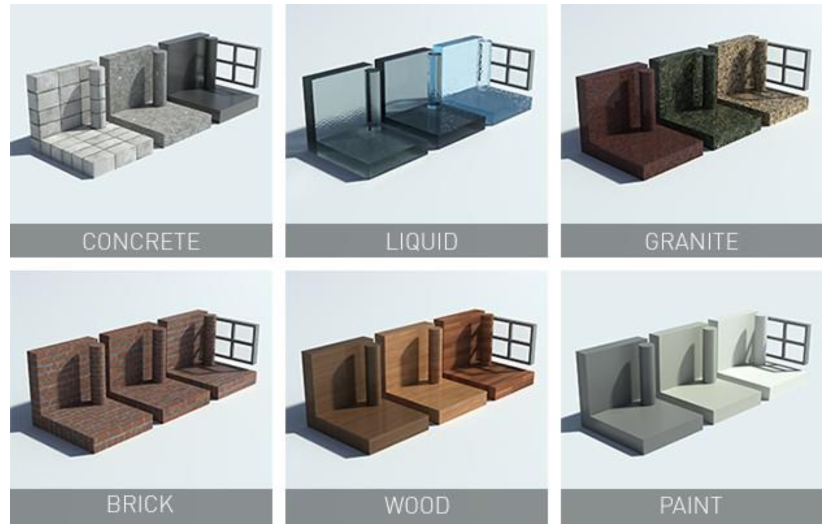
Computer Graphics

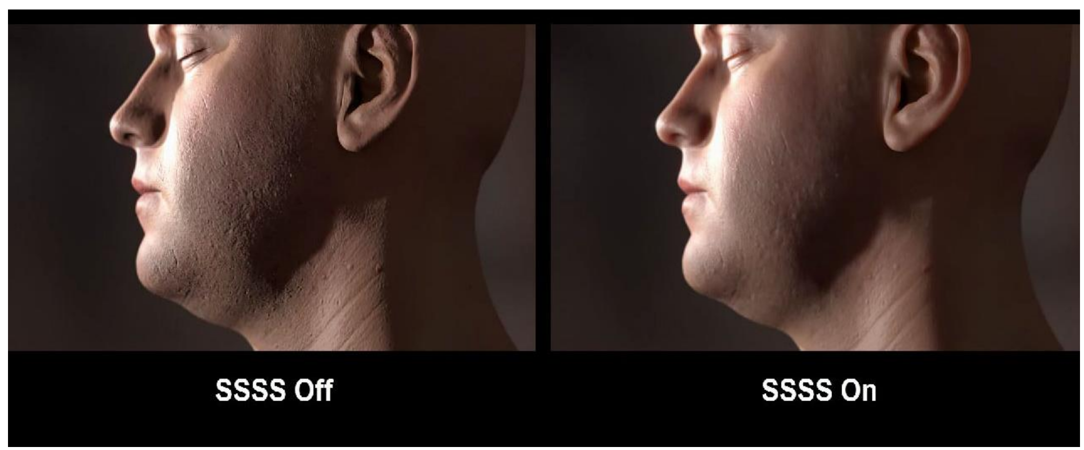
- Material Models -

Philipp Slusallek

Material Samples

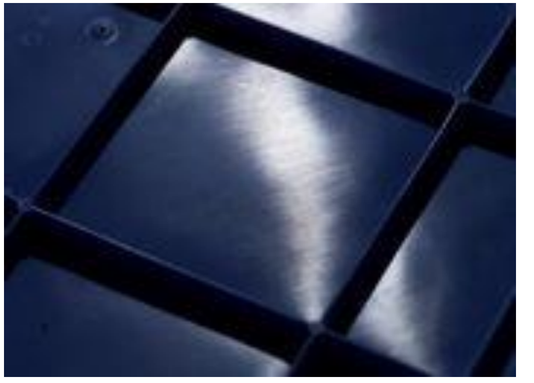
- **How do materials reflect light?**
  - At the same point or in neighborhood (subsurface scattering)





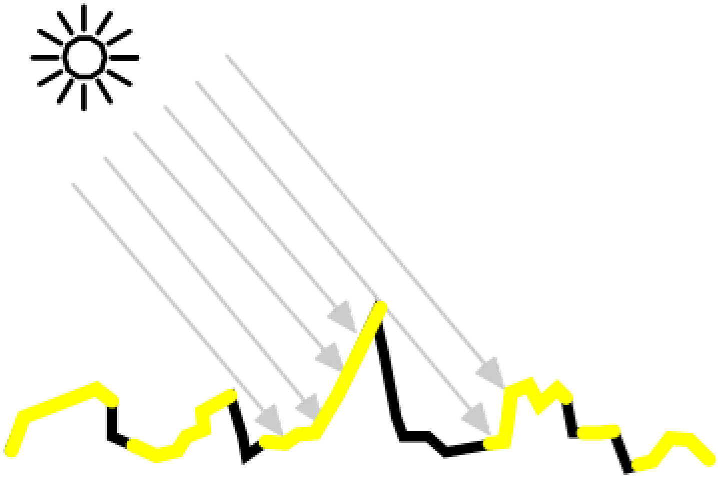
Material Samples

- **Anisotropic surfaces**



Material Samples

- **Complex surface meso-structure**



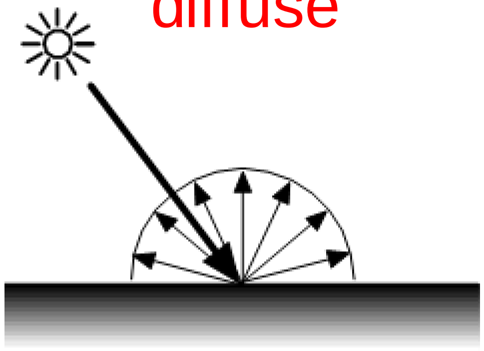
Material Samples

- **Lots of details: Fibers**

Material Samples

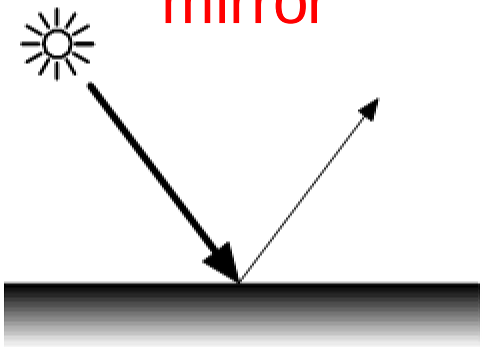
- **Typical material types: Photos of samples with light source at exactly the same position**

diffuse



glossy

mirror



How to describe materials?

- **Surface roughness**
  - Cause of different reflection properties (often in combination):
    - Perfectly smooth: Mirror reflection
    - Slightly rough: Glossy highlights, approx. in direction of reflection
    - Very rough: Diffuse reflection, light reflected many times in material, looses directionality
    - Combination of the above
- **Geometry**
  - Macro structure: Described as explicit geometry (e.g. triangles)
  - Micro structure: Captured in scattering function (BRDF)
  - Meso structure: Difficult to handle: integrate into BRDF (offline simulation), use geometry and simulate (online)
- **Representation of reflection properties**
  - Bidirectional reflection distribution function (BRDF)
    - For reflections at a single point (approx.)
  - More complex scattering functions (e.g. subsurface scattering)
- **Goal: Relightable representation of appearance**

Rendering Equation

- **Reflection equation**

$$L_o(x, \omega_o) = \int_{\Omega_+} f_r(\omega_i, x, \omega_o) L_i(x, \omega_i) cos\theta_i d\omega_i$$

- **BRDF Definition**
  - Ratio of *reflected* radiance to *incident* irradiance

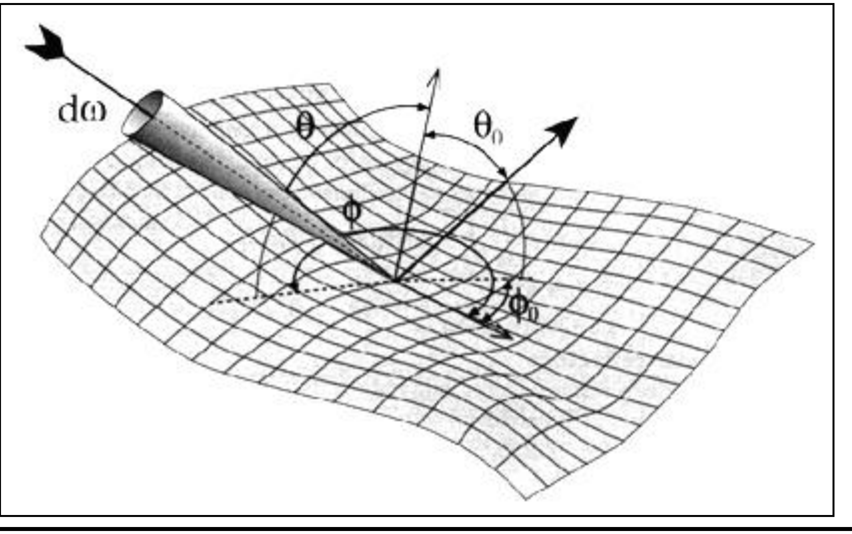
$$f_r(\omega_i, x, \omega_o) = \frac{dL_o(x, \omega_o)}{dE_i(x, \omega_i)}$$

Units: $\left[\frac{1}{sr}\right]$

BRDF

- **BRDF describes surface reflection**
  - for light incident from direction $\boldsymbol{\omega_i} = (\boldsymbol{\theta_i}, \boldsymbol{\varphi_i})$
  - observed from direction $\boldsymbol{\omega_o} = (\boldsymbol{\theta_o}, \boldsymbol{\varphi_o})$
- **Bidirectional**
  - Depends on 2 directions $\omega_i,\ \omega_o$ and position x (a 6-D function)

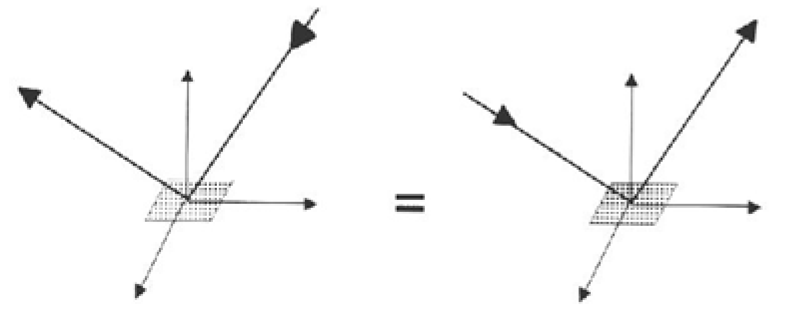
$$f_r(\omega_i, x, \omega_o) = \frac{dL_o(x, \omega_o)}{dE_i(x, \omega_i)} = \frac{dL_o(x, \omega_o)}{L_i(x, \omega_i) cos\theta_i d\omega_i}$$



BRDF Properties

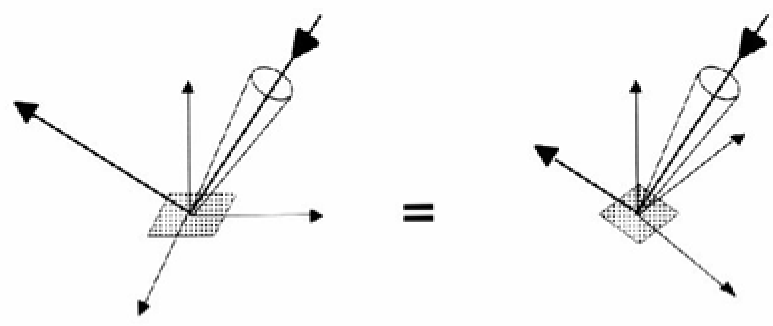
- **Helmholtz reciprocity principle**
  - BRDF remains unchanged if incident and reflected directions are interchanged
  - Due to physical principle of time reversal

$$f_r(\omega_i, \omega_o) = f_r(\omega_o, \omega_i)$$



- **No surface structure: Isotropic BRDF**
  - Reflectivity independent of rotation around surface normal
  - BRDF has only 3 instead of 4 directional degrees of freedom

$$f_r(x, \theta_i, \theta_o, \varphi_o - \varphi_i)$$



BRDF Properties

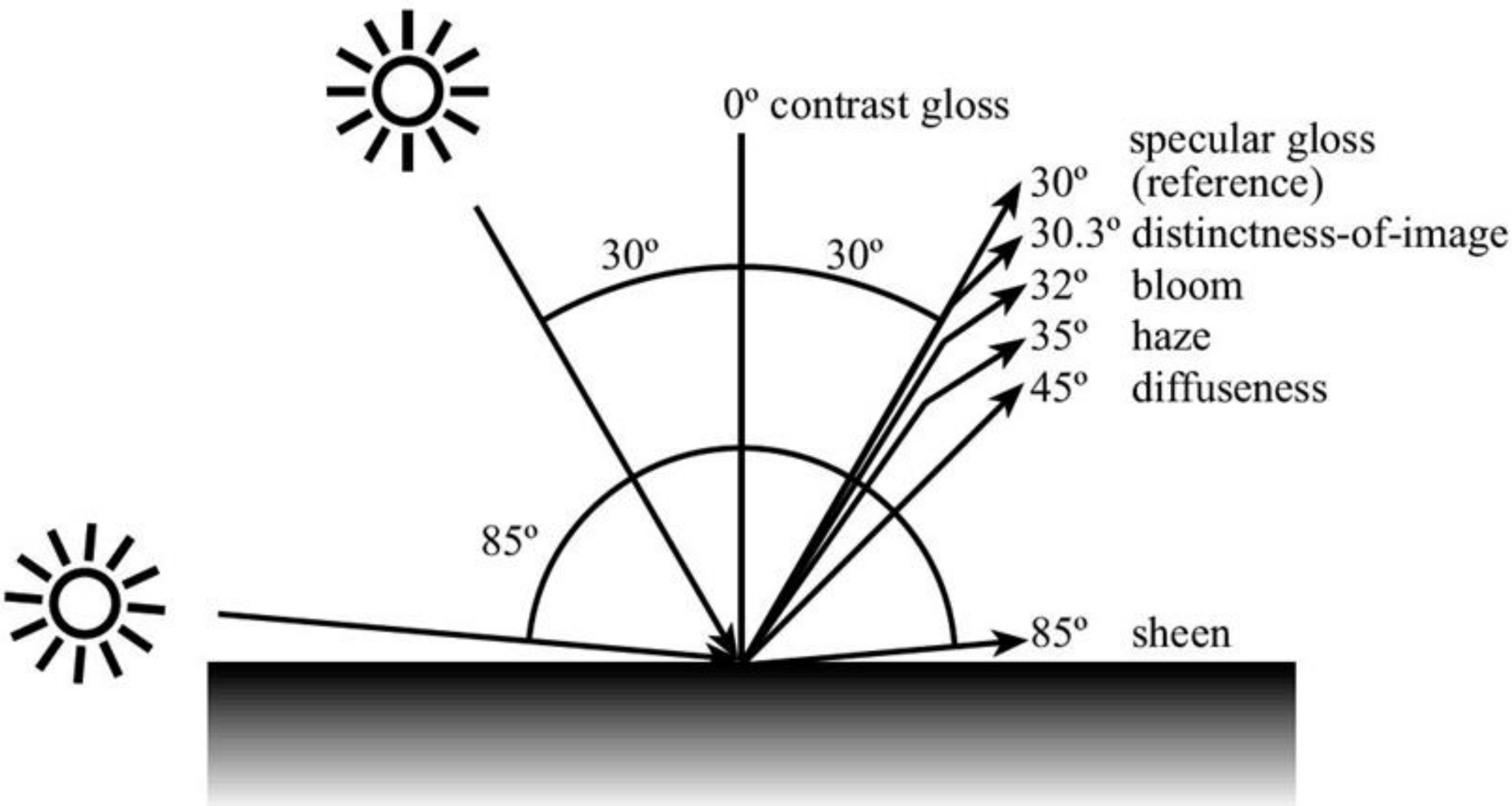
- **Characteristics**
  - BRDF units
    - Inverse steradian: sr^{-1} (not really intuitive)
  - Range of values: distribution function is positive, can be infinite
    - From 0 (no reflection in that direction, perfectly black)
    - to ∞ (perfect reflection into exactly one direction, δ-function, mirror)
  - Energy conservation law
    - Absorption physically unavoidable and assuming no self-emission
    - Integral of f_r over *outgoing* directions integrates to less than one
      - For any incoming direction

$$\int_{\Omega_+} f_r(\omega_i, x, \omega_o) cos\theta_o d\omega_o \leq 1, \qquad \forall \omega_i$$

- **Reflection only at the point of entry ($\boldsymbol{x_i = x_o}$)**
  - Ignoring subsurface scattering (SSS)

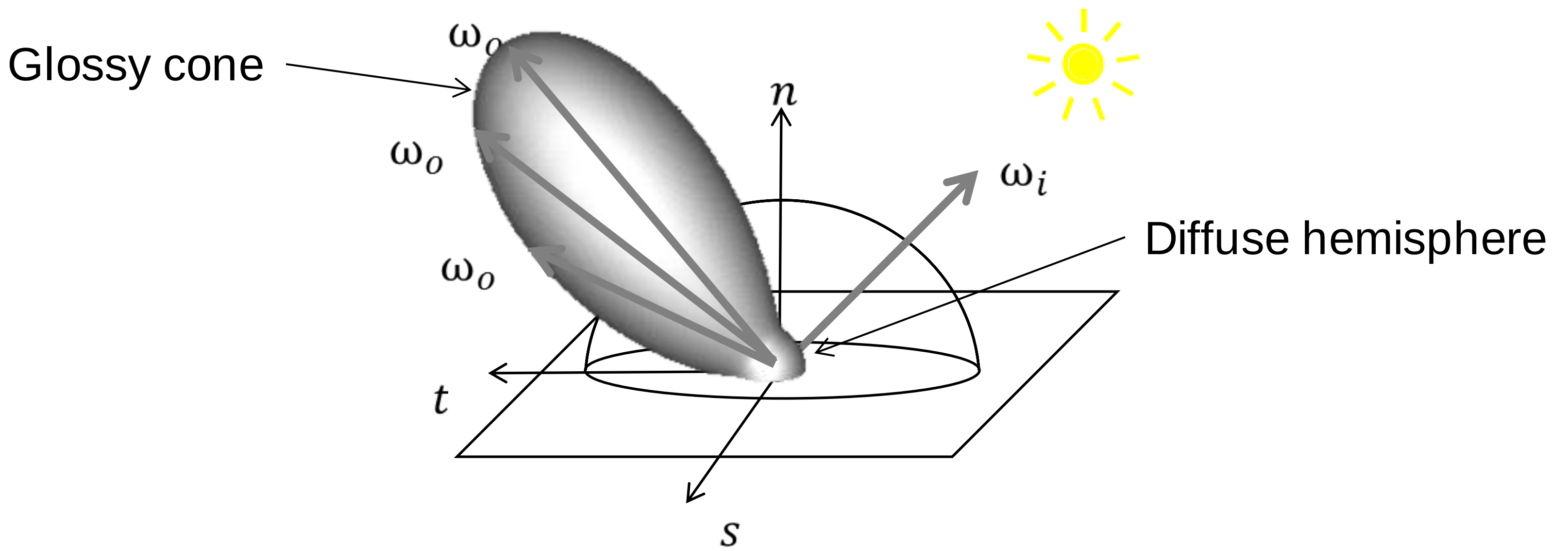
Standardized Gloss Model

- **Industry often uses only a subset of BRDF values**
  - Reflection only measured at discrete set of angles in plane of incidence (not typically used in graphics)



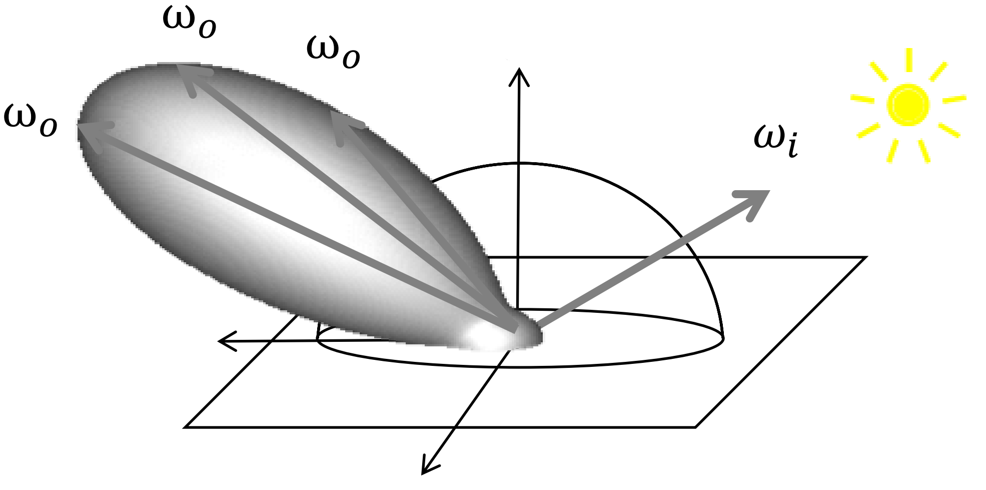
Reflection on an Opaque Surface

- **BRDF is often shown as a slice of the 6D function**
  - Given point x and given incident direction ω_i
    - Show 3D polar plot (intensity as length of vector from origin)
  - Often consists of some mostly diffuse component (here small)
    - and a somewhat glossy component (here rather large)



Reflection on an Opaque Surface

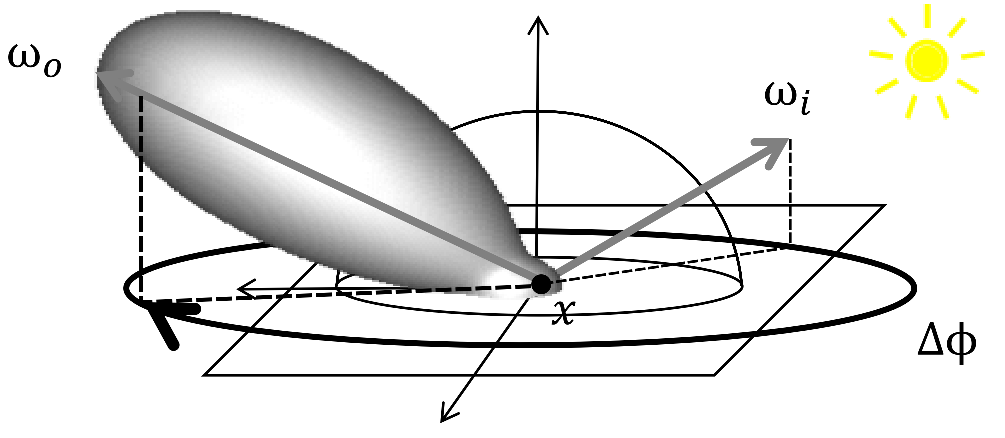
- **BRDF slice varies with incident direction**
  - and possibly location



Homog. & Isotropic BRDF – 3D

- **Invariant with respect to rotation about the normal**
  - Homogeneous and isotropic across surface
  - Only depends on azimuth difference to incoming angle

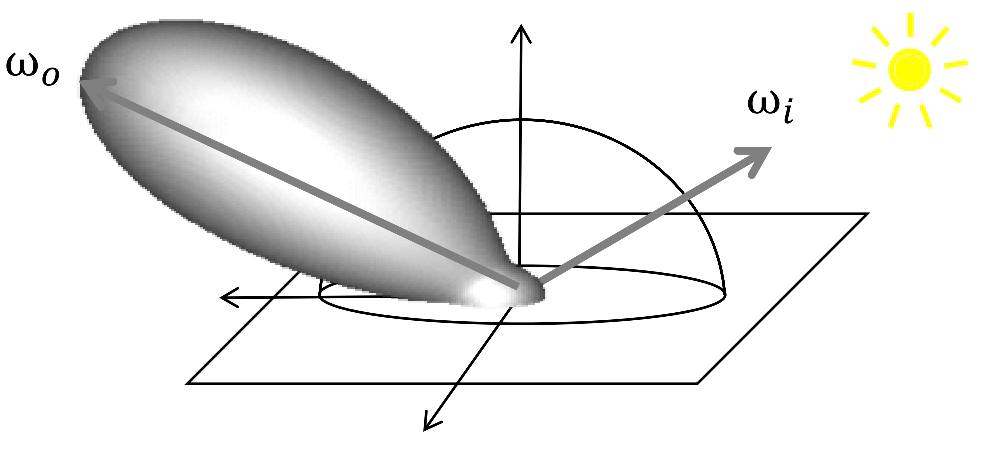
$$f_r\big((\theta_i, \varphi_i) \to (\theta_o, \varphi_o)\big) \Longrightarrow$$
$$f_r(\theta_i \to \theta_o, (\varphi_i - \varphi_o)) = f_r(\theta_i \to \theta_o, \Delta\varphi)$$



Homogeneous BRDF – 4D

- **Homogeneous bidirectional reflectance distribution function**
  - Ratio of reflected radiance to incident irradiance
  - Independent of position

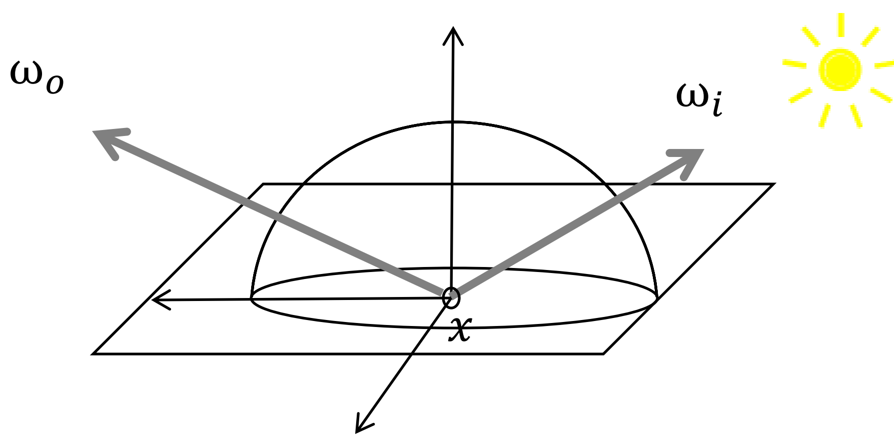
$$f_r(\omega_i \to \omega_o) = \frac{dL_o(\omega_o)}{dE_i(\omega_i)}$$



Spatially Varying BRDF – 6D

- **Heterogeneous materials (standard model for BRDF)**
  - Dependent on position, and two directions
  - Reflection at the point of incidence

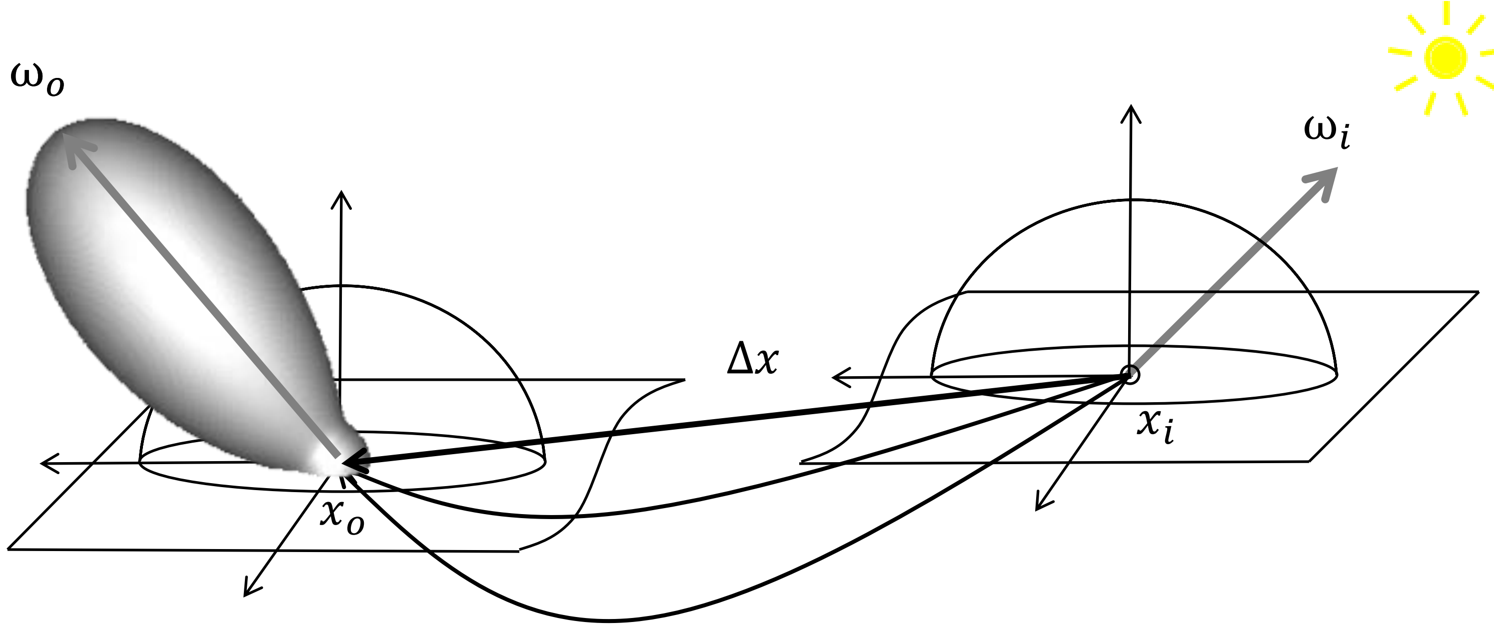
$$f_r(x, \omega_i \to \omega_o)$$



Homogeneous BSSRDF – 6D

- **Homogeneous bidirectional scattering surface reflectance distribution function**
  - Assumes a homogeneous and flat surface
  - Only depends on the difference vector to the outgoing point

$$f_r(\Delta x, \omega_i \rightarrow \omega_o)$$



BSSRDF – 8D

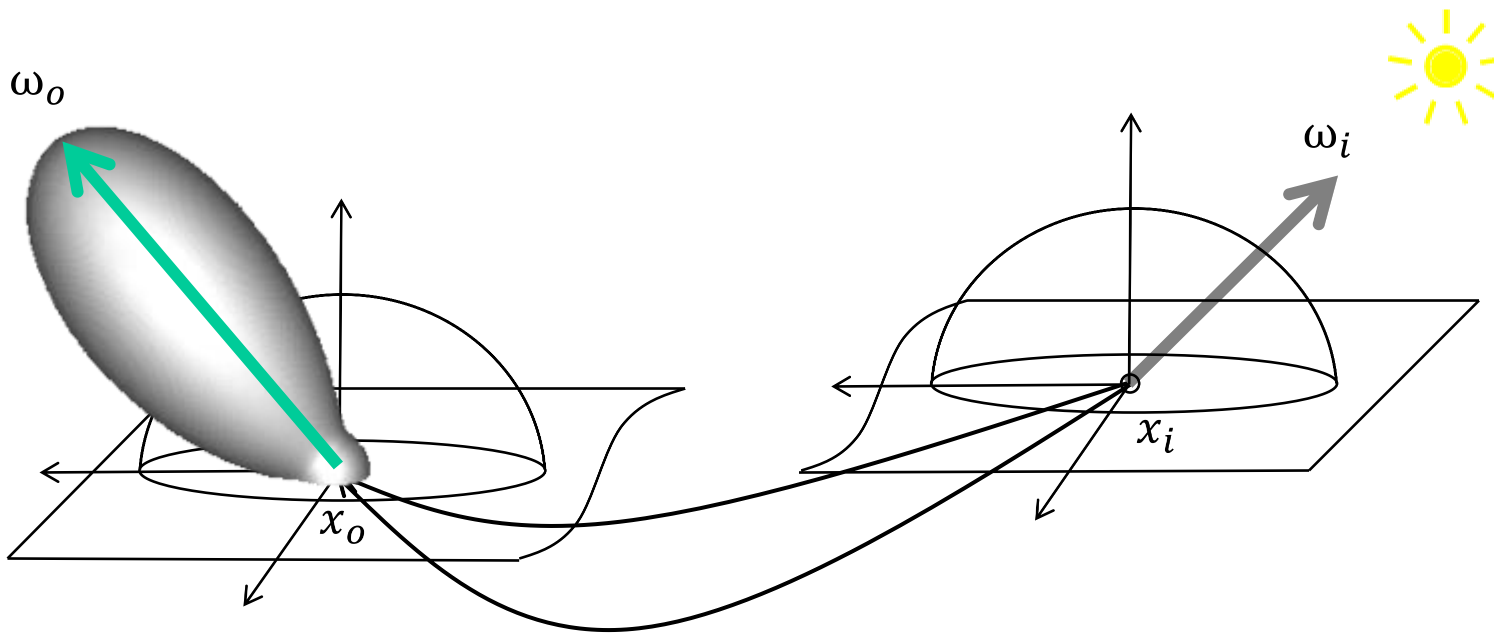
- **Bidirectional scattering surface reflectance distribution function**

$$f_r((x_i, \omega_i) \rightarrow (x_o, \omega_o))$$

Generalization – 9D

- **Generalizations**
  - Add wavelength dependence

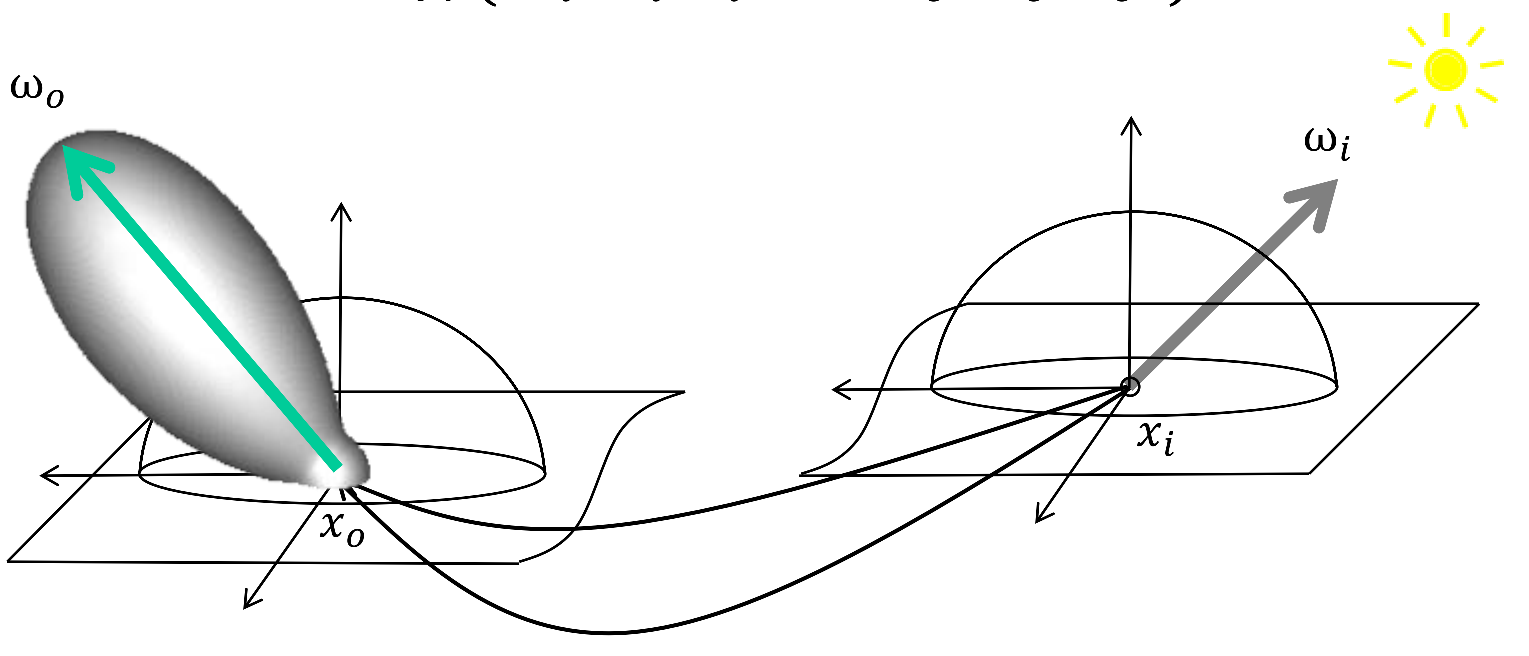
$$f_r(\lambda, (x_i, \omega_i) \to (x_o, \omega_o))$$



Generalization – 10D

- **Generalizations**
  - Add wavelength dependence
  - Add **fluorescence**
    - Change to longer wavelength during scattering

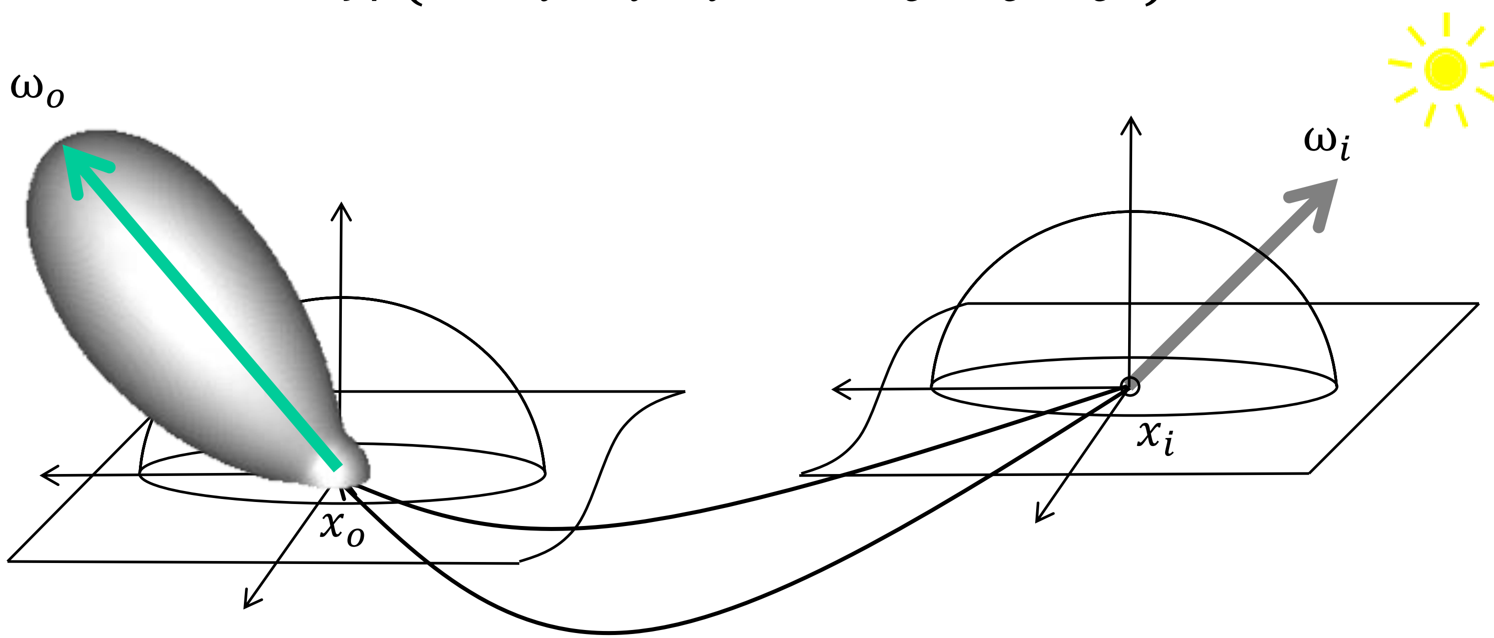
$$f_r\big((x_i, \omega_i, \lambda_i) \rightarrow (x_o, \omega_o, \lambda_o)\big)$$



Generalization – 11D

- **Generalizations**
  - Add wavelength dependence
  - Add fluorescence (change to longer wavelength for reflection)
  - Time varying surface characteristics

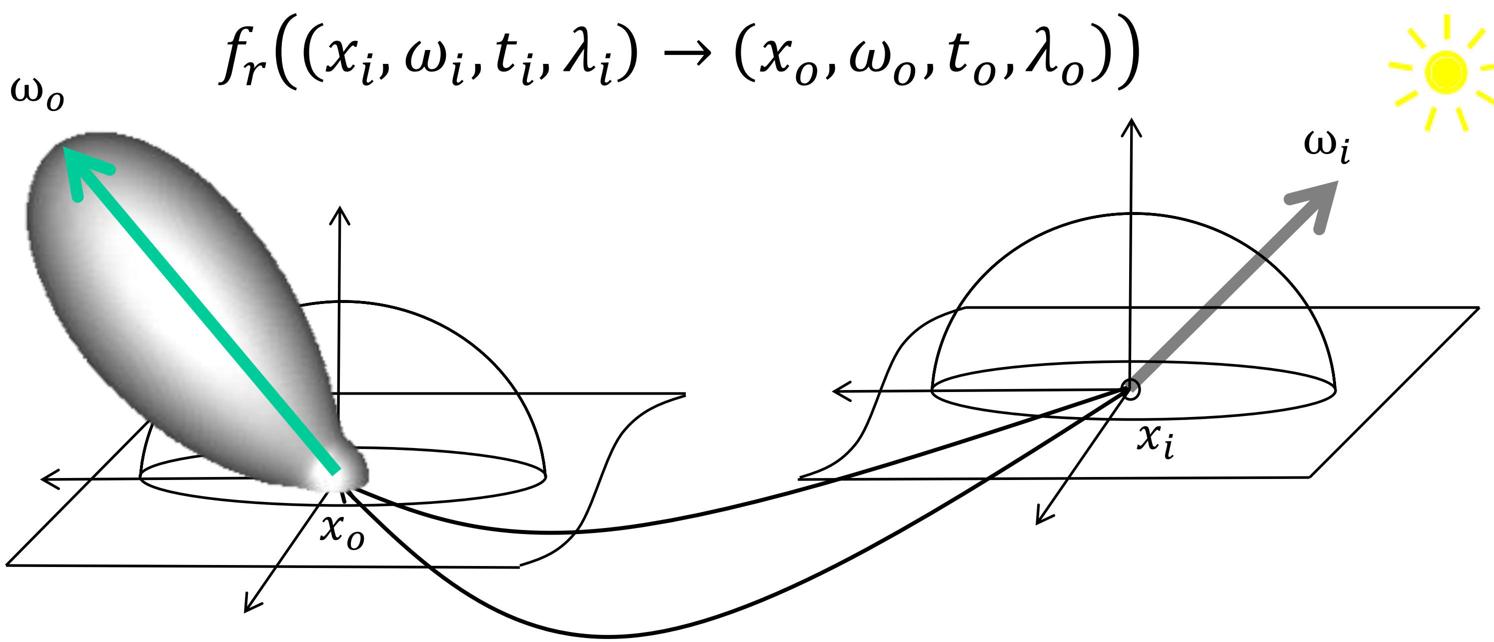
$$f_r\big(t, (x_i, \omega_i, \lambda_i) \to (x_o, \omega_o, \lambda_o)\big)$$



Generalization – 12D

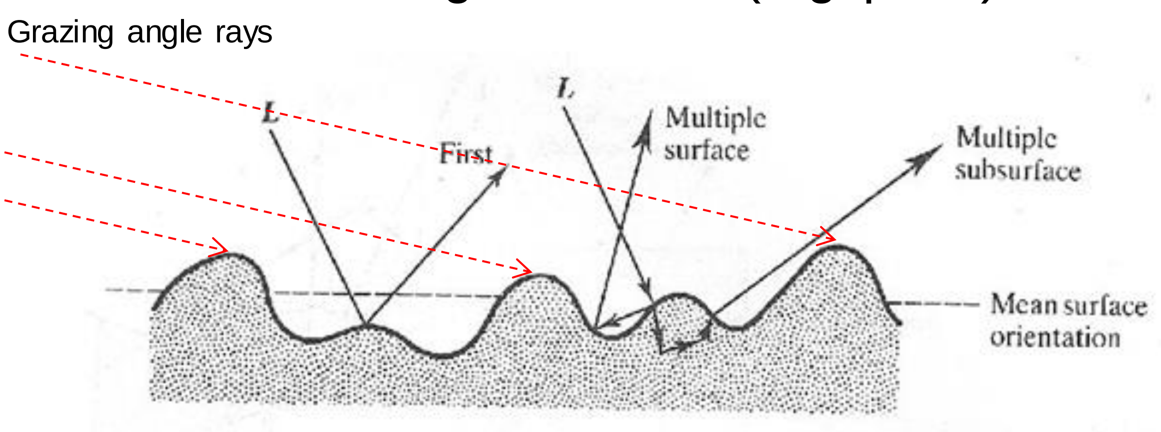
- **Generalizations**
  - Add wavelength dependence
  - Add fluorescence (change to longer wavelength for reflection)
  - Time varying surface characteristics
  - **Phosphorescence**
    - Temporal storage of light

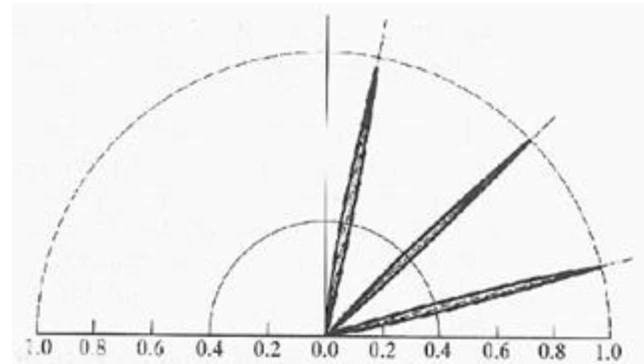
$$f_r\big((x_i, \omega_i, t_i, \lambda_i) \rightarrow (x_o, \omega_o, t_o, \lambda_o)\big)$$



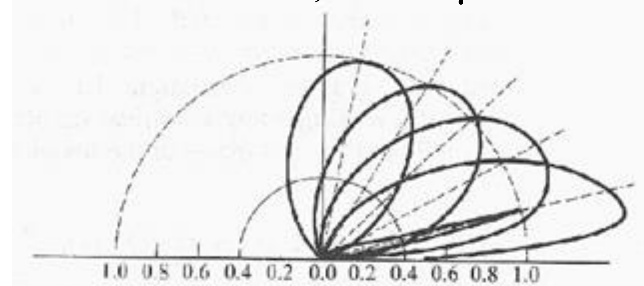
Reflectance

- **Reflectance may vary with**
  - Illumination angle
  - Viewing angle
  - Wavelength
  - (Polarization, ...)
- **Variations due to**
  - Absorption
  - Surface micro-geometry
  - Index of refraction / dielectric constant
  - Scattering in material (e.g. paint)

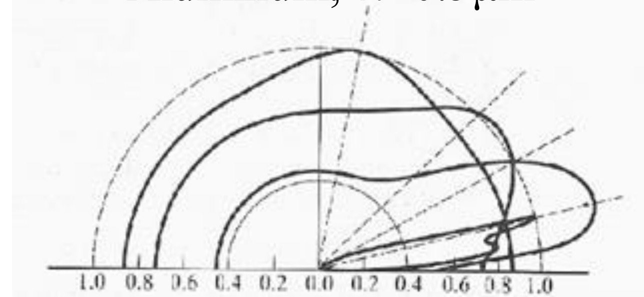




Aluminum; λ=2.0μm



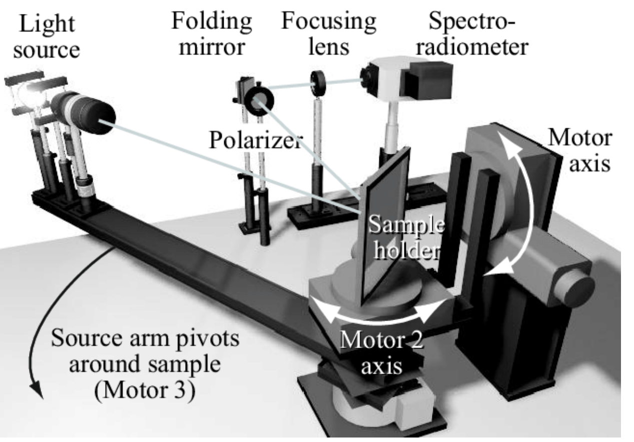
Aluminum; λ=0.5μm

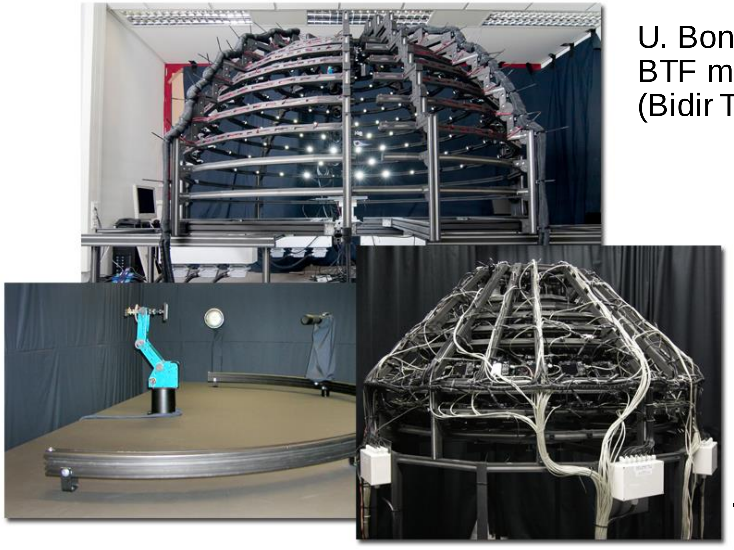


Magnesium oxide; λ=0.5μm

BRDF Measurement

- **Gonio-Reflectometer**
- **BRDF measurement**
  - Point light source position (θ_i, φ_i)
  - Light detector position (θ_o, φ_o)
- **4 directional degrees of freedom**
- **BRDF representation (large!!!)**
  - *m* (in) * *n* (out) directional samples
  - Additional position (e.g. image → 6D)



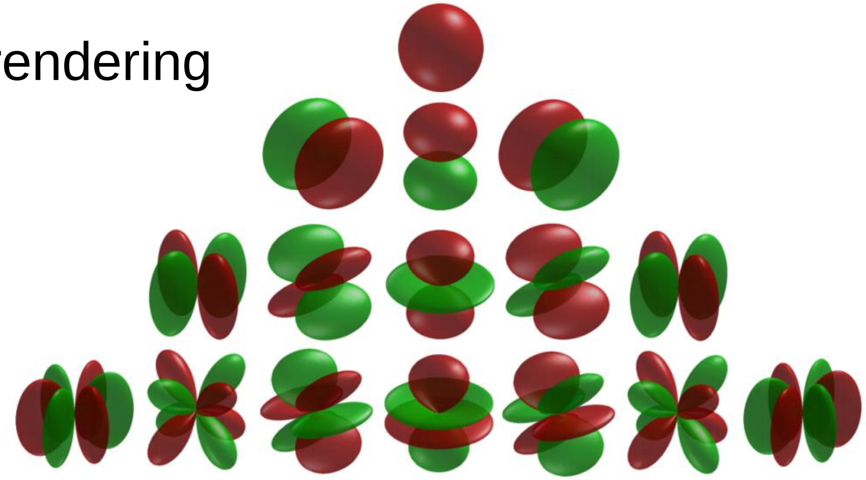


U. Bonn,
BTF measurement,
(Bidir Texture Func.)

Stanford
light gantry

Rendering from Measured BRDF

- **Linearity, superposition principle**
  - Continuous illumin.: integrating light distribution against BRDF
  - Sampled illumination: superimposing many point light sources
- **Interpolation**
  - Look-up of BRDF values during rendering
  - Sampled BRDF must be filtered
- **BRDF Modeling**
  - Fitting of parameterized BRDF models to measured data
    - Continuous, analytic function
    - No interpolation
    - Typically fast evaluation

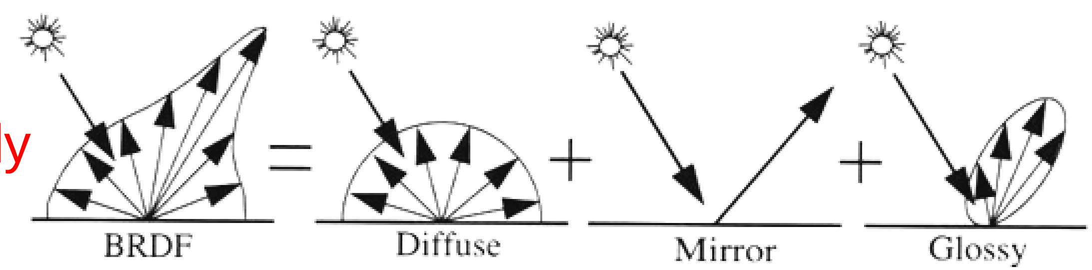


Spherical Harmonics
Red is positive, green negative [Wikipedia]

- **Representation in a basis**
  - Often: Spherical harmonics (ortho-normal basis on sphere)
    - Or BTFs (bidirectional texture function)
  - Mathematically elegant filtering, illumination-BRDF integration

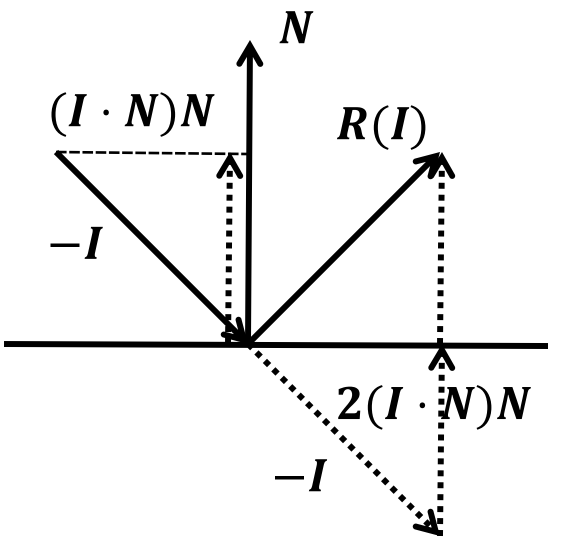
BRDF Modeling

- **Phenomenological approach (not physically correct)**
  - Description of visual surface appearance
  - Composition of different terms:
- **Ideal diffuse reflection +**
  - Lambert's law, interactions within material
  - Matte surfaces
- **Ideal specular/mirror reflection +**
  - Reflection law
  - Mirror surfaces
- **Glossy reflection**
  - "Directional diffuse", reflection on surface that is somewhat rough
  - Shiny surface
  - Glossy highlights
  - Sometimes incorrectly called "specular"

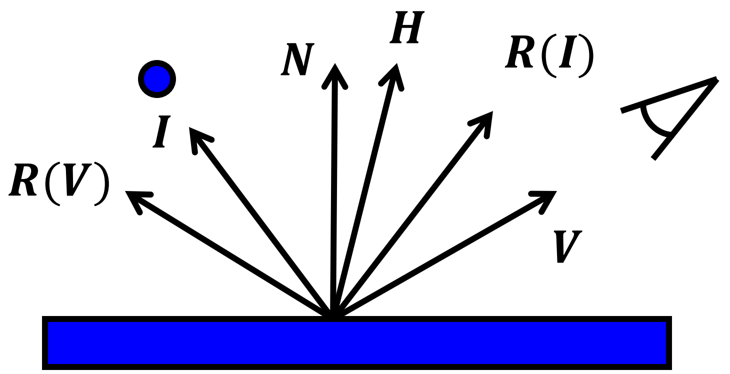


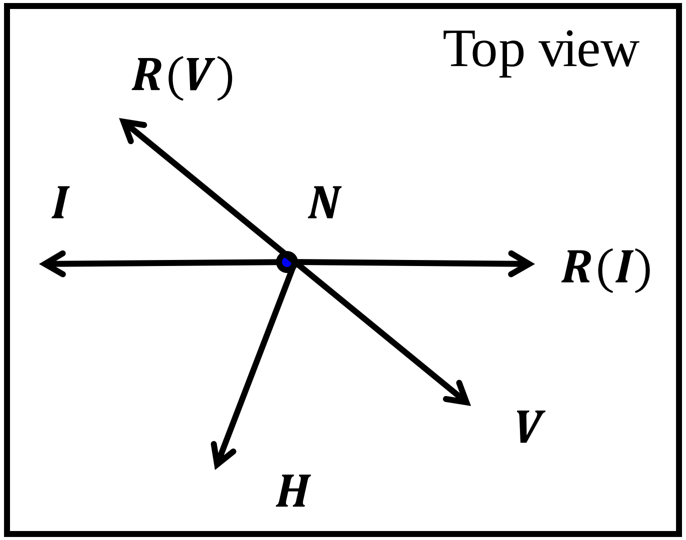
Reflection Geometry

- **Direction vectors (normalize):**
  - N: Surface normal
  - I: Light source direction vector
  - V: Viewpoint direction vector
  - $R(I)$: Reflection vector
    - $R(I) = -I + 2(I \cdot N)N$
  - H: Halfway vector
    - $H = (I + V) / |I + V|$



- **Tangential surface: local plane**

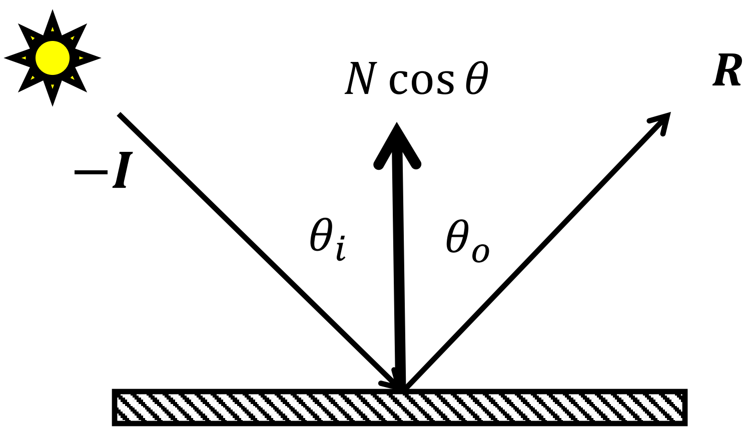




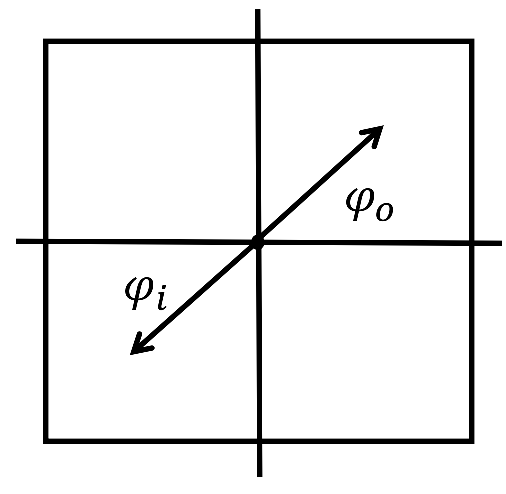
Ideal Specular (Mirror) Reflection

- **Angle of reflectance equal to angle of incidence**
- **Reflected vector in a plane with incident ray and surface normal vector**

$$R + I = 2\cos\boldsymbol{\theta}\, N = 2(I \cdot N)N \Longrightarrow$$
$$R(I) = -I + 2(I \cdot N)\, N$$



$\theta_i = \theta_o$



$\varphi_o = \varphi_i + 180°$

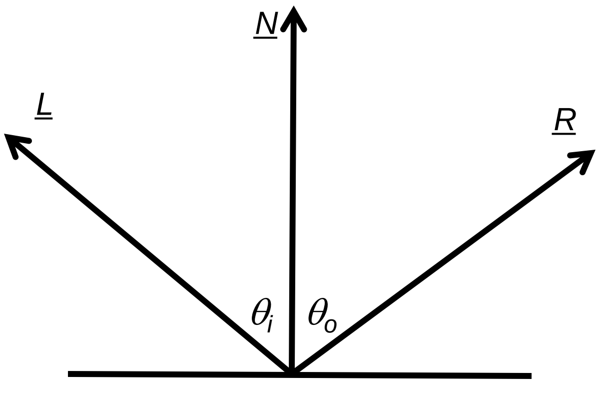
Mirror BRDF

- **Dirac Delta function $\boldsymbol{\delta(x)}$**
  - $\boldsymbol{\delta(x)}$: zero everywhere except at $x = 0$
  - Unit integral iff domain contains $x = 0$ (else zero)

$$f_{r,m}(\omega_i, x, \omega_o) = \rho_s(\theta_i) \frac{\delta(cos\theta_i - cos\theta_o)}{\cos\theta_i} \delta(\varphi_i - \varphi_o \pm \pi)$$

$$L_o(x, \omega_o) = \int_{\Omega_+} f_{r,m}(\omega_i, x, \omega_o) L_i(x, \omega_i) cos\theta_i d\omega_i = \rho_s(\theta_o) L_i(x, \theta_o, \varphi_o \pm \pi)$$

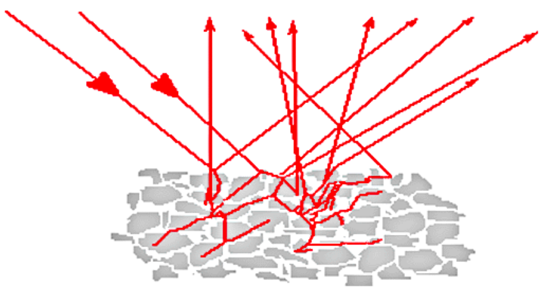
- **Specular reflectance ρ_s**
  - Ratio of reflected radiance in specular direction and incoming radiance
  - Dimensionless quantity between 0 and 1



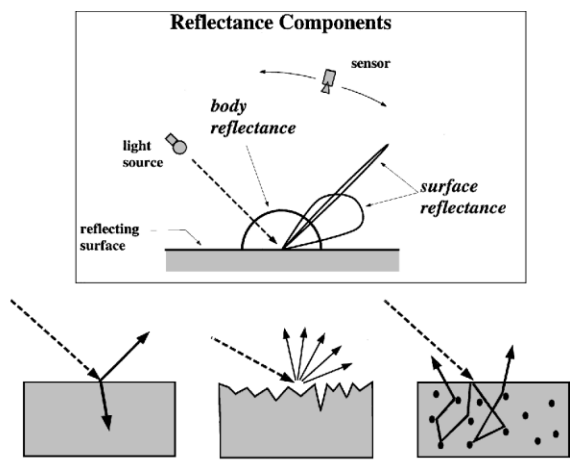
$$\rho_s(x, \theta_i) = \frac{L_o(x, \theta_o)}{L_i(x, \theta_o)}$$

“Diffuse” Reflection

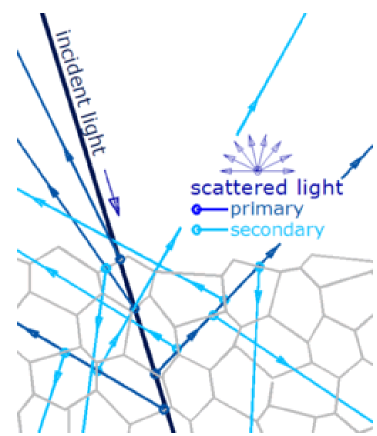
- **Theoretical explanation**
  – Multiple scattering within the material (at very short range)
- **Experimental realization**
  – Pressed magnesium oxide powder (or foam/snow)
    - Random mixture of tiny, highly reflective surfaces
  – Almost never valid at grazing angles of incidence
  – Paint manufacturers attempt to create ideal diffuse paints



Highly reflective particles
(e.g. magnesium oxide, plaster
paper fibers)



specular glossy diffuse



Highly reflective/refractive
foam-like materials

Diffuse Reflection Model

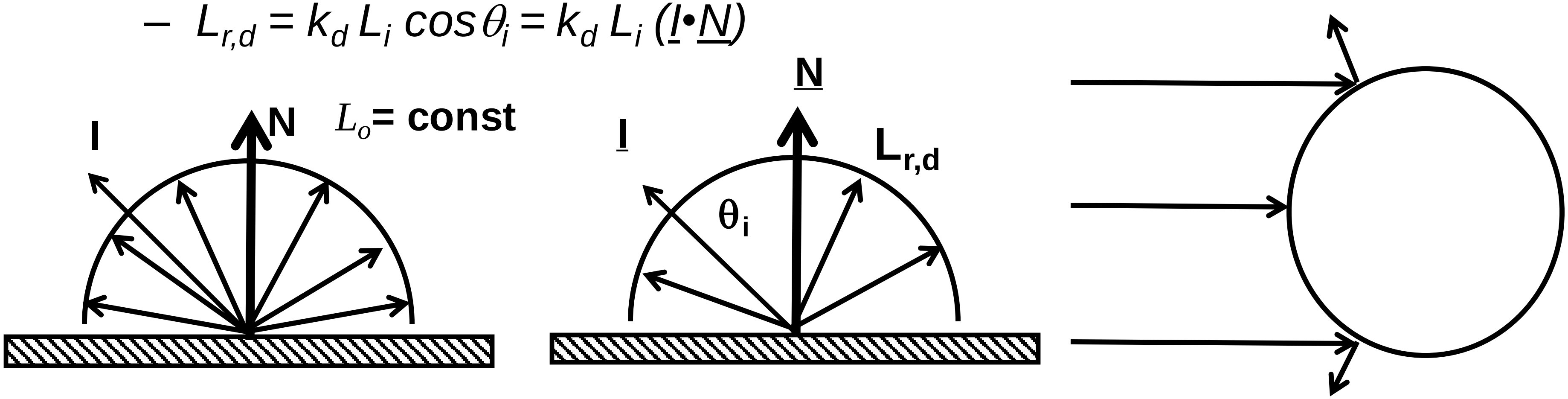
- **Light equally likely to be reflected in any output direction (independent of input direction, idealized)**
- **Constant BRDF**

$$f_{r,d}(\omega_i, x, \omega_o) = k_d = const = \rho_d / \pi [sr] \quad \text{with } \rho_r \in [0,1]$$

$$L_o(x, \omega_o) = k_d \int_{\Omega_+} L_i(x, \omega_i) \cos\theta_i \, d\omega_i = k_d E = \frac{\rho_d}{\pi [sr]} E$$

  – ρ_d: diffuse reflection coefficient, material property [1/sr]

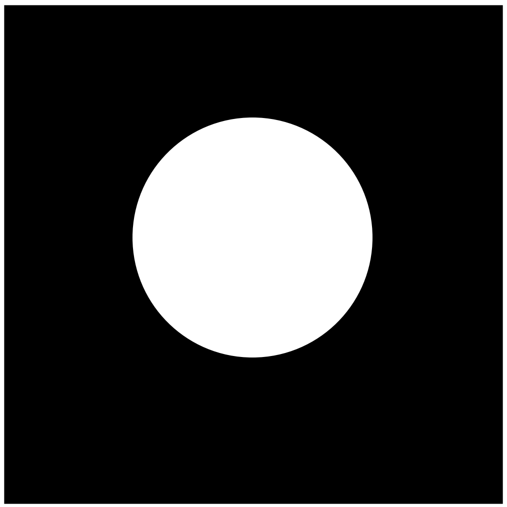
- **For each point light source**
  – $L_{r,d} = k_d L_i \cos\theta_i = k_d L_i (\underline{I} \bullet \underline{N})$



Lambertian Objects

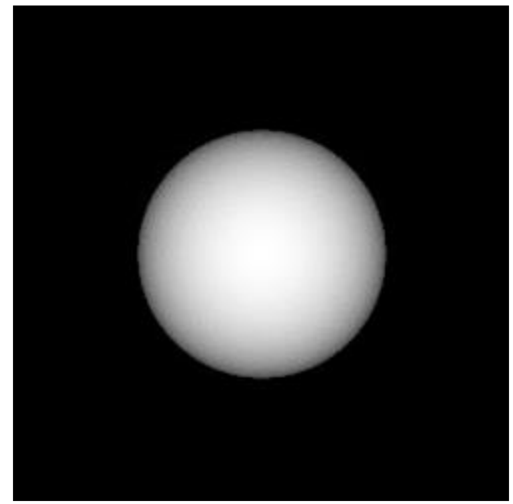
Self-luminous
spherical Lambertian light source

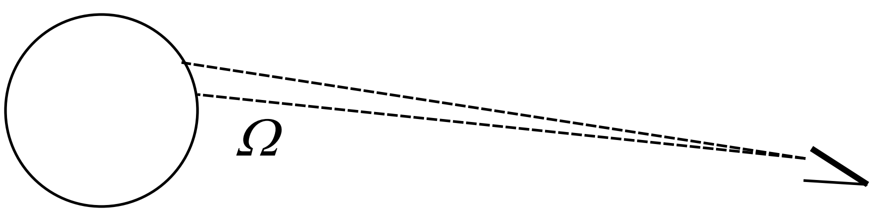
$\Phi_0 \propto L_0 \cdot \Omega$

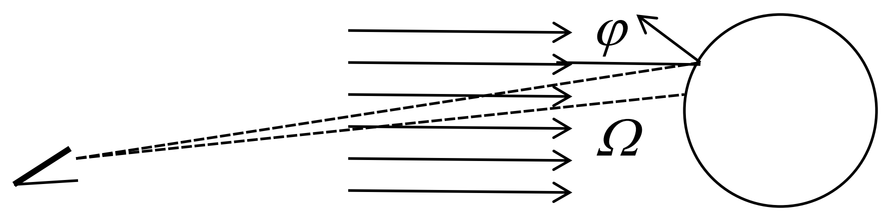


Eye-light illuminated
spherical Lambertian reflector

$\Phi_1 \propto L_i \cdot cos\,\theta \cdot \Omega$

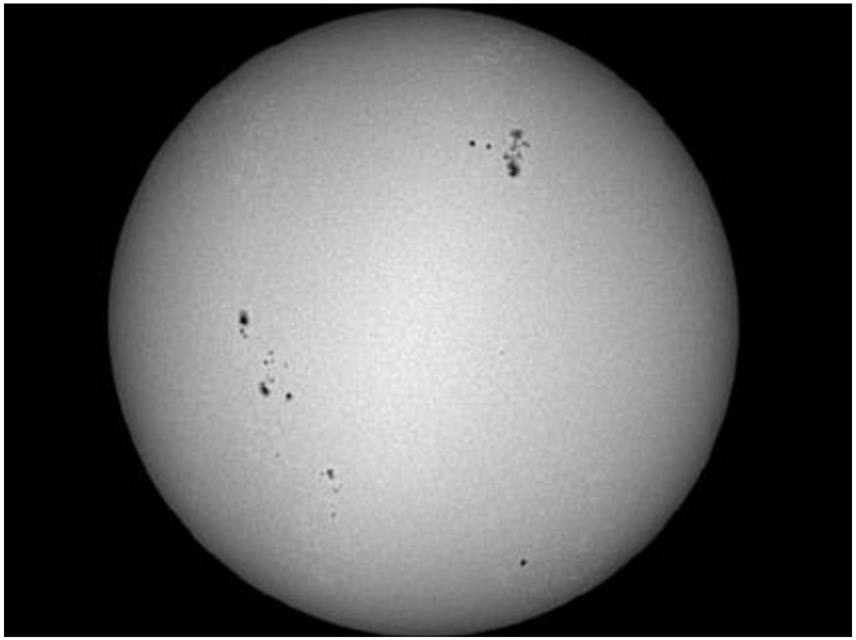






Lambertian Objects (?)

The Sun



- Some absorption in photosphere
- Path length through photosphere longer from the Sun's rim

The Moon

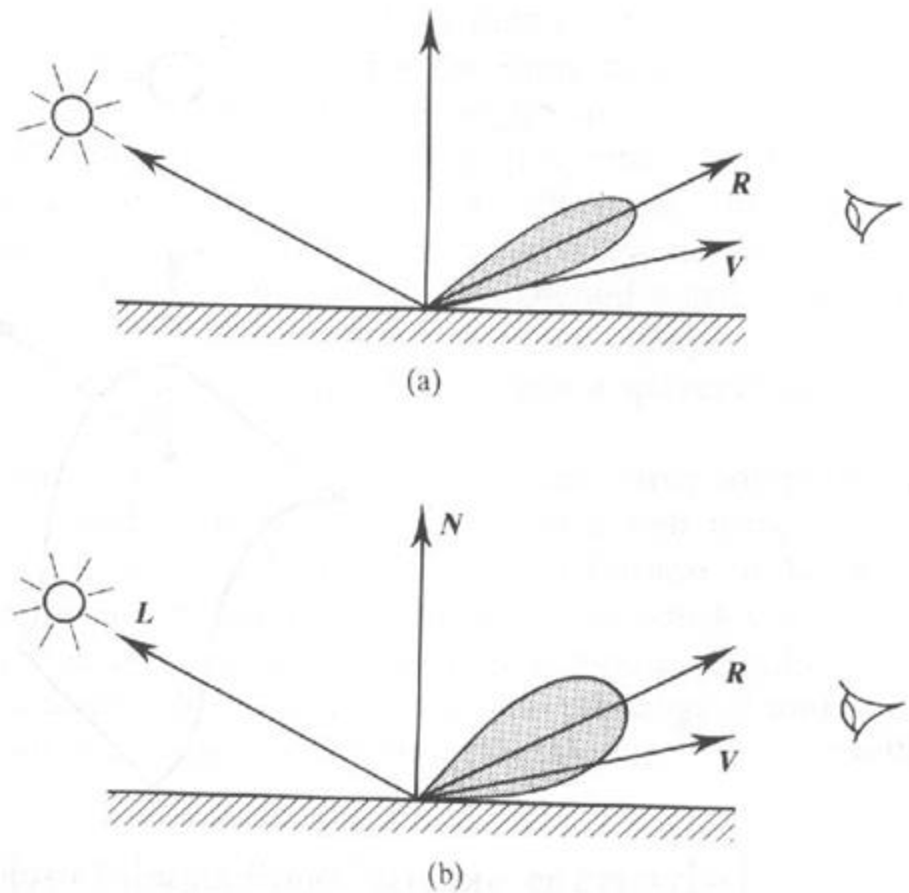
- Surface covered with fine dust
- Dust visible best from slanted viewing angle

$\Rightarrow$ Neither the Sun nor the Moon are Lambertian

Glossy Reflection

- **Due to surface roughness**
- **Empirical models (phenomenological)**
  - Phong
  - Blinn-Phong
- **Physically-based models**
  - Blinn
  - Cook & Torrance

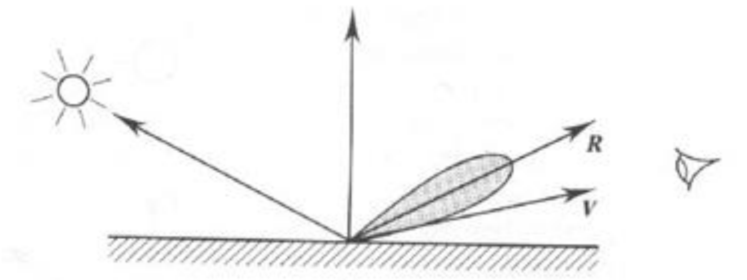
- **Sometimes incorrectly called "specular"**



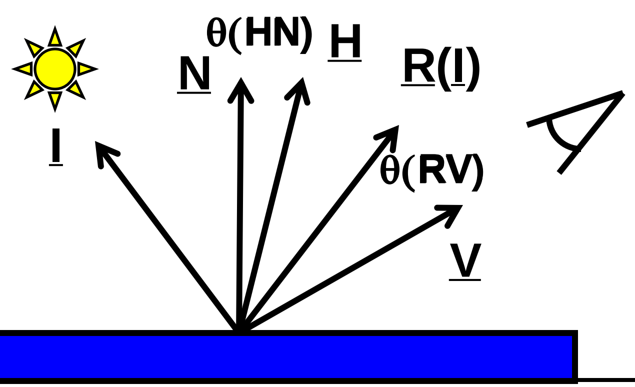
Phong Glossy Reflection Model

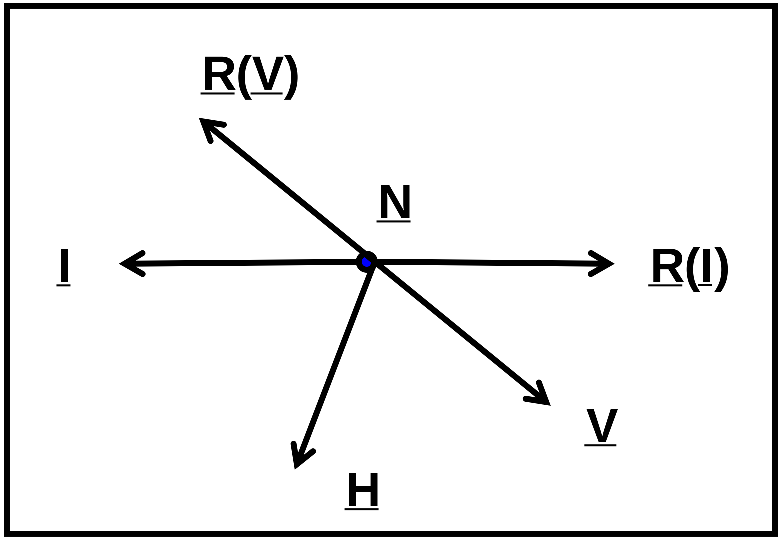
- **Simple experimental description: Cosine power lobe**

$$f_r(\omega_i, x, \omega_o) = k_s(R(I) \cdot V)^{k_e} / I \cdot N$$



- Take angle to reflection direction to some
  - $L_{r,s} = L_i k_s \cos^{ke} \theta_{RV}$
- **Issues**
  - Not energy conserving/reciprocal
  - Plastic-like appearance
- **Dot product & power**
  - Still widely used in CG

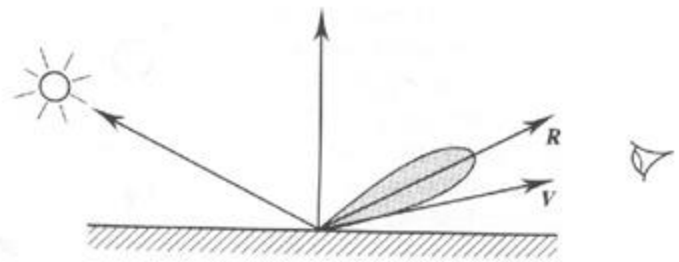




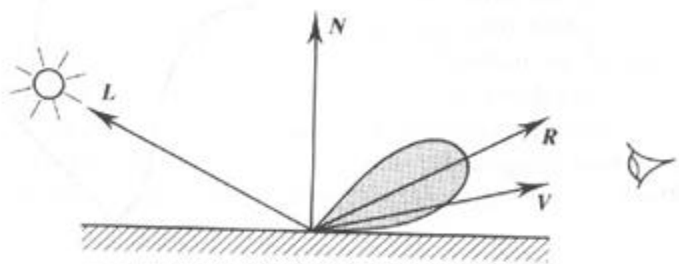
Phong Exponent k_e

$$f_r(\omega_i, x, \omega_o) = k_s (R(I) \cdot V)^{k_e} \,/\, I \cdot N$$

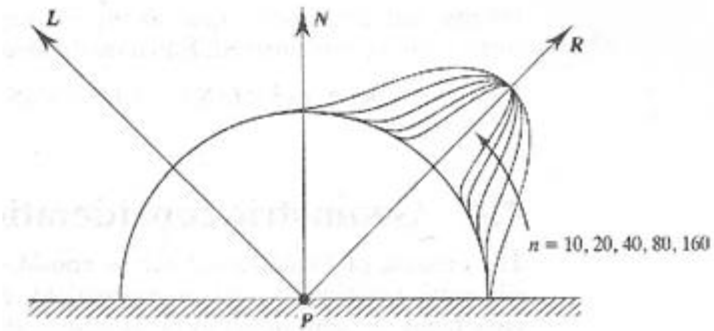
- **Determines size of highlight**

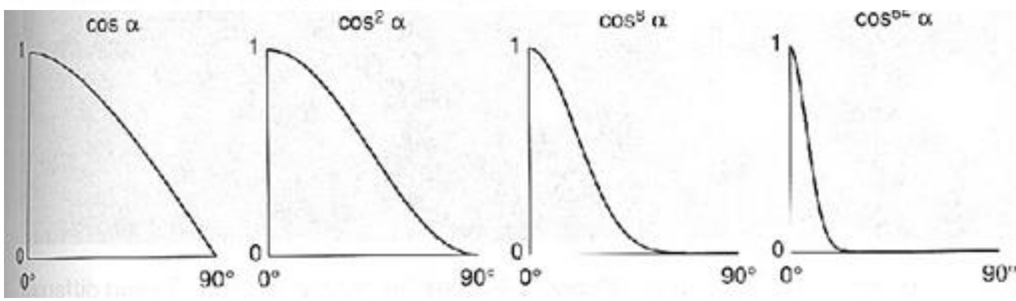


(a)



(b)



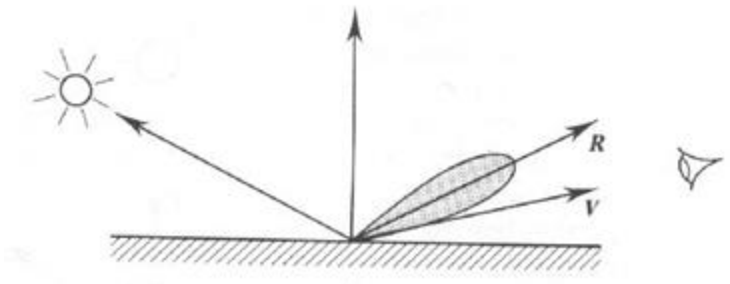


- **Beware: Non-zero contribution into the material !!!**
  - Cosine is non-zero between -90 and 90 degrees

Blinn-Phong Glossy Reflection

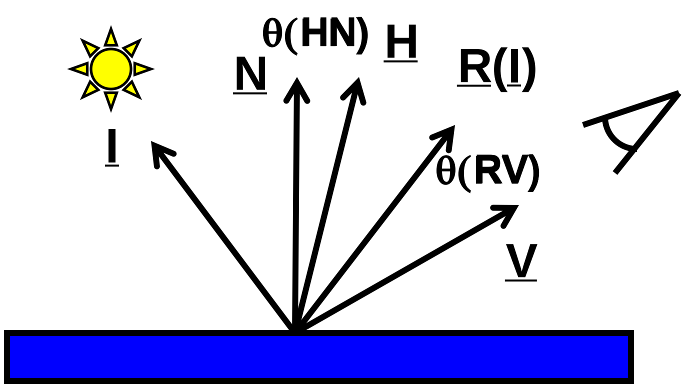
- **Same idea: Cosine power lobe**

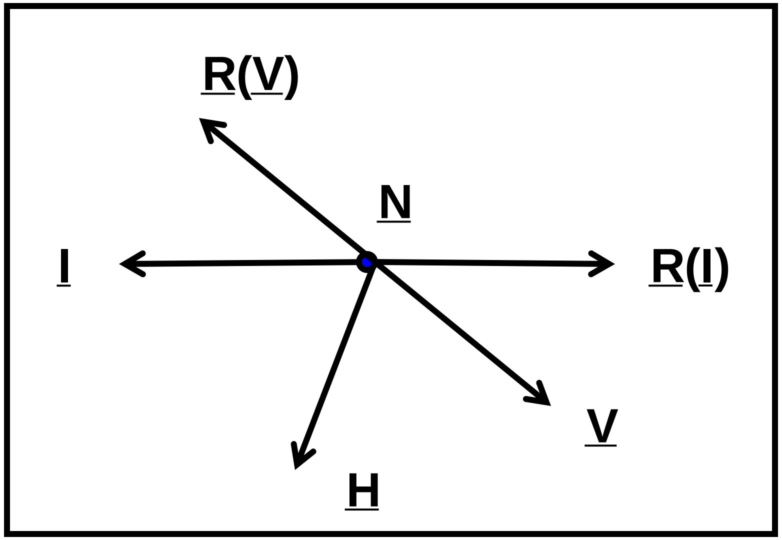
$$f_r(\omega_i, x, \omega_o) = k_s (H \cdot N)^{k_e} / I \cdot N$$



  – $L_{r,s} = L_i \, k_s \cos^{ke} \theta_{HN}$

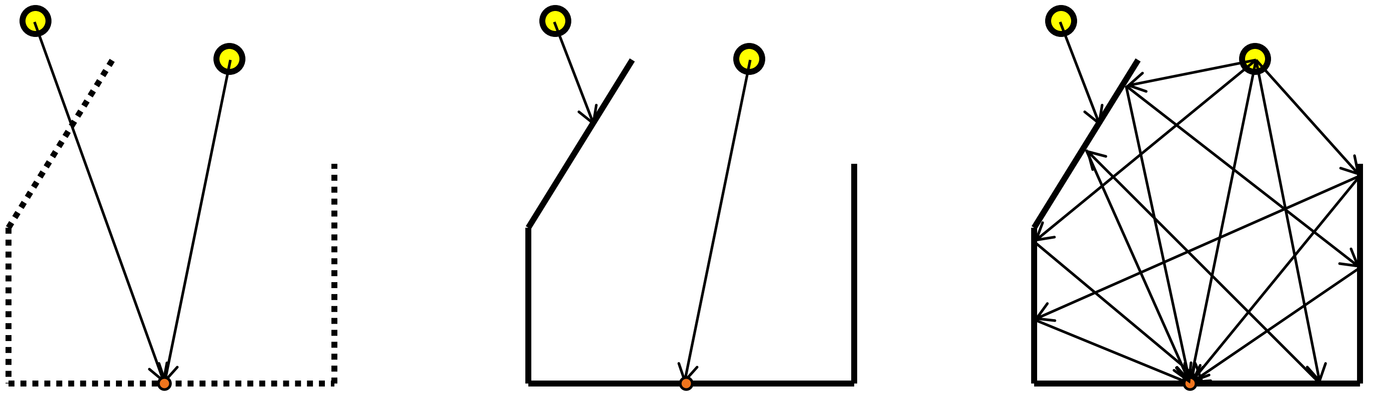
- **Dot product & power**
  – $\theta_{RV} \rightarrow \theta_{HN}$
  – Special case: Light source, viewer far away
    - *I*, *R* constant: *H* constant
    - θ_{HN} less expensive to compute





Different Types of Illumination

- **Three types of illumination computations in CG**



Local
(without shadows,
used in rasterization)

Direct
(with shadows)

Global
(with all interreflecions)

- **Ambient Illumination**
  - Global illumination is costly to compute
  - Indirect illumination (through interreflections) is typically smooth
    - ➔Approximate via a constant term $L_{i,a}$ (incoming ambient illum.)
  - Has no incoming direction, provide ambient reflection term k_a
    - Often chosen to be the same as the diffuse term $k_a = k_d$

$$L_o(x, \omega_o) = k_a L_{i,a}$$

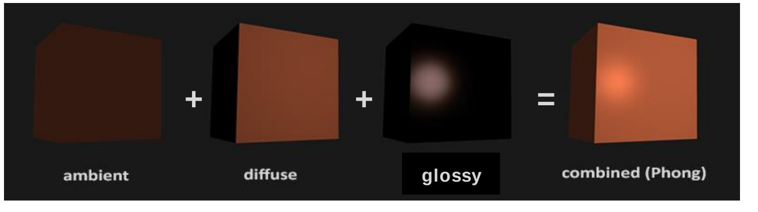
Full Phong Reflection Model

- **Phong illumination model for *multiple* point light sources**

$$L_r = k_a L_{i,a} + k_d \sum_l L_l (I_l \cdot N) + k_s \sum_l L_l (R(I_l) \cdot V)^{k_e} \ (Phong)$$

$$L_r = k_a L_{i,a} + k_d \sum_l L_l (I_l \cdot N) + k_s \sum_l L_l (H_l \cdot N)^{k_e} \ (Blinn)$$

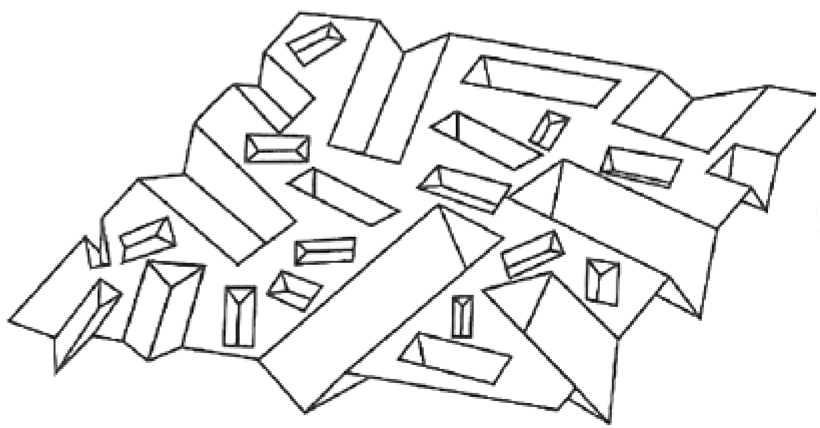
  - Diffuse reflection (contribution only depends on incoming cosine)
  - Ambient and glossy reflection (Phong or Blinn-Phong)
- **Typically: Color of specular reflection k_s is white**
  - Often separate specular and diffuse color (common extension, OGL)
- **Empirical reflection model!**
  - Contradicts physics
  - Purely local illumination
    - Only direct light from the light sources + constant ambient term

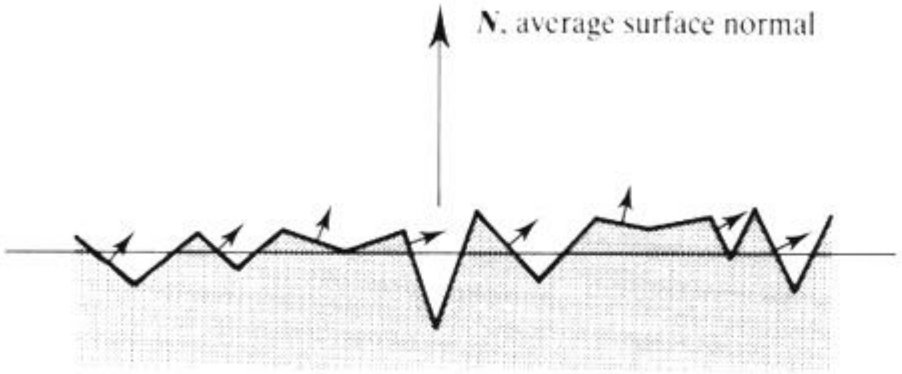


- **Optimization: Lights & viewer assumed to be far away**

Microfacet BRDF Model

- **Physically-Inspired Models**
  - Isotropic microfacet collection
  - Microfacets assumed as perfectly smooth reflectors
- **BRDF**
  - Distribution of microfacets
    - Often probabilistic distribution of orientation or V-groove assumption
  - Planar reflection properties
  - Self-masking, shadowing



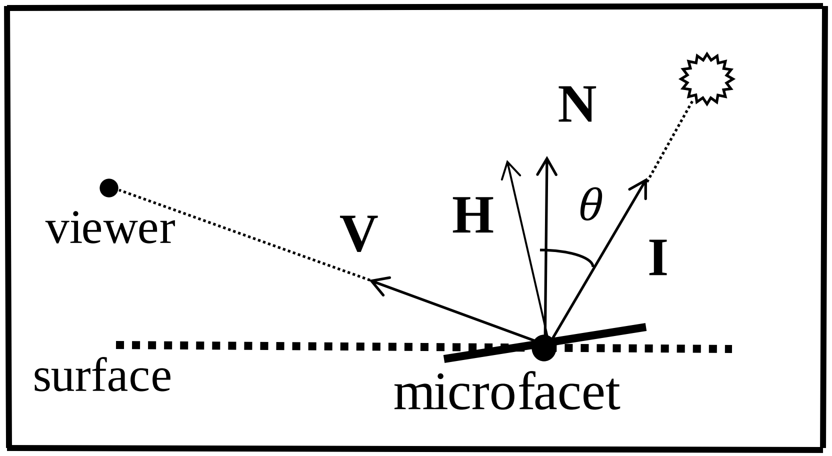


Ward Reflection Model

- **BRDF**

$$f_r = \frac{\rho_d}{\pi} + \frac{\rho_s}{\sqrt{(I \cdot N)(V \cdot N)}} \frac{\exp\left(-\frac{tan^2 \angle H, N}{\sigma^2}\right)}{4\pi\sigma^2}$$

  - σ standard deviation (RMS) of surface slope
  - Simple expansion to anisotropic model (σ_x, σ_y)
  - Empirical, not physics-based
- **Inspired by notion of reflecting microfacets**
  - Convincing results
  - Good match to measured data



Cook-Torrance Reflection Model

- **Cook-Torrance reflectance model**
  - Is based on the *microfacet* model
  - BRDF is defined as the sum of a diffuse and a glossy component:

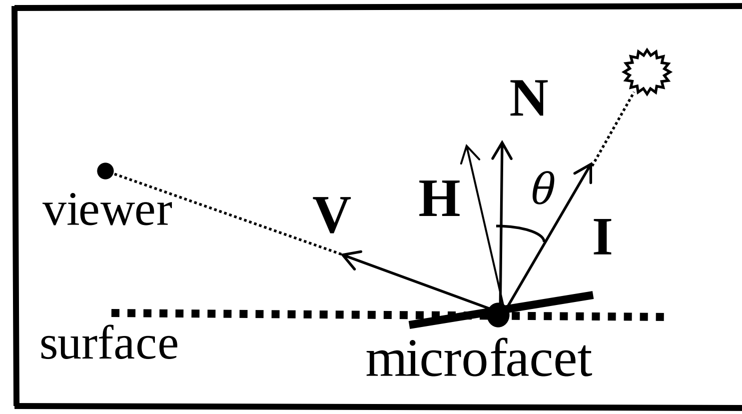
$$f_r = \kappa_d \rho_d + \kappa_g \rho_g; \qquad \rho_d + \rho_g \leq 1$$

where ρ_g and ρ_d are the glossy and diffuse coefficients.

  - Derivation of the glossy component κ_g is based on a physically derived theoretical reflectance model
  - (The original paper talks about "specular" instead of "glossy" as the glossy reflection originates from averaging the specular reflections of many microfacets)

Cook-Torrance Specular Term

$$\kappa_s = \frac{F_\lambda D G}{\pi (N \cdot V)(N \cdot I)}$$



- **D : Distribution function of microfacet orientations**
- **G : Geometrical attenuation factor**
  - represents self-masking and shadowing effects of microfacets
- **F_λ : Fresnel term**
  - computed by Fresnel equation
  - Fraction of specularly reflected light for each planar microfacet
- **N·V : Proportional to visible surface area**
- **N·I : Proportional to illuminated surface area**

Electric Conductors (e.g. Metals)

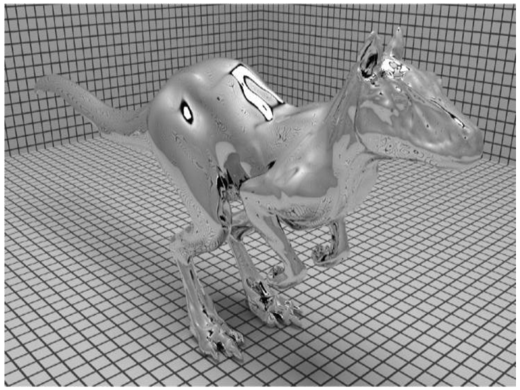
- **Assume ideally smooth surface**
- **Perfect specular reflection of light, rest is absorbed**
- **Reflectance is defined by Fresnel formula based on:**
  - Index of refraction η
  - Absorption coefficient κ
  - Both wavelength dependent

Object	η	k
Gold	0.370	2.820
Silver	0.177	3.638
Copper	0.617	2.63
Steel	2.485	3.433

- **Given for parallel and perpendicular polarized light**

$$r_{\parallel}^2 = \frac{(\eta^2 + k^2)\cos\theta_i^2 - 2\eta\cos\theta_i + 1}{(\eta^2 + k^2)\cos\theta_i^2 + 2\eta\cos\theta_i + 1}$$

$$r_{\perp}^2 = \frac{(\eta^2 + k^2) - 2\eta\cos\theta_i + \cos\theta_i^2}{(\eta^2 + k^2) + 2\eta\cos\theta_i + \cos\theta_i^2}.$$

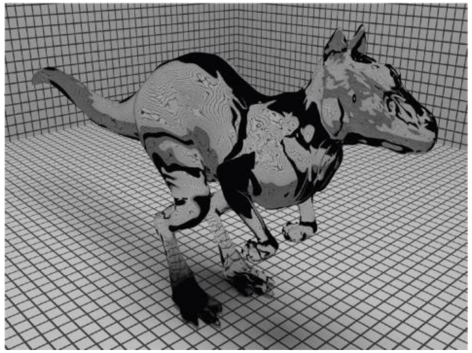


  - θ_i, θ_t: Angle between ray & plane, incident & transmitted
- **For unpolarized light:**

$$F_r = \frac{1}{2}(r_{\parallel}^2 + r_{\perp}^2)$$

Dielectrics (e.g. Glass)

- **Assume ideally smooth surface**
- **Non-reflected light is perfectly transmitted: *1 - F_r***
  - They do not conduct electricity
- **Fresnel formula depends on:**
  - Refr. index: speed of light in vacuum vs. medium
  - Refractive index in incident medium $\eta_i = c_0 / c_i$
  - Refractive index in transmitted medium $\eta_t = c_0 / c_t$



- **Given for parallel and perpendicular polarized light**

$$r_{\parallel} = \frac{\eta_t \cos\theta_i - \eta_i \cos\theta_t}{\eta_t \cos\theta_i + \eta_i \cos\theta_t}$$

$$r_{\perp} = \frac{\eta_i \cos\theta_i - \eta_t \cos\theta_t}{\eta_i \cos\theta_i + \eta_t \cos\theta_t},$$

Medium	Index of refraction η
Vacuum	1.0
Air at sea level	1.00029
Ice	1.31
Water (20° C)	1.333
Fused quartz	1.46
Glass	1.5–1.6
Sapphire	1.77
Diamond	2.42

- **For unpolarized light:**

$$F_r = \frac{1}{2}(r_{\parallel}^2 + r_{\perp}^2)$$

Microfacet Distribution Functions

- **Isotropic Distributions** $D(\omega) \Rightarrow D(\alpha) \quad \alpha = \angle N, H$
  - α : angle to average normal of surface
  - m : average slope of the microfacets

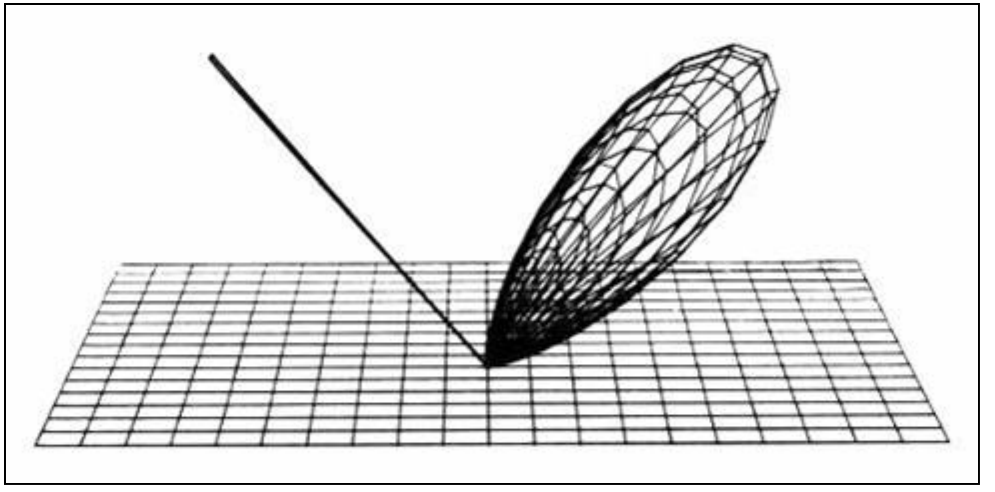
- **Blinn:** $$D(\alpha) = cos^{\frac{ln\,2}{ls\,cos\,m_\alpha}}$$

- **Torrance-Sparrow** $$D(\alpha) = e^{-\left(\frac{\alpha}{m}\right)^2}$$
  - Gaussian

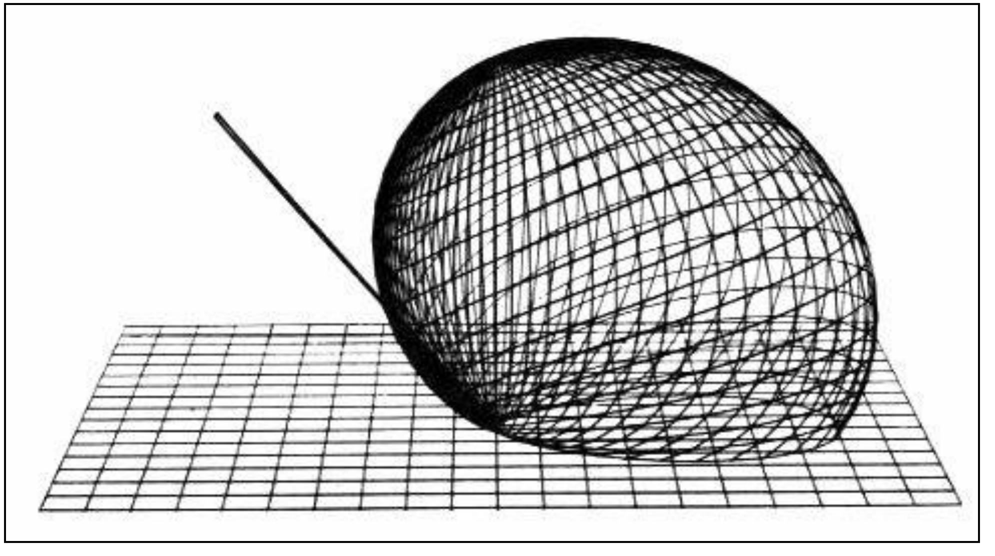
- **Beckmann** $$D(\alpha) = \frac{1}{\pi m^2 cos^4 \alpha} e^{-(\frac{tan\alpha}{m})^2}$$
  - Used by Cook-Torrance

Beckman Microfacet Distribution

m=0.2

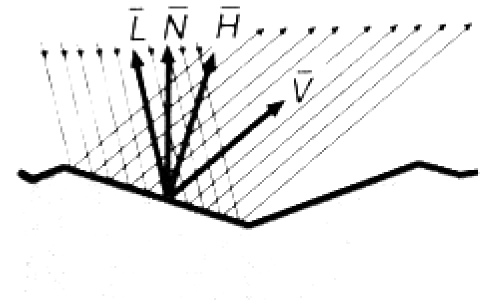


m=0.6



Geometric Attenuation Factor

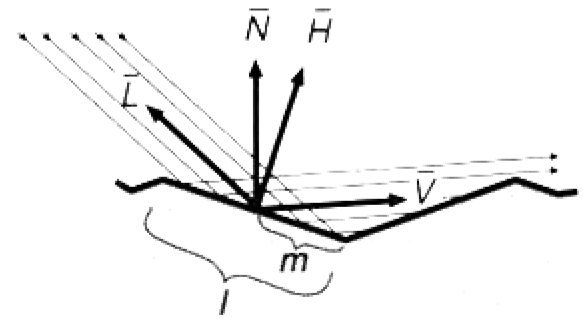
- **V-shaped grooves**
- **Fully illuminated and visible**



$$G = 1$$

- **Partial masking of reflected light**

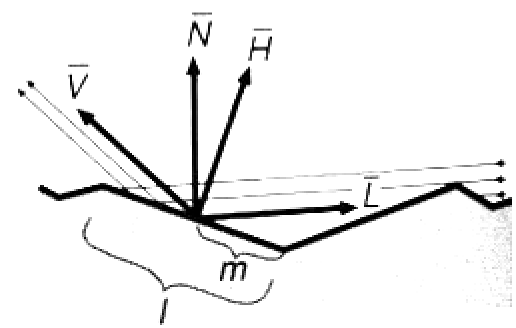
$$G = \frac{2(\underline{N} \cdot \underline{H})(\underline{N} \cdot \underline{V})}{(\underline{V} \cdot \underline{H})}$$



- **Partial shadowing of incident light**

$$G = \frac{2(\underline{N} \cdot \underline{H})(\underline{N} \cdot \underline{I})}{(\underline{V} \cdot \underline{H})}$$

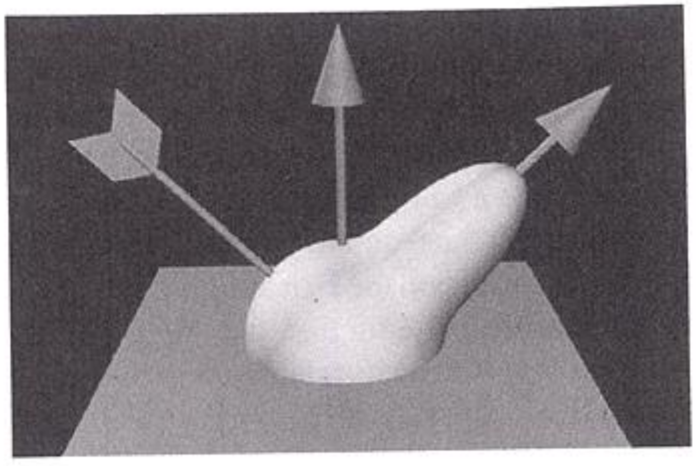
- **Final**



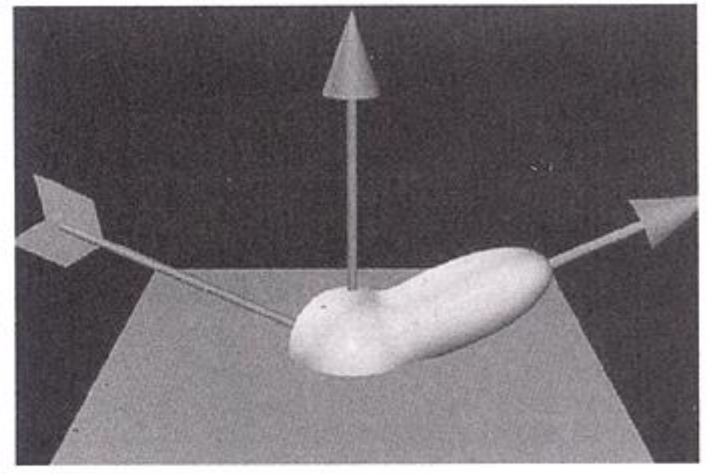
$$G = min\left\{1, \frac{2(\underline{N} \cdot \underline{H})(\underline{N} \cdot \underline{V})}{(\underline{V} \cdot \underline{H})}, \frac{2(\underline{N} \cdot \underline{H})(\underline{N} \cdot \underline{I})}{(\underline{V} \cdot \underline{H})}\right\}$$

Comparison Phong vs. Torrance

Phong:

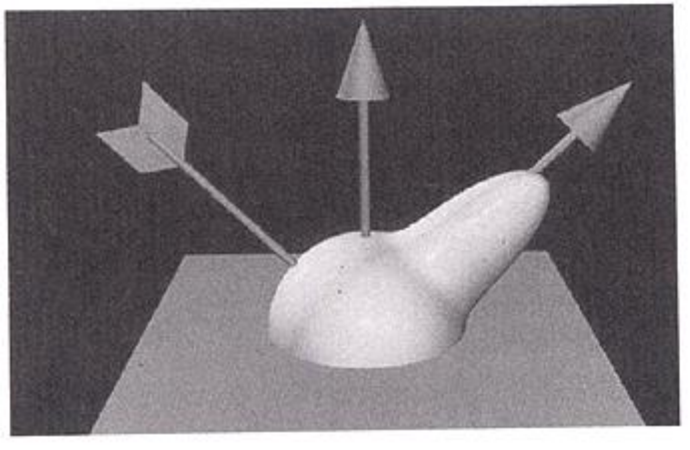


(a)

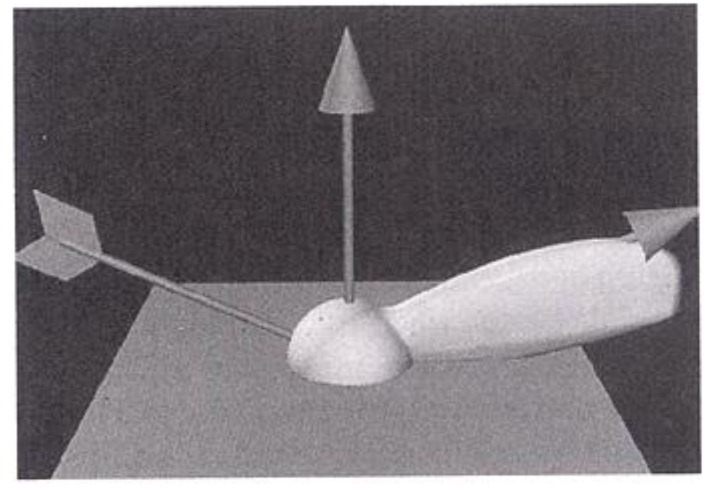


(b)

Torrance:



(c)



(d)

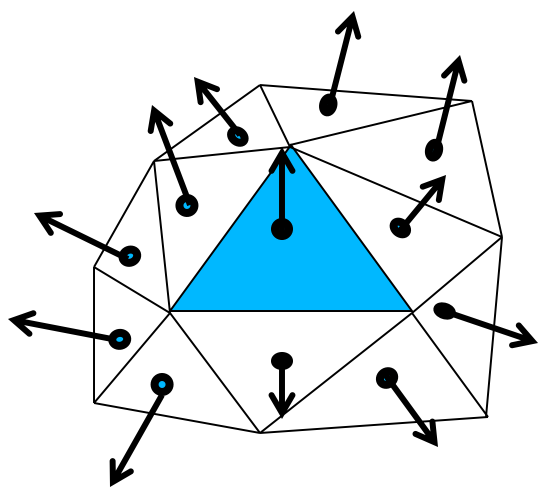
SHADING

What is Shading?

- **Shading**
  - Computation of reflected light (radiance) at every pixel
  - In ray tracing typically computed at every hit point
  - In rasterization computed per triangle, vertex, pixel, or sample
- **What is required for shading**
  - Position of shaded point
  - Position of viewpoint
  - Position of light source and its description/parameters
  - Surface normal / local coordinate frame at shaded point
  - Reflectance model (BRDF)

Flat Shading Model

- **Most simple: Constant Shading**
  - Fixed color per polygon/triangle
- **Shading Model: Flat Shading**
  - Single per-surface normal
  - Single color per polygon
  - Evaluated at one of the vertices (➔ OpenGL) or at center



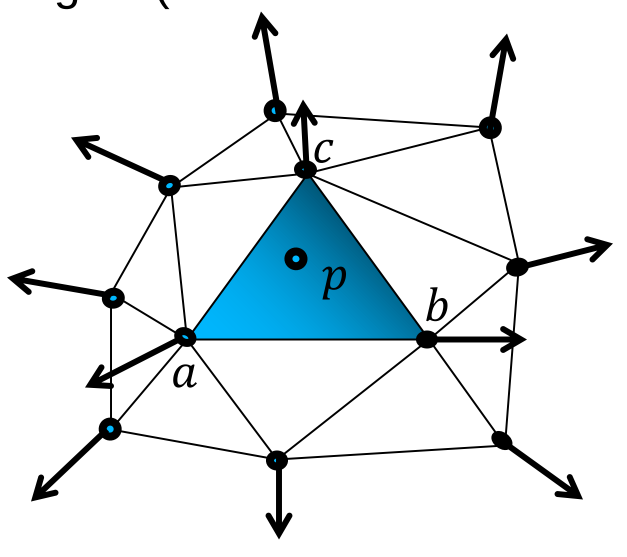
[wikipedia]

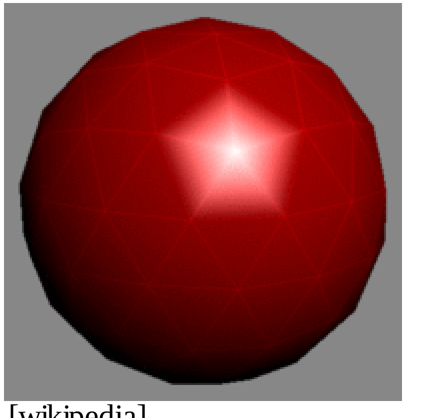
Gouraud Shading Model

- **Shading Model: Gouraud Shading**
  - Computed only at vertices (with per-vertex normal)
    - Normal can be computed from adjacent triangle normals
  - Linear interpolation of the shaded colors
    - Computed at all vertices and interpolated
  - Often results in shading artifacts along edges
    - **Mach Banding** (i.e. discontinuous 1st derivative)
    - Flickering of highlights (when one of the normal generates strong reflection)

$$L_x \sim f_r(\omega_o, n_x, \omega_i) L_i \cos\theta_i$$
$$L_p = \lambda_1 L_a + \lambda_2 L_b + \lambda_3 L_c$$

  - **Barycentric interpolation** within triangle





[wikipedia]

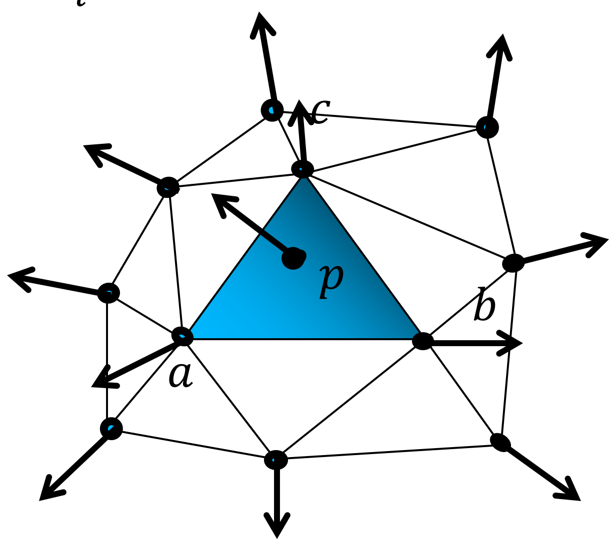
Phong Shading Model

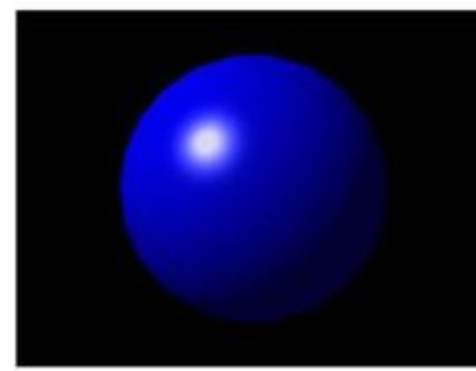
- **Shading Model: Phong Shading**
  - Linear interpolation of the surface normal
  - Shading is evaluated at every point separately
  - Smoother but still off due to hit point offset from apparent surface

$$n_p = \frac{\lambda_1 n_1 + \lambda_2 n_2 + \lambda_3 n_3}{\| \lambda_1 n_1 + \lambda_2 n_2 + \lambda_3 n_3 \|}$$

$$L_p \sim f_r(\omega_o, n_p, \omega_i) L_i \cos\theta_i$$

  - Barycentric interpolation of normal within the triangle
  - With subsequent renormalization

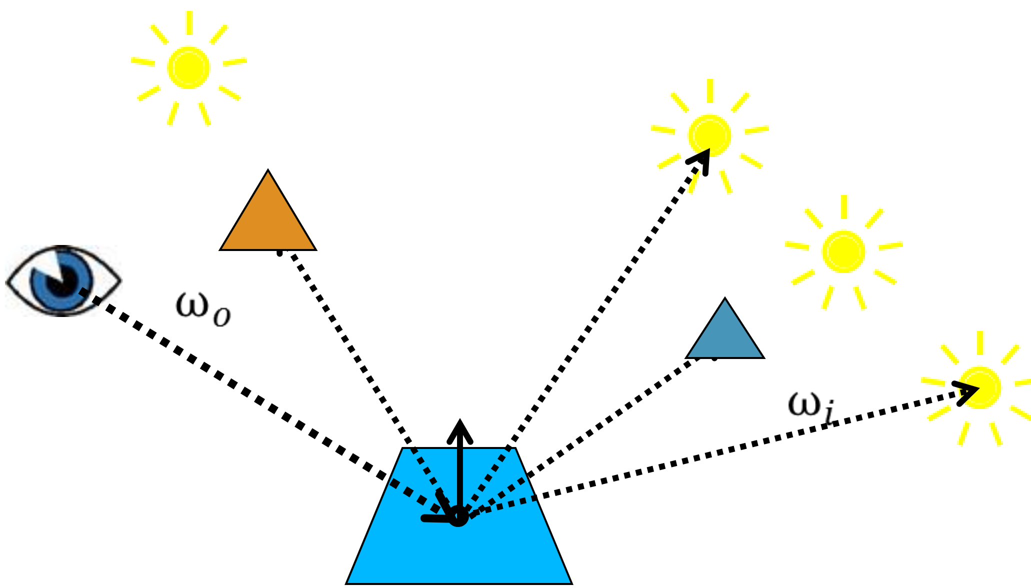




[wikipedia]

Occlusion / Shadows

- **The point on the surface might be in shadow**
  - Rasterization (OpenGL):
    - Not easily done
    - Can use shadow map or shadow volumes (➔ later)
  - Ray tracing
    - Simply trace ray to light source and test for occlusion



Area Light sources

- **Typically approximated by sampling**
  - Replacing area with some point light sources
    - Often randomly sampled