Computer Graphics

- Light Transport -

Philipp Slusallek
LIGHT
What is Light?

• Electro-magnetic wave propagating at speed of light

Electric field

Magnetic field
What is Light?

[Visible spectrum diagram from Wikipedia]
What is Light?

- **Ray**
  - Linear propagation
  - Geometrical optics / ray optics

- **Vector**
  - Polarization
  - *Jones Calculus*: matrix representation,
  - Has been used in graphics with extended ray model

- **Wave**
  - Diffraction, interference
  - *Maxwell equations*: propagation of light
  - Partial simulation possible using extended ray model, e.g. radar

- **Particle**
  - Light comes in discrete energy quanta: photons
  - *Quantum theory*: interaction of light with matter

- **Field**
  - Electromagnetic force: exchange of virtual photons
  - *Quantum Electrodynamics (QED)*: interaction between particles
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Light in Computer Graphics

• **Based on human visual perception**
  – Focused on macroscopic geometry (→ Reflection Models)
  – Only tristimulus color model (e.g., RGB, → Human Visual System)
  – Psycho-physics: tone mapping, compression, … (→ RIS course)

• **Ray optic assumptions**
  – Macroscopic objects (micro scale geometry → BRDF)
  – Incoherent light (no laser; focus on power – not amplitude)
  – No attenuation in free space (no participating media)
  – Linear propagation
  – Light: scalar, real-valued quantity
  – Superposition principle: light contributions add up, do not interact

• **Limitations**
  – No microscopic structures (≈ λ), no volumetric effects (for now)
  – No polarization, no coherent light (e.g., laser, radar)
  – No diffraction, interference, dispersion, etc. …
Angle and Solid Angle

• The angle $\theta$ (in radians) subtended by a curve in the plane is the length of the corresponding arc on the unit circle: $l = \theta \cdot r = 1$

• The solid angle $\Omega$, $d\omega$ subtended by an object is the surface area of its projection onto the unit sphere
  – Units for solid angle: steradian [sr] (dimensionless, $\leq 4\pi$)
Solid Angle in Spherical Coords

- **Infinitesimally (!) small solid angle** $d\omega$
  - In spherical coords $(d\theta, d\Phi)$:
  - $du = r \, d\theta$
  - $dv = r' \, d\Phi = r \sin \theta \, d\Phi$
  - $dA = du \, dv = r^2 \sin \theta \, d\theta d\Phi$
  - $d\omega = dA/r^2 = \sin \theta \, d\theta d\Phi$

- **Finite solid angle**
  - Integration of area, e.g.

\[
\Omega = \int_{\phi_0}^{\phi_1} d\phi \int_{\theta_0(\phi)}^{\theta_1(\phi)} \sin \theta \, d\theta
\]
Solid Angle for a Surface

- The solid angle subtended by a small surface patch $S$ with area $dA$ is obtained by (i) projecting it orthogonal to the vector $r$ from the origin:
  $$dA \cos \theta$$
  and (ii) dividing by the squared distance to the origin:
  $$d\omega = \frac{dA \cos \theta}{r^2}$$

$$\Omega = \iiint_S \frac{\hat{r} \cdot \hat{n}}{r^3} dA$$
Radiometry

• **Definition:**
  - Radiometry is the science of measuring radiant energy transfer. Radiometric quantities have physical meaning and can be directly measured using proper equipment such as spectral radiometers.

• **Radiometric Quantities**
  - **Energy** \([\text{J}]\)  
    
    \[ Q \quad \text{(#Photons x Energy} = n \cdot h\nu) \]
  - **Radiant power** \([\text{watt} = \text{J/s}]\)  
    
    \[ \Phi \quad \text{(Total Flux)} \]
  - **Intensity** \([\text{watt/sr}]\)  
    
    \[ I \quad \text{(Flux from a point per s. angle)} \]
  - **Irradiance** \([\text{watt/m}^2]\)  
    
    \[ E \quad \text{(Incoming flux per area)} \]
  - **Radiosity** \([\text{watt/m}^2]\)  
    
    \[ B \quad \text{(Outgoing flux per area)} \]
  - **Radiance** \([\text{watt/(m}^2 \text{ sr)}]\)  
    
    \[ L \quad \text{(Flux per area & proj. s. angle)} \]
Radiometric Quantities: Radiance

• Radiance is used to describe radiant energy transfer

• Radiance \( L \) is defined as
  – The power (flux) traveling through areas \( dA \) around some point \( x \)
  – In a specified direction \( \omega = (\theta, \varphi) \)
  – Per unit area perpendicular to the direction of travel
  – Per unit solid angle
    ➔ # photons through area and cone times their energy per second

• Thus, the differential power \( d^2 \Phi \) radiated through the differential solid angle \( d\omega \), from the projected differential area \( dA \cos \theta \) is:

\[
d^2 \Phi = L(x, \omega)dA(x) \cos \theta \ d\omega
\]
Radiometric Quantities: Irradiance

- **Irradiance** $E$ is defined as the **total power per unit area** (flux density) incident onto a surface. To obtain the total flux incident to $dA$, the **incoming** radiance $L_i$ is integrated over the upper hemisphere $\Omega_+$ above the surface:

\[
E \equiv \frac{d\Phi}{dA} = \int_{\Omega_+} L_i(x, \omega) \cos \theta \, d\omega \, dA
\]

\[
E(x) = \int_{\Omega_+} L_i(x, \omega) \cos \theta \, d\omega = \int_0^{2\pi} \int_0^{\frac{\pi}{2}} L_i(x, \omega) \cos \theta \sin \theta \, d\theta \, d\phi
\]
Radiometric Quantities: Radiosity

- **Radiosity** $B$ is defined as the **total power per unit area** (flux density) leaving a surface. To obtain the total flux leaving some area $dA$, the **outgoing** radiance $L_o$ is integrated over the upper hemisphere $\Omega_+$:

\[
B \equiv \frac{d\Phi}{dA} = \left[ \int_{\Omega_+} L_o(x, \omega) \cos \theta \, d\omega \right] dA
\]

\[
B(x) = \int_{\Omega_+} L_o(x, \omega) \cos \theta \, d\omega = \int_0^{2\pi} \int_0^{\frac{\pi}{2}} L_o(x, \omega) \cos \theta \sin \theta \, d\theta d\phi
\]
Spectral Properties

• **Wavelength**
  – Light is composed of electromagnetic waves
  – These waves have different frequencies (and wavelengths)
  – Most transfer quantities are continuous functions over the spectrum

• **In graphics**
  – Each measurement $L(x,\omega)$ is for a discrete band of wavelength only
    - Often $R$(ed, long), $G$(reen, medium), $B$(lue, short) (but see later)
Photometry

- The human eye is sensitive to a limited range of wavelengths
  - Roughly from 380 nm to 780 nm
- Our visual system responds differently to different wavelengths
  - Can be characterized by the \textit{Luminous Efficiency Function} \( V(\lambda) \)
  - Represents the average human spectral response
  - Separate curves exist for light and dark adaptation of the eye
- Photometric quantities are derived from radiometric quantities by \textit{integrating} them against this function
- More details later \( \rightarrow \) Human Visual System
### Radiometry vs. Photometry

<table>
<thead>
<tr>
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**Units:**
- **W** (watt)
- **lm** (lumens)
- **lm/m²** (lux)
- **cd/m²** (candela)
- **cd (candela)**
Perception of Light

The eye detects radiance

Solid angle of a rod = resolution \( (\approx 1 \text{ arcminute}^2) \)

Projected rod size = area \( A \)

Angular extent of pupil aperture \( (r \leq 4 \text{ mm}) \) = solid angle

Flux proportional to area and solid angle

Radiance = flux per unit area per unit solid angle

The eye detects radiance

As \( l \) increases:

\[ \Phi_0 = L \cdot l^2 \cdot \Omega \cdot \pi \frac{r^2}{l^2} = L \cdot \text{const} \]
Brightness Perception

- $A' > A$: area of sun covers more than one rod:
  - photon flux per rod stays constant
- $A' < A$: photon flux per rod decreases

Where does the Sun turn into a star?
- Depends on apparent Sun disc size on retina
- Photon flux per rod stays the same on Mercury, Earth or Neptune
- Photon flux per rod decreases when $\Omega' < 1\ \text{arcminute}^2$ (≈ beyond Neptune)
The radiance in the direction of a light ray remains constant as it propagates along the ray.
Point Light Source

- **Point light with isotropic (same in all dir.) radiance**
  - Power (total flux) of a point light source
    - $\Phi_g =$ Power of the light source [watt]
  - Intensity of a light source (radiance cannot be defined, no area)
    - $I = \Phi_g / 4\pi$ [watt/sr]
  - Irradiance on a sphere with radius $r$ around light source:
    - $E_r = \Phi_g / (4\pi r^2)$ [watt/m$^2$]
  - Irradiance on some other surface $A$

$$E(x) = \frac{d\Phi_g}{dA} = \frac{d\Phi_g}{d\omega} \frac{d\omega}{dA} = I \frac{d\omega}{dA}$$

$$= \frac{\Phi_g}{4\pi} \cdot dA \cos \theta$$

$$= \frac{\Phi_g}{4\pi} \cdot \frac{dA \cos \theta}{r^2 dA}$$

$$= \frac{\Phi_g}{4\pi} \cdot \frac{\cos \theta}{r^2} = \frac{\Phi_g}{4\pi r^2} \cdot \cos \theta$$
Inverse Square Law

- **Irradiance** $E$: power per m$^2$
  - Illuminating quantity
- **Distance-dependent**
  - Double distance from emitter: area of sphere is four times bigger
- **Irradiance falls off with inverse of squared distance**
  - Only for point light sources (!)
Light Source Specifications

- **Power (total flux)**
  - Emitted energy / time
- **Active emission size**
  - Point, line, area, volume
- **Spectral distribution**
  - Thermal, line spectrum
- **Directional distribution**
  - Goniometric diagram
Sky Light

- Sun
  - Point source (approx.)
  - White light (by def.)

- Sky
  - Area source
  - Scattering: blue

- Horizon
  - Brighter
  - Haze: whitish

- Overcast sky
  - Multiple scattering in clouds
  - Uniform grey

- Several sky models are available

Courtesy Lynch & Livingston
LIGHT TRANSPORT
Light Transport in a Scene

- Scene
  - Lights (emitters)
  - Object surfaces (partially absorbing)

- Illuminated object surfaces become emitters, too!
  - Radiosity = Irradiance minus absorbed photons flux density
    - Radiosity: photons per second per m$^2$ leaving surface
    - Irradiance: photons per second per m$^2$ incident on surface
    - But also need to look at directional distribution

- Light bounces between all mutually visible surfaces

- Invariance of radiance in free space
  - No absorption in-between objects

- Dynamic energy equilibrium in a scene
  - Emitted photons = absorbed photons (+ escaping photons)
    → Global Illumination, discussed in RIS lecture
Surface Radiance

\[ L(x, \omega_o) = L_e(x, \omega_o) + \int_{\Omega_+} f_r(\omega_i, x, \omega_o) L_i(x, \omega_i) \cos \theta_i \, d\omega_i \]

• **Visible surface radiance**
  – Surface position
  – Outgoing direction

• **Incoming illumination direction**

• **Emission**

• **Reflected light**
  – Incoming radiance from all directions
  – Direction-dependent reflectance
    (BRDF: bidirectional reflectance distribution function)
Rendering Equation

- Most important equation for graphics
  - Expresses energy equilibrium in scene

\[ L(x, \omega_o) = L_e(x, \omega_o) + \int_{\Omega_+} f_r(\omega_i, x, \omega_o) L_i(x, \omega_i) \cos \theta_i \, d\omega_i \]

  total radiance = emitted + reflected radiance

- First term: Emission from the surface itself
  - Non-zero only for light sources

- Second term: reflected radiance
  - Integral over all possible incoming directions of radiance times angle-dependent surface reflection function

- Fredholm integral equation of 2nd kind
  - Difficulty: Unknown radiance appears both on the left-hand side and inside the integral
  - Numerical methods necessary to compute approximate solution
RE: Integrating over Surfaces

- **Outgoing illumination at a point**

  \[ L(x, \omega_o) = L_e(x, \omega_o) + L_r(x, \omega_o) \]

  \[ L(x, \omega_o) = L_e(x, \omega_o) + \int_{\Omega_+} f_r(\omega_i, x, \omega_o) L_i(x, \omega_i) \cos \theta_i \, d\omega_i \]

- **Linking with other surface points**
  - Incoming radiance at \( x \) is outgoing radiance at \( y \)

  \[ L_i(x, \omega_i) = L(y, -\omega_i) = L(RT(x, \omega_i), -\omega_i) \]

  - **Ray-Tracing operator:** \( RT(x, \omega_i) = y \)
Integrating over Surfaces

- **Outgoing illumination at a point**

\[ L(x, \omega_o) = L_e(x, \omega_o) + \int_{\Omega_+} f_r(\omega_i, x, \omega_o) L_i(x, \omega_i) \cos \theta_i \, d\omega_i \]

- **Re-parameterization over surfaces \( S \)**

\[ d\omega_i = \frac{\cos \theta_y}{\|x - y\|^2} \, dA_y \]

\[ L(x, \omega_o) = L_e(x, \omega_o) + \int_{y \in S} f_r(\omega(x, y), x, \omega_o) L_i(x, \omega(x, y)) V(x, y) \frac{\cos \theta_i \cos \theta_y}{\|x - y\|^2} \, dA_y \]
Integrating over Surfaces

\[ L(x, \omega_o) \]
\[ = L_e(x, \omega_o) + \int_{y \in S} f_r(\omega(x, y), x, \omega_o)L_i(x, \omega(x, y))V(x, y)\frac{\cos \theta_i \cos \theta_y}{\|x - y\|^2} dA_y \]

- **Geometry term:** \( G(x, y) = V(x, y)\frac{\cos \theta_i \cos \theta_y}{\|x - y\|^2} \)

- **Visibility term:** \( V(x, y) = \begin{cases} 1, & \text{if visible} \\ 0, & \text{otherwise} \end{cases} \)

- **Integration over all surfaces:** \( \int_{y \in S} \cdots dA_y \)

\[ L(x, \omega_o) = L_e(x, \omega_o) + \int_{y \in S} f_r(\omega(x, y), x, \omega_o)L_i(x, \omega(x, y))G(x, y)dA_y \]
• Approximations based only on empirical foundations
  – An example: rasterization e.g. in OpenGL (→ later)

• Using RGB instead of full spectrum
  – Follows roughly the eye’s sensitivity (L, \( f_r \) are 3D vectors for RGB)

• Sampling hemisphere only at discrete directions
  – Simplifies integration to a summation (only directly to light sources)

• Reflection function model (BRDF, see later)
  – Approximation by parameterized functions
    • **Diffuse**: light reflected uniformly in all directions
    • **Specular**: perfect reflection/refraction direction
    • **Glossy**: mirror reflection, but from a rough surface
    • And mixture thereof
Ray Tracing

\[ L(x, \omega_o) = L_e(x, \omega_o) + \int_{\Omega_+} f_r(\omega_i, x, \omega_o)L_i(x, \omega_i) \cos \theta_i \, d\omega_i \]

- **Simple ray tracing**
  - Illumination from discrete point light sources only – **direct illumination only**
    - Integral \( \rightarrow \) sum of contributions from each light
    - **No global illumination**
  - Evaluates angle-dependent reflectance function (BRDF) – **shading process**

- **Advanced ray tracing techniques**
  - Recursive ray tracing
    - Multiple reflections/refractions (e.g. for specular surfaces)
  - Ray tracing for global illumination
    - Stochastic sampling (Monte Carlo methods) \( \rightarrow \) RIS course
Different Types of Illumination

• Three types of illumination computations in CG

  - Local
    (without shadows, used in rasterization)

  - Direct
    (with shadows)

  - Global
    (with all interreflections)

• Ambient Illumination
  – Global illumination is costly to compute
  – Indirect illumination (through interreflections) is typically smooth
    ➔ Approximate via a constant term $L_{i,a}$ (incoming ambient illum.)
  – Has no incoming direction, provide ambient reflection term $k_a$
    • Often chosen to be the same as the diffuse term ($k_a = k_d$)

$$L_o(x, \omega_o) = k_a L_{i,a}$$