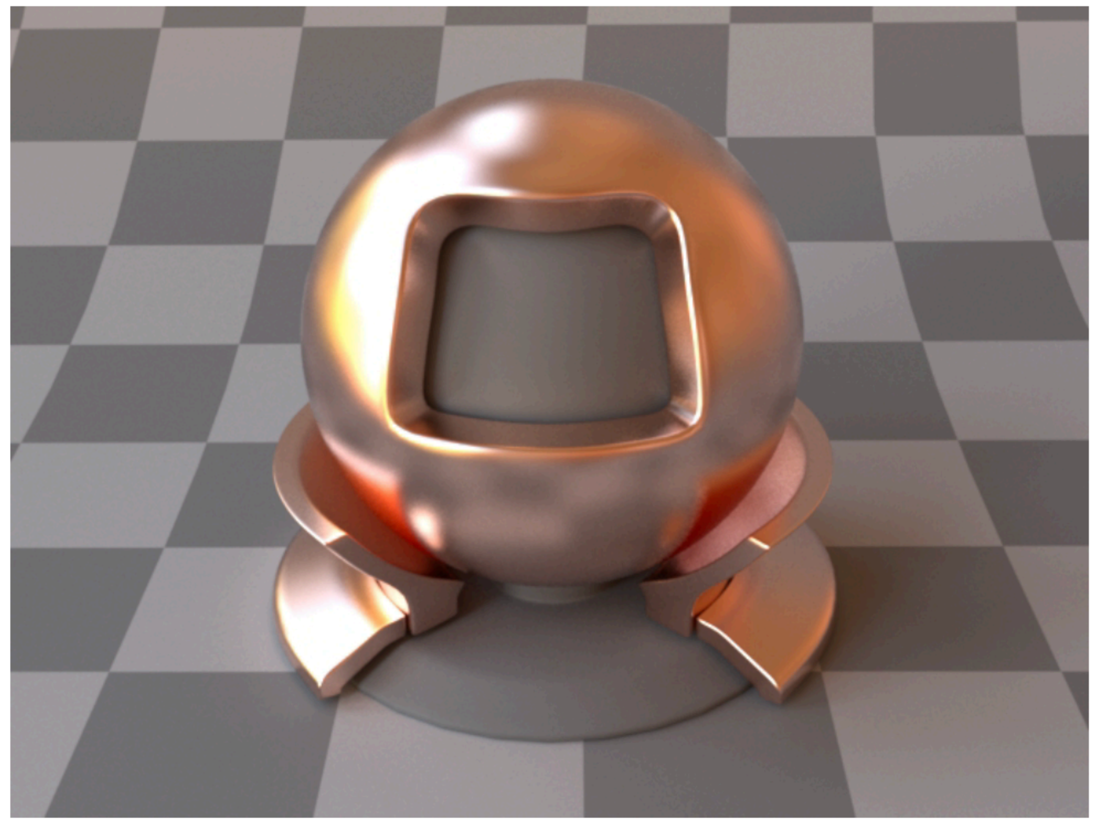
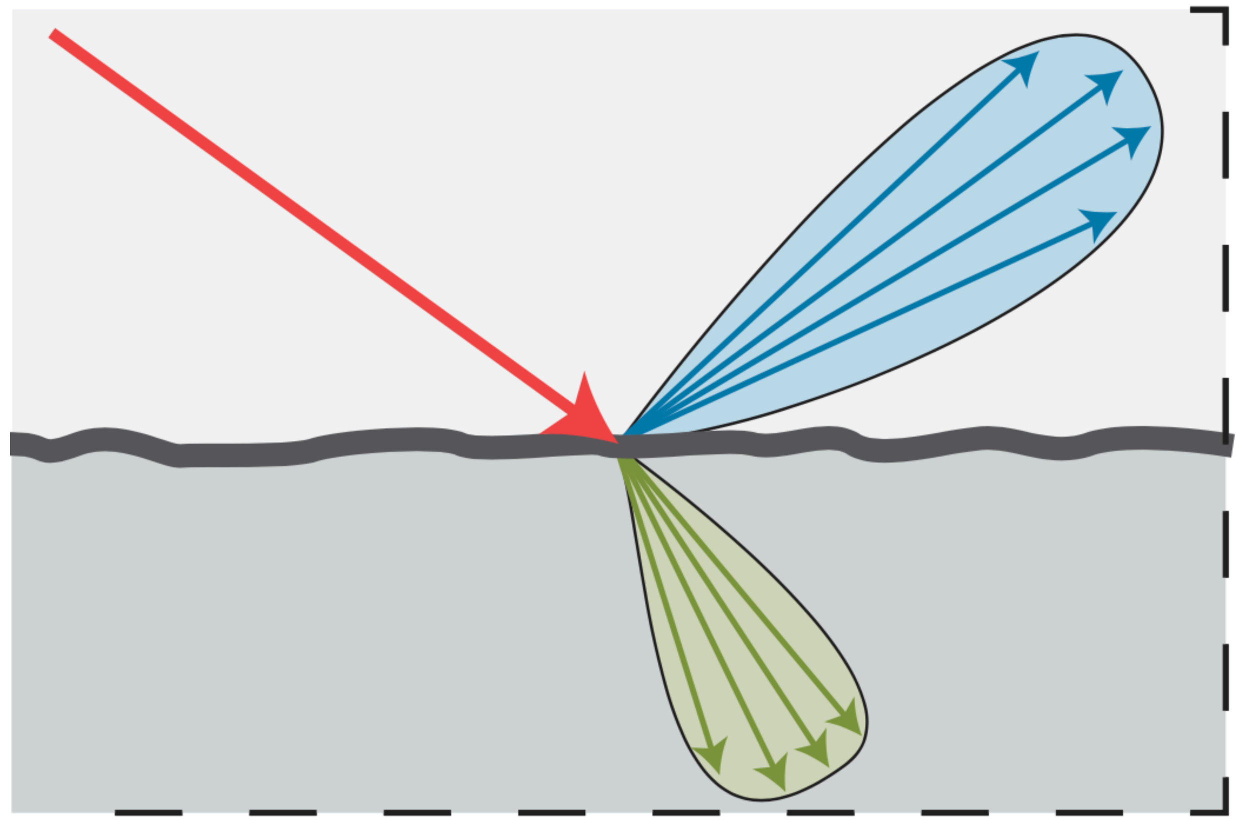
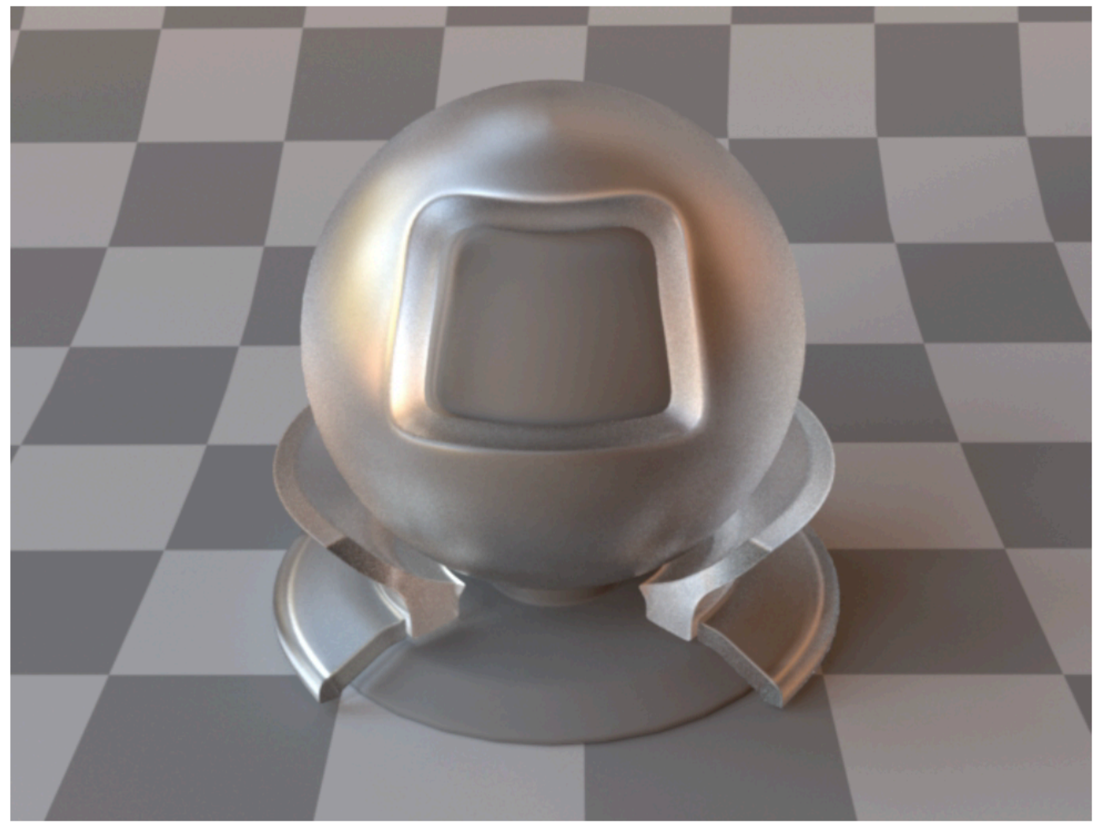



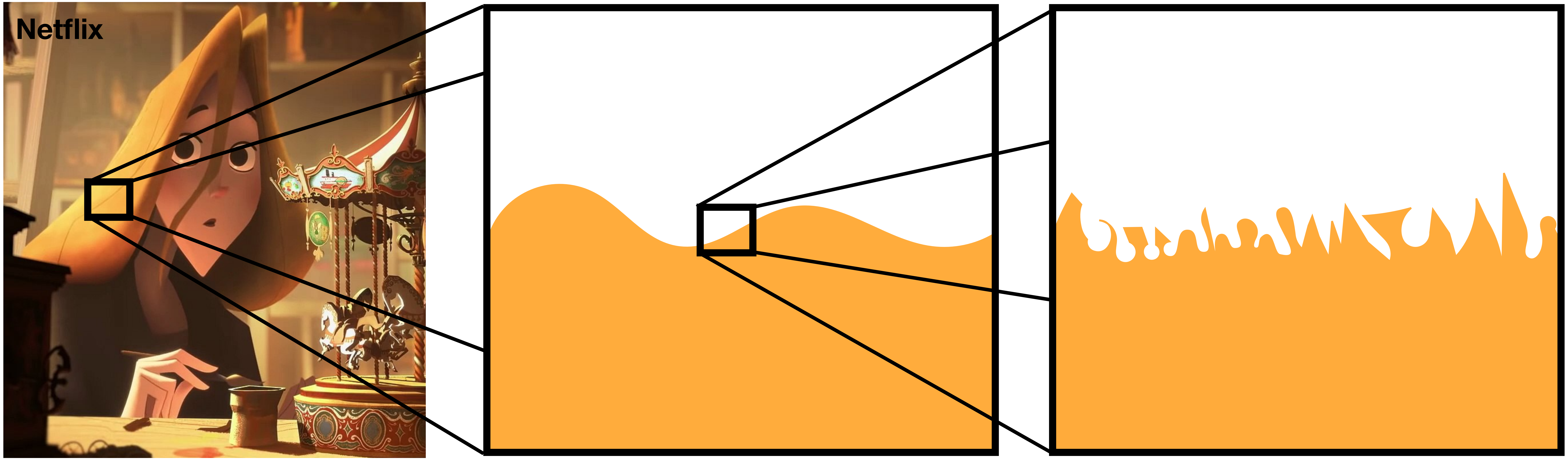
Rough conducting material

Rough dielectric material

Three Levels of Detail

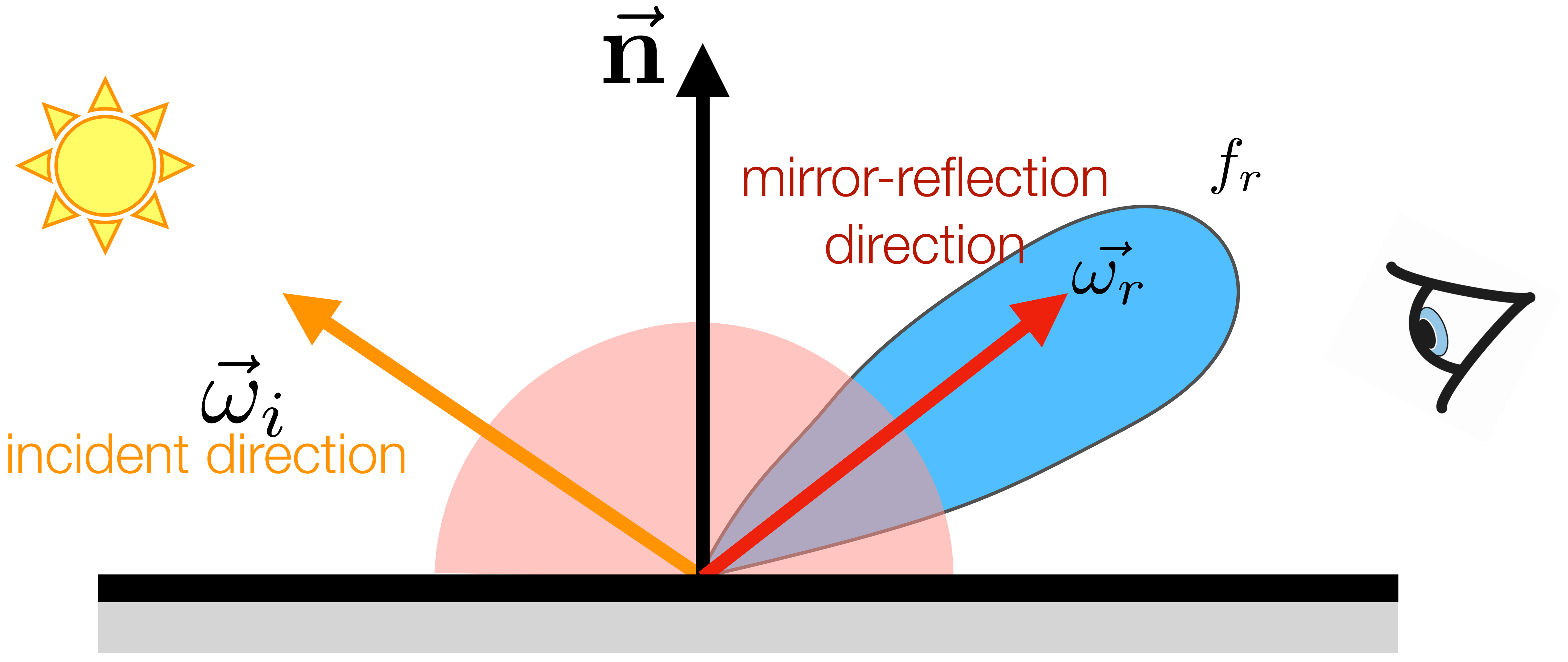
Key Idea: transition from individual interactions to statistical averages



Macro Scale	Meso Scale	Micro Scale
Scene geometry	Detail at intermediate scale	Roughness

Phong BRDF

Reflection direction distributed over an exponentiated cosine lobe:



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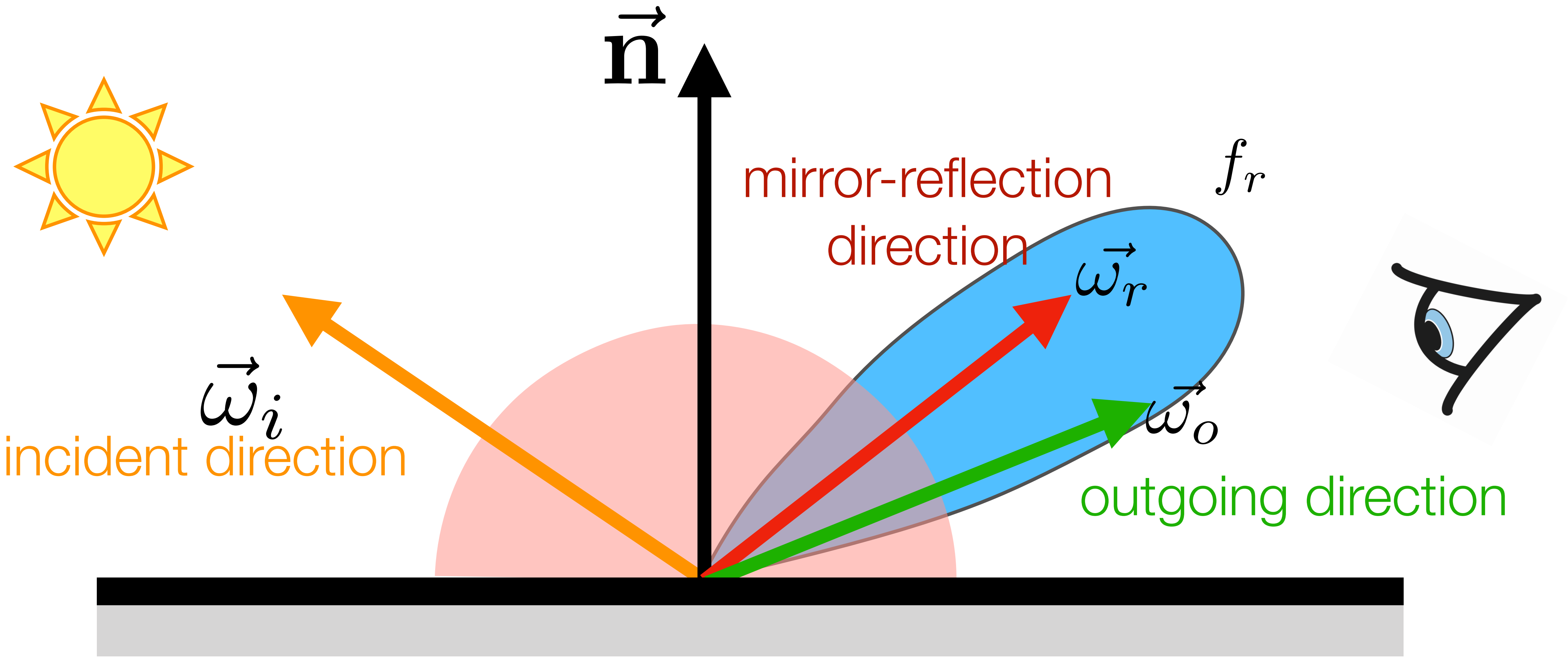
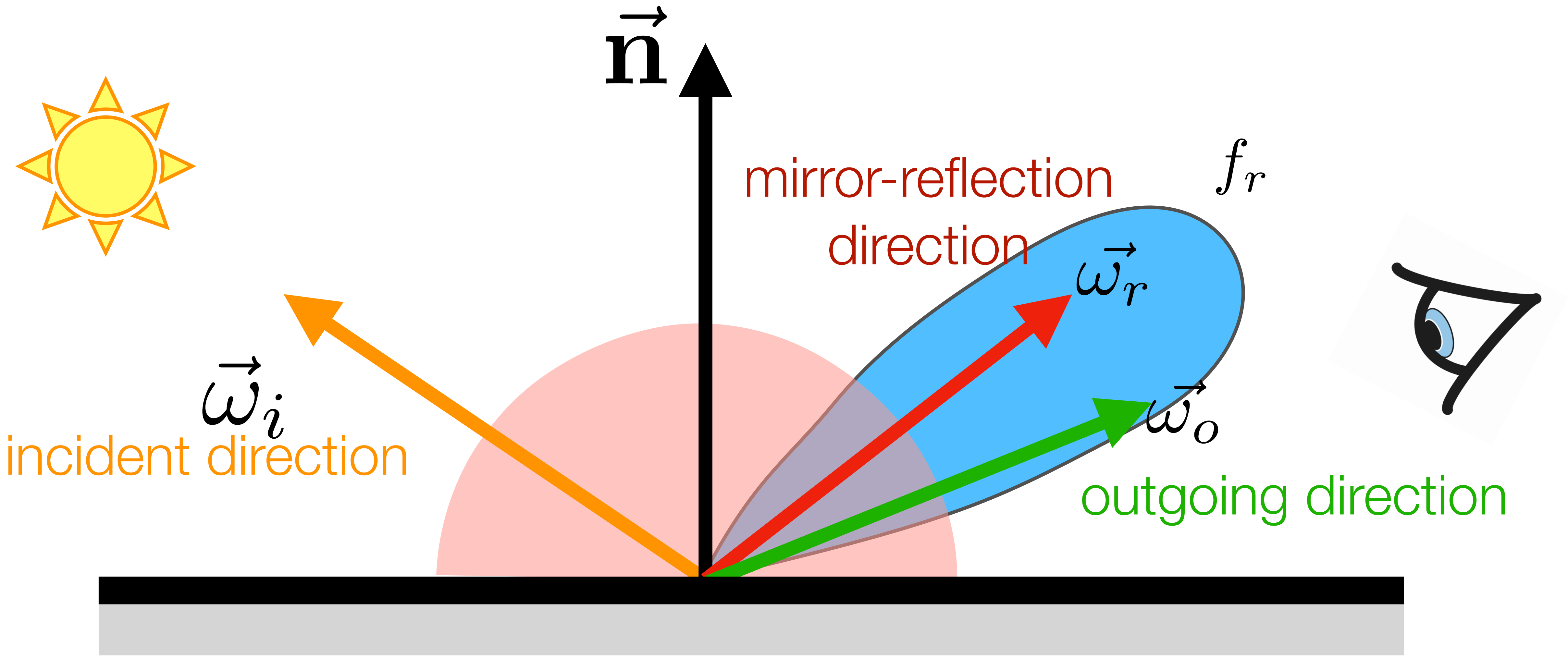


Phong BRDF

Reflection direction distributed over an exponentiated cosine lobe:

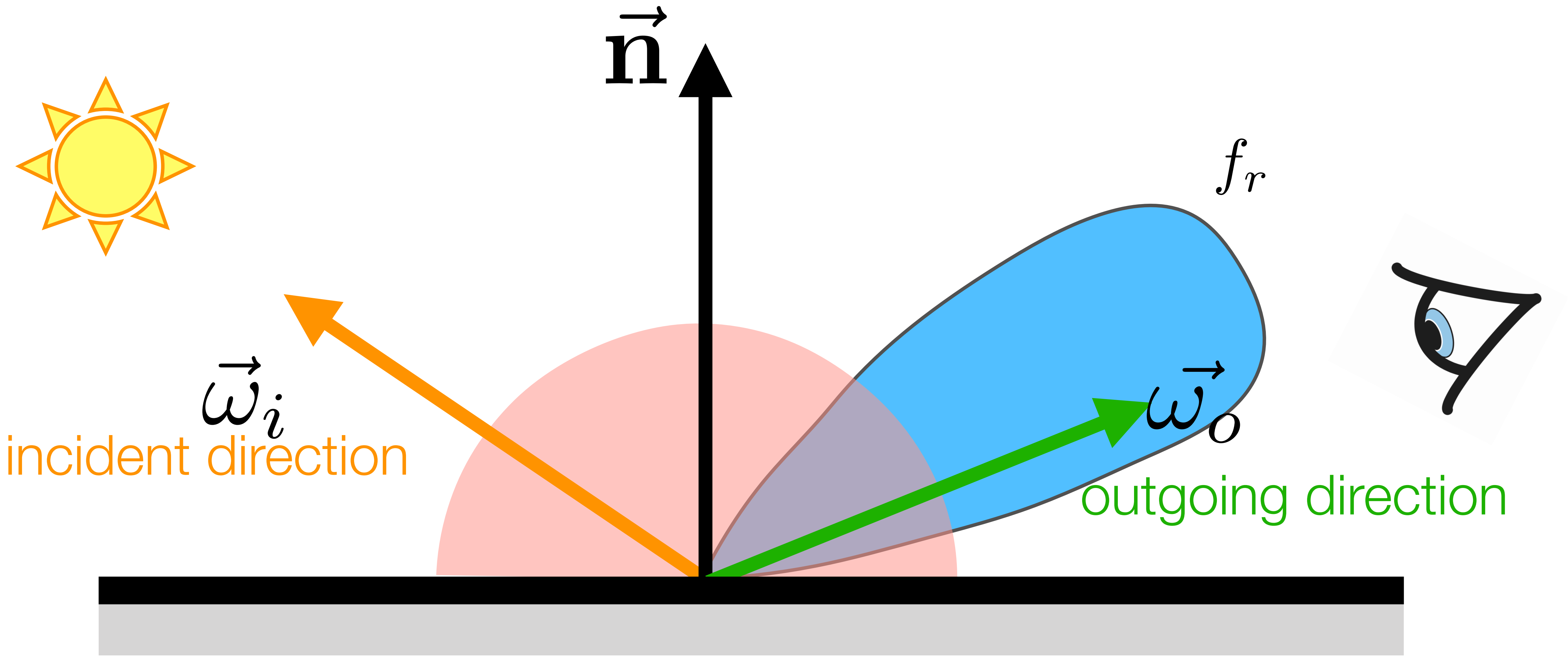
$$f_r(\vec{\omega}_o, \vec{\omega}_i) = \frac{e+2}{2\pi}(\vec{\omega_r} \cdot \vec{\omega_o})^e$$

$$\vec{\omega_r} = (2\vec{\mathbf{n}}(\vec{\mathbf{n}} \cdot \vec{\omega}_i) - \vec{\omega}_i)$$



Blinn-Phong BRDF

Reflection direction distributed over an exponentiated cosine lobe:



Blinn-Phong BRDF

Reflection direction distributed over an exponentiated cosine lobe:

$$\vec{\omega}_h = \frac{\vec{\omega}_i + \vec{\omega}_o}{||\vec{\omega}_i + \vec{\omega}_o||}$$

$$f_r(\vec{\omega}_o, \vec{\omega}_i) = \frac{e+2}{2\pi}(\vec{\omega}_h \cdot \vec{\mathbf{n}})^e$$

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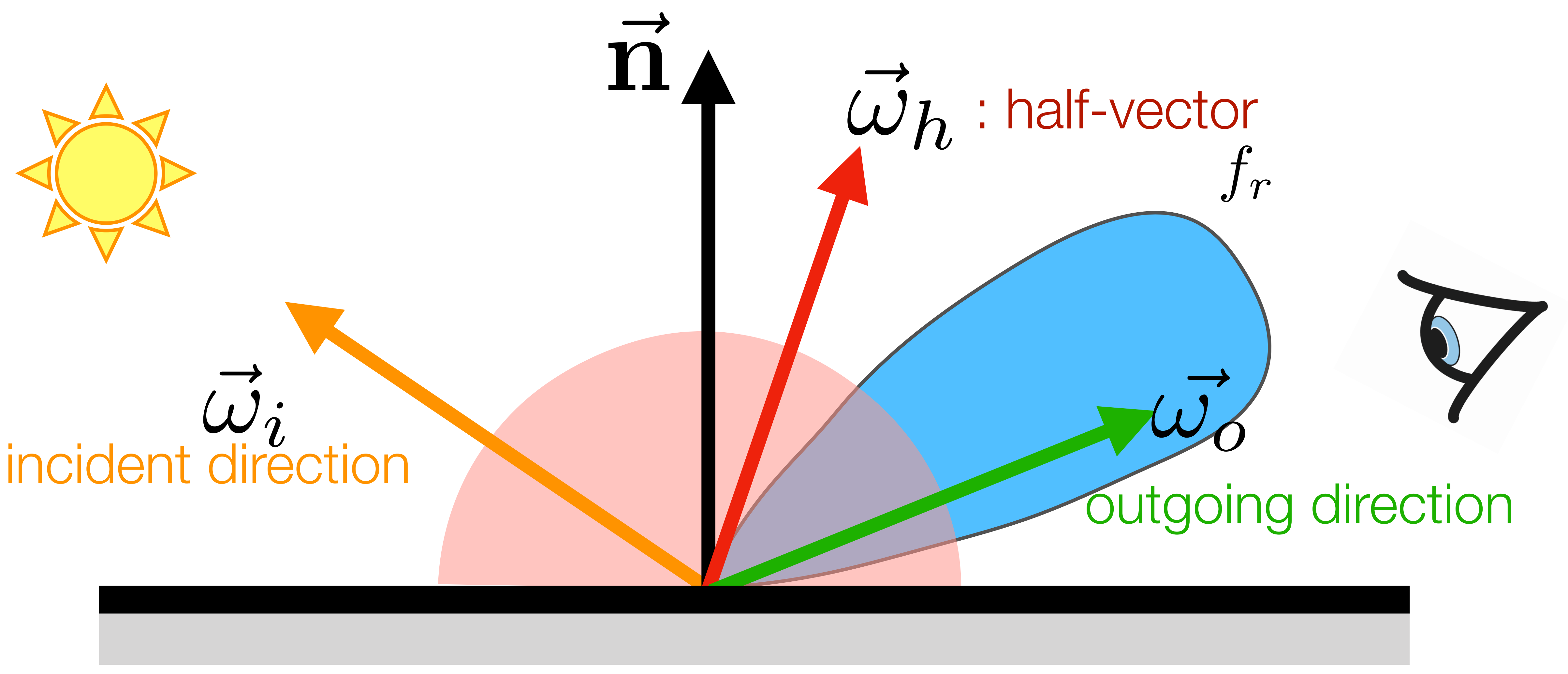

Rough Surfaces

Empirical glossy models have limitations:

- not physically-based
- (often) not reciprocal

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- (often) no Fresnel effects
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- (often) no Fresnel effects
- cannot accurately model appearance of many glossy surfaces

Blinn-Phong was first step in the right direction

Can do Better

Microfacet Theory

Microfacet Theory

In geometric-optics-based approaches, rough surfaces can be modeled as a collection of small microfacets.

Surfaces comprised of microfacets are often modeled as heightfields, where the distribution of facet orientations is described statistically

Microfacet Theory

Assume surface consists of tiny facets

Assume that the differential area being viewed/illuminated is relatively large compared to the size of microfacets

A facet can be perfectly specular or diffuse

Torrance-Sparrow Model

Developed by Torrance & Sparrow in 1967

Originally used in the physics community

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- added ambient and diffuse terms

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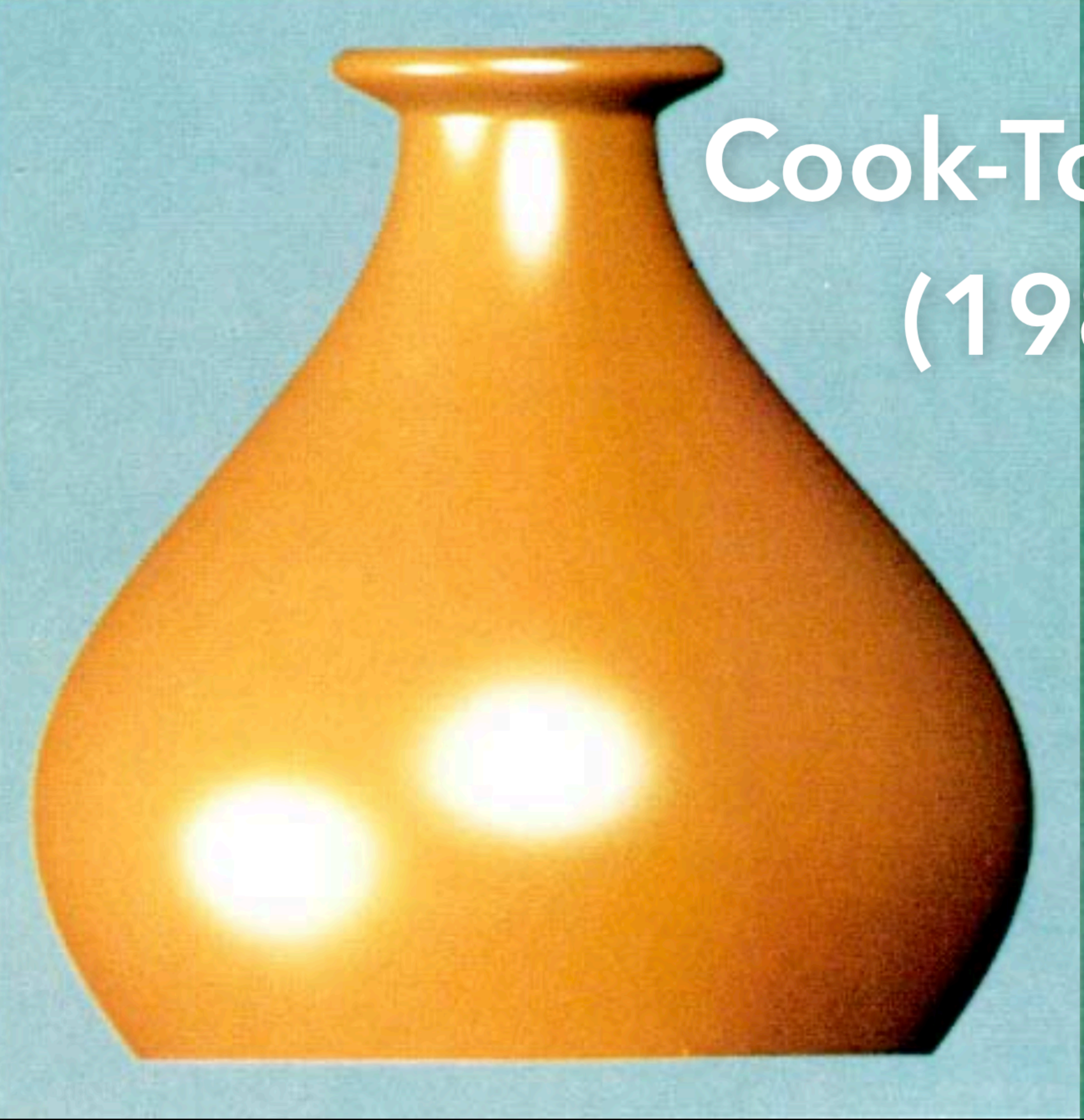
- added ambient and diffuse terms

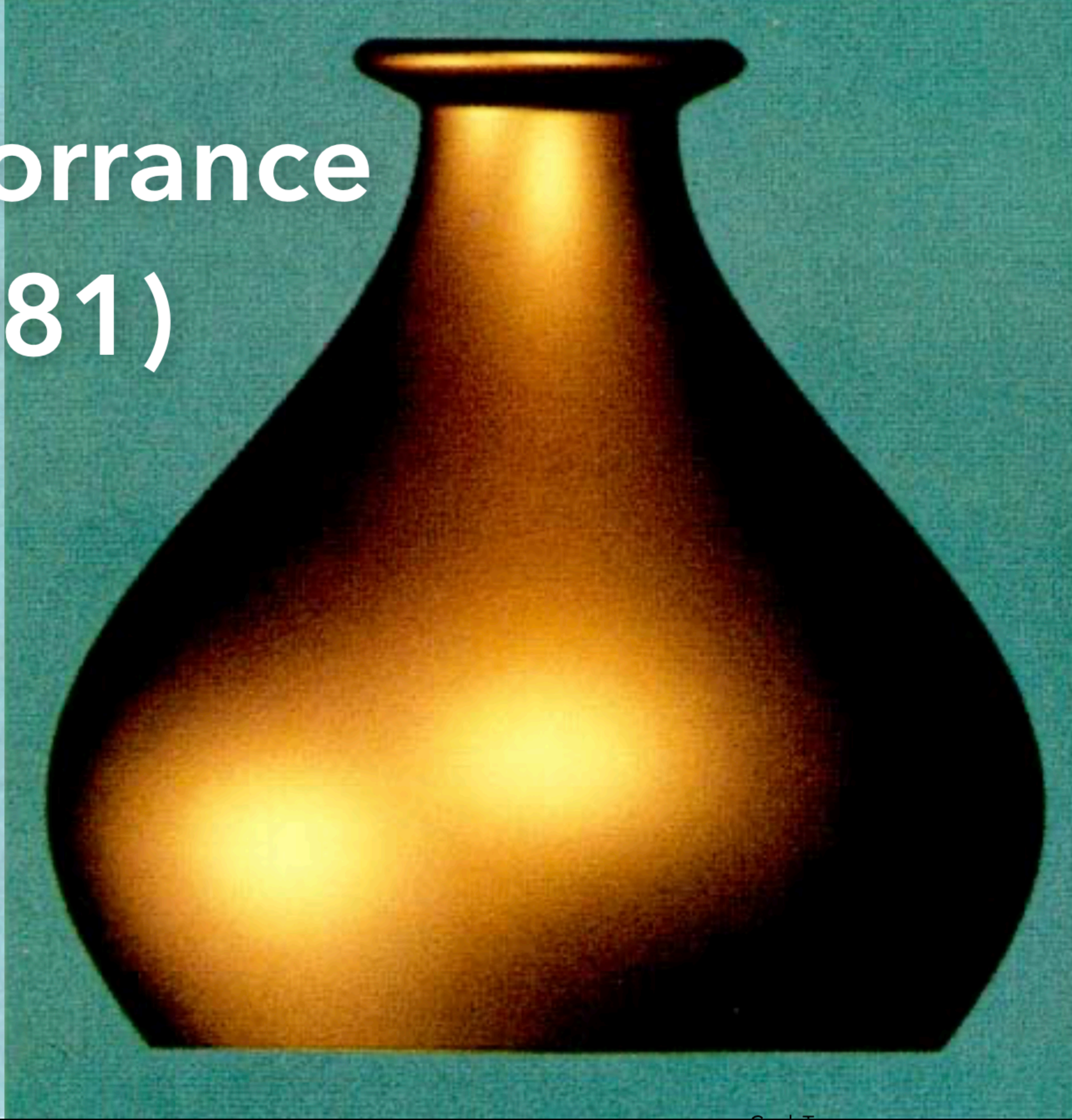
Explain off-specular peaks

Assumes surface is composed of many micro-grooves, each of which is a perfect mirror

Cook-Torrance (1981)



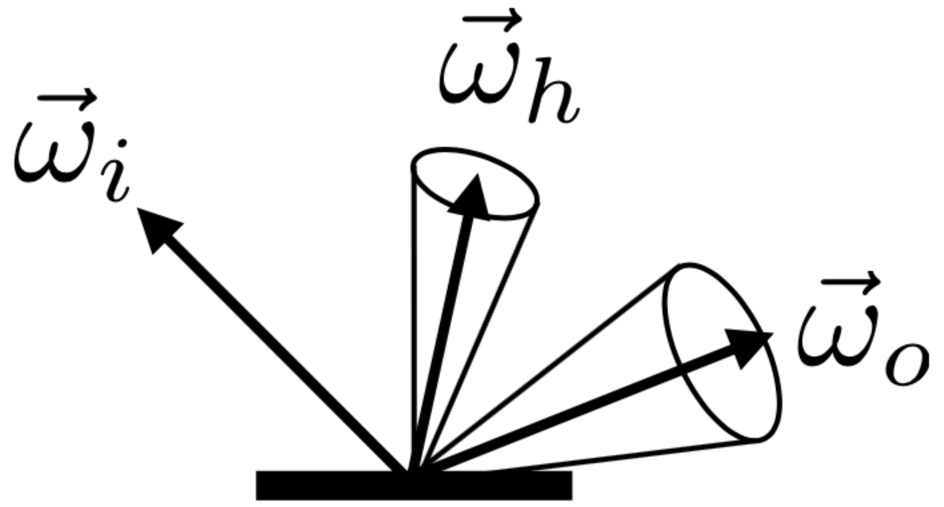
Copper-colored plastic



Copper

General Microfacet Model

$$f(\vec{\omega}_i, \vec{\omega}_o) = \frac{F(\vec{\omega}_h, \vec{\omega}_o) \cdot D(\vec{\omega}_h) \cdot G(\vec{\omega}_i, \vec{\omega}_o)}{4|(\vec{\omega}_i \cdot \mathbf{\vec{n}})(\vec{\omega}_o \cdot \mathbf{\vec{n}})|}$$



$$\vec{\omega}_h = \frac{\vec{\omega}_i + \vec{\omega}_o}{\|\vec{\omega}_i + \vec{\omega}_o\|}$$

General Microfacet Model

Fresnel coefficient

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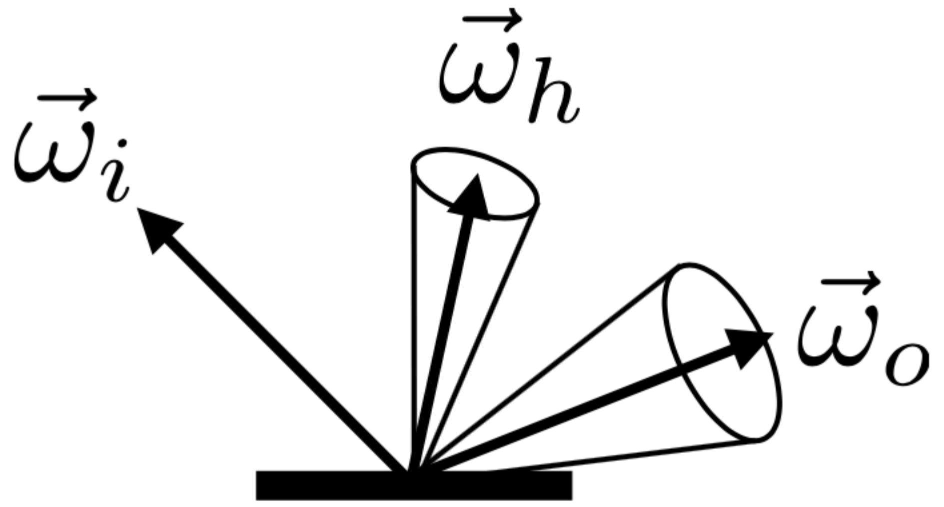

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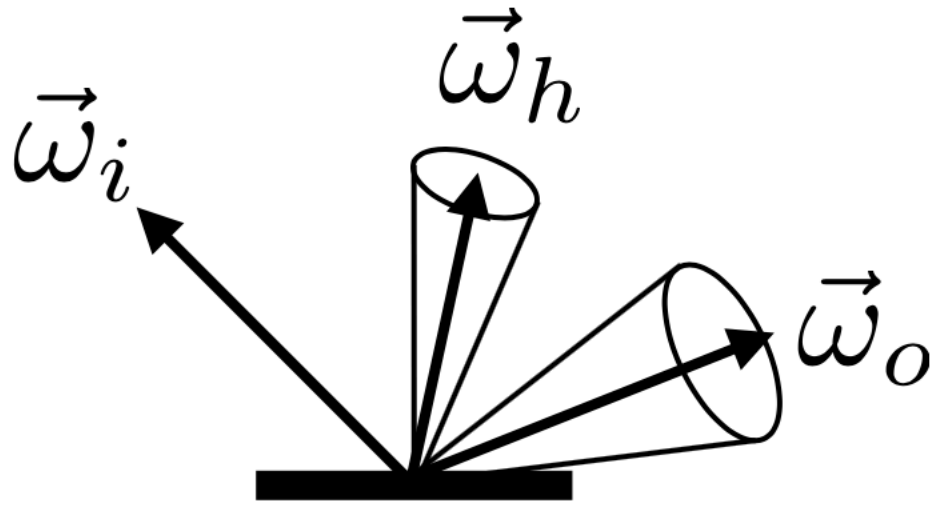
General Microfacet Model

Fresnel coefficient

Microfacet distribution

$$f(\vec{\omega}_i, \vec{\omega}_o) = \frac{F(\vec{\omega}_h, \vec{\omega}_o) \cdot D(\vec{\omega}_h) \cdot G(\vec{\omega}_i, \vec{\omega}_o)}{4|(\vec{\omega}_i \cdot \vec{\mathbf{n}})(\vec{\omega}_o \cdot \vec{\mathbf{n}})|}$$



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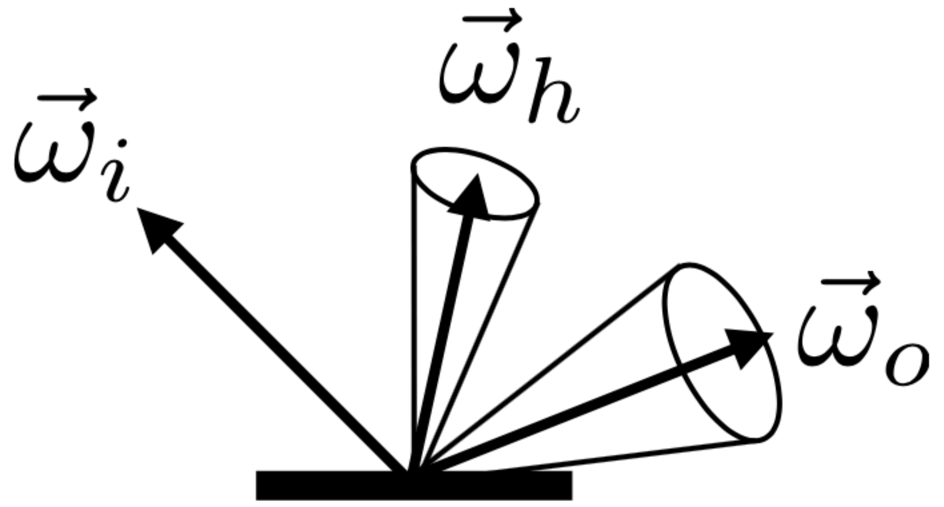
General Microfacet Model

Fresnel coefficient

Microfacet distribution

shadowing/masking

$$f(\vec{\omega}_i, \vec{\omega}_o) = \frac{F(\vec{\omega}_h, \vec{\omega}_o) \cdot D(\vec{\omega}_h) \cdot G(\vec{\omega}_i, \vec{\omega}_o)}{4|(\vec{\omega}_i \cdot \mathbf{\vec{n}})(\vec{\omega}_o \cdot \mathbf{\vec{n}})|}$$



$$\vec{\omega}_h = \frac{\vec{\omega}_i + \vec{\omega}_o}{\|\vec{\omega}_i + \vec{\omega}_o\|}$$

General Microfacet Model

Fresnel coefficient

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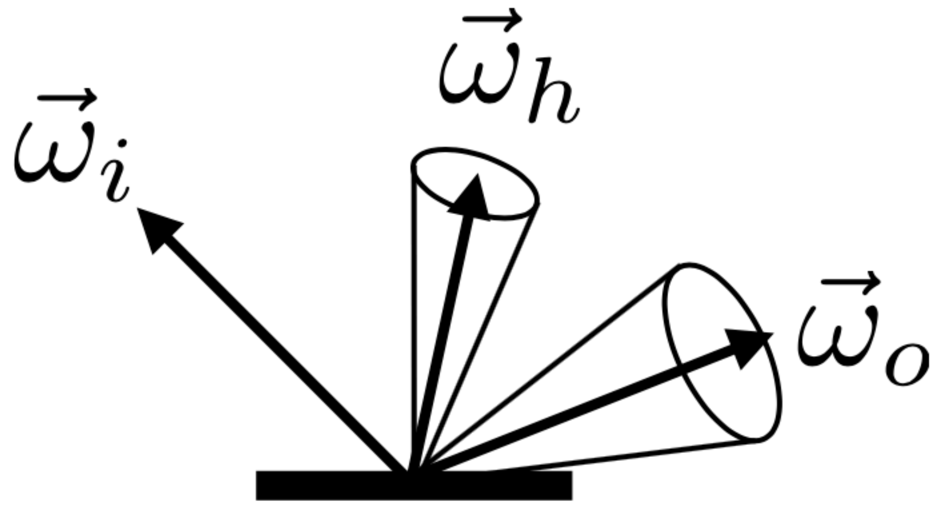

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Fresnel Term

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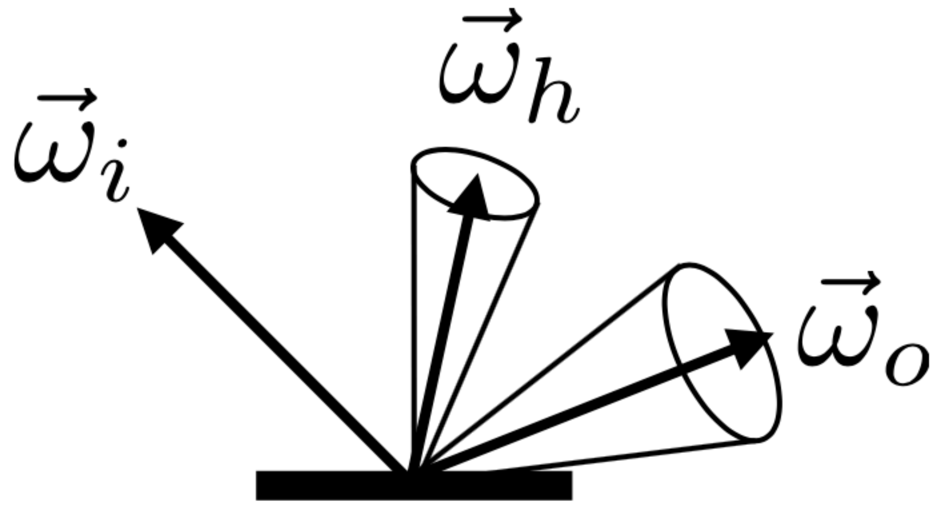

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Microfacet Distribution

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How much of the surface reflects?

Microfacet Distribution

What fraction of the surface participates in the reflection?

1) difficult to say (need an actual micro surface to compute this, tedious..)

2) Solve using principles of statistical physics

- Is there anything general we can say about the surface when there are many bumps?

Microfacet Distribution

Fraction of facets facing each direction

Probability density function over projected solid angle (must be normalized):

$$\int_{\mathcal{H}^2} D(\vec{\omega}_h) \cos\theta_h d\vec{\omega}_h = 1$$

Beckmann-Spizzichino Model

The slopes follow a Gaussian distribution

Let's express slope distribution w.r.t. directions

$$D(\vec{\omega}_h) = \qquad \exp^{-\frac{tan^2\theta_h}{\alpha^2}}$$

Beckmann-Spizzichino Model

The slopes follow a Gaussian distribution

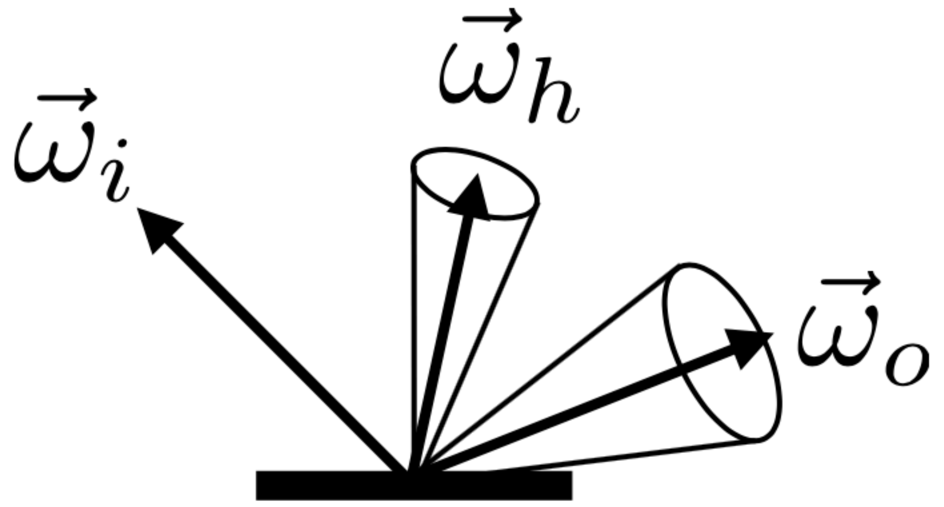
Let's express slope distribution w.r.t. directions

$$D(\vec{\omega}_h) = \frac{1}{\pi\alpha^2\cos^4\theta_h}\exp^{-\frac{tan^2\theta_h}{\alpha^2}}$$

General Microfacet Model

shadowing/masking

$$f(\vec{\omega}_i, \vec{\omega}_o) = \frac{F(\vec{\omega}_h, \vec{\omega}_o) \cdot D(\vec{\omega}_h) \cdot G(\vec{\omega}_i, \vec{\omega}_o)}{4|(\vec{\omega}_i \cdot \mathbf{\vec{n}})(\vec{\omega}_o \cdot \mathbf{\vec{n}})|}$$



$$\vec{\omega}_h = \frac{\vec{\omega}_i + \vec{\omega}_o}{\|\vec{\omega}_i + \vec{\omega}_o\|}$$

Microfacet Distribution: Masking effect

Microfacet Distribution: Masking effect

The microfacet of interest not visible to the viewer due to occlusions

Masking effect:

Microfacet Distribution: Shadowing effect

Light does not reach the microfacet

Shadowing effect:

Microfacet Distribution: Shadowing/Masking

Light bounces among the facets before reaching the viewer

Microfacet Distribution: Shadowing/Masking

Light bounces among the facets before reaching the viewer

Microfacet Distribution: Interreflection

Light bounces among the facets before reaching the viewer

Reading

- PBRT Section 8.4
- GGX Distribution, Walter et al. (EGSR 2007)
- Isotropic and anisotropic microfacet distributions
- Oren–Nayar model, a "directed-diffuse" microfacet model, with perfectly diffuse (rather than specular) microfacets.
- Ashikhmin-Shirley model, allowing for anisotropic reflectance, along with a diffuse substrate under a specular surface


Isotropic microfacet distribution
Anisotropic microfacet distribution
PBRT v3 [2016]

Acknowledgements

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