Computer Graphics

- Volume Rendering -

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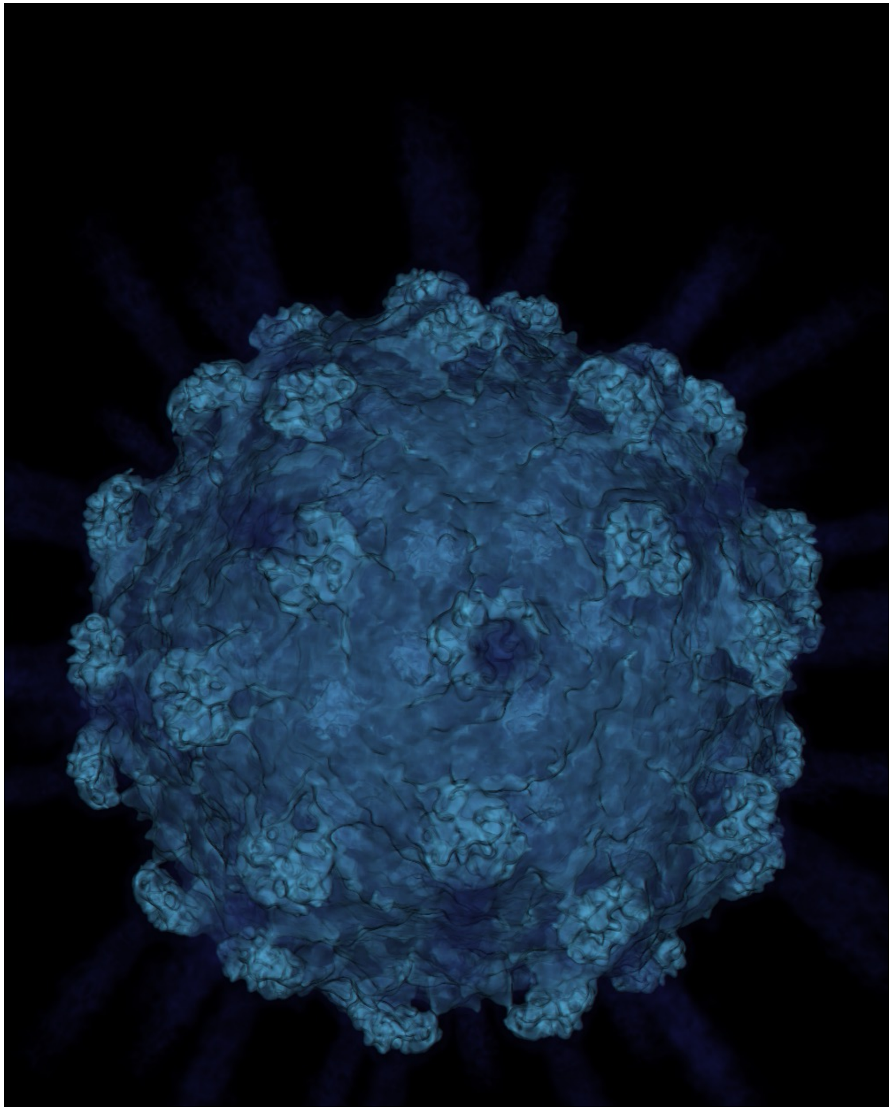
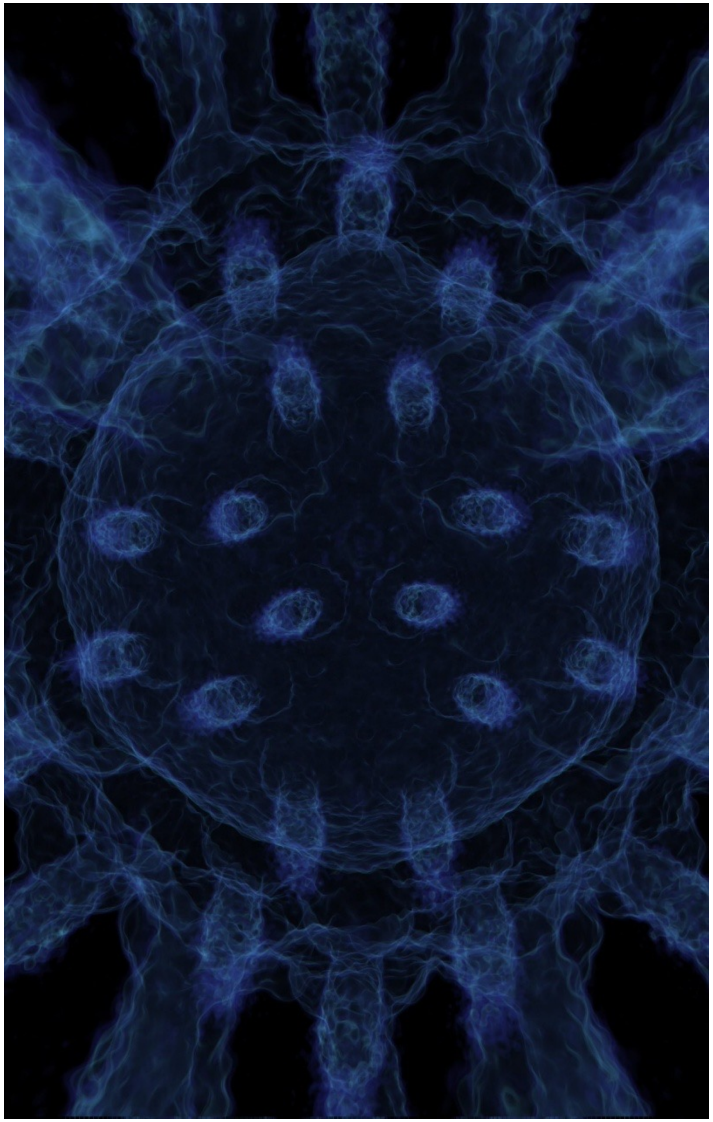
Overview

- **Motivation**
- **Volume Representation**
- **Indirect Volume Rendering**
- **Volume Classification**
- **Direct Volume Rendering**

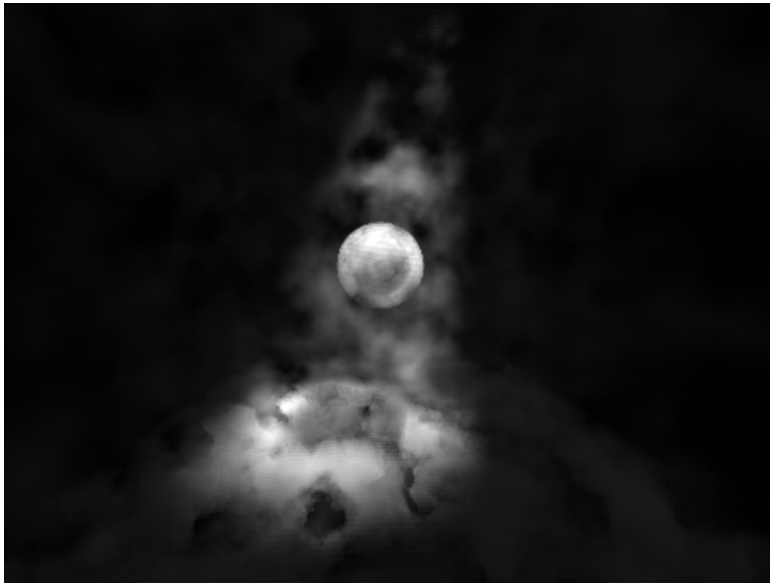
Applications: Bioinformatics



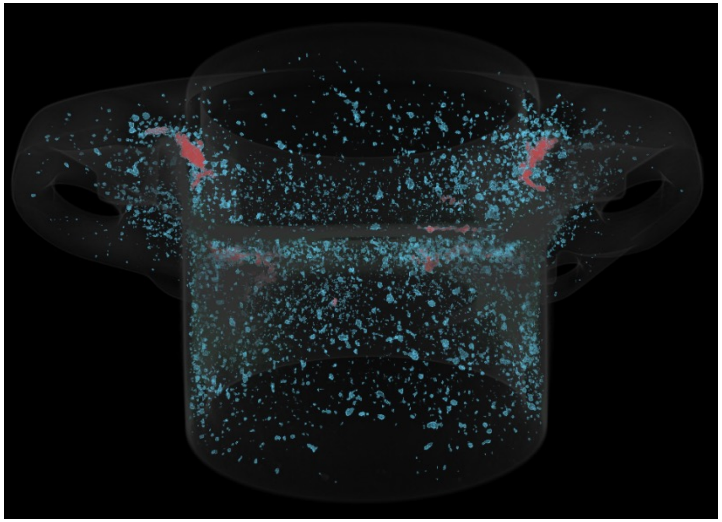


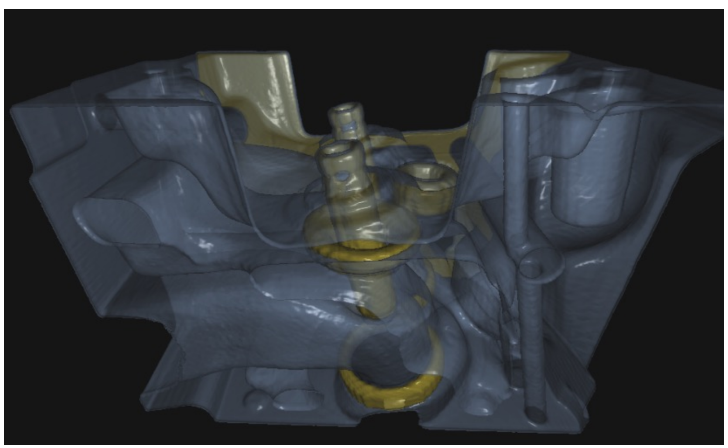
Applications: Entertainment

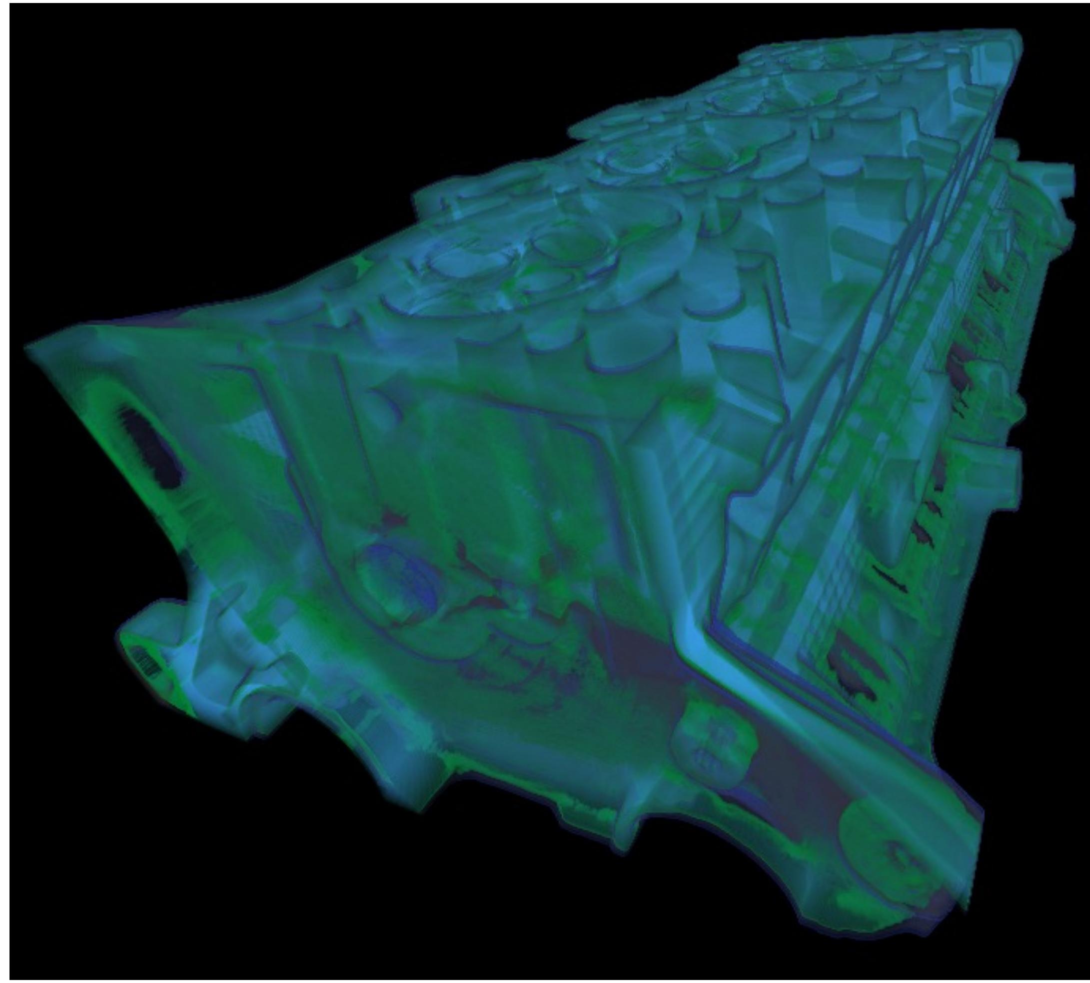
Image by [Salama 07]



Applications: Industrial







Applications: Medical

Applications: Simulations

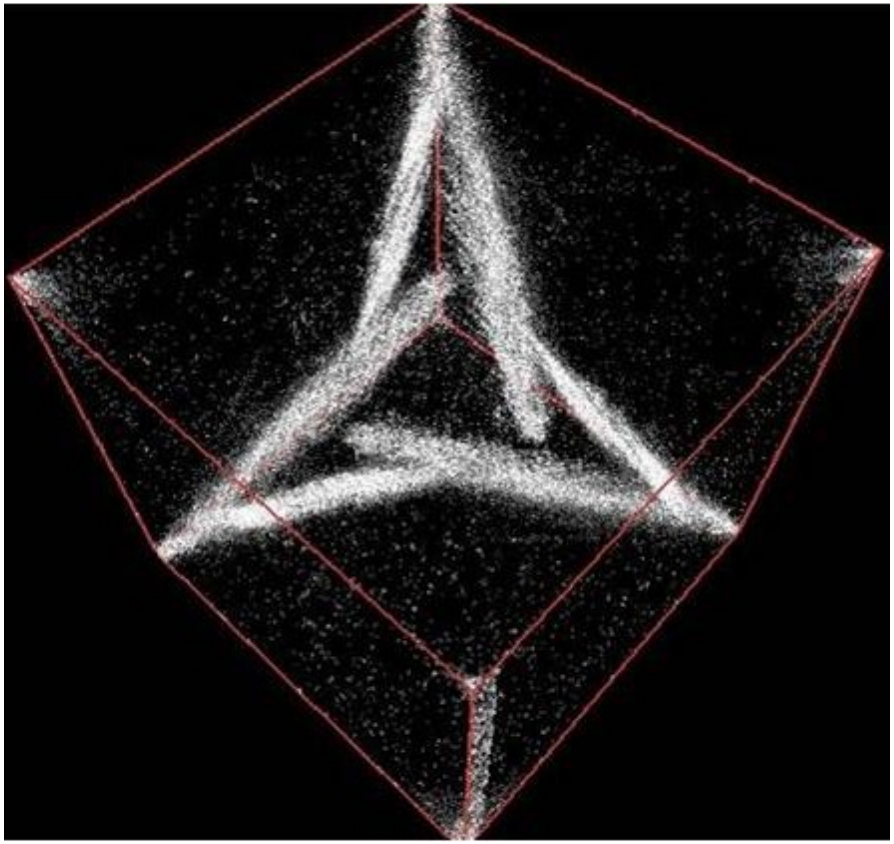
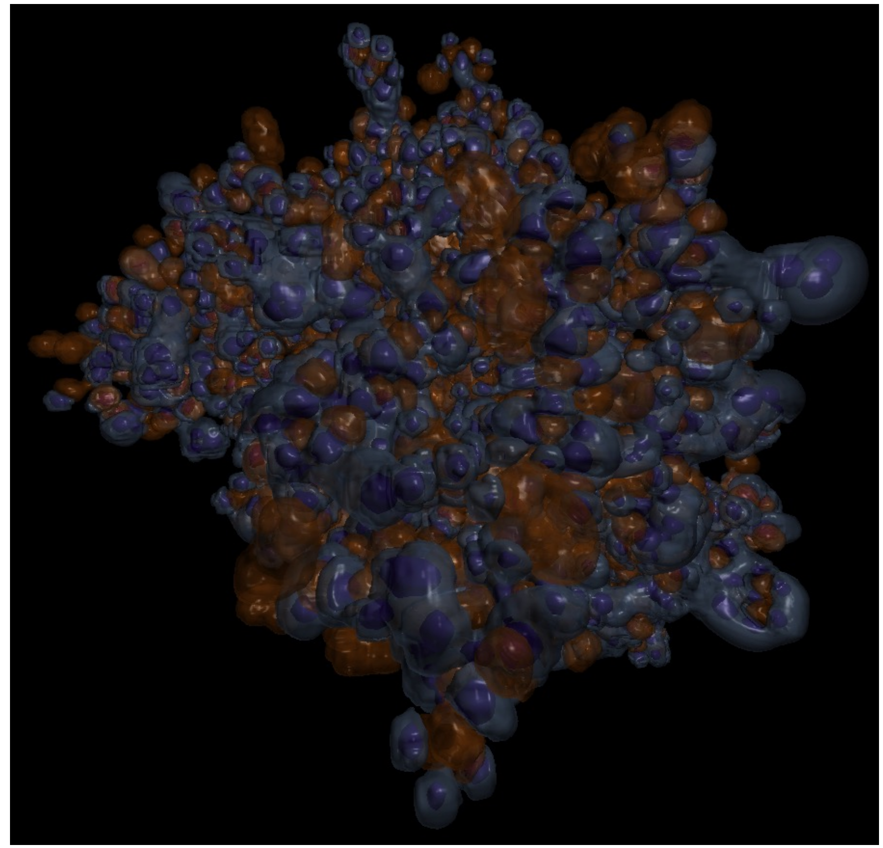


Image by [RTVG 08]



Volume Processing Pipeline

- **Acquisition**
  - Measurement or computation of the data
- **Filtering**
  - Picking desired features, cleaning, noise-reduction, re-sampling, reconstruction, classification, ...
- **Mapping**
  - Map N-dimensional data to visual primitives
- **Rendering**
  - Generate the image
- **Post-processing**
  - Enhancements (gamma correction, tone mapping)

Volume Acquisition

- **Measurements**
  - Computer Tomography (CT, X-Ray),
  - Magnetic Resonance Imaging (MRI, e-spin)
  - Positron-Emission Tomography (PET)
  - Ultrasound, sonar
  - Electron microscopy
  - Confocal microscopy
  - Cryo-EM/Light-Tomography
  - Seismic exploration
  - …
- **Simulations**
  - Essentially everything > 2D
- **Visualization of mathematical objects**

Filtering

- **Raw data usually unsuitable**
  - Cleaning & repairing
  - Noise reduction and removal
  - Correcting incomplete, out-of-scale values
  - Selection of relevant aspects
    - Lots of information and features in a 3D volume
    - Potentially hiding/obscuring each other
  - Classification
- **Adaptation of format**
  - Re-sampling (often to Cartesian grids)
- **Transformations**
  - Volume reconstructing of 3D data from projection

Mapping

- **Create something visible**
  - Interpretation of measurement values
  - Mapping to geometric primitives
  - Mapping to parameters (colors, absorption coefficients, ...)

- **Rendering**
  - Surface extraction vs. direct volume rendering
  - Single volume vs multiple (possibly overlapping)
  - Object-based vs. image-based rendering
    - Forward- or backward mappings (rasterization/RT)

Volume Rendering

- **Our input?**
  - Representation of volume
- **Our output?**
  - Colors for given samples (pixels)
- **Our tasks?**
  - Map “weird values” to optical properties
  - “Project 1D/2D/3D/nD data values within a 3D context to 2D image plane”

VOLUME ACQUISITION AND REPRESENTATION

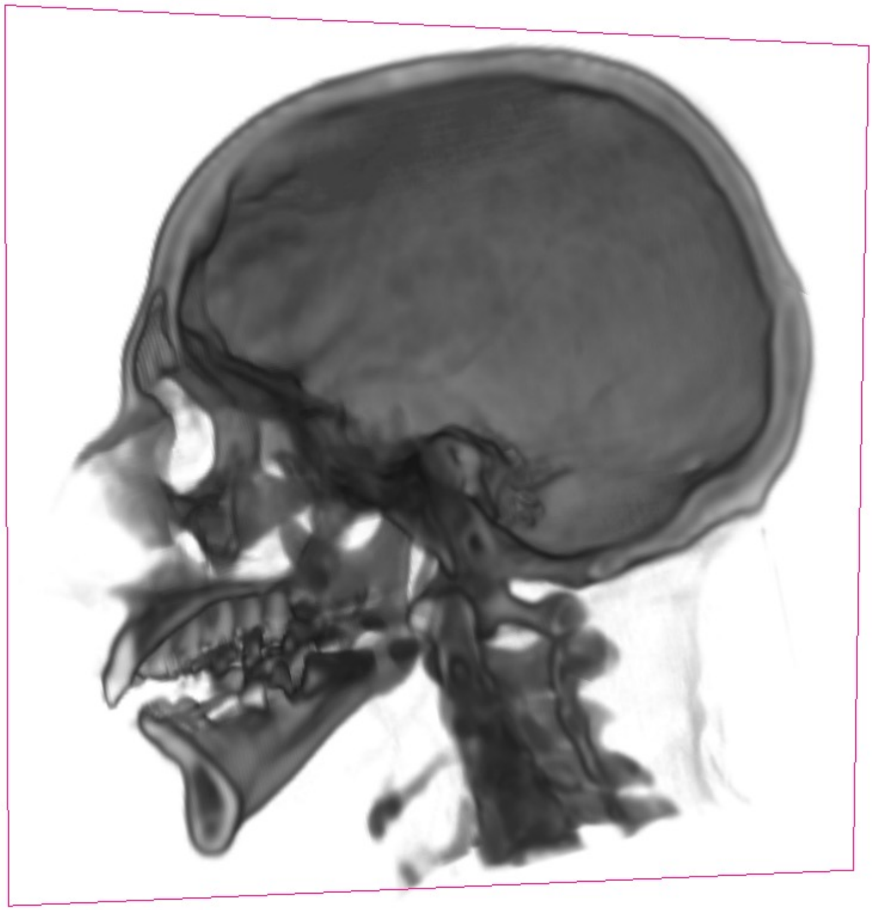
Data Acquisition

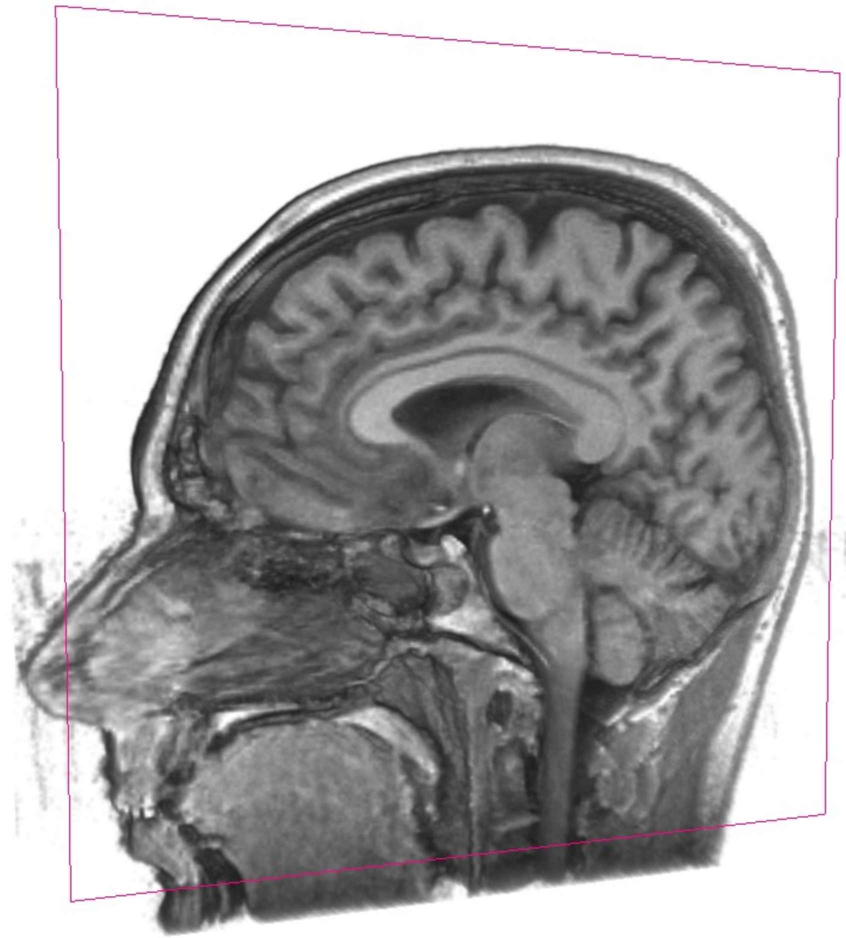
- **Simulated Data**
  - Fluid dynamics
  - Heat transfer
  - etc…
  - Generally: “Scientific Visualization”

- **Measured Data**
  - CT (Computed Tomography) scanner
    - Reconstructed from rotated series of two-dimensional X-ray images
    - Good contrast between high and low density media (e.g., fat and bones)
  - MRI (Magnetic Resonance Imaging)
    - Based on magnetic/spin response of hydrogen atoms in water
    - Better contrast between different soft tissues (e.g., brain, muscles, heart)
  - PET (Positron Emission Tomography)
  - And many others (also here on campus, e.g., material science)

Data Acquisition

- **CT vs. MRI**



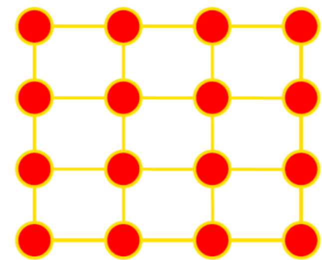


Volume Representations

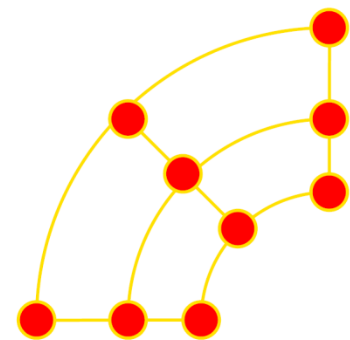
- **Definition**
  - 3D field of values: Essentially a 3D scalar or color texture
  - Sometimes higher dimensional data (e.g., vector/tensor fields)
- **Sampled representation**
  - 3D lattice of sample points (akin to an image but in 3D)
    - Typically, equal-distance in each directions
  - Generally, point cloud in space
  - Ideally, neighborhood information (topology)
  - Data values at these locations
- **Procedural**
  - Mathematical description of values in space
  - Sum of Gaussians (e.g., in quantum mechanics)
  - Perlin noise (e.g., for non-homogeneous fog)
  - Always convertible to sampled representation
    - But with loss of information

Volume Organization

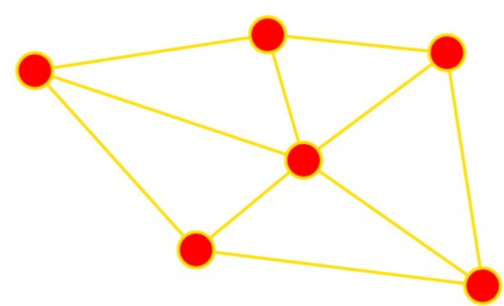
- **Rectilinear Grids**
  - Common for scanned data
  - May have different spacing



- **Curvilinear Grids**
  - Warped rectilinear grids

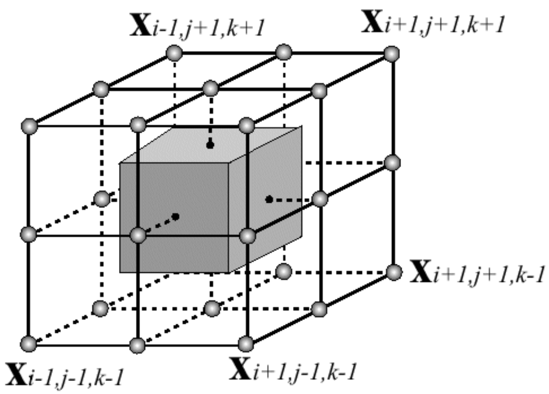


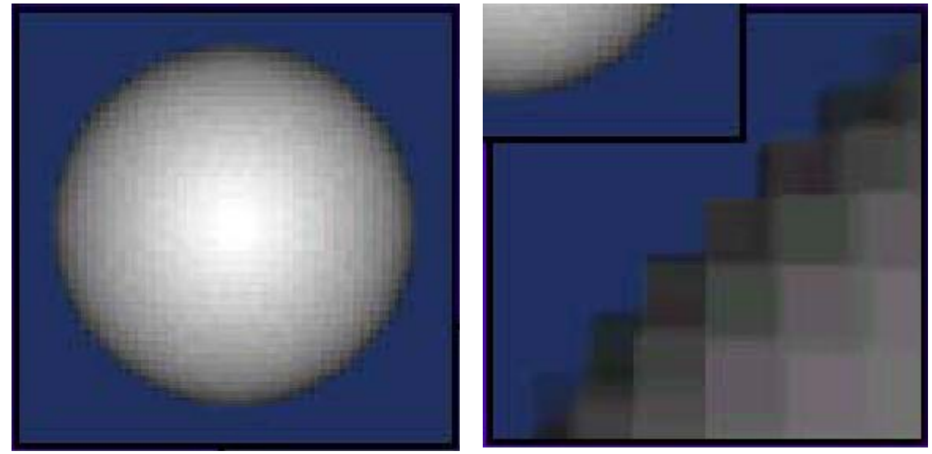
- **Unstructured Meshes**
  - Common for simulated data
  - E.g., tetrahedral meshes
- **Point clouds**
  - No topological/connection information
    - Neighborhood computed on the fly



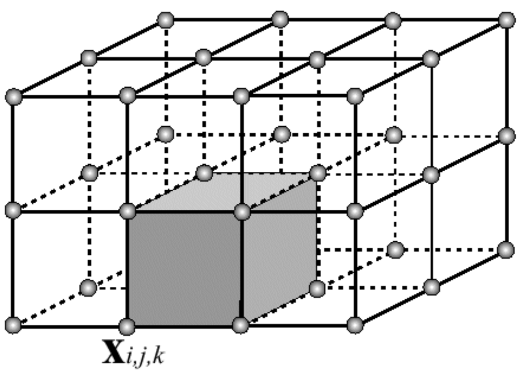
Reconstruction Filter

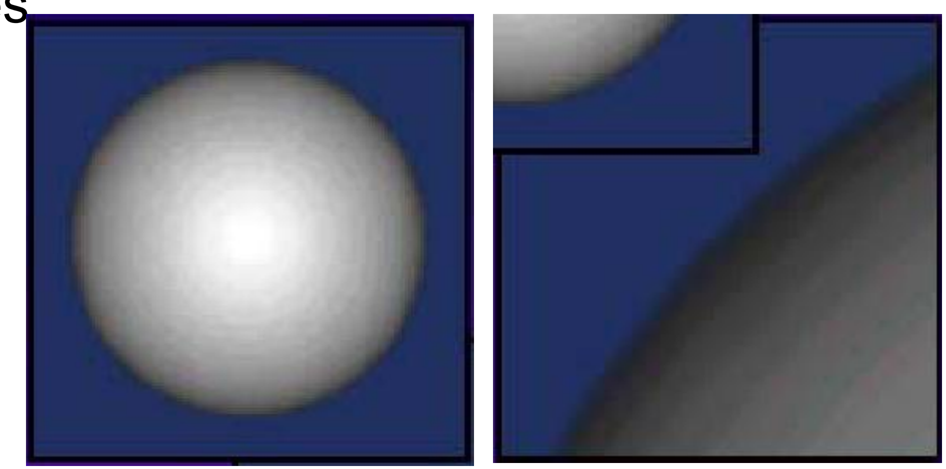
- **Nearest Neighbor**
  - Cell-centered sample values





- **Tri-Linear Interpolation**
  - Node-centered sample values

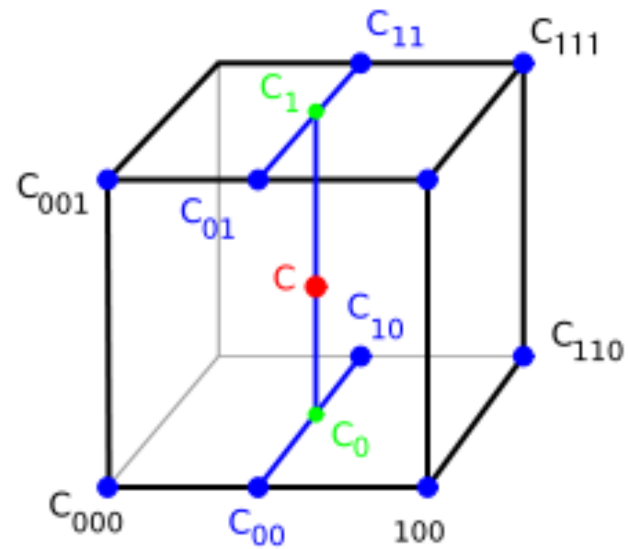


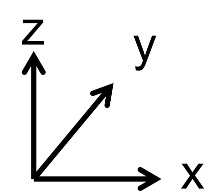


Tri-Linear Interpolation

- **Compute Coefficients**
  - $wx = (x - x0) / (x1 - x0)$
  - $wy = (y - y0) / (y1 - y0)$
  - $wz = (z - z0) / (z1 - z0)$

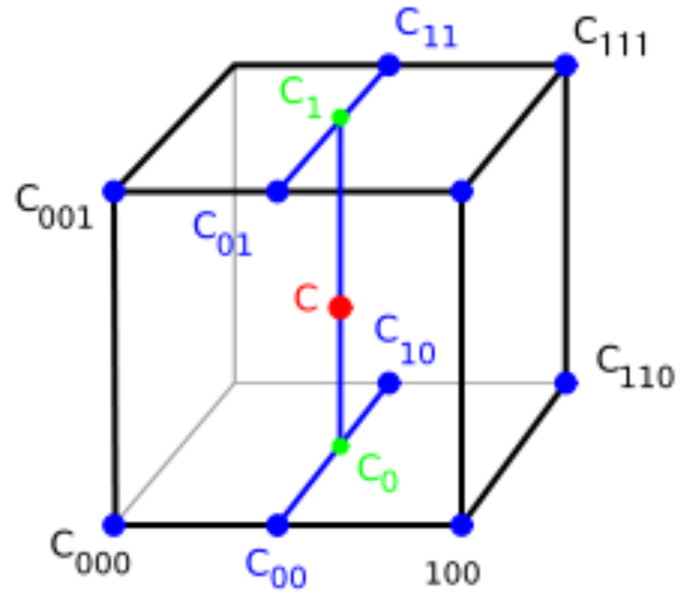
- **3-D Scalar Field per Voxel**
  - $f(x, y, z) = (1 - wz)\,(1 - wy)\,(1 - wx)\ c000$
  - $+ (1 - wz)\,(1 - wy)\ wx\ c100$
  - $+ (1 - wz)\ wy\ (1 - wx)\ c010$
  - $+ (1 - wz)\ wy\ wx\ c110$
  - $+ wz\ (1 - wy)\,(1 - wx)\ c001$
  - $+ wz\ (1 - wy)\ wx\ c101$
  - $+ wz\ wy\ (1 - wx)\ c011$
  - $+ wz\ wy\ wx\ c111$

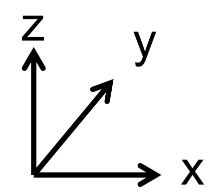




Tri-Linear Interpolation

- **Successive Linear Interpolations**
  - Along X
    - c00 = (1 - wx) c000 + wx c100
    - c01 = (1 - wx) c001 + wx c101
    - c10 = (1 - wx) c010 + wx c110
    - c11 = (1 - wx) c011 + wx c111
  - Along Y
    - c0 = (1 - wy) c00 + wy c10
    - c1 = (1 - wy) c01 + wy c11
  - Along Z
    - c = (1 - wz) c0 + wz c1





- **Order of dimensions does not matter**

VOLUME MAPPING

Mapping / Classification

- **Definition**
  - Map scalar data values to optical properties
  - E.g.
    - Optical density
    - Albedo
    - Emission

- **Instances**
  - Analytical function
  - Discrete representation
    - Array of sample colors corresponding to sample data values
    - Interpolate colors for data values in between given data points

Mapping / Classification

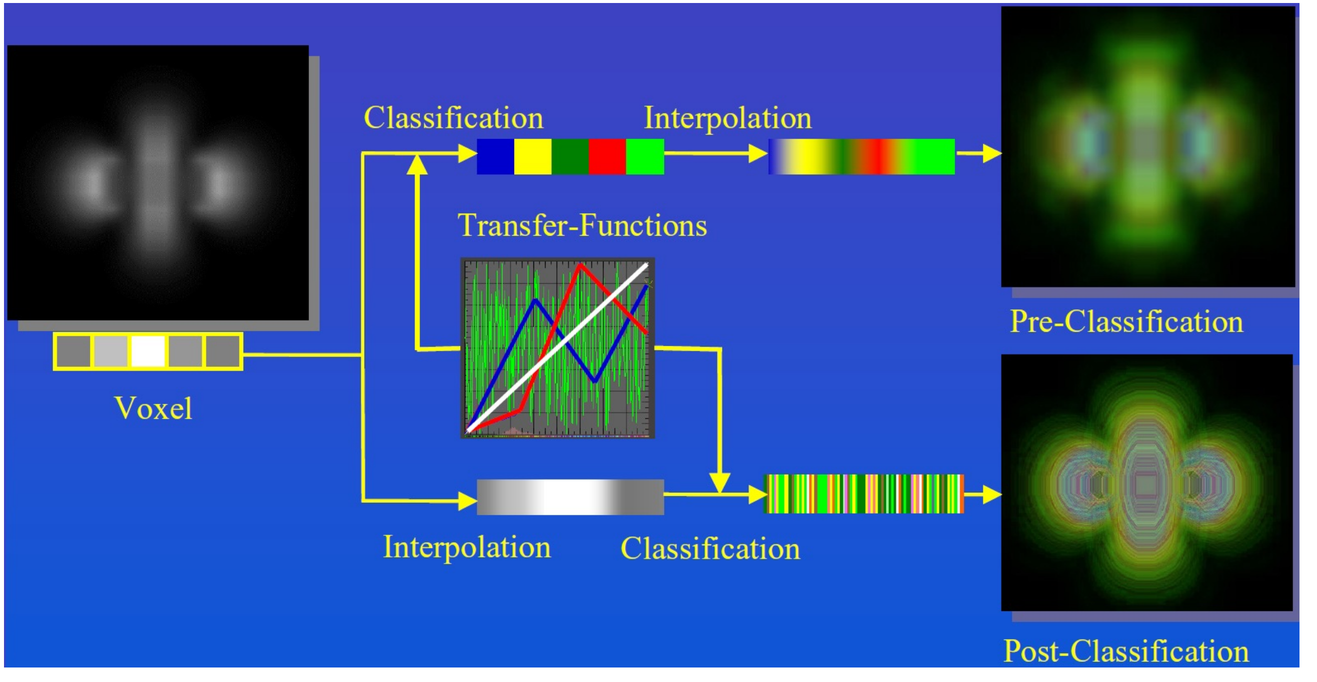
- **Physical Mapping**
  - Physically-based mapping via optical properties of material
    - Concentration of soot to optical density, albedo, etc…
    - Temperature to emitted blackbody radiation
  - Allows for realistic rendering, often intuitively interpretable by us

Mapping / Classification

- **Empirical or task-specific mapping (Transfer Function)**
  - User-defined mapping from data to colors
    - Typically stored as an array of sample correspondences (color map transfer function)
  - Mapping may have no physical interpretation
    - Assigning color to pressure, electrostatic potential, electron density, …
    - Use of ideas from *visualization*
  - Highlight specific features of the data
    - Isolate bones from fat

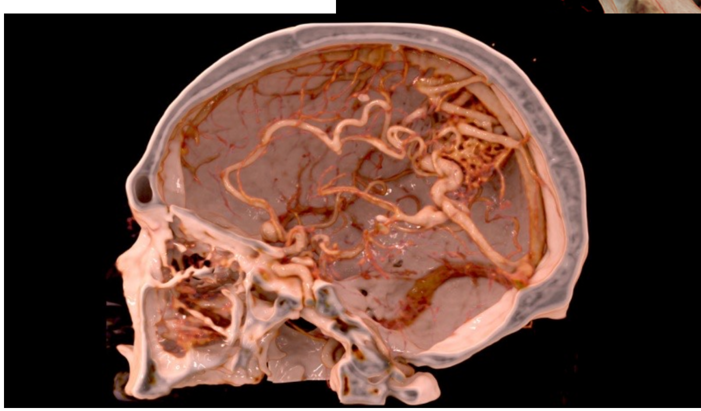
Pre/Post-Classification

- **Pre-Classification**
  - First classify data values in sample cells
  - Then interpolate classified optical properties
- **Post-Classification**
  - First interpolate data values, then classify interpolated values



Cinematic Rendering

- **Deutsche Zukunftspreis 2017**
  - Klaus Engel & Robert Schneider, Siemens Healthineers

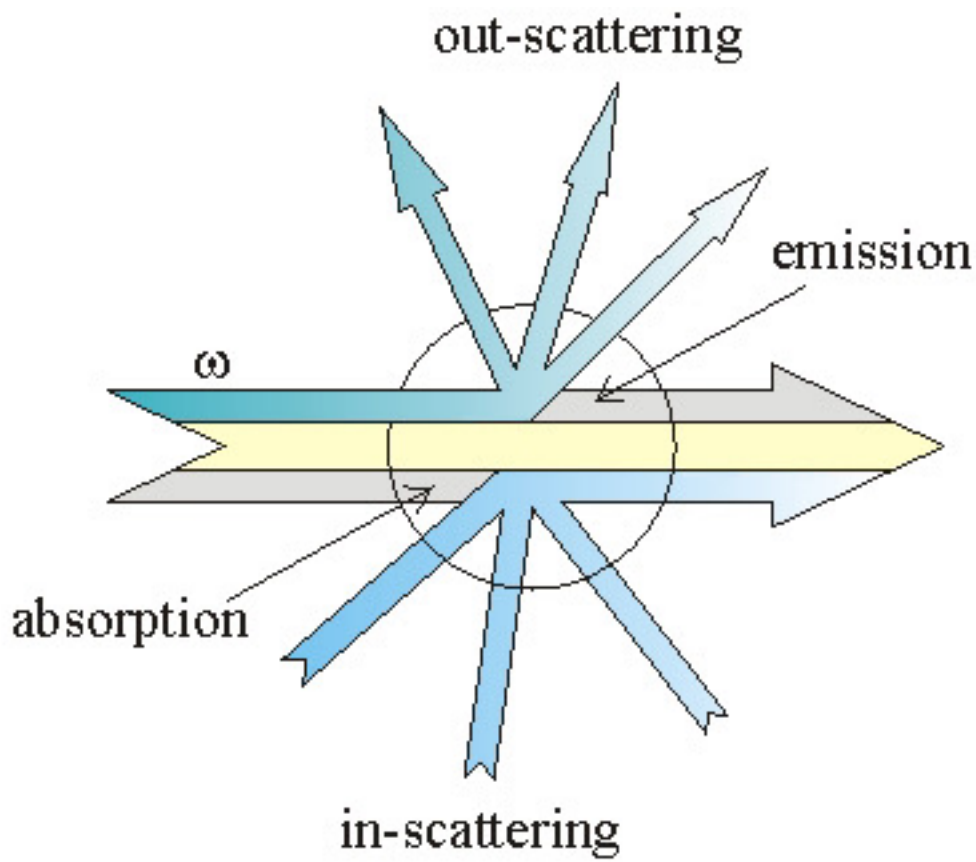


DIRECT VOLUME RENDERING

Direct Volume Rendering

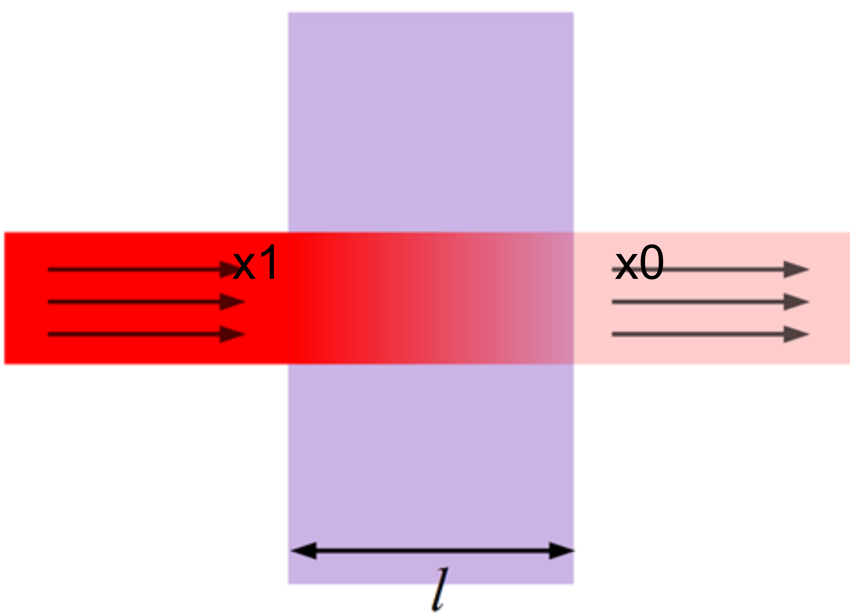
- **Definition**
  - Directly render the volumetric data (only) as translucent material

Scattering in a Volume



Beer's Law

- **Volumetric Attenuation**
  - Assume constant optical density κ_{01}
  - Transmittance:
    - $T(x_0, x_1) = e^{-\kappa_{01}(x_1 - x_0)}$
  - Transmitted radiance:
    - $L_o(x_0, \omega) = T(x_0, x_1)\, L_o(x_1, \omega)$



Analytical Form

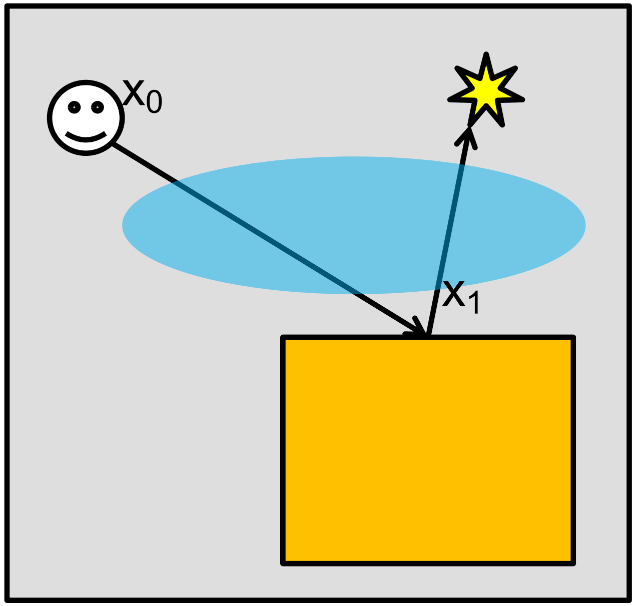
- **Volumetric Attenuation**
  - Assume constant optical density κ_{01}(extinction coefficient)
  - Transmittance: $T(x_0, x_1) = e^{-\kappa_{01}(x_1 - x_0)}$
  - Transmitted radiance: $T(x_0, x_1)\ L_o(x_1, \omega)$

- **Volumetric Contribution/Emission**
  - Also assume (constant) volume radiance $L_v(x, \omega)$ [Watt/(sr m^3)]
  - Contributed radiance: $\big(1 - T(x_0, x_1)\big)L_v(x_{01}, \omega)$

- **Volumetric Equation**
  - Radiance reaching the observer
    - Emission within segment + transmitted background radiance
  - $L_o(x_0, \omega) = \big(1 - T(x_0, x_1)\big)L_v(x_{01}, \omega) + T(x_0, x_1)L_o(x_1, \omega)$

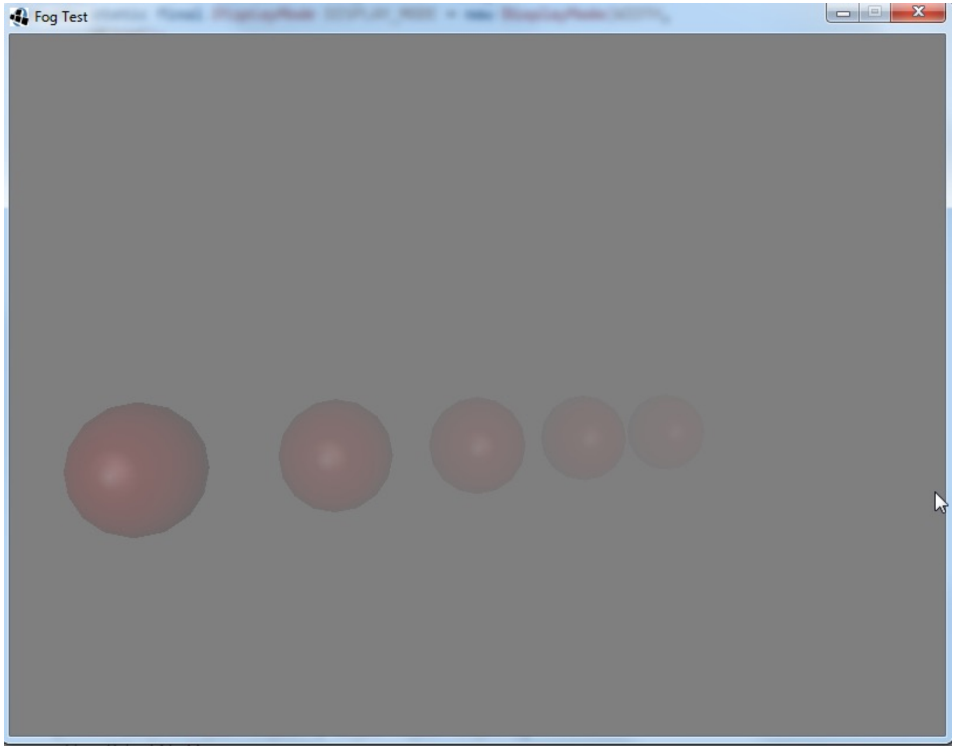
Ambient Homogenous Fog

- **Constant-Optical Density**
- **Volumetric Contributions**
  - Assume constant volumetric albedo $\rho_v(x)$
  - Assume constant ambient lighting L_a (everywhere, no shadowing)
  - Leads to constant volume radiance $L_v(x, \omega) = L_a\, \rho_v$
- **Pervasive Fog**
  - Entry at camera, exit at intersection, or inf.
- **Algorithm**
  - Compute surface illumination $L_o(x_1, \omega)$
    - Modulate shadow visibility by transmittance between surface and light source
  - Compute volume transmittance $T(x_0, x_1)$ and attenuate surface radiance
  - Add contributions from volume radiance



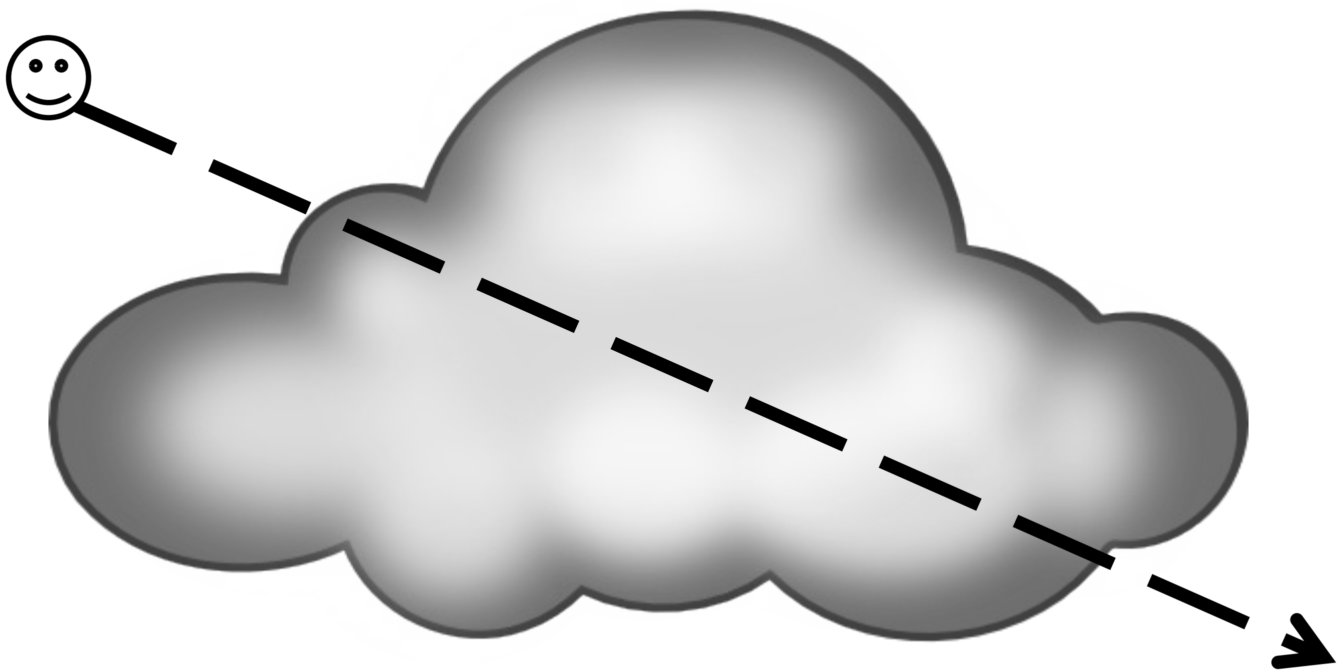
Ambient Homogeneous Fog

- **Pros**
  - Simple
  - Efficient
- **Cons**
  - No true light contributions
  - No volumetric shadows



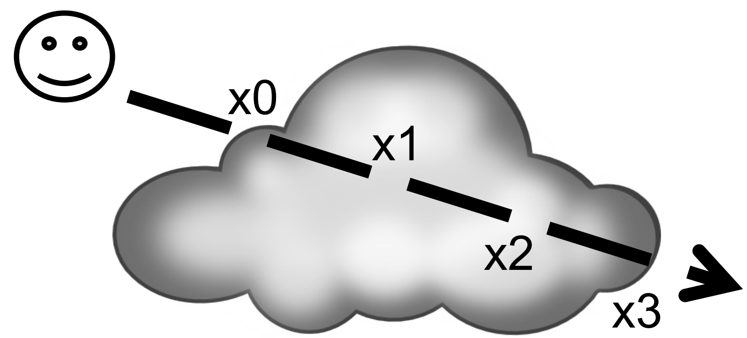
Ray-Marching

- **Riemann Summation**
  - Non-constant optical density / non-constant volume radiance
  - Sample volume at discrete locations
  - Assume constant density and volume radiance in each interval



Ray-Marching

- **Homogeneous Segments**
  - $L_o(x_0,\omega) = \left(1 - e^{-\kappa_{01}\Delta x}\right)L_v(x_{01},\omega) + e^{-\kappa_{01}\Delta x}L_o(x_1,\omega)$
  - $L_o(x_1,\omega) = \left(1 - e^{-\kappa_{12}\Delta x}\right)L_v(x_{12},\omega) + e^{-\kappa_{12}\Delta x}L_o(x_2,\omega)$
  - $L_o(x_2,\omega) = \dots$



- **Recursive Substitution**

$$L_o(x_0,\omega) = \left(1 - e^{-\kappa_{01}\Delta x}\right)L_v(x_{01},\omega) + e^{-\kappa_{01}\Delta x}\left(\left(1 - e^{-\kappa_{12}\Delta x}\right)L_v(x_{12},\omega) + e^{-\kappa_{12}\Delta x}(\dots)\right)$$

$$= \left(1 - e^{-\kappa_{01}\Delta x}\right)L_v(x_{01},\omega) + e^{-\kappa_{01}\Delta x}\left(1 - e^{-\kappa_{12}\Delta x}\right)L_v(x_{12},\omega) + e^{-\kappa_{01}\Delta x}e^{-\kappa_{12}\Delta x}(\dots)$$

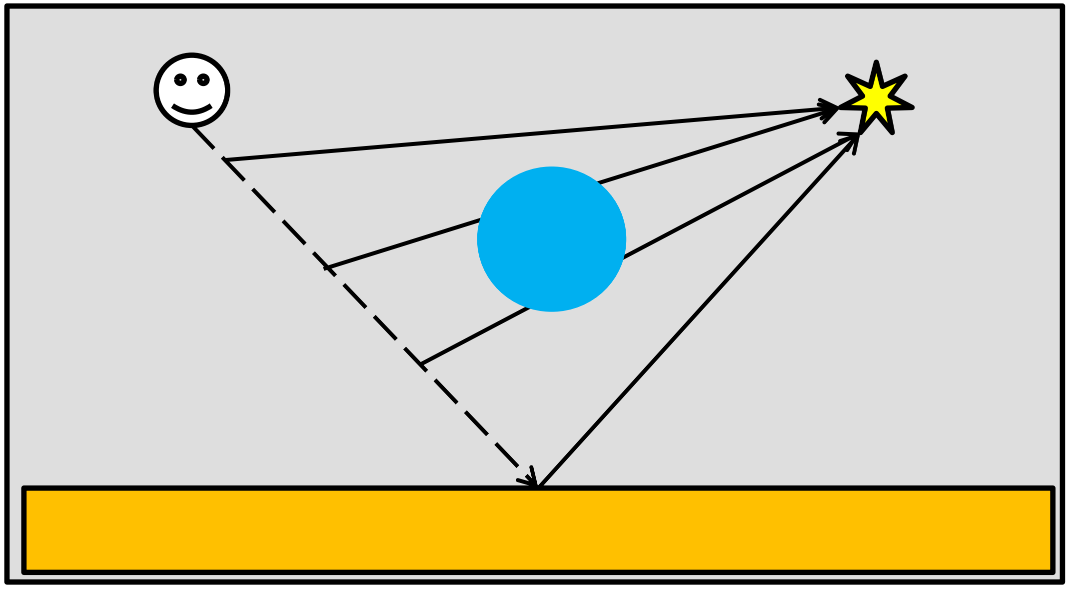
$$= \sum_{i=0}^{n-1}\left(\prod_{j=0}^{i-1} e^{-\kappa_{j,j+1}\Delta x}\right)\left(1 - e^{-\kappa_{i,i+1}\Delta x}\right)L_v\left(x_{i,i+1},\omega\right) + \left(\prod_{j=0}^{n-1} e^{-\kappa_{j,j+1}\Delta x}\right)L_o(x_n,\omega)$$

Ray-Marching (front to back)

- **L = 0;**
- **T = 1;**
- **t = 0; // t_enter;**
- **while(t < t_exit)**
  - dt = min(t_step, t_exit - t);
  - P = ray.origin + (t + dt/2) * ray.direction;
  - b = exp(- volume.density(P) * dt);
  - L += T * (1 - b) * Lv(P);
  - T *= b;
  - // Optional early termination
  - t += t_step;
- **L += T * trace(ray.origin + t_exit * ray.direction, ray.direction);**
- **return L;**

Homogeneous Fog

- **Constant-optical density**
- **Non-constant volume radiance**
  - Similar to surface reflected radiance (i.e., rendering equation)
  - Use phase function $\rho(x, \Delta\omega)$, (e.g., $\frac{\rho_v}{4\,\pi}$) instead of BRDF*cosine
  - Modulate shadow visibility by transmittance



Homogeneous Fog

- **E.g., Anisotropic Point Light**
  - Modulate visibility at surfaces by transmittance

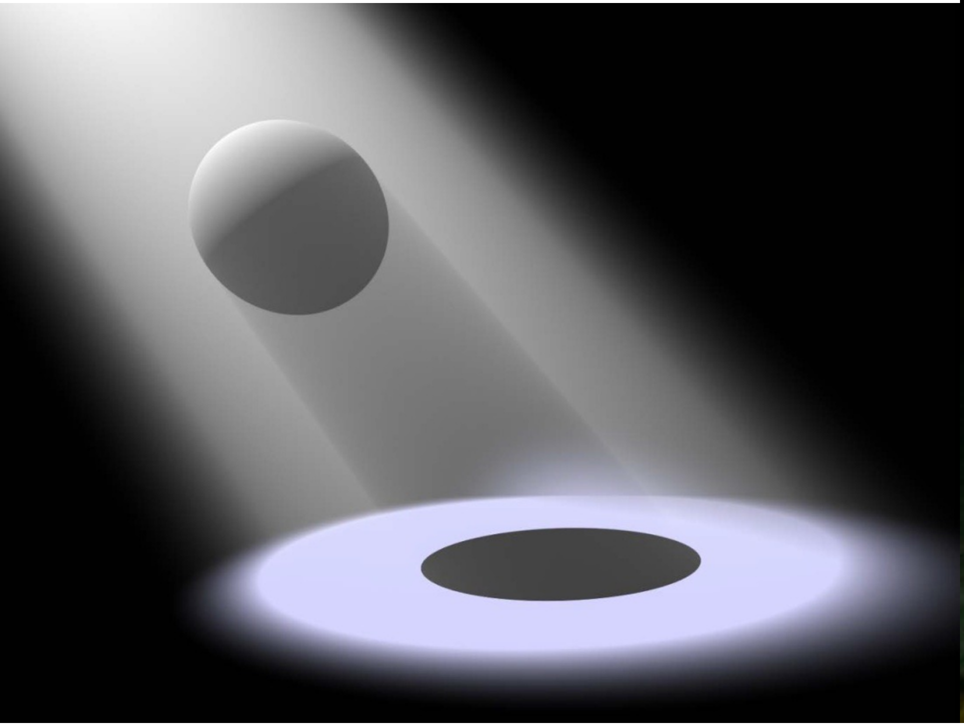
$$L_{rl}(x,\omega_o) = \frac{I(-\omega)}{\|x-y\|^2} V(x,y) T(x,y) f_r(\omega(x,y),x,\omega_o)\cos\theta_i$$

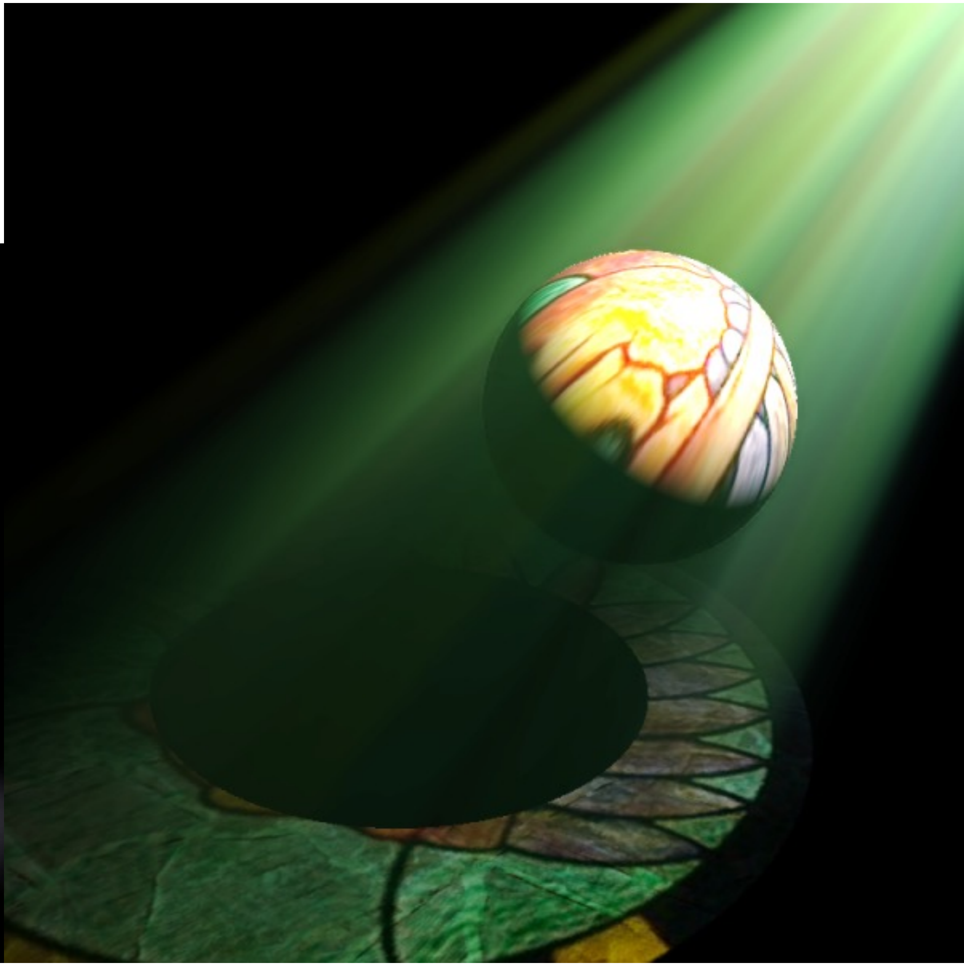
  - Modulate visibility at each volume sample by transmittance

$$L_v(x,\omega_o) = \frac{I(-\omega)}{\|x-y\|^2} V(x,y) T(x,y)\,\frac{\rho_v}{4\,\pi}$$

Homogeneous Fog

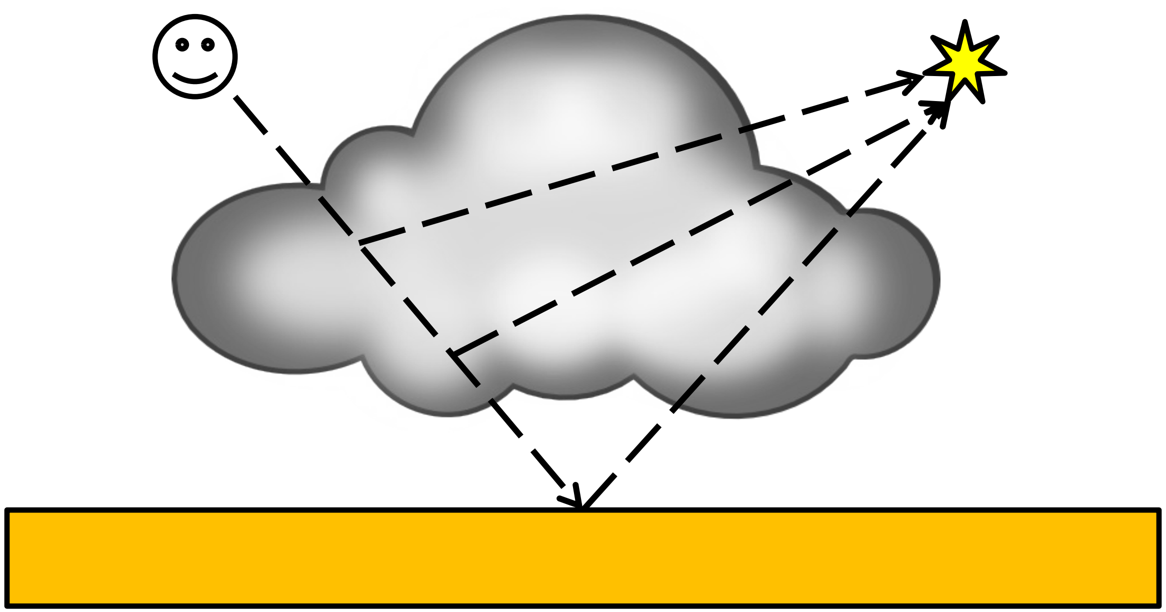
- **Inverse Square Law**
- **Volumetric Shadows**
- **Projective Light**





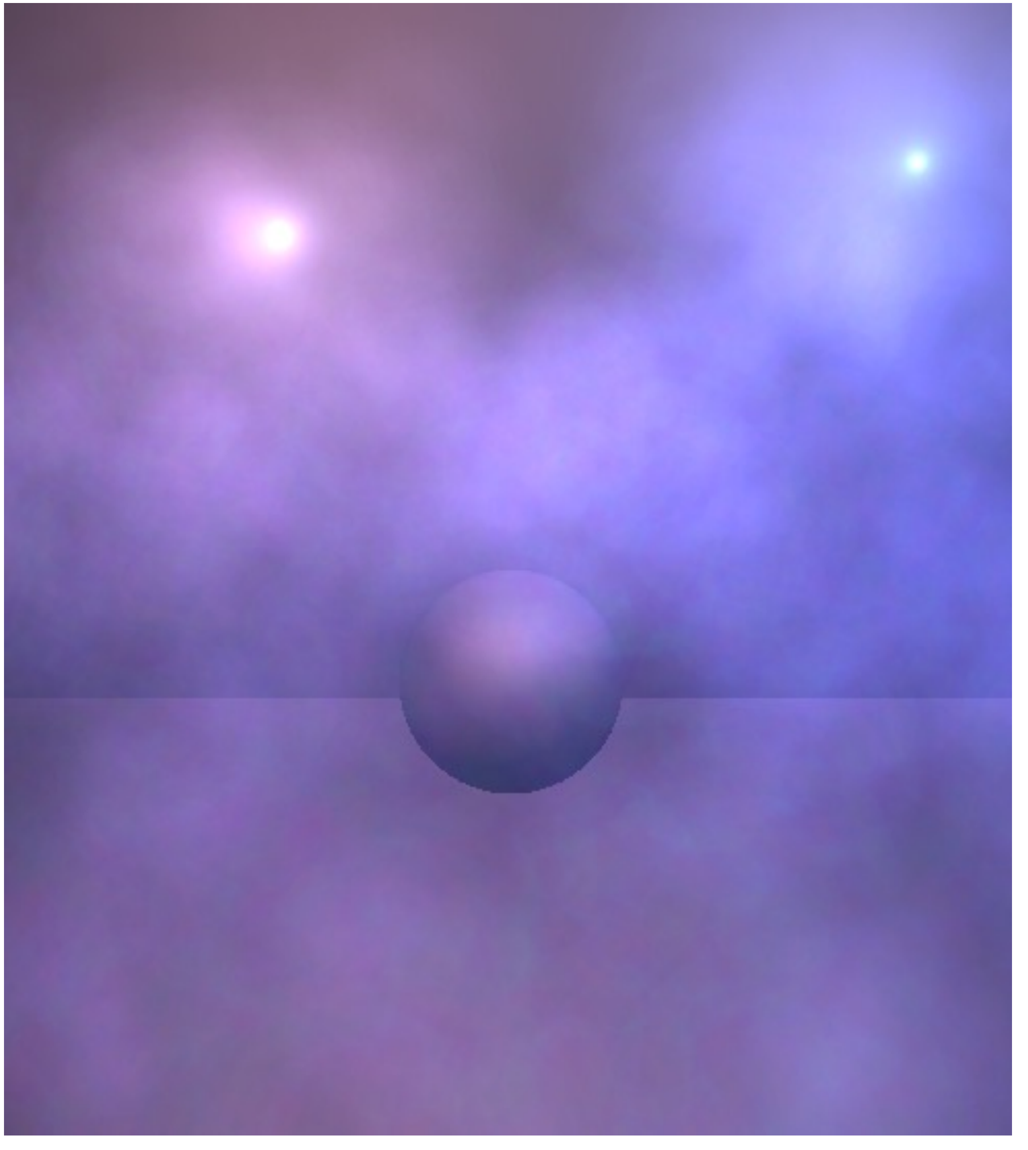
Heterogeneous Fog

- **Assumptions**
  - Non-constant-optical density
  - Non-constant volume radiance
- **Shadow visibility modulated by transmittance**
  - Ray-marched shadow rays at surface
  - Ray-marched shadow rays at each volume sample!!



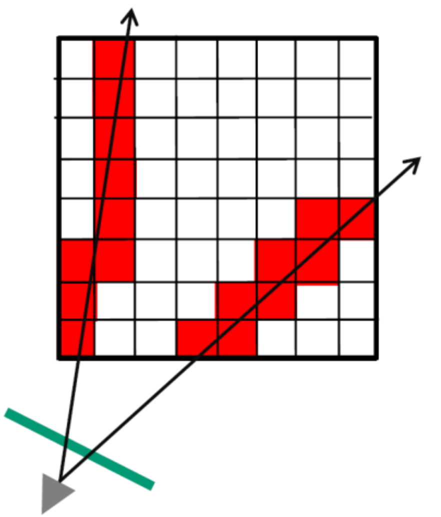
$$T(x_0, x_n) = \prod_{j=0}^{n-1} e^{-\kappa_{j,j+1}\Delta x}$$

Heterogeneous Fog



Ray-Casting

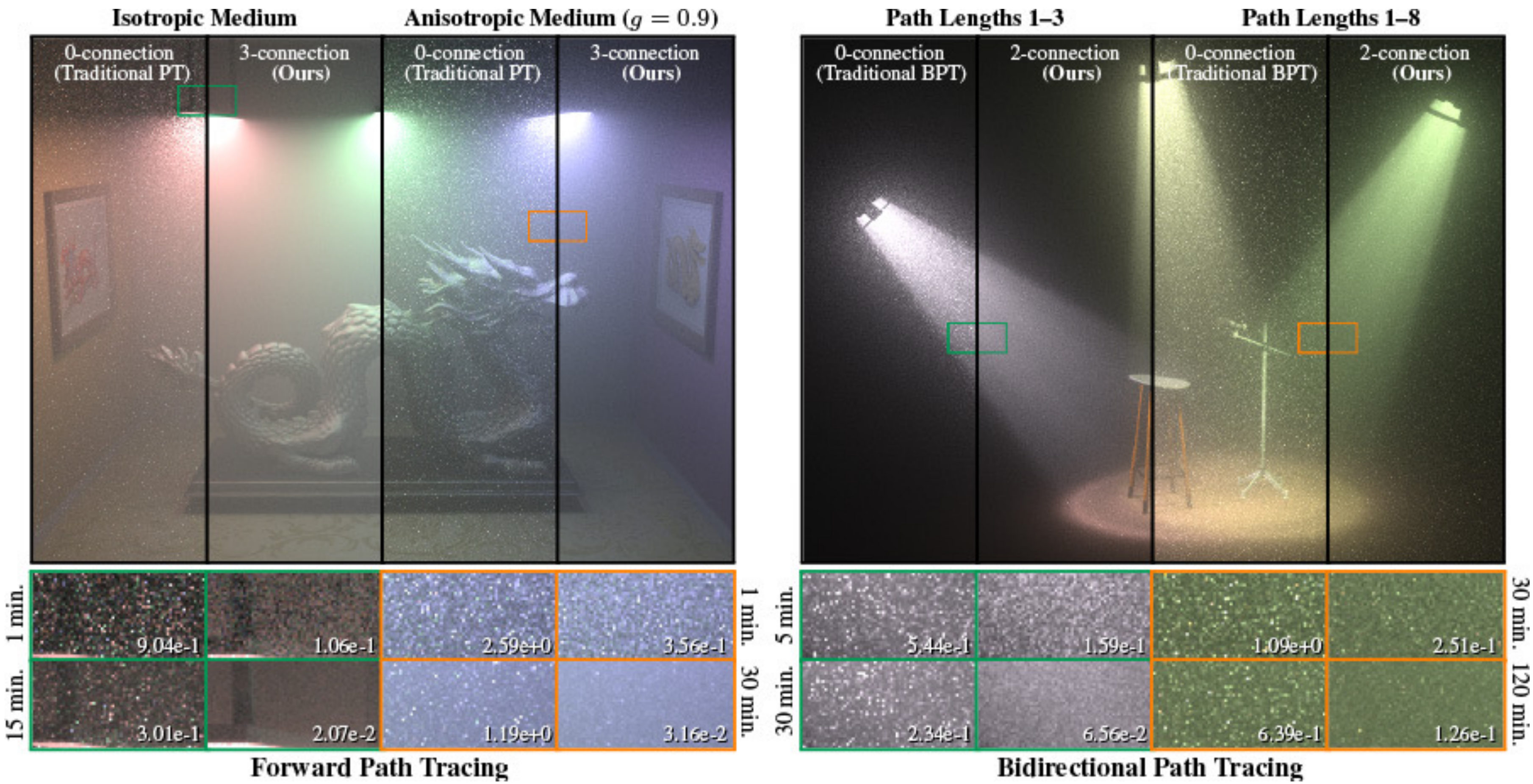
- **Early Ray Termination**
  - Abort ray-marching when subsequent contributions are negligible
  - if (T < epsilon) return L;
  - Very effective in dense volumes
  - Also avoids ray-marching to infinity
- **Grid Traversal**
  - 3-D DDA
  - Ray-marching



- **Adaptive Marching**
  - Bulk integration over homogeneous regions (e.g., octree, bricks)
  - Pre-compute and store maximum step size separately
  - Increasing step size with decreasing accumulated transmittance
  - Vertex Connection and Merging & Joint Path Sampling [Siggraph'14]

Full Volumetric Light Simulation

- **Taking into account multiple scattering in the volume**



Full Volumetric Light Simulation

- **Including Shadows, Caustics, etc.**