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Lehrstuhl für Computer Graphik
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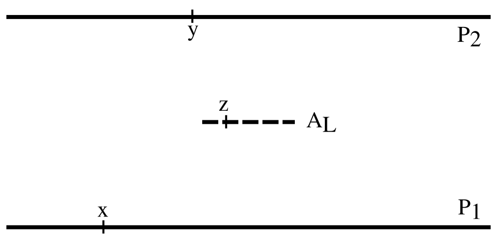
Realistic Image Synthesis (SS 2018)
Assignment 6

Submission deadline for the exercises: June 28, 2018

An Analytical Integration Problem

Consider the simple scene depicted in the figure below. It consists of three parallel planes.



The light (dashed line) is a square with surface area A_L between the two other planes, at an unknown distance. Both sides of the light are diffuse emitters, that is, the emitted radiance $L_e(z, \omega)$ is a constant for all points and directions, denoted by L_e in the following. For simplicity, we also assume that the light source itself is transparent, that is, it does not reflect or absorb light.
The plane P_2 is a non-emissive, opaque surface with a perfectly white Lambertian BSDF: $f_2(y, \omega_i, \omega_o) = \frac{1}{\pi}$
The plane P_1 is a non-emissive black-body, that is, $f_1(x, \omega_i, \omega_o) = 0$
Both planes are infinitely large. The space between the planes is "filled" with vacuum.

6.1 Radiometric Quantities (4+4+8+4+4)

a.) Compute the total flux emitted by the light source. Keep in mind that the light emits in both directions.

b.) Provide a formula for the irradiance at a point $y \in P_2$. Keep in mind that P_1 is a black-body. You do not have to compute the exact solution.

c.) Provide a formula for the irradiance at a point $x \in P_1$. Keep in mind that P_2 reflects light and the light source is transparent. You do not have to compute the exact solution.

d.) Compute the total flux reaching P_2.

e.) Compute the total flux reaching P_1

6.2 Estimators and Variance (4+4+8+20+2)

We want to estimate the irradiance at a point $x \in P_1$ via Monte Carlo integration. We are able to sample directions from the cosine-weighted hemisphere, and points uniformly distributed on the light source.

a.) Write down the primary estimator $I_1(x, z)$ for the direct illumination arriving at x, using a random point z on the light source.

b.) Write down the primary estimator $I_2(x, \omega, z)$ for the indirect illumination arriving at x, using a random direction ω from the hemisphere at x and a random point z on the light source.

c.) Show that the expected value of the sum of both estimators is indeed the quantity you computed in part (c) of the previous exercise:

$$E[I_1(x) + I_2(x, \omega, z)] = \text{(c)}$$

d.) Compute the variance:

$$V[I_1(x) + I_2(x, \omega, z)] = ?$$

e.) Is there a choice of A_L such that the variance will be zero?

Procedure of Submitting

Write your solutions and submit them on June 28, 2018, before the lecture. You can also e-mail the solution as a pdf to your tutor.