Computer Graphics

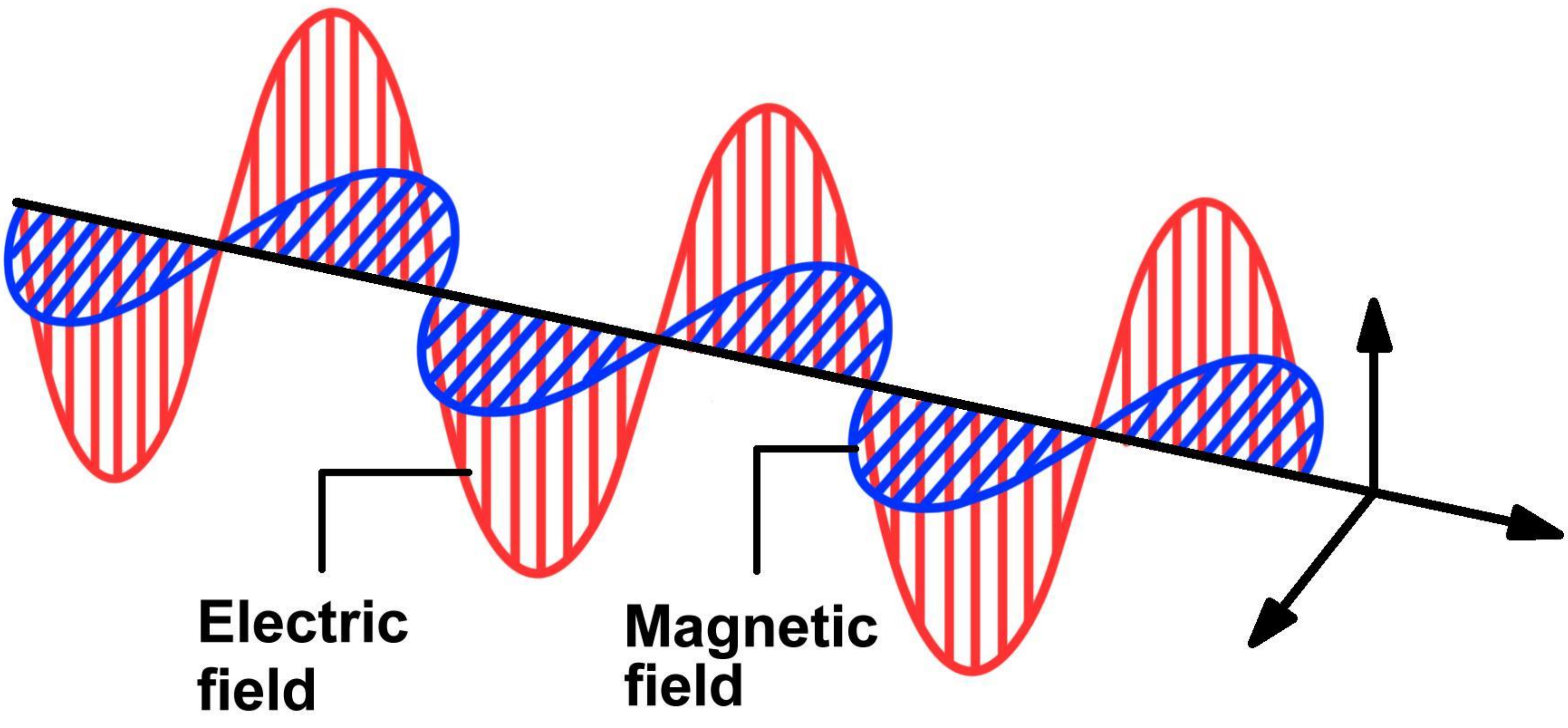
- Light Transport -

Philipp Slusallek

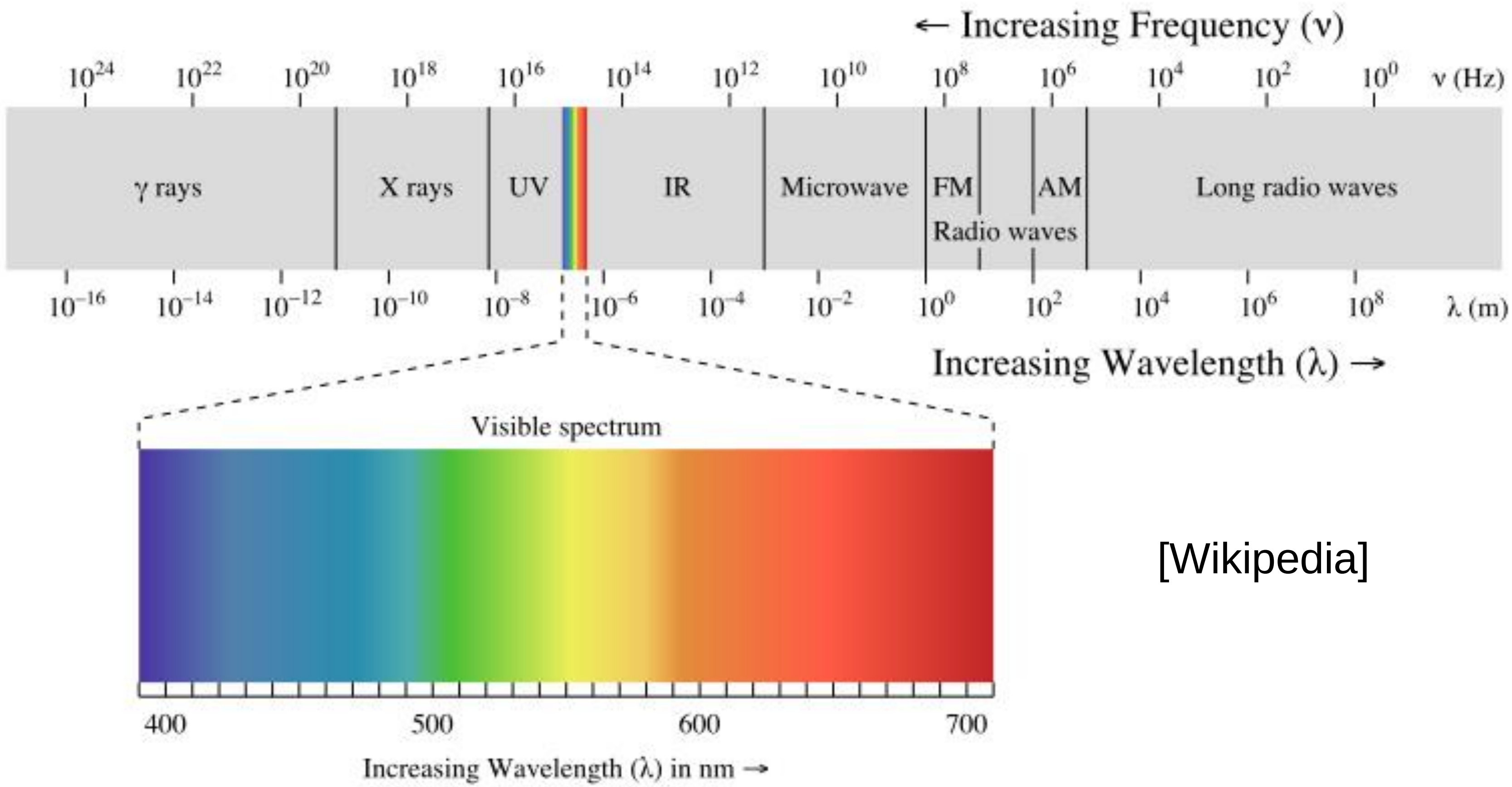
LIGHT

What is Light ?

- **Electro-magnetic wave propagating at speed of light**



What is Light ?



[Wikipedia]

What is Light ?

- **Ray**
  - Linear propagation
  - Geometrical optics / ray optics
- **Vector**
  - Polarization
  - **Jones Calculus**: matrix representation,
  - Has been used in graphics with extended ray model
- **Wave**
  - Diffraction, interference
  - **Maxwell equations**: propagation of light
  - Partial simulation possible using extended ray model, e.g. radar
- **Particle**
  - Light comes in discrete energy quanta: photons
  - **Quantum theory**: interaction of light with matter
- **Field**
  - Electromagnetic force: exchange of virtual photons
  - **Quantum Electrodynamics (QED)**: interaction between particles

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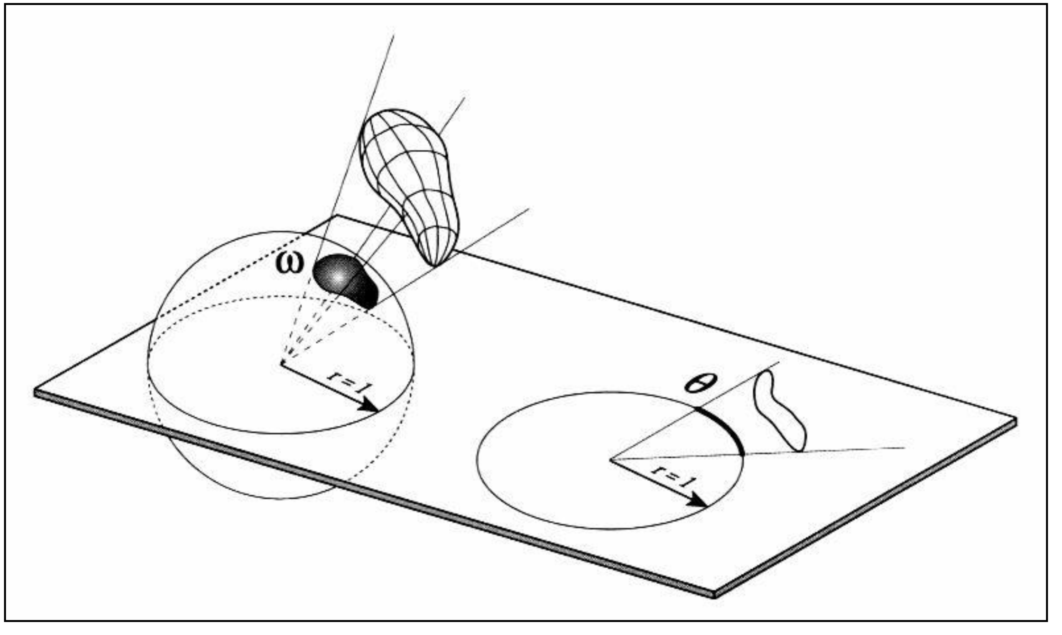
Light in Computer Graphics

- **Based on human visual perception**
  - Focused on macroscopic geometry (→ Reflection Models)
  - Only tristimulus color model (e.g., RGB, → Human Visual System)
  - Psycho-physics: tone mapping, compression, … (→ RIS course)
- **Ray optic assumptions**
  - Macroscopic objects (micro scale geometry → BRDF)
  - Incoherent light (no laser; focus on power – not amplitude)
  - No attenuation in free space (no participating media)
  - Linear propagation
  - Light: scalar, real-valued quantity
  - Superposition principle: light contributions add up, do not interact
- **Limitations**
  - No microscopic structures (≈ λ), no volumetric effects (for now)
  - No polarization, no coherent light (e.g., laser, radar)
  - No diffraction, interference, dispersion, etc. …

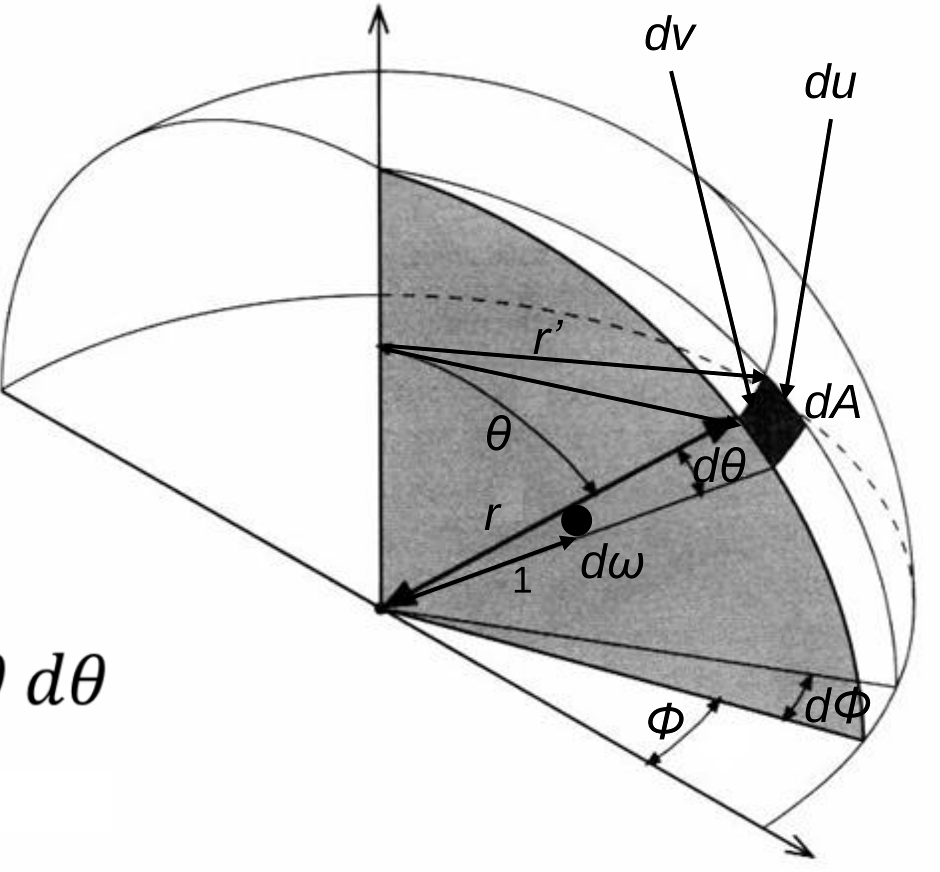
Angle and Solid Angle

- **The** **angle** ***θ*** **(in radians) subtended by a curve in the plane is the length of the corresponding arc on the unit circle:** ***l = θ r = 1***
- **The** **solid angle** ***Ω*, *dω*** **subtended by an object is the surface area of its projection onto the unit sphere**
  - Units for solid angle: steradian [sr] (dimensionless, $\leq 4\pi$)



Solid Angle in Spherical Coords

- **Infinitesimally (!) small solid angle *dω***
  - In spherical coords ($d\theta$, $d\Phi$):
  - $du = r\,d\theta$
  - $dv = r'\,d\Phi = r\sin\theta\,d\Phi$
  - $dA = du\,dv = r^2\sin\theta\,d\theta d\Phi$
  - $d\omega = dA/r^2 = \sin\theta\,d\theta d\Phi$



- **Finite solid angle**
  - Integration of area, e.g.

$$\Omega = \int_{\phi_0}^{\phi_1} d\phi \int_{\theta_0(\phi)}^{\theta_1(\phi)} \sin\theta\,d\theta$$

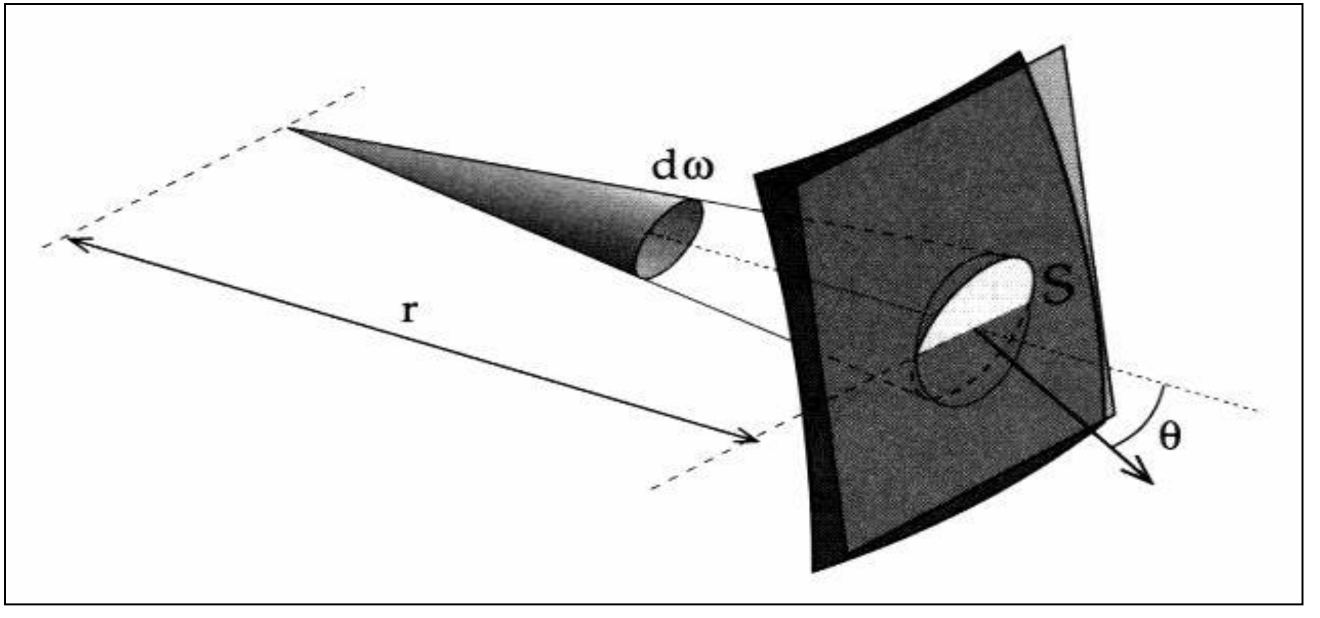
Solid Angle for a Surface

- The solid angle subtended by a small surface patch *S* with area *dA* is obtained by (i) projecting it orthogonal to the vector *r* from the origin:

$$dA \cos\theta$$

and (ii) dividing by the squared distance to the origin: $\mathrm{d}\omega = \frac{\mathrm{d}A \cos\theta}{r^2}$

$$\Omega = \iint_S \frac{\vec{r}\cdot\vec{n}}{r^3} dA$$



Radiometry

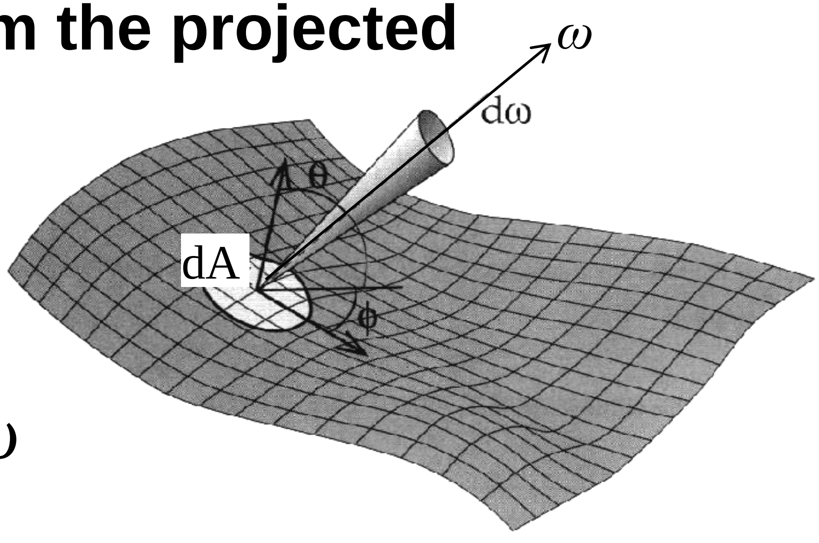
- **Definition:**
  - Radiometry is the science of measuring radiant energy transfer. Radiometric quantities have physical meaning and can be directly measured using proper equipment such as spectral radiometers.

- **Radiometric Quantities**
  - Energy [J] *Q* (#Photons x Energy = $n \cdot h\nu$)
  - Radiant power [watt = J/s] Φ (Total Flux)
  - Intensity [watt/sr] *I* (Flux from a point per s.angle)
  - Irradiance [watt/m^2] *E* (Incoming flux per area)
  - Radiosity [watt/m^2] *B* (Outgoing flux per area)
  - Radiance [watt/(m^2 sr)] *L* (Flux per area & proj. s. angle)

Radiometric Quantities: Radiance

- **Radiance is used to describe radiant energy transfer**
- **Radiance *L* is defined as**
  - The power (flux) traveling through areas dA around some point *x*
  - In a specified direction $\omega = (\theta, \varphi)$
  - Per unit area perpendicular to the direction of travel
  - Per unit solid angle

  ➔ # photons through area and cone times their energy per second
- **Thus, the differential power $d^2\Phi$ radiated through the differential solid angle $d\omega$, from the projected differential area $dA\cos\theta$ is:**



$$d^2\Phi = L(x,\omega)dA(x)\cos\theta\, d\omega$$

Radiometric Quantities: Irradiance

- **Irradiance E** is defined as the **total power per unit area** (flux density) incident onto a surface. To obtain the total flux incident to *dA*, the incoming radiance L_i is integrated over the upper hemisphere Ω_+ above the surface**:**

$$E \equiv \frac{d\Phi}{dA}$$

$$d\Phi = \left[\int_{\Omega_+} L_i(x,\omega)\cos\theta \, d\omega\right] dA$$

$$E(x) = \int_{\Omega_+} L_i(x,\omega)\cos\theta \, d\omega = \int_0^{2\pi}\int_0^{\frac{\pi}{2}} L_i(x,\omega)\cos\theta\sin\theta \, d\theta d\phi$$

Radiometric Quantities: Radiosity

- **Radiosity B** is defined as the **total power per unit area** (flux density) leaving a surface. To obtain the total flux leaving some area *dA*, the outgoing radiance L_o is integrated over the upper hemisphere Ω_+**:**

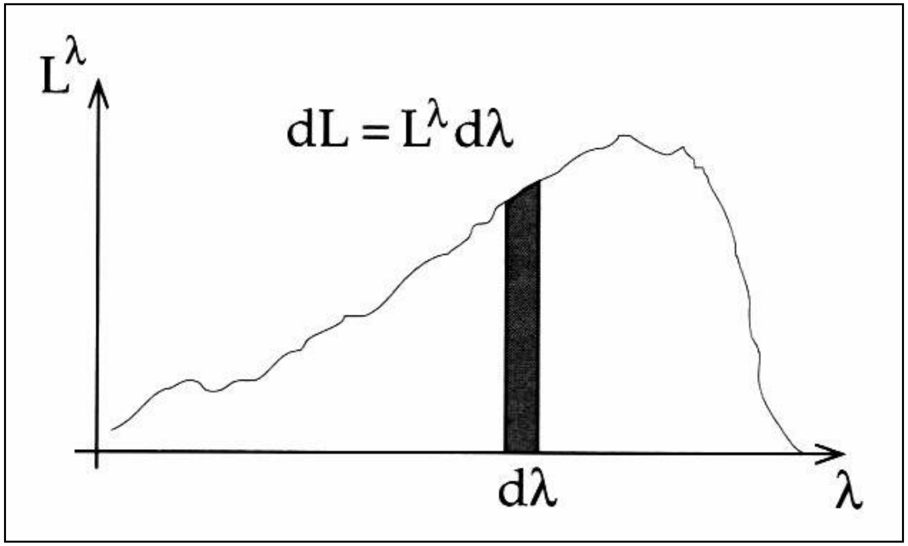
$$B \equiv \frac{d\Phi}{dA}$$

$$d\Phi = \left[\int_{\Omega_+} L_o(x,\omega)\cos\theta\, d\omega\right] dA$$

$$B(x) = \int_{\Omega_+} L_o(x,\omega)\cos\theta\, d\omega = \int_0^{2\pi}\int_0^{\frac{\pi}{2}} L_o(x,\omega)\cos\theta\sin\theta\, d\theta d\phi$$

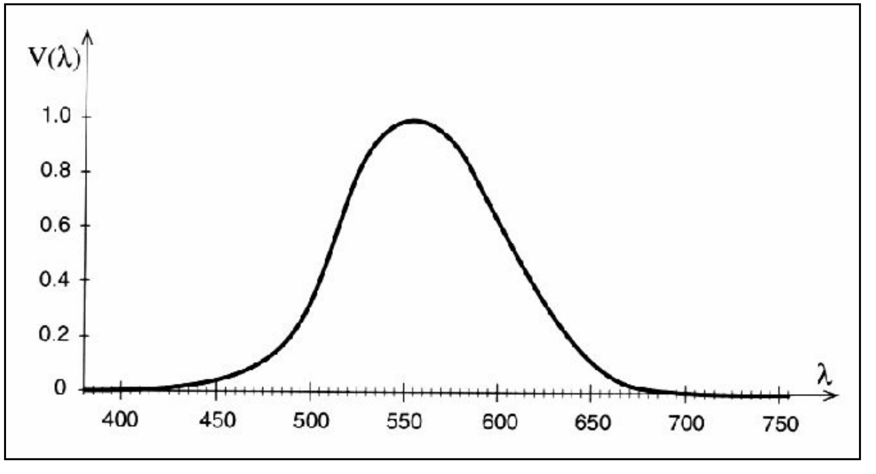
Spectral Properties

- **Wavelength**
  - Light is composed of electromagnetic waves
  - These waves have different frequencies (and wavelengths)
  - Most transfer quantities are continuous functions over the spectrum
- **In graphics**
  - Each measurement *L(x,ω)* is for a discrete band of wavelength only
    - Often R(ed, long), G(reen, medium), B(lue, short) (but see later)



Photometry

– The human eye is sensitive to a limited range of wavelengths
  - Roughly from 380 nm to 780 nm
– Our visual system responds differently to different wavelengths
  - Can be characterized by the *Luminous Efficiency Function* V(λ)
  - Represents the average human spectral response
  - Separate curves exist for light and dark adaptation of the eye
– Photometric quantities are derived from radiometric quantities by *integrating* them against this function
– More details later → Human Visual System

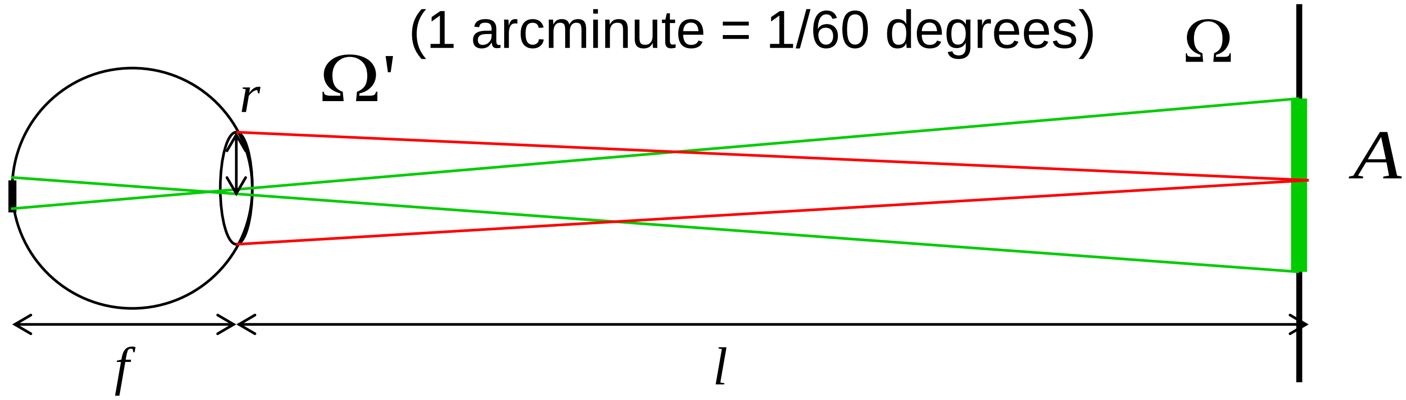


Radiometry vs. Photometry

Radiometry (physics-based quantities)		→	Photometry (perception-based quantities)	
W	Radiant power	→	Luminous power	lm (lumens)
W/m²	Radiosity	→	Luminosity	lm/m² (lux)
	Irradiance	→	Illuminance	
W/m²/sr	Radiance	→	Luminance	cd/m² (lm/m²/sr)
W/sr	Radiant intensity	→	Luminous intensity	cd (candela)

English	**German**	→	**English**	**German**
Radiant power	Strahlungsleistung	→	Luminous power	Lichtstrom
Radiosity	Spezifische Ausstrahlung	→	Luminosity	Leuchtkraft
Irradiance	Bestrahlungsstärke	→	Illuminance	Beleuchtungsstärke
Radiance	Strahldichte	→	Luminance	Leuchtdichte
Radiant intensity	Strahlstärke	→	Luminous intensity	Lichtstärke

Perception of Light



photons / second = **flux = energy / time = power** (Φ)

rod sensitive to flux

Solid angle of a rod = **resolution** (≈ 1 arcminute[2]) Ω

projected rod size = **area A** $A \approx l^2 \cdot \Omega$

angular extent of pupil aperture (r ≤ 4 mm) = **solid angle** $\Omega' \approx \pi \cdot r^2 / l^2$

flux proportional to area and solid angle $\Phi = L\ \mathrm{A}\ \Omega'$

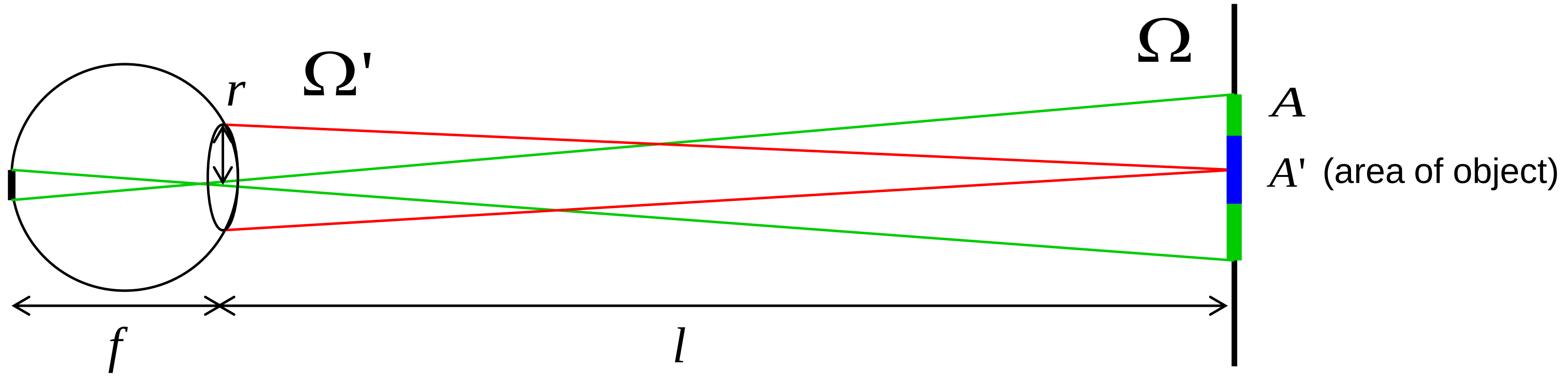
radiance = flux per unit area per unit solid angle $L = \dfrac{\Phi}{\Omega' \cdot A}$

The eye detects radiance

As *l* increases:

$$\Phi_0 = L \cdot l^2 \cdot \Omega \cdot \pi \frac{r^2}{l^2} = L \cdot \mathrm{const}$$

Brightness Perception



- $A' > A$: area of sun covers more than one rod: photon flux per rod stays constant
- $A' < A$: photon flux per rod decreases

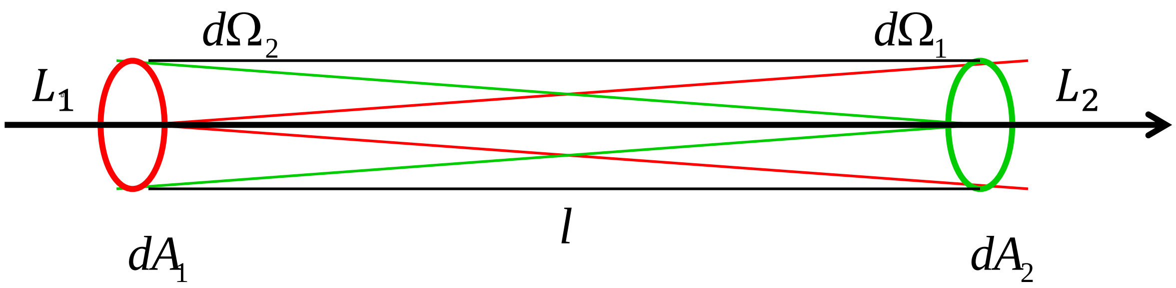
Where does the Sun turn into a star ?

– Depends on apparent Sun disc size on retina

– Photon flux per rod stays the same on Mercury, Earth or Neptune

– Photon flux per rod decreases when $\Omega' < 1$ arcminute^2 (~ beyond Neptune)

Radiance in Space



Flux leaving surface 1 must be equal to flux arriving on surface 2

$$L_1 d\Omega_1 dA_1 = L_2 d\Omega_2 dA_2$$

From geometry follows $d\Omega_1 = \frac{dA_2}{l^2}$ $\quad d\Omega_2 = \frac{dA_1}{l^2}$

Ray throughput T: $T = d\Omega_1 \cdot dA_1 = d\Omega_2 \cdot dA_2 = \frac{dA_1 \cdot dA_2}{l^2}$

$$L_1 = L_2$$

The **radiance** in the direction of a light ray **remains constant** as it propagates along the ray

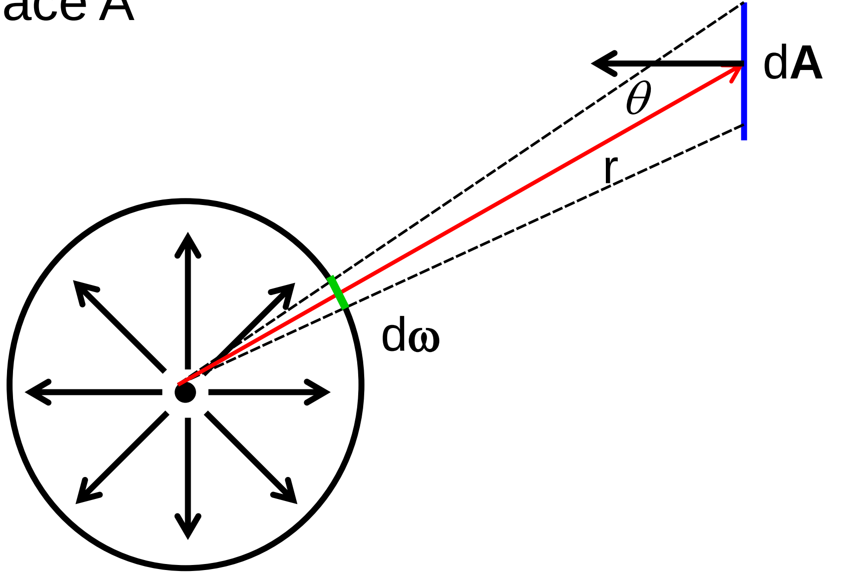
Point Light Source

- **Point light with *isotropic* (same in all dir.) radiance**
  - Power (total flux) of a point light source
    - Φ_g = Power of the light source [watt]
  - Intensity of a light source (radiance cannot be defined, no area)
    - $I = \Phi_g / 4\pi$ [watt/sr]
  - Irradiance on a sphere with radius r around light source:
    - $E_r = \Phi_g / (4 \pi r^2)$ [watt/m²]
  - Irradiance on some other surface A

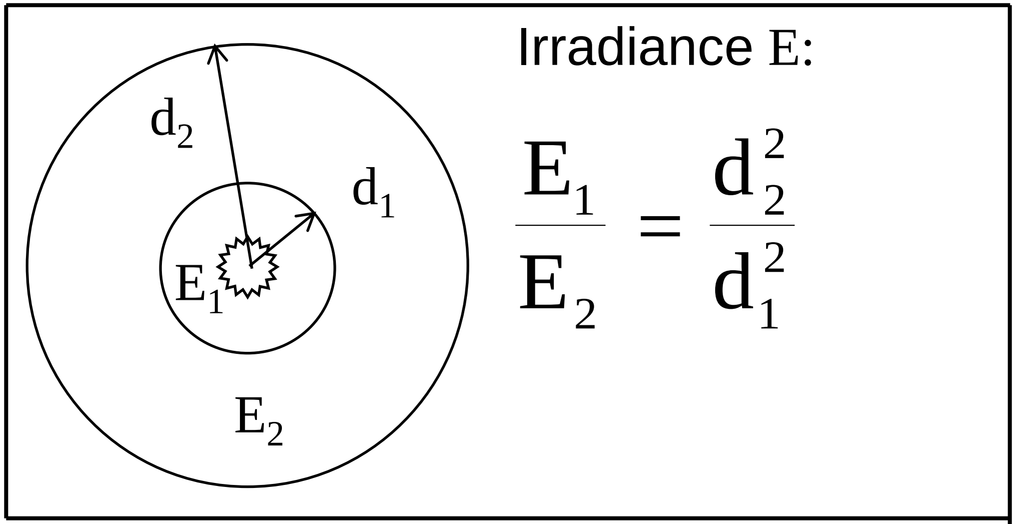
$$E(x) = \frac{d\Phi_g}{dA} = \frac{d\Phi_g}{d\omega}\frac{d\omega}{dA} = I\frac{d\omega}{dA}$$

$$= \frac{\Phi_g}{4\pi} \cdot \frac{dA\cos\theta}{r^2 dA}$$

$$= \frac{\Phi_g}{4\pi} \cdot \frac{\cos\theta}{r^2} = \frac{\Phi_g}{4\pi r^2} \cdot \cos\theta$$



Inverse Square Law



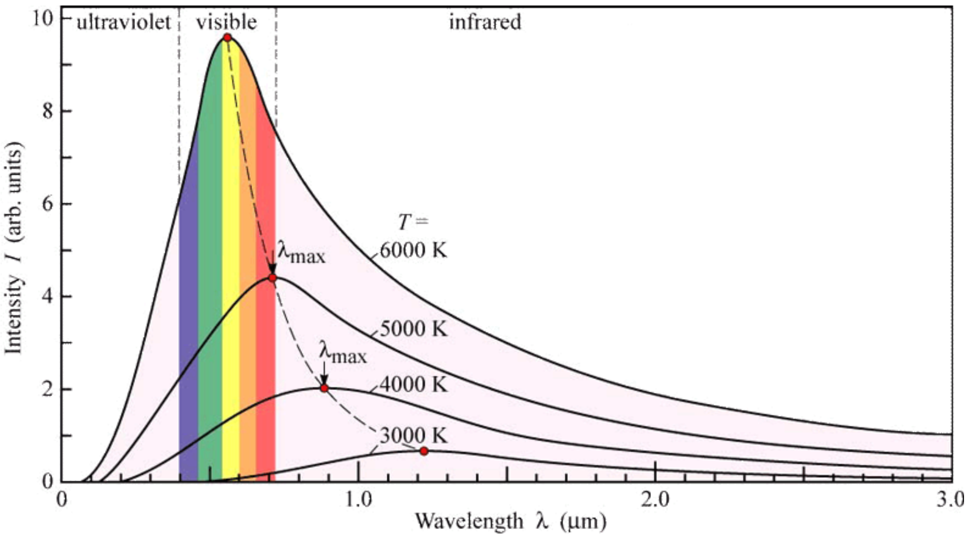
$$\frac{E_1}{E_2} = \frac{d_2^2}{d_1^2}$$

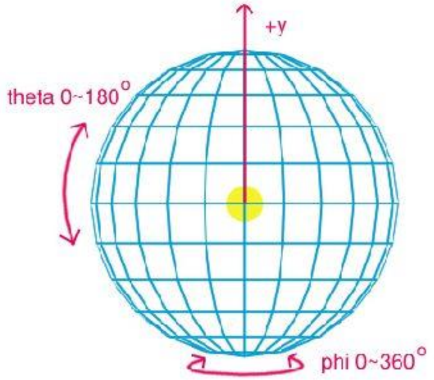
- **Irradiance *E*: power per m^2**
  - Illuminating quantity
- **Distance-dependent**
  - Double distance from emitter: area of sphere is four times bigger
- **Irradiance falls off with inverse of squared distance**
  - Only for point light sources (!)

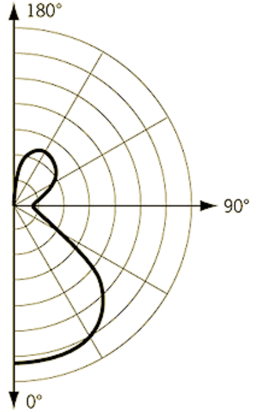
Light Source Specifications

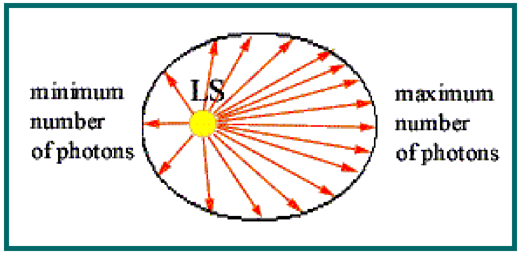
- **Power (total flux)**
  - Emitted energy / time
- **Active emission size**
  - Point, line, area, volume
- **Spectral distribution**
  - Thermal, line spectrum
- **Directional distribution**
  - Goniometric diagram

Black body radiation (see later)









Sky Light

- **Sun**
  - Point source (approx.)
  - White light (by def.)
- **Sky**
  - Area source
  - Scattering: blue
- **Horizon**
  - Brighter
  - Haze: whitish
- **Overcast sky**
  - Multiple scattering in clouds
  - Uniform grey
- **Several sky models are available**

Courtesy Lynch & Livingston

LIGHT TRANSPORT

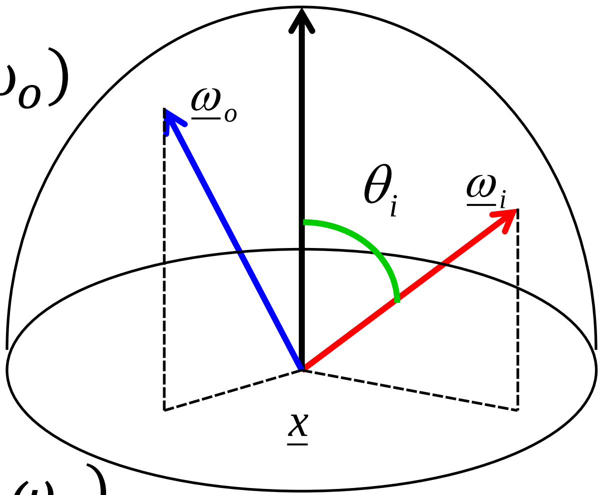
Light Transport in a Scene

- **Scene**
  - Lights (emitters)
  - Object surfaces (partially absorbing)
- **Illuminated object surfaces become emitters, too!**
  - Radiosity = Irradiance minus absorbed photons flux density
    - Radiosity: photons per second per m^2 leaving surface
    - Irradiance: photons per second per m^2 incident on surface
    - But also need to look at directional distribution
- **Light bounces between all mutually visible surfaces**
- **Invariance of radiance in free space**
  - No absorption in-between objects
- **Dynamic energy equilibrium in a scene**
  - Emitted photons = absorbed photons (+ escaping photons)
    → Global Illumination, discussed in RIS lecture

Surface Radiance

$$L(x, \omega_o) = L_e(x, \omega_o) + \int_{\Omega_+} f_r(\omega_i, x, \omega_o) L_i(x, \omega_i) \cos\theta_i \, d\omega_i$$

- **Visible surface radiance** $L(x, \omega_o)$
  - Surface position x
  - Outgoing direction ω_o
- **Incoming illumination direction** ω_i



- **Emission** $L_e(x, \omega_o)$
- **Reflected light**
  - Incoming radiance from all directions $L_i(x, \omega_i)$
  - Direction-dependent reflectance (BRDF: bidirectional reflectance distribution function) $f_r(\omega_i, x, \omega_o)$

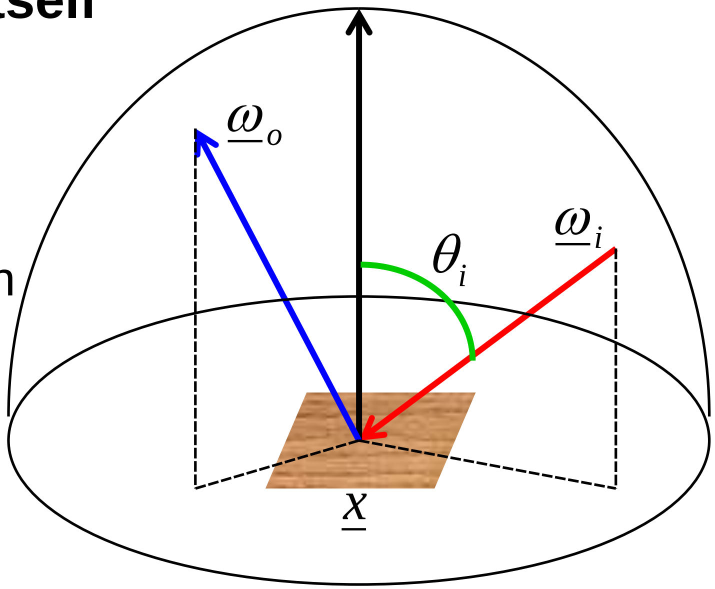
Rendering Equation

- **Most important equation for graphics**
  - Expresses energy equilibrium in scene

$$L(x,\omega_o) = L_e(x,\omega_o) + \int_{\Omega_+} f_r(\omega_i,x,\omega_o) L_i(x,\omega_i) \cos\theta_i \, d\omega_i$$

total radiance = emitted + reflected radiance

- **First term: Emission from the surface itself**
  - Non-zero only for light sources
- **Second term: reflected radiance**
  - Integral over all possible incoming directions of radiance times angle-dependent surface reflection function
- **Fredholm integral equation of 2nd kind**
  - **Difficulty: Unknown radiance appears both on the left-hand side and inside the integral**
  - Numerical methods necessary to compute approximate solution



RE: Integrating over Surfaces

- **Outgoing illumination at a point**

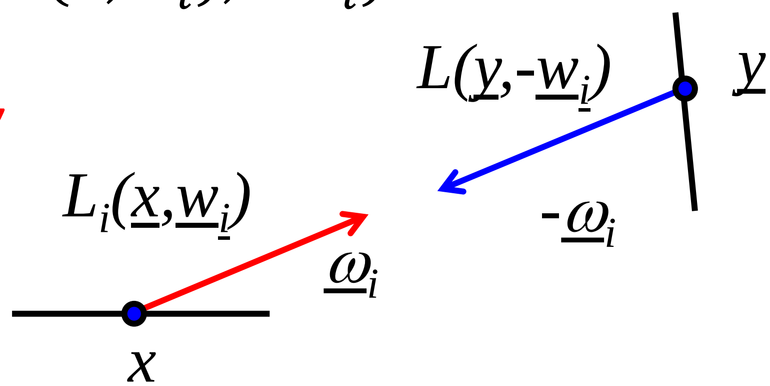
$$L(x, \omega_o) = L_e(x, \omega_o) + L_r(x, \omega_o)$$

$$L(x, \omega_o) = L_e(x, \omega_o) + \int_{\Omega_+} f_r(\omega_i, x, \omega_o) L_i(x, \omega_i) \cos\theta_i \, d\omega_i$$

- **Linking with other surface points**
  - Incoming radiance at x is outgoing radiance at y

$$L_i(x, \omega_i) = L(y, -\omega_i) = L(RT(x, \omega_i), -\omega_i)$$

  - **Ray-Tracing operator:** $RT(x, \omega_i) = y$



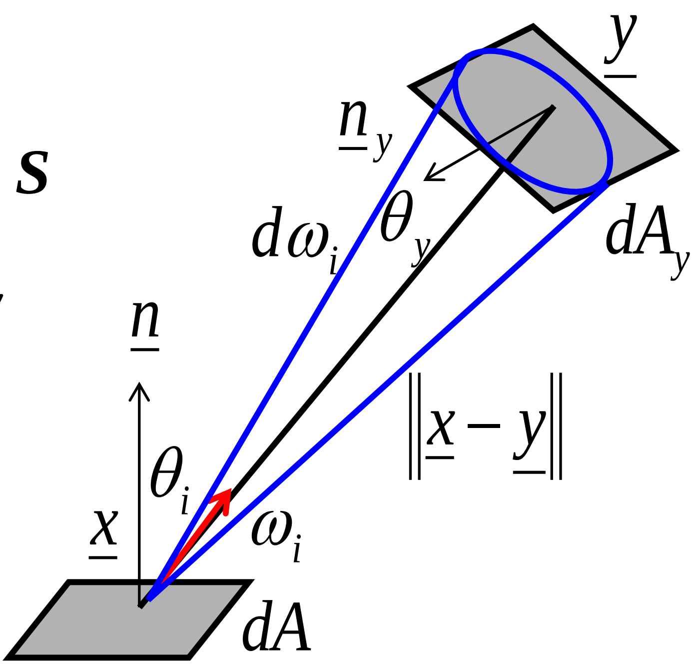
Integrating over Surfaces

- **Outgoing illumination at a point**

$$L(x,\omega_o) = L_e(x,\omega_o) + \int_{\Omega_+} f_r(\omega_i, x, \omega_o) L_i(x,\omega_i)\cos\theta_i \, d\omega_i$$

- **Re-parameterization over surfaces *S***

$$d\omega_i = \frac{\cos\theta_y}{\|x-y\|^2} dA_y$$



$$L(x,\omega_o) = L_e(x,\omega_o) + \int_{y\in S} f_r(\omega(x,y), x, \omega_o) L_i(x, \omega(x,y)) V(x,y) \frac{\cos\theta_i \cos\theta_y}{\|x-y\|^2} dA_y$$

Integrating over Surfaces

$$L(x, \omega_o)$$
$$= L_e(x, \omega_o) + \int_{y \in S} f_r(\omega(x,y), x, \omega_o) L_i(x, \omega(x,y)) V(x,y) \frac{\cos\theta_i \cos\theta_y}{\|x-y\|^2} dA_y$$

- **Geometry term:** $G(\mathrm{x},\mathrm{y}) = V(x,y)\frac{\cos\theta_i \cos\theta_y}{\|x-y\|^2}$
- **Visibility term:** $V(x,y) = \begin{cases} 1, & if\ visible \\ 0, & otherwise \end{cases}$
- **Integration over all surfaces:** $\int_{y \in S} \cdots dA_y$

$$L(x, \omega_o) = L_e(x, \omega_o) + \int_{y \in S} f_r(\omega(x,y), x, \omega_o) L_i\big(x, \omega(x,y)\big) G(x,y) dA_y$$

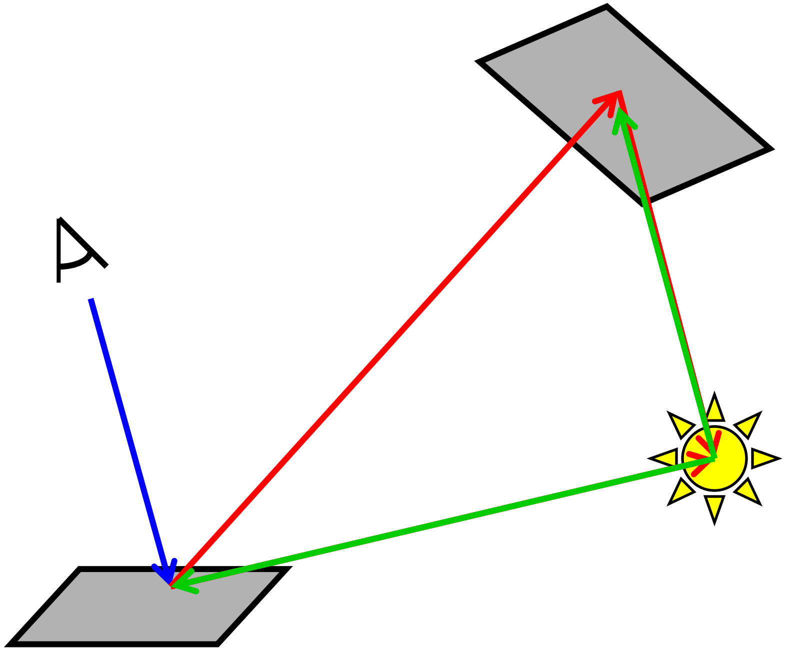
Rendering Equation: Approximations

- **Approximations based only on empirical foundations**
  - An example: rasterization e.g. in OpenGL (→ later)
- **Using RGB instead of full spectrum**
  - Follows roughly the eye's sensitivity (L, f_r are 3D vectors for RGB)
- **Sampling hemisphere only at discrete directions**
  - Simplifies integration to a summation (only directly to light sources)
- **Reflection function model (BRDF, see later)**
  - Approximation by parameterized functions
    - Diffuse: light reflected uniformly in all directions
    - Specular: perfect reflection/refraction direction
    - Glossy: mirror reflection, but from a rough surface
    - And mixture thereof

Ray Tracing

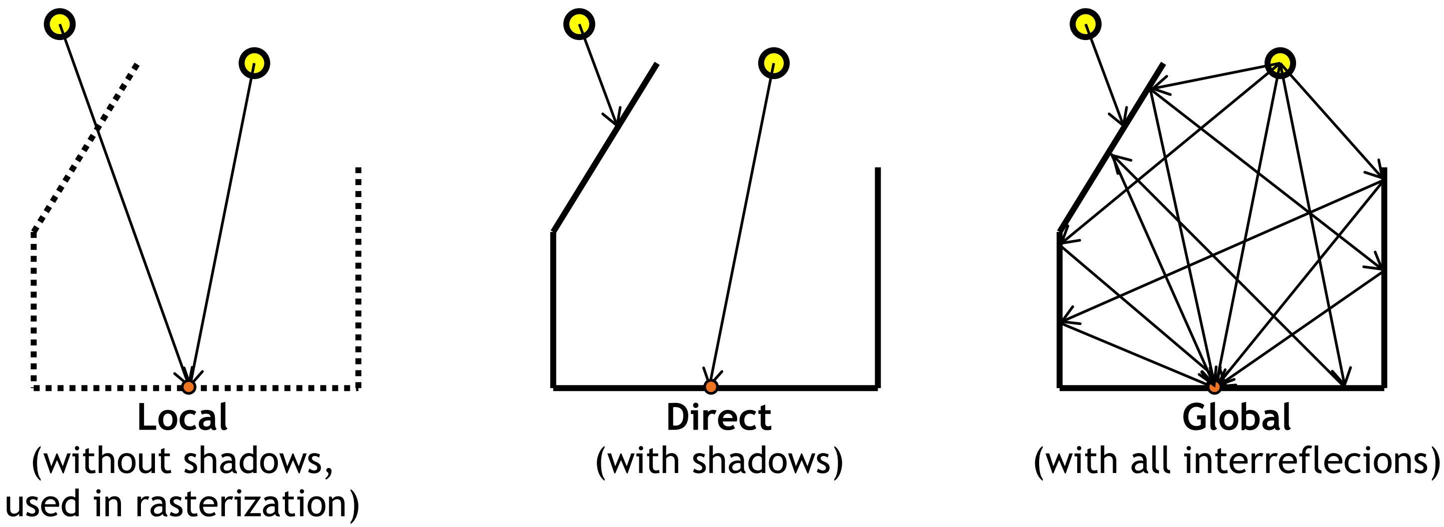
$$L(x,\omega_o) = L_e(x,\omega_o) + \int_{\Omega_+} f_r(\omega_i,x,\omega_o)L_i(x,\omega_i)\cos\theta_i \, d\omega_i$$

- **Simple ray tracing**
  - Illumination from discrete point light sources only – direct illumination only
    - Integral → sum of contributions from each light
    - No global illumination
  - Evaluates angle-dependent reflectance function (BRDF) – **shading process**
- **Advanced ray tracing techniques**
  - Recursive ray tracing
    - Multiple reflections/refractions (e.g. for specular surfaces)
  - Ray tracing for global illumination
    - Stochastic sampling (Monte Carlo methods) → RIS course



Different Types of Illumination

- **Three types of illumination computations in CG**



- **Ambient Illumination**
  - Global illumination is costly to compute
  - Indirect illumination (through interreflections) is typically smooth
    ➔Approximate via a constant term $L_{i,a}$ (incoming ambient illum.)
  - Has no incoming direction, provide ambient reflection term k_a
    - Often chosen to be the same as the diffuse term ($k_a = k_d$)

$$L_o(x, \omega_o) = k_a L_{i,a}$$