Computer Graphics

- Light Transport -

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Overview

• So far
  – Nuts and bolts of ray tracing

• Today
  – Light
    • Physics behind ray tracing
    • Physical light quantities
    • Perception of light
    • Light sources
  – Light transport simulation

• Next lecture
  – Reflectance properties
  – Shading
LIGHT
What is Light?

[astronomynotes.com]

Electromagnetic Wave

Magnetic field
Electric field
What is Light?

[Diagram of the electromagnetic spectrum]

Visible spectrum

Increasing Wavelength (λ) →

[Visible spectrum graph]

Increasing Wavelength (λ) in nm →

[Additional information from Wikipedia]
What is Light?

• Ray
  – Linear propagation
  – Geometrical optics

• Vector
  – Polarization
  – Jones Calculus: matrix representation

• Wave
  – Diffraction, interference
  – Maxwell equations: propagation of light

• Particle
  – Light comes in discrete energy quanta: photons
  – Quantum theory: interaction of light with matter

• Field
  – Electromagnetic force: exchange of virtual photons
  – Quantum Electrodynamics (QED): interaction between particles
What is Light?

- **Ray**
  - Linear propagation
  - Geometrical optics

- **Vector**
  - Polarization
  - **Jones Calculus**: matrix representation

- **Wave**
  - Diffraction, interference
  - **Maxwell equations**: propagation of light

- **Particle**
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- **Field**
  - Electromagnetic force: exchange of virtual photons
  - **Quantum Electrodynamics (QED)**: interaction between particles
Light in Computer Graphics

• **Based on human visual perception**
  – Macroscopic geometry
  – Tristimulus color model
  – Psycho-physics: tone mapping, compression, …

• **Ray optics**
  – Macroscopic objects
  – Incoherent light
  – Light: scalar, real-valued quantity
  – Linear propagation
  – Superposition principle: light contributions add up linearly
  – No attenuation in free space

• **Limitations**
  – Microscopic structures (≈ λ): diffraction, interference
  – Polarization
  – Dispersion
Angle and Solid Angle

- The angle $\theta$ (in radians) subtended by a curve in the plane is the length of the corresponding arc on the unit circle: $l = \theta$ $r = \theta$

- The solid angle $\Omega$, $d\omega$ subtended by an object is the surface area of its projection onto the unit sphere
  - Units for measuring solid angle: steradian [sr] (dimensionless)
Solid Angle in Spherical Coords

- **Infinitesimally small solid angle** $d\omega$
  - $du = r \, d\theta$
  - $dv = r' \, d\Phi = r \, \sin \theta \, d\Phi$
  - $dA = du \, dv = r^2 \, \sin \theta \, d\theta \, d\Phi$
  - $d\omega = dA / r^2 = \sin \theta \, d\theta \, d\Phi$

- **Finite solid angle**

\[
\Omega = \int_{\phi_0}^{\phi_1} \int_{\theta_0(\phi)}^{\theta_1(\phi)} \sin \theta \, d\theta \, d\phi
\]
Solid Angle for a Surface

• The solid angle subtended by a small surface patch $S$ with area $dA$ is obtained (i) by projecting it orthogonal to the vector $r$ to the origin:

$$dA \cos \theta$$

and (ii) dividing by the distance to the origin squared:

$$d\omega = \frac{dA \cos \theta}{r^2}$$

$$\Omega = \iint_S \frac{\hat{r} \cdot \hat{n}}{r^3} dA$$
Radiometry

• Definition:
  – Radiometry is the science of measuring radiant energy transfers. Radiometric quantities have physical meaning and can be directly measured using proper equipment such as spectral photometers.

• Radiometric Quantities
  – Energy [J] \( Q \) (Photons Energy = \( n \cdot h\nu \))
  – Radiant power [watt = J/s] \( \Phi \) (Total Flux)
  – Intensity [watt/sr] \( I \)
  – Irradiance [watt/m\(^2\)] \( E \)
  – Radiosity [watt/m\(^2\)] \( B \)
  – Radiance [watt/(m\(^2\) sr)] \( L \)
Radiant flux

Flux: Total energy in a region per unit time
Intensity

\[ \Phi = \int_{\Omega} I \cdot d\omega \]

Intensity: Flux density per solid angle

\[ I = \frac{d\Phi}{d\omega} \]
Irradiance: Incoming flux density per surface

\[ \Phi = \int_A E \cdot dA \]

\[ E = \frac{d\Phi}{dA} \]
Radiosity:

\[ \Phi = \int_A B \cdot dA \]

\[ B = \frac{d\Phi}{dA} \]

Radiosity: Outgoing flux density per surface
Radiance

\[ \Phi = \int_{\Omega} \int_{A} L \cdot d\omega dA_{\perp} = \int_{\Omega} \int_{A} L \cos\theta \cdot d\omega dA_{\perp} \]

\[ L = \frac{d^2 \Phi}{d\omega dA_{\perp}} = \frac{d^2 \Phi}{d\omega dA \cos\theta} \]

Radiosity: Outgoing flux density per surface per solid angle
Radiance

\[ \Phi = \int_{\Omega} I \cdot d\omega \]

\[ I = \int_A L \cos \theta \cdot dA \]

\[ \Phi = \int_A E \cdot dA \]

\[ E = \int_{\Omega} L \cos \theta \cdot d\omega \]
Radiometric Quantities: Radiance

• Radiance is used to describe radiant energy transfer
• Radiance $L$ is defined as
  – The power (flux) traveling at some point $x$
  – In a specified direction $\omega = (\theta, \varphi)$
  – Per unit area perpendicular to the direction of travel
  – Per unit solid angle

• Thus, the differential power $d^2\Phi$ radiated through the differential solid angle $d\omega$, from the projected differential area $dA \cos \theta$ is:

$$d^2\Phi = L(x, \omega)dA \cos \theta d\omega$$
Radiometric Quantities: Irradiance

- Irradiance $E$ is defined as the **total power per unit area** (flux density) incident onto a surface. To obtain the total flux incident to $dA$, the incoming radiance $L_i$ is integrated over the upper hemisphere $\Omega_+$ above the surface:

$$E \equiv \frac{d\Phi}{dA}$$

$$d\Phi = \left[ \int_{\Omega_+} L_i(x, \omega) \cos \theta \ d\omega \right] dA$$

$$E = \int_{\Omega_+} L_i(x, \omega) \cos \theta \ d\omega = \int_0^{\pi/2} \int_0^{2\pi} L_i(x, \omega) \cos \theta \sin \theta \ d\theta d\phi$$
Radiometric Quantities: Radiosity

- **Radiosity** $B$ is defined as the **total power per unit area** (flux density) **exitant from** a surface. To obtain the total flux incident to $dA$, the **outgoing** radiance $L_o$ is integrated over the upper hemisphere $\Omega_+$ above the surface:

\[
B \equiv \frac{d\Phi}{dA} \\
\frac{d\Phi}{dA} = \left[ \int_{\Omega_+} L_o(x,\omega) \cos \theta \, d\omega \right] \, dA \\
B = \int_{\Omega_+} L_o(x,\omega) \cos \theta \, d\omega = \int_0^{\pi/2} \int_0^{2\pi} L_o(x,\omega) \cos \theta \sin \theta \, d\theta \, d\phi
\]
Spectral Properties

- **Wavelength**
  - Light is composed of electromagnetic waves
  - These waves have different frequencies and wavelengths
  - Most transfer quantities are continuous functions of wavelength

- **In graphics**
  - Each measurement $L(x, \omega)$ is for a discrete band of wavelength only
    - Often some abstract R, G, B (but see later)
Photometry

- The human eye is sensitive to a limited range of wavelengths
  - Roughly from 380 nm to 780 nm
- Our visual system responds differently to different wavelengths
  - Can be characterized by the *Luminous Efficiency Function* $V(\lambda)$
  - Represents the average human spectral response
  - Separate curves exist for light and dark adaptation of the eye
- Photometric quantities are derived from radiometric quantities by *integrating* them against this function
Radiometry vs. Photometry

<table>
<thead>
<tr>
<th>Radiometry</th>
<th>→</th>
<th>Photometry</th>
</tr>
</thead>
<tbody>
<tr>
<td>W</td>
<td>Radiant power → Luminous power</td>
<td>Lumens (lm)</td>
</tr>
<tr>
<td>W/m²</td>
<td>Radiosity → Luminosity</td>
<td>Lux (lm/m²)</td>
</tr>
<tr>
<td></td>
<td>Irradiance → Illuminance</td>
<td></td>
</tr>
<tr>
<td>W/m²/sr</td>
<td>Radiance → Luminance</td>
<td>cd/m² (lm/m²/sr)</td>
</tr>
</tbody>
</table>
Perception of Light

The eye detects radiance

\[ \text{radiance} = \text{flux per unit area per unit solid angle} \]

\[ r \] \[ \text{angular extent of rod} = \text{resolution} \approx 1 \text{ arcminute}^2 \]

\[ A \] \[ \text{projected rod size} = \text{area} \]

\[ \Omega \] \[ \text{angular extent of pupil aperture} \leq 4 \text{ mm} = \text{solid angle} \]

\[ \Phi = L \cdot A \cdot \Omega' \]

\[ L = \frac{\Phi}{\Omega' \cdot A} \]

\[ \Phi_0 = L \cdot l^2 \cdot \Omega \cdot \pi \cdot \frac{r^2}{l^2} = L \cdot \text{const} \]
Brightness Perception

- $A' > A$ : photon flux per rod stays constant
- $A' < A$ : photon flux per rod decreases

Where does the Sun turn into a star?
- Depends on apparent Sun disc size on retina
- Photon flux per rod stays the same on Mercury, Earth or Neptune
- Photon flux per rod decreases when $\Omega' < 1$ arcminute$^2$ (beyond Neptune)
Radiometry in ray tracing

Radiance

dA \, d\omega

dA \, d\omega
Radiometry in ray tracing

Each pixel:

\[ L_1 = L_2 \]

\[ \text{color}[p_1] = \text{color}[p_2] \]

Total area:

\[ \Phi_1 = \int_{\Omega} \int_{A_1} L \cdot d\omega dA_\perp \approx \int_{A_1} E \cdot dA = ES_1 \]

\[ \Phi_2 = \int_{\Omega} \int_{A_2} L \cdot d\omega dA_\perp \approx \int_{A_2} E \cdot dA = ES_2 \]
Radiometry in ray tracing

Each pixel:

\[ L_1 = L_2 \]

\[ \text{color}[p_1] = L_1 \frac{S_1}{S_{p_1}} \]

\[ \text{color}[p_2] = L_2 \frac{S_2}{S_{p_2}} \]

\[ \frac{S_1}{S_2} \sim \left( \frac{d_2}{d_1} \right)^2 \]
LIGHT TRANSPORT
Light Transport in a Scene

- **Scene**
  - Lights (emitters)
  - Object surfaces (partially absorbing)
Light Transport in a Scene

- **Illuminated object surfaces become emitters, too!**
  - Radiosity = Irradiance – absorbed photons flux density
    - Radiosity: photons per second per $m^2$ leaving surface
    - Irradiance: photons per second per $m^2$ incident on surface
Light Transport in a Scene

- Light bounces between all mutually visible surfaces
- Dynamic energy equilibrium
  - Emitted photons = absorbed photons (+ escaping photons)
  → Global Illumination
Light Transport in a Scene

- **Light interaction with surfaces**
  - Incident angle
  - Material
    - BRDF: bidirectional reflectance distribution function

\[
\frac{L_i}{L_o} = ?
\]
Light Transport in a Scene

• **Outgoing radiance proportional to:**

  – Incoming radiance \( L_o \sim L_i \)
  – Incident angle \( L_o \sim \cos \theta \)
  – Material reflectance (BRDF) \( L_o \sim f_r \)
  – Material self emission \( L_o \sim L_e \)

• **Rendering equation:**

\[
L_o = L_e + \int_{\Omega_+} f_r L_i \cos \theta_i \, d\omega_i
\]
(Surface) Rendering Equation

\[ L_o(x, \omega_o) = L_e(x, \omega_o) + \int_{\Omega^+} f_r(\omega_i, x, \omega_o)L_i(x, \omega_i) \cos \theta_i \, d\omega_i \]

- **Visible surface radiance**
  - Surface position
  - Outgoing direction
- **Incoming illumination direction**
- **Self-emission**
- **Reflected light**
  - Incoming radiance from all directions
  - Direction-dependent reflectance
    (BRDF: bidirectional reflectance distribution function)
(Surface) Rendering Equation

- **Fredholm integral equation of 2nd kind**
  - Unknown radiance appears both on the left-hand side and inside the integral
  - Numerical methods necessary to compute approximate solution

\[
L_A(x, \omega_1) = L_{Ae}(x, \omega_1) + \int_{\Omega_+} f_A(\omega, x, \omega_1)L(x, \omega) \cos \theta \, d\omega
\]

\[
L_B(y, \omega_2) = L_{Be}(y, \omega_2) + \int_{\Omega_+} f_B(\omega, y, \omega_2)L(y, \omega) \cos \theta \, d\omega
\]
(Surface) Rendering Equation

- Reparameterization over surfaces
  - Represent receiver’s $d\omega$ as emitter’s $dA$.

\[
L_B(y, \omega_2) = L_{Be}(y, \omega_2) + \int_{\Omega_+} f_B(\omega, y, \omega_2) L(x, \omega) \cos \theta \, d\omega
\]

\[
L_B(y, \omega_2) = L_{Be}(y, \omega_2) + \int_A f_B(\omega_{yx}, y, \omega_2) \cos \theta \, dA
\]
(Surface) Rendering Equation

- Reparameterization over surfaces
  - Represent receiver’s $d\omega$ as emitter’s $dA$
  - Check visibility

\[
L_B(y, \omega_2) = L_{Be}(y, \omega_2) + \int_{\Omega_+} f_B(\omega, y, \omega_2)L(x, \omega) \cos \theta \ d\omega
\]

\[
d\omega = \frac{\cos \theta_A}{\|x - y\|^2} dA
\]

\[
V(x, y) = \begin{cases} 
0 & \iff RT(x, \omega_1) \neq y \\
1 & \iff RT(x, \omega_1) = y
\end{cases}
\]

\[
L_B(y, \omega_2) = L_{Be}(y, \omega_2) + \int_A f_B(\omega_{yx}, y, \omega_2)L(y, \omega_{yx})V(x, y)\frac{\cos \theta_B \cos \theta_A}{\|x - y\|^2} dA
\]
LIGHT SOURCES
Light Specifications

- **Emitted Power $\Phi_e$**
  - Total brightness
- **Spectral Distribution**
  - Continuous thermal spectrum
  - Discrete spectral lines
- **Approximation**
  - RGB color
Light Types

Ambient Light

Directional Light

Point Light

Spot Light
Ambient Light

• **Omnidirectional Constant Illumination**
  – Identical incident radiance from all directions

\[
L_{rl}(x, \omega_o) = L_a \int_{\Omega_+} f_r(\omega_i, x, \omega_o) \cos \theta_i \, d\omega_i = L_a \rho_r(x, \omega_o)
\]

• **Not Physically Plausible**
  – Crude approximation to indirect illumination
Point Light

- **Sphere of Radius** $r$
  - Surface area: $4 \pi r^2$

- **Irradiance on Surrounding Sphere**
  - $E_r = \Phi_e / (4 \pi r^2)$

- **Quadratic Surface Area**
  - Double distance from emitter: sphere area four times bigger

- **Inverse Square Law**
  - Irradiance falls off with inverse of squared distance

\[
\frac{E_1}{E_2} = \frac{r_2^2}{r_1^2}
\]
Isotropic Point Light

- **Emitted Intensity**
  \[ I = \frac{\Phi_e}{4\pi} \]

- **Irradiance on Surface dA**
  \[ E(x) = \frac{d\Phi_e}{dA} = \frac{d\Phi_e}{d\omega} \frac{d\omega}{dA} = I \frac{d\omega}{dA} = \frac{dA \cos \theta}{r^2 dA} = \frac{I \cos \theta}{r^2} \]

- **Illumination**
  \[ L_{rl}(x, \omega_o) = \frac{I}{||x - y||^2} V(x, y) f_r(\omega(x, y), x, \omega_o) \cos \theta_i \]

- **Extrinsic Parameters**
  - Position
Anisotropic Point Light

- **Emitted Intensity**
  - $I(\omega) = \Phi_e P(\omega)$

- **Directional Distribution**
  - Tabulated
    - Goniometric diagram
  - Analytical
    - E.g. Warn (un-normalized)
      - Zero if dot product < 0
      - $P(\omega) = (\omega \cdot \omega_l)^n$

- **Extrinsic Parameters**
  - Position
  - Forward vector $\omega_l$

- **Illumination**

$$L_{rl}(x, \omega_o) = \frac{I(-\omega)}{\|x - y\|^2} V(x, y) f_r(\omega(x, y), x, \omega_o) \cos \theta_i$$
Spot Light

- **Restricted Directional Distribution**
  - If $\angle \omega, \omega_l < \theta_c$ then $P(\omega) = (\omega \cdot \omega_l)^n$
  - Else $P(\omega) = 0$
  - With cut-off angle $\theta_c$
Projective Light
Projective Light

• Unit direction from light center to surface point
• Find light-screen coordinates from ray direction
  – Light-space coords: dot product with light basis vectors
  – Like for a perspective camera, but in reverse
• \( P(\omega) = \text{color/intensity at corresponding coordinates} \)
• Extrinsic Parameters
  – Position
  – Forward vector
  – Up vector
Projective Light

• Examples
Directional Light

• **Set point light to infinity**
  – In the limit, all light rays have parallel directions

• **Illumination**
  \[
  L_{rl}(x, \omega_o) = B(y)V(x, \omega_i)f_r(\omega_i, x, \omega_o) \cos \theta_i
  \]

• **Extrinsic Parameters**
  – Forward vector
Sky Light

- **Sun**
  - Point source (approx.)
  - White light (by def.)

- **Sky**
  - Area source
  - Scattering: blue

- **Horizon**
  - Brighter
  - Haze: whitish

- **Overcast sky**
  - Multiple scattering in clouds
  - Uniform grey

Courtesy Lynch & Livingston
PRACTICAL APPROXIMATION
Rendering Equation: Approximations

\[ L_0(x, \omega_o) = L_e(x, \omega_o) + \int_{\Omega^+} f_r(\omega_i, x, \omega_o) L_i(x, \omega_i) \cos \theta_i \ d\omega_i \]

- **Approximations based only on empirical foundations**
  - An example: polygon rendering in OpenGL
- **Using RGB instead of full spectrum**
  - Follows roughly the eye’s sensitivity
- **Sampling hemisphere along finite, discrete directions**
  - Simplifies integration to summation
- **Reflection function model**
  - Parameterized function
    - Ambient: constant, non-directional, background light
    - Diffuse: light reflected uniformly in all directions
    - Specular: light of higher intensity in mirror-reflection direction
Wrap Up

• **Physical Quantities in Rendering**
  – Radiance
  – Radiosity
  – Irradiance
  – Intensity

• **Light Perception**

• **Light Sources**

• **Rendering Equation**
  – Integral equation
  – Balance of radiance