

Computer Graphics

- Texturing -

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Overview

- **Last time**
 - Shading
 - BRDFs (will continue Monday)
- **Today**
 - Texture definition
 - Image textures
 - Procedural textures
 - Texture mapping
- **Next lecture**
 - Alias & signal processing

Texture

- **Textures modify the input for shading computations**
 - Either via (painted) images textures or procedural functions
- **Example texture maps for**
 - Reflectance, normals, shadow reflections, ...



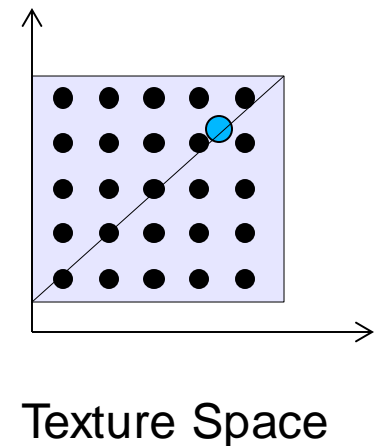
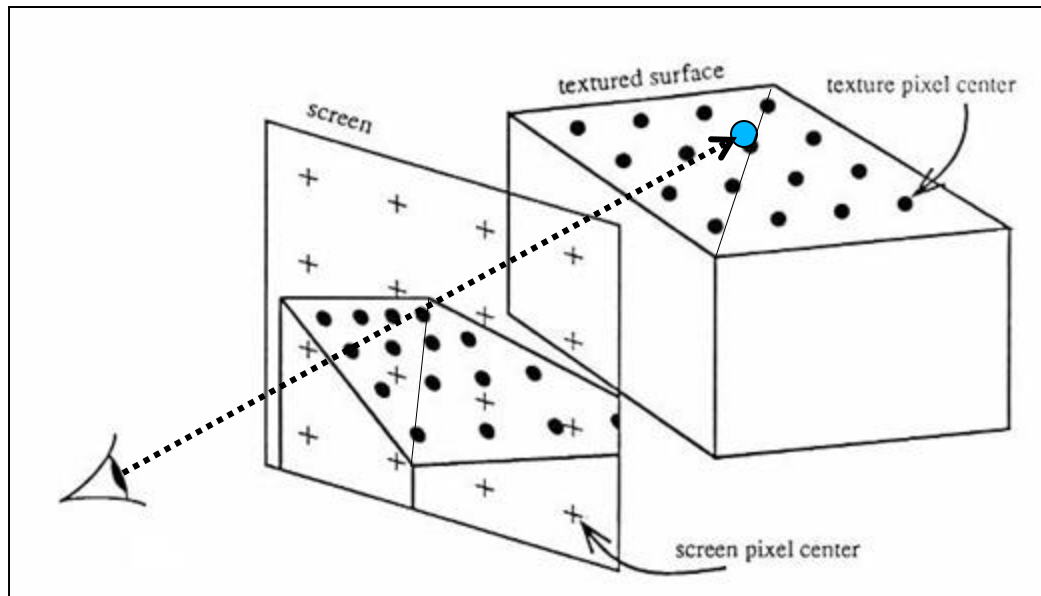
Definition: Textures

- **Texture maps texture coordinates to shading values**
 - Input: 1D/2D/3D texture coordinates
 - Explicitly given or derived via other data (e.g. position, direction, ...)
 - Output: Scalar or vector value
 - **Modified values in shading computations**
 - Reflectance
 - Changes the diffuse or specular reflection coefficient (k_d, k_s)
 - Geometry and Normal (important for lighting)
 - Displacement mapping $P' = P + \Delta P$
 - Normal mapping $N' = N + \Delta N$
 - Bump mapping $N' = N(P + tN)$
 - Opacity
 - Modulating transparency (e.g. for fences in games)
 - Illumination
 - Light maps, environment mapping, reflection mapping
-

IMAGE TEXTURES

Reconstruction Filter

- **Image texture**
 - Discrete set of sample values (given at texel centers!)
- **In general**
 - Hit point does not exactly hit a texture sample
- **Still want to reconstruct a continuous function**
 - Use reconstruction filter to find color for hit point



Nearest Neighbor

- **Local Coordinates**

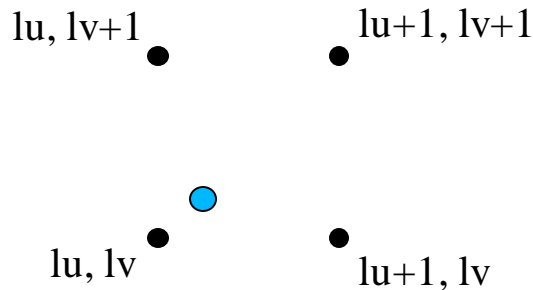
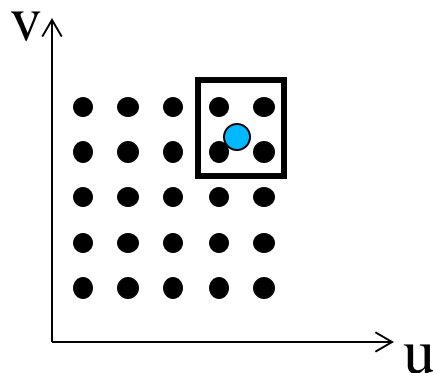
- Assuming cell-centered samples
- $u = t_u * \text{resU};$
- $v = t_v * \text{resV};$

- **Lattice Coordinates**

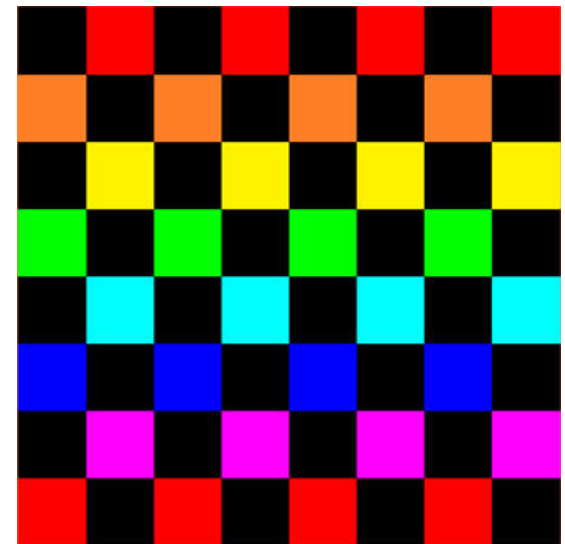
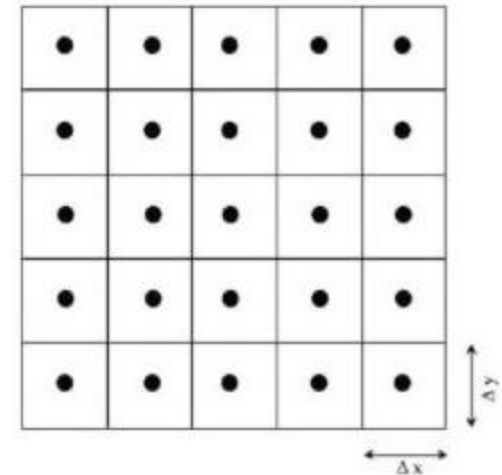
- $l_u = \min(\lfloor u \rfloor, \text{resU} - 1);$
- $l_v = \min(\lfloor v \rfloor, \text{resV} - 1);$

- **Texture Value**

- `return image[lu, lv];`



Pixel centred registration



Bilinear Interpolation

- **Local Coordinates**

- Assuming node-centered samples
- $u = t_u * (\text{resU} - 1);$
- $v = t_v * (\text{resV} - 1);$

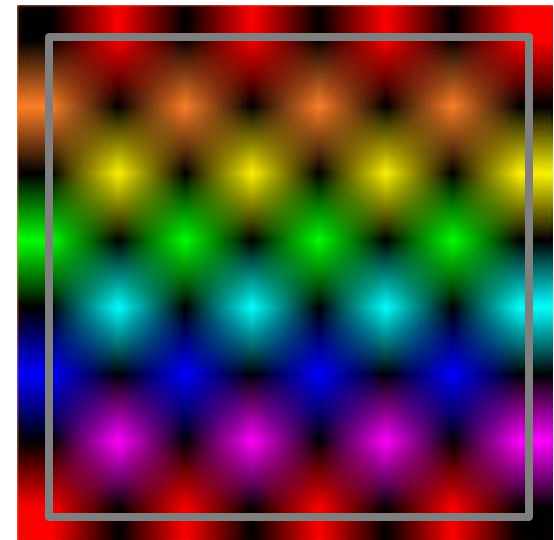
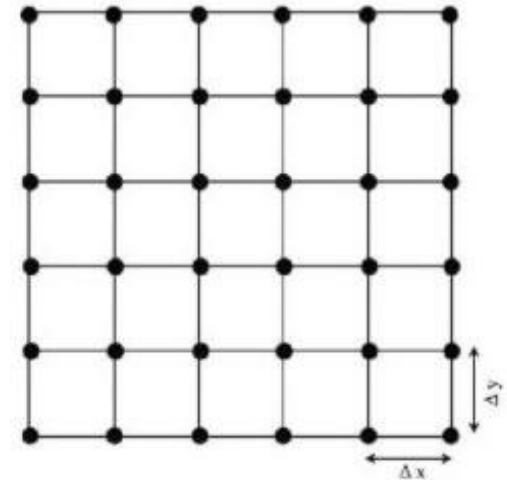
- **Fractional Coordinates**

- $f_u = u - \lfloor u \rfloor;$
- $f_v = v - \lfloor v \rfloor;$

- **Texture Value**

- $\text{return } (1-f_u)(1-f_v) \text{image}[\lfloor u \rfloor, \lfloor v \rfloor]$
+ $(1-f_u)(f_v) \text{image}[\lfloor u \rfloor, \lfloor v \rfloor + 1]$
+ $(f_u)(1-f_v) \text{image}[\lfloor u \rfloor + 1, \lfloor v \rfloor]$
+ $(f_u)(f_v) \text{image}[\lfloor u \rfloor + 1, \lfloor v \rfloor + 1]$

Grid node registration



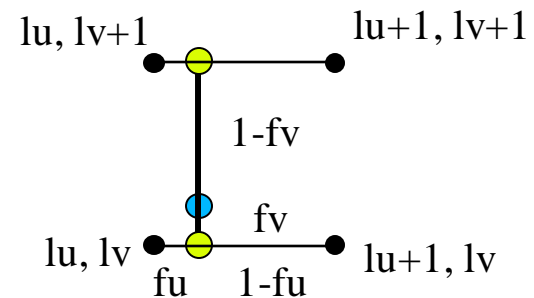
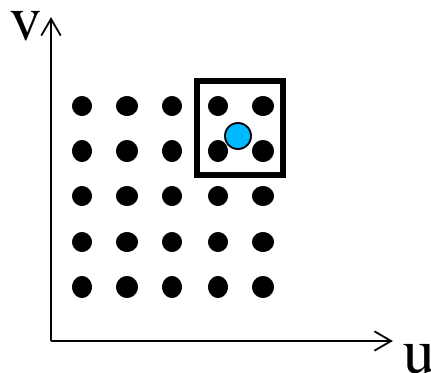
Bilinear Interpolation

- **Successive Linear Interpolations**

- $u0 = (1-fv) \text{ image}[\lfloor u \rfloor, \lfloor v \rfloor] + (fv) \text{ image}[\lfloor u \rfloor, \lfloor v \rfloor + 1];$

- $u1 = (1-fv) \text{ image}[\lfloor u \rfloor + 1, \lfloor v \rfloor] + (fv) \text{ image}[\lfloor u \rfloor + 1, \lfloor v \rfloor + 1];$

- $\text{return } (1-fu) u0 + (fu) u1;$



Nearest vs. Bilinear Interpolation



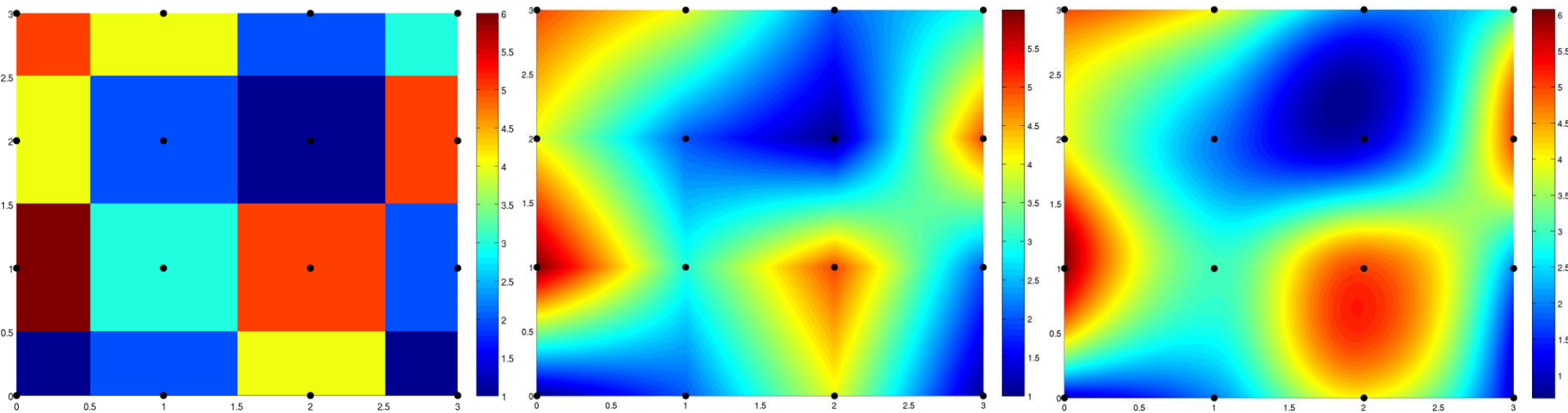
GL_NEAREST



GL_LINEAR

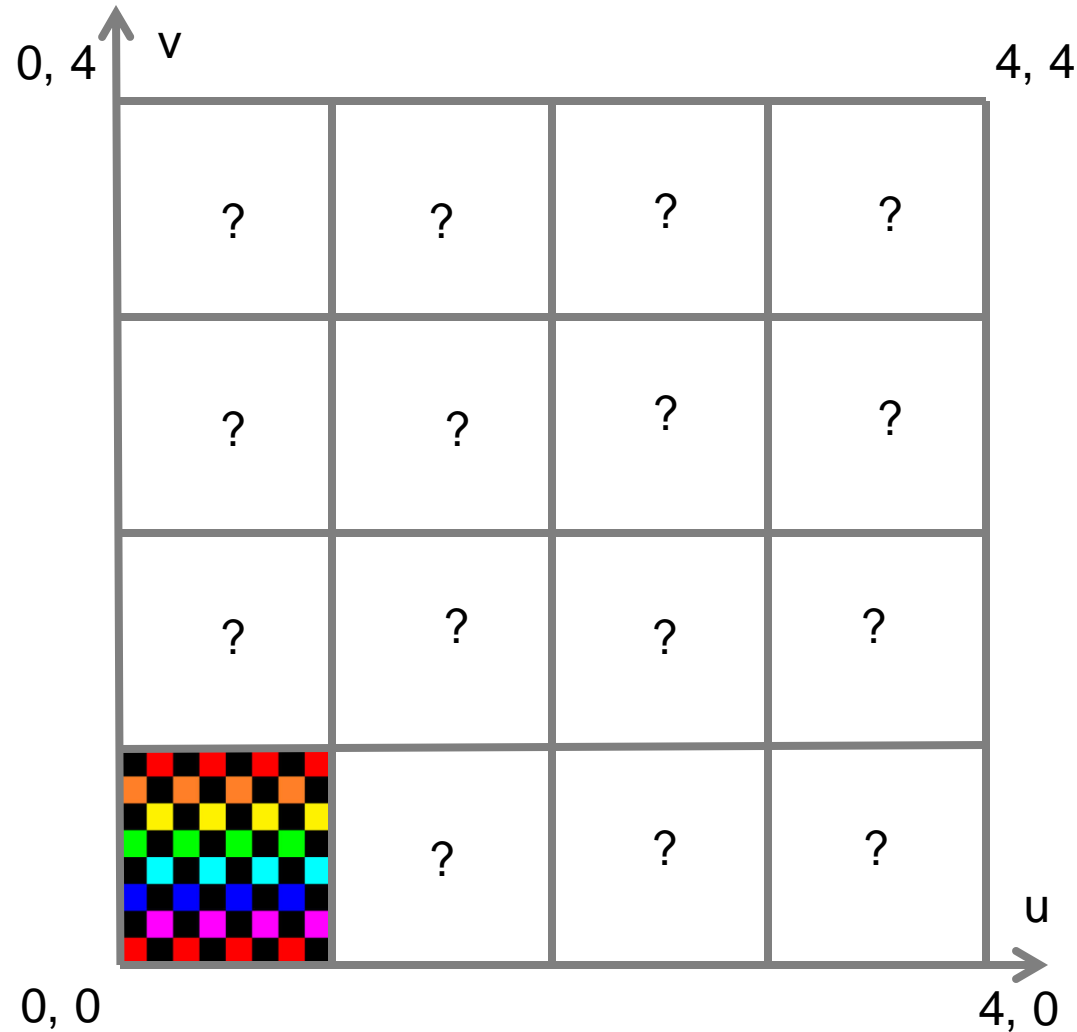
Bicubic Interpolation

- **Properties**
 - Assuming node-centered samples
 - Essentially based on cubic splines (see later)
- **Pros**
 - Even smoother
- **Cons**
 - More complex & expensive (4x4 kernel)
 - Overshoot



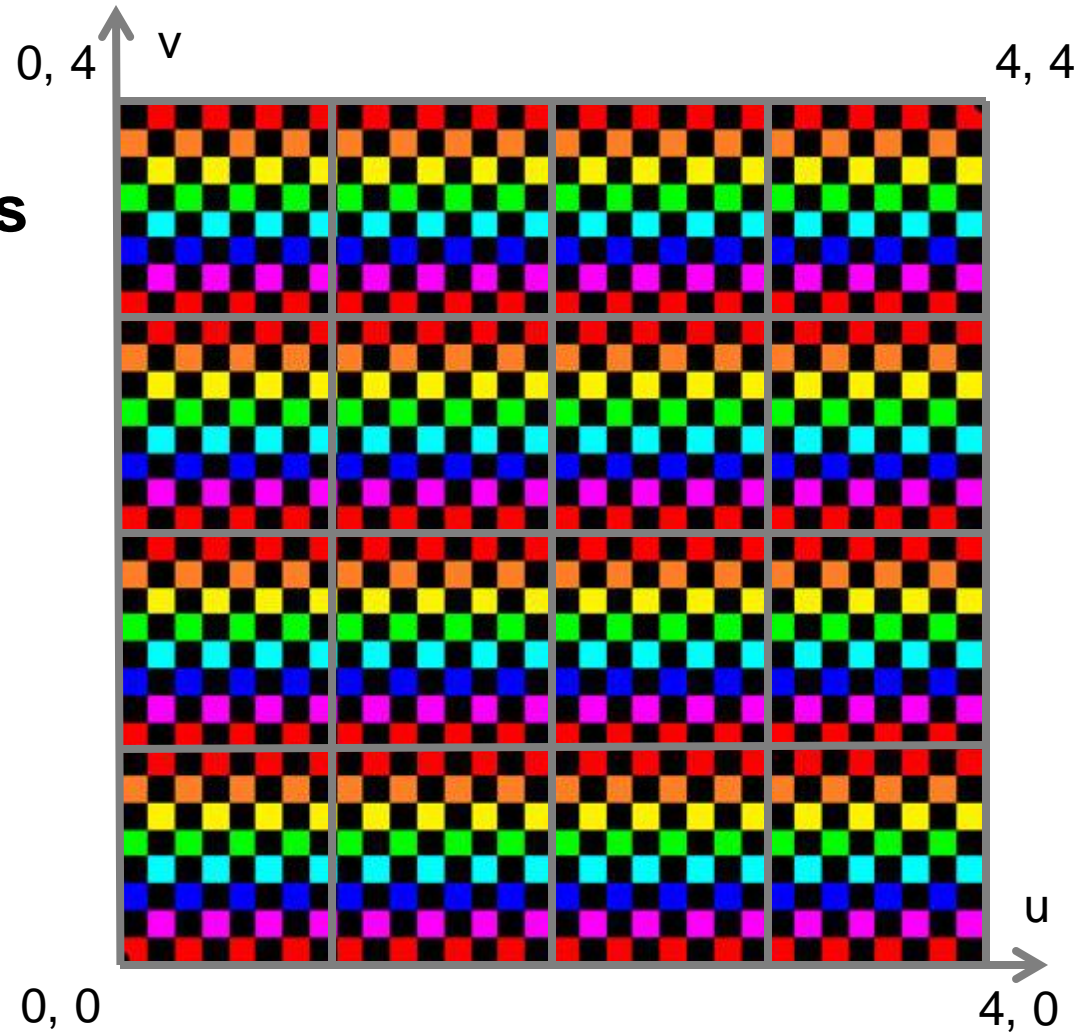
Wrap Mode

- **Texture Coordinates**
 - (u, v) in $[0, 1] \times [0, 1]$
- **What if?**
 - (u, v) not in unit square?



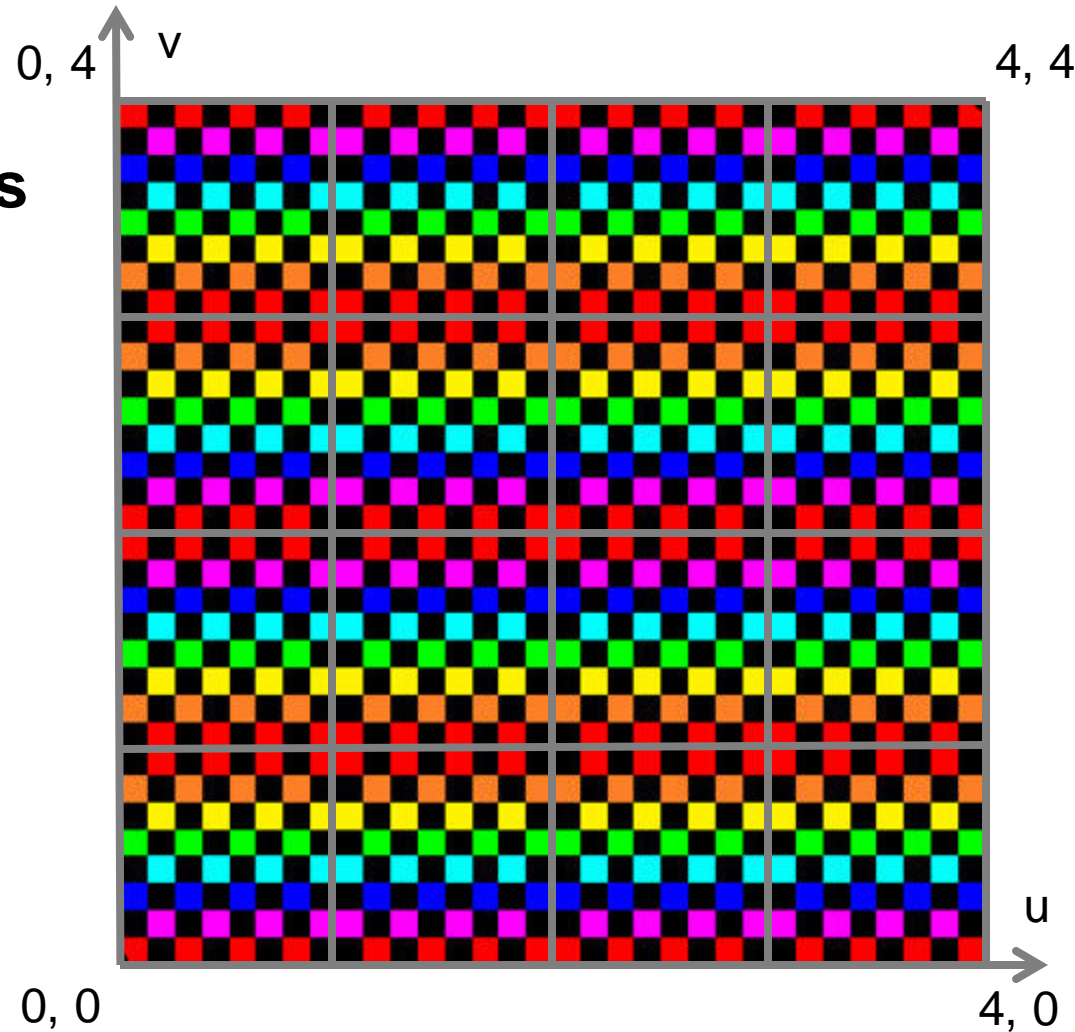
Wrap Mode

- Repeat
- Fractional Coordinates
 - $t_u = u - \lfloor u \rfloor$
 - $t_v = v - \lfloor v \rfloor$



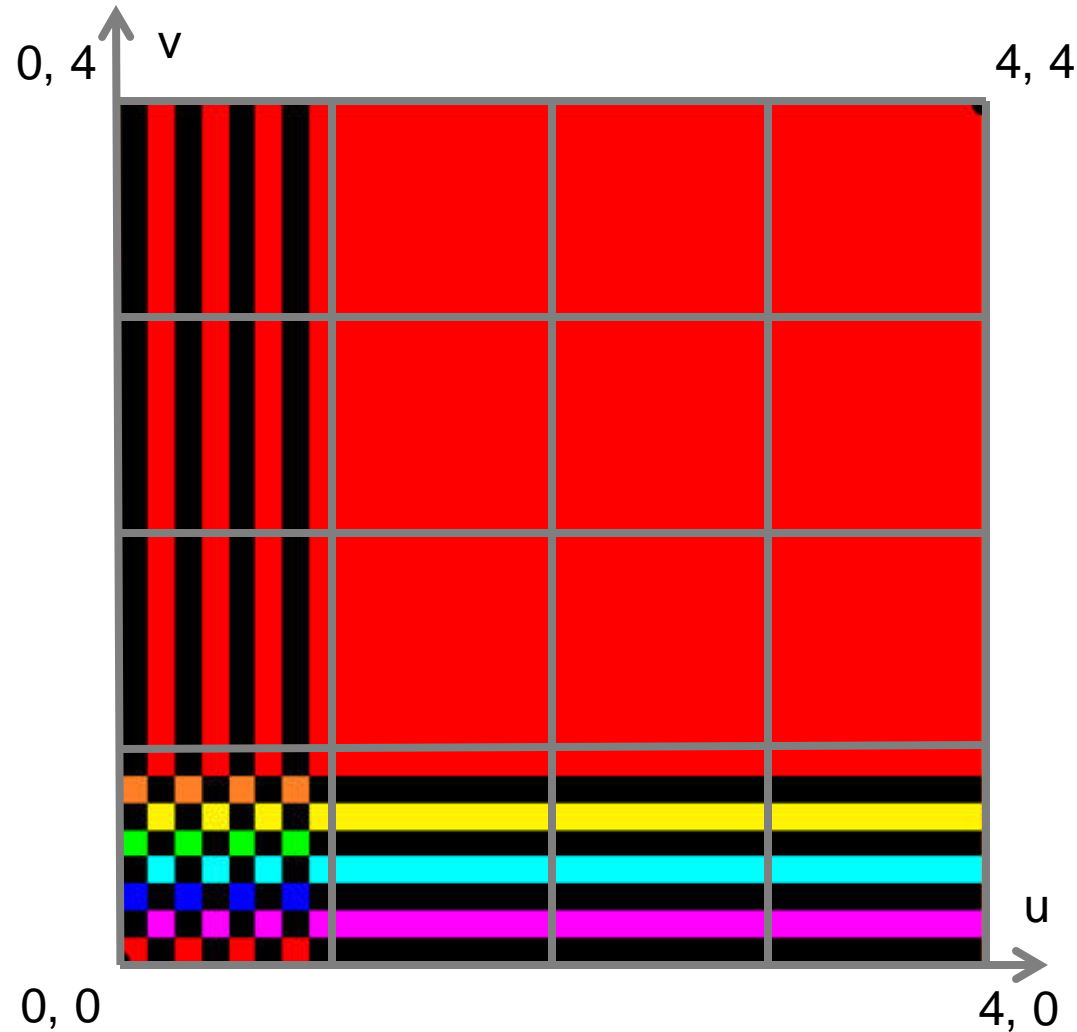
Wrap Mode

- **Mirror**
- **Fractional Coordinates**
 - $t_u = u - \lfloor u \rfloor$
 - $t_v = v - \lfloor v \rfloor$
- **Lattice Coordinates**
 - $l_u = \lfloor u \rfloor$
 - $l_v = \lfloor v \rfloor$
- **Mirror if Odd**
 - if $(l_u \% 2 == 1)$
 $t_u = 1 - t_u$
 - if $(l_v \% 2 == 1)$
 $t_v = 1 - t_v$



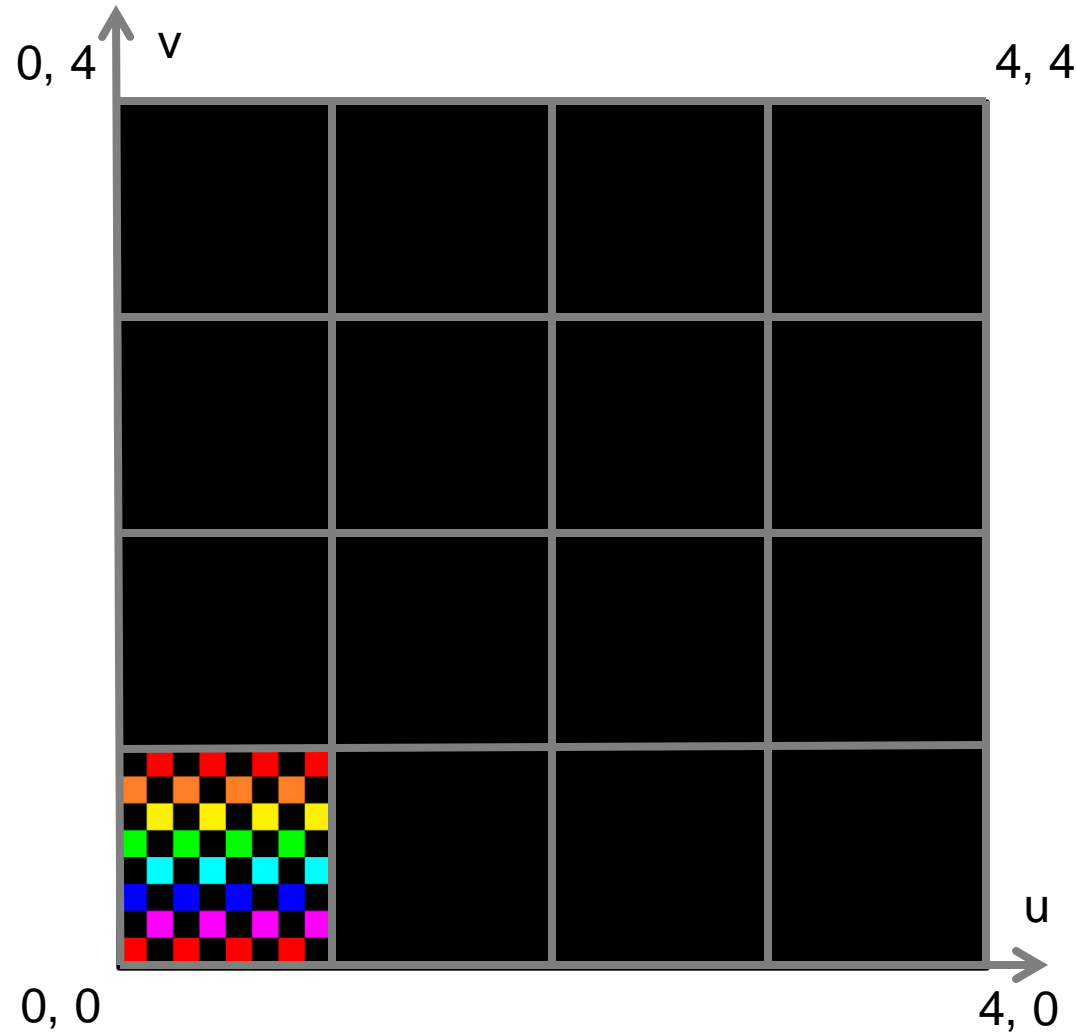
Wrap Mode

- **Clamp**
- **Clamp u to $[0, 1]$**
if $(u < 0)$ $tu = 0$;
else if $(u > 1)$ $tu = 1$;
else $tu = u$;
- **Clamp v to $[0, 1]$**
if $(v < 0)$ $tv = 0$;
else if $(v > 1)$ $tv = 1$;
else $tv = v$;



Wrap Mode

- **Border**
- **Check Bounds**
if ($u < 0 \parallel u > 1$
 $\parallel v < 0 \parallel v > 1$)
 return backgroundColor;
else
 $tu = u$;
 $tv = v$;



Wrap Mode

- **Comparison**
 - With OpenGL texture modes



GL_REPEAT



GL_MIRRORED_REPEAT



GL_CLAMP_TO_EDGE



GL_CLAMP_TO_BORDER

Discussion: Image Textures

- **Pros**

- Simple generation
 - Painted, simulation, ...
- Simple acquisition
 - Photos, videos

- **Cons**

- Illumination “frozen” during acquisition
 - Limited resolution
 - Susceptible to aliasing
 - High memory requirements (often HUGE for films, 100s of GB)
 - Issues when mapping 2D image onto 3D object
-

PROCEDURAL TEXTURES

Discussion: Procedural Textures

- **Cons**

- Sometimes hard to achieve specific effect
- Possibly non-trivial programming

- **Pros**

- Flexibility & parametric control
 - Unlimited resolution
 - Anti-aliasing possible
 - Low memory requirements
 - May be directly defined as 3D “image” mapped to 3D geometry
 - Low-cost visual complexity
-

2D Checkerboard Function

- **Lattice Coordinates**

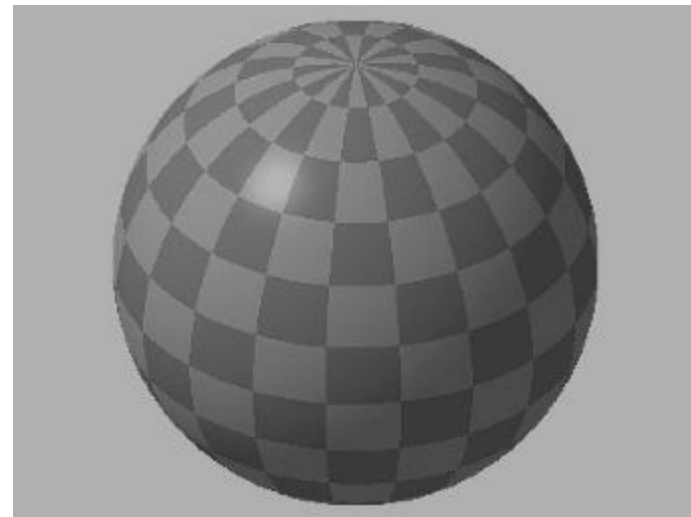
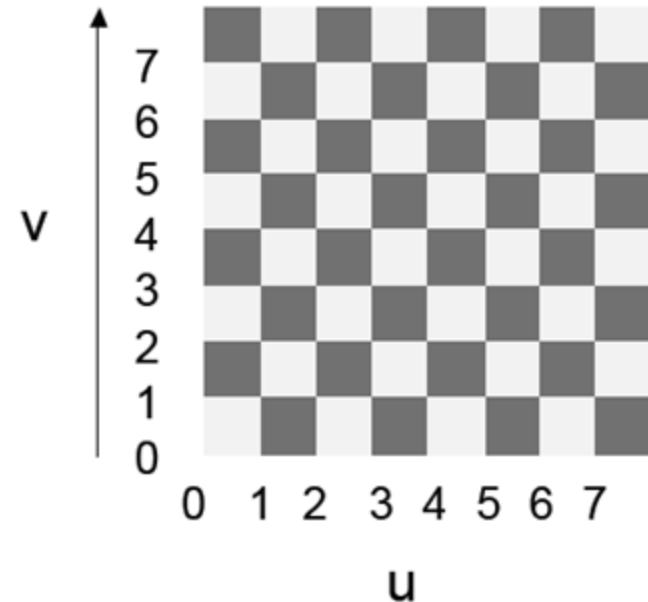
- $lu = \lfloor u \rfloor$
- $lv = \lfloor v \rfloor$

- **Compute Parity**

- $parity = (lu + lv) \% 2;$

- **Return Color**

- if ($parity == 1$)
 - return color1;
- else
 - return color0;



3D Checkerboard - Solid Texture

- **Lattice Coordinates**

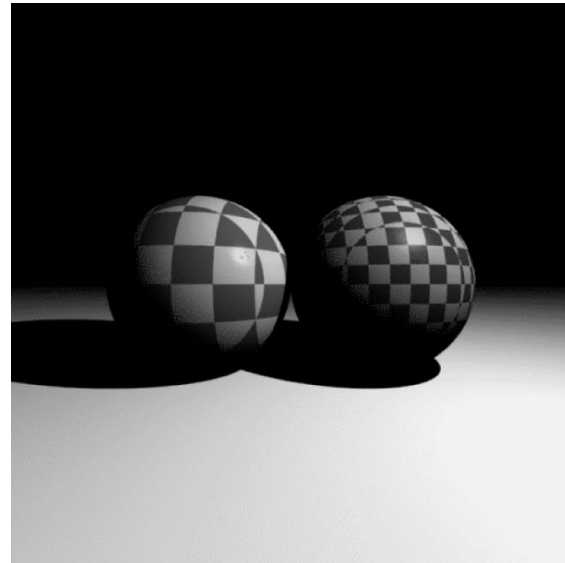
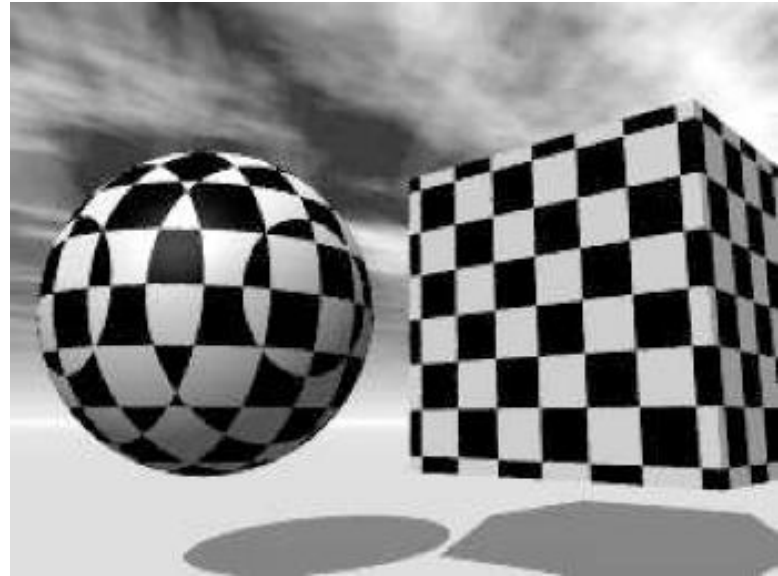
- $lu = \lfloor u \rfloor$
- $lv = \lfloor v \rfloor$
- $lw = \lfloor w \rfloor$

- **Compute Parity**

- $parity = (lu + lv + lw) \% 2;$

- **Return Color**

- if ($parity == 1$)
 - return color1;
- else
 - return color0;



Tile

- **Fractional Coordinates**

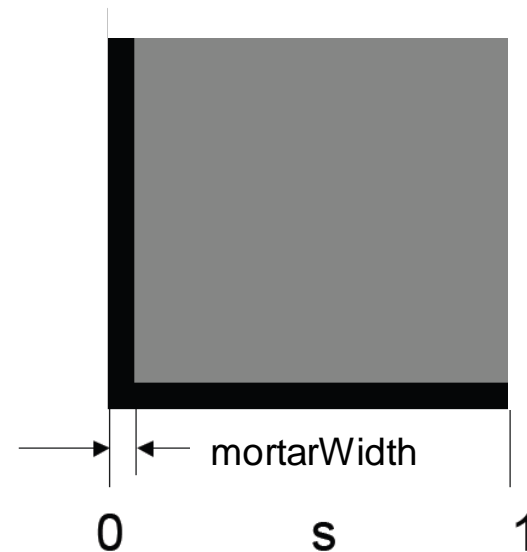
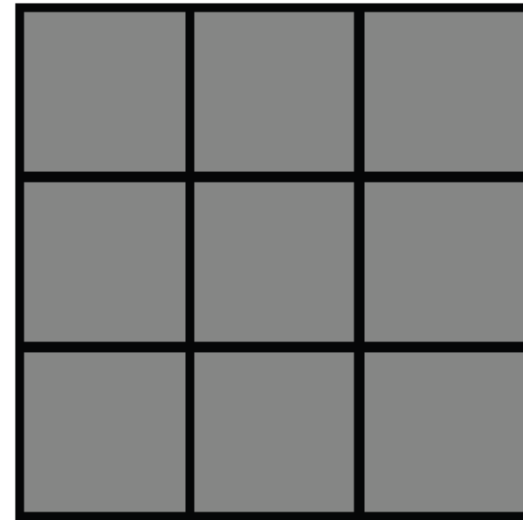
- $fu = u - \lfloor u \rfloor$
- $fv = v - \lfloor v \rfloor$

- **Compute Booleans**

- $bu = fu < \text{mortarWidth};$
- $bv = fv < \text{mortarWidth};$

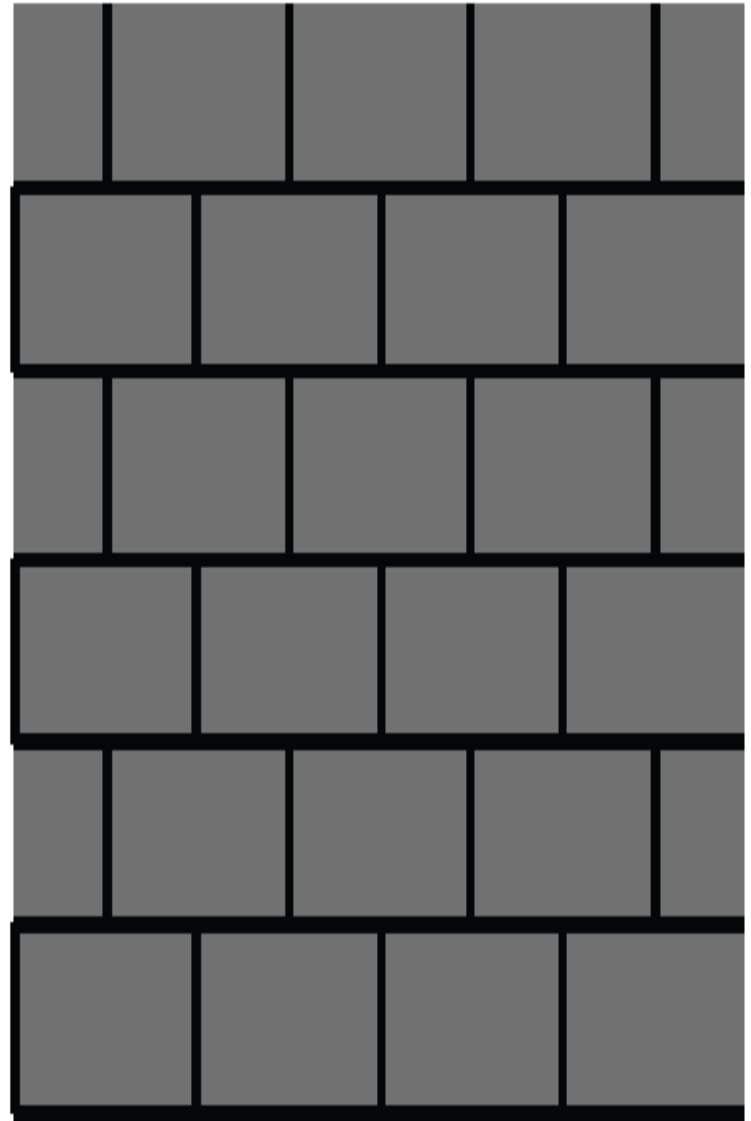
- **Return Color**

- if $(bu \parallel bv)$
 - return mortarColor;
- else
 - return tileColor;

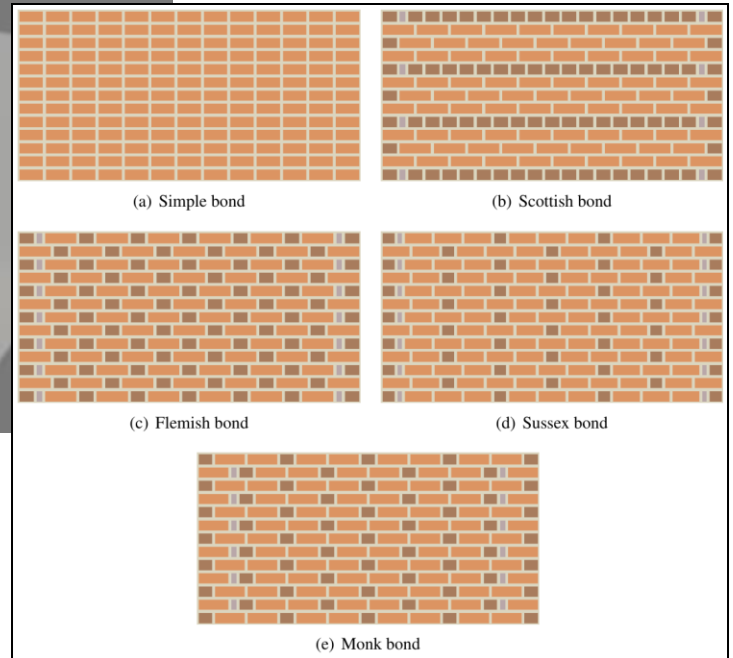
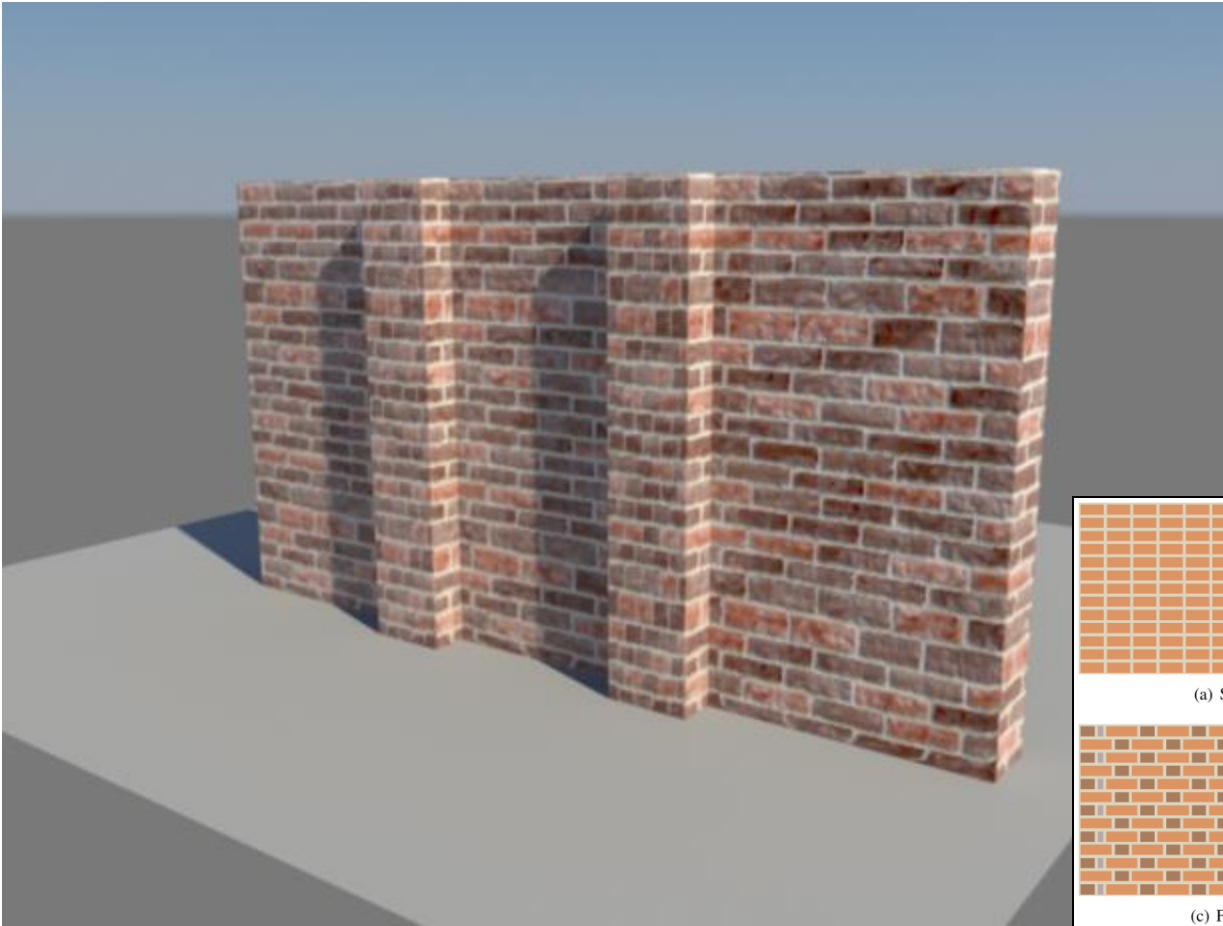


Brick

- **Shift Column for Odd Rows**
 - $\text{parity} = \lfloor v \rfloor \% 2;$
 - $u -= \text{parity} * 0.5;$
- **Fractional Coordinates**
 - $f_u = u - \lfloor u \rfloor$
 - $f_v = v - \lfloor v \rfloor$
- **Compute Booleans**
 - $\text{bu} = f_u < \text{mortarWidth};$
 - $\text{bv} = f_v < \text{mortarWidth};$
- **Return Color**
 - if ($\text{bu} \parallel \text{bv}$)
 - return mortarColor;
 - else
 - return brickColor;

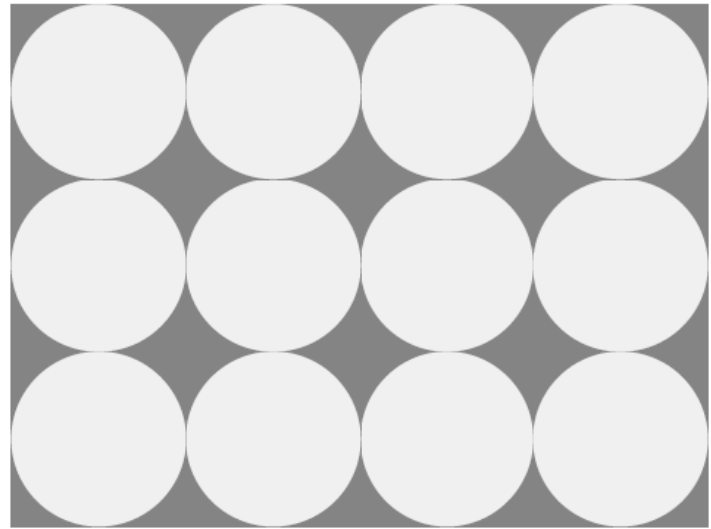


More Variation

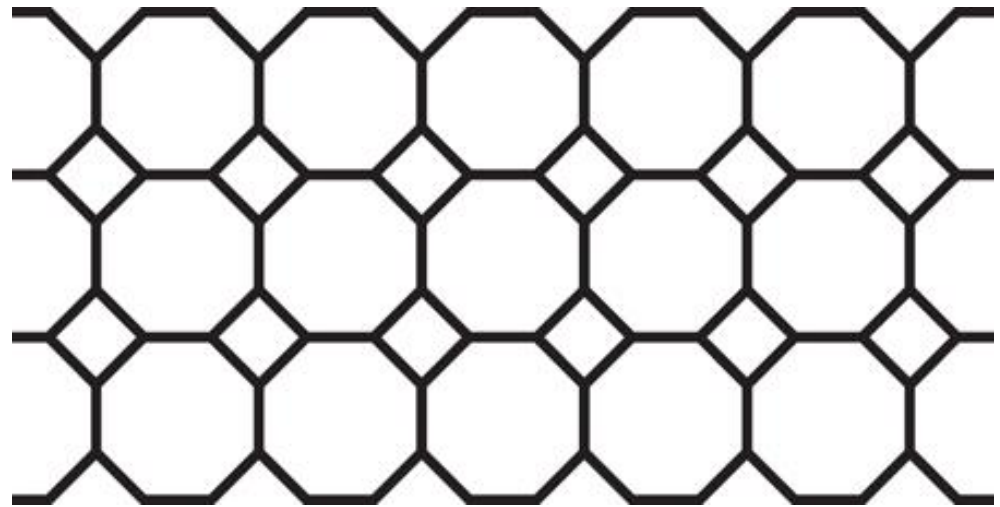


Other Patterns

- **Circular Tiles**



- **Octagonal Tiles**



- **Use your imagination!**
-

Perlin Noise

- **Natural Patterns**

- Similarity between patches at different locations
 - Repetitiveness, coherence (e.g., skin of a tiger or zebra)
- Similarity on different resolution scales
 - Self-similarity
- But never completely identical
 - Additional disturbances, turbulence, noise

- **Mimic Statistical Properties**

- Purely empirical approach
- Looks convincing, but has nothing to do with material's physics

- **Perlin Noise is essential for adding “natural” details**

- Used in many texture functions
-

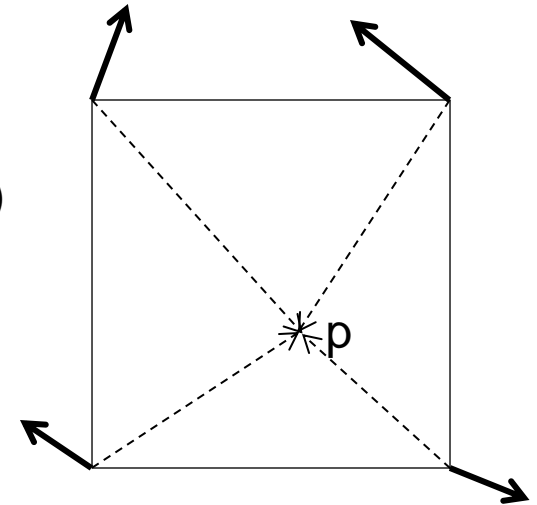
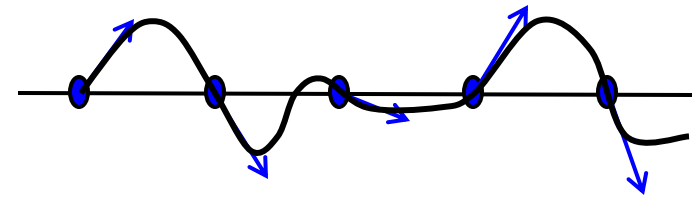
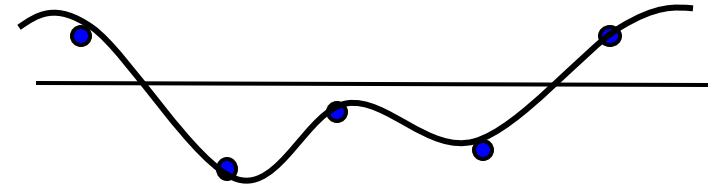
Perlin Noise

- Natural Fractals



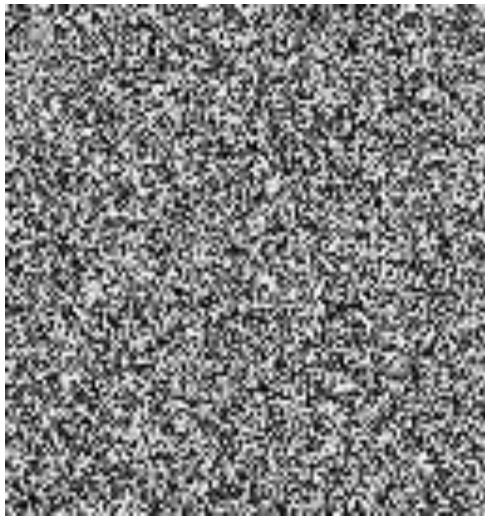
Noise Function

- **Noise(x, y, z)**
 - Statistical invariance under rotation
 - Statistical invariance under translation
 - Roughly fixed frequency of ~ 1 Hz
- **Integer Lattice (i, j, k)**
 - Value noise
 - Random value at lattice points
 - Gradient noise
 - Random gradient vector at lattice point
 - Interpolation
 - Bi-/tri-linear or cubic (Hermite spline, \rightarrow later)
 - Hash function to map vertices to values
 - Randomized look up
 - Virtually infinite extent and variation with finite array of values

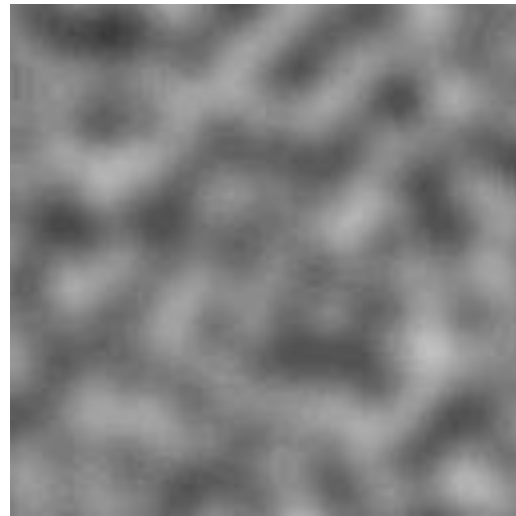


Noise vs. Noise

- **Value Noise vs. Gradient Noise**
 - Gradient noise has lower regularity artifacts
 - More high frequencies in noise spectrum
- **Random Values vs. Perlin Noise**
 - Stochastic vs. deterministic



Random values
at each pixel



Gradient noise

Turbulence Function

- **Noise Function**

- Single spike in frequency spectrum (single frequency, see later)

- **Natural Textures**

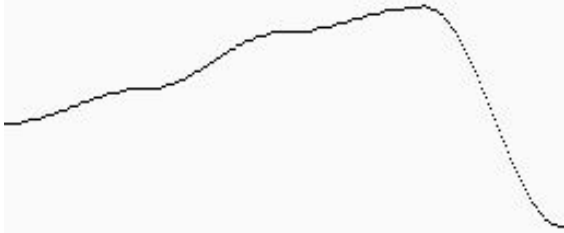
- Mix of different frequencies
- Decreasing amplitude for high frequencies

- **Turbulence from Noise**

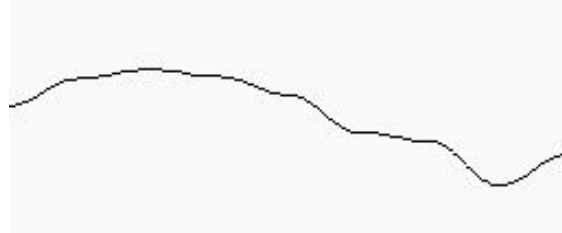
- $Turbulence(x) = \sum_{i=0}^k |a_i * noise(f_i x)|$
 - Frequency: $f_i = 2^i$
 - Amplitude: $a_i = 1 / p^i$
 - Persistence: p typically $p=2$
 - Power spectrum : $a_i = 1 / f_i$
 - Brownian motion: $a_i = 1 / f_i^2$
 - Summation truncation
 - 1st term: $noise(x)$
 - 2nd term: $noise(2x)/2$
 - ...
 - Until period $(1/f_k) < 2$ pixel-size (band limit, see later)
-

Synthesis of Turbulence (1-D)

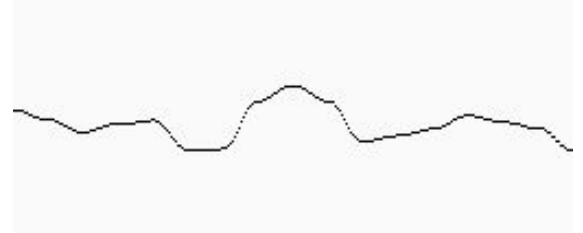
Amplitude : 128
frequency : 4



Amplitude : 64
frequency : 8



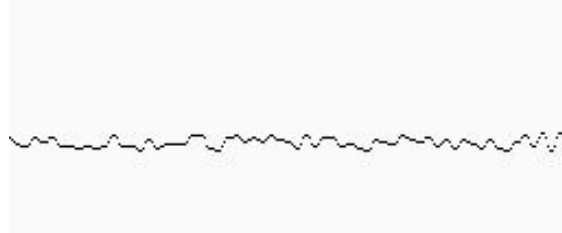
Amplitude : 32
frequency : 16



Amplitude : 16
frequency : 32



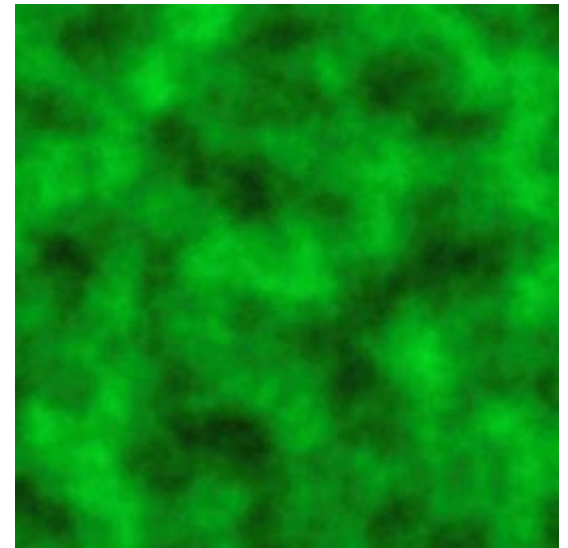
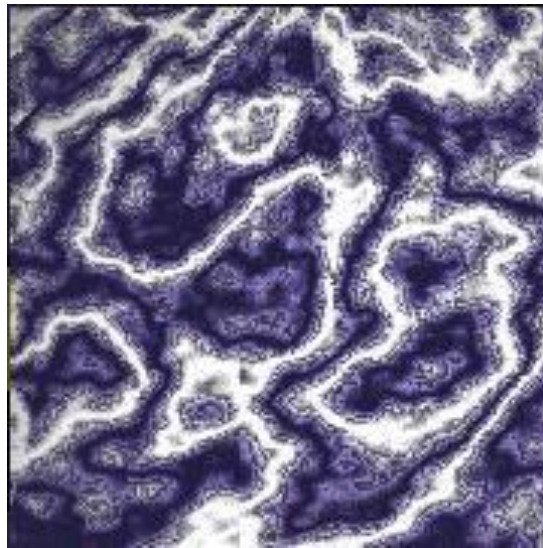
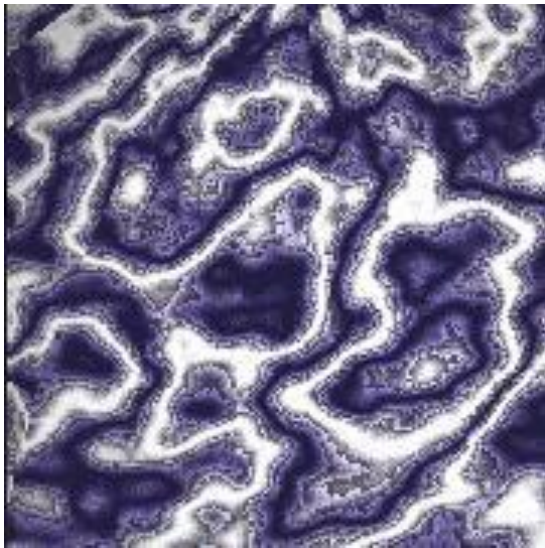
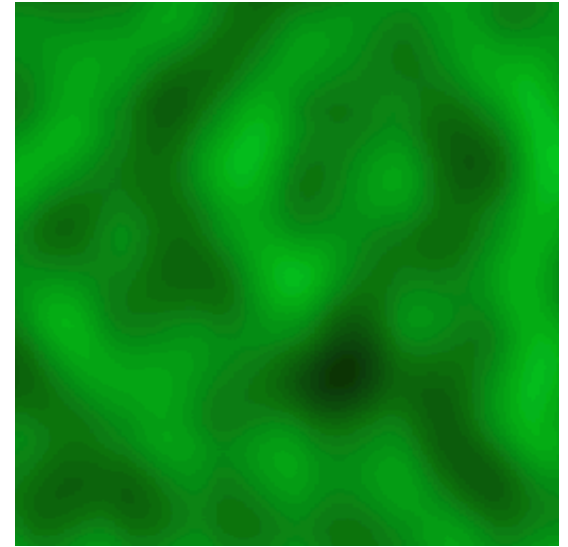
Amplitude : 8
frequency : 64



Sum of Noise Functions = (Perlin Noise)



Synthesis of Turbulence (2-D)



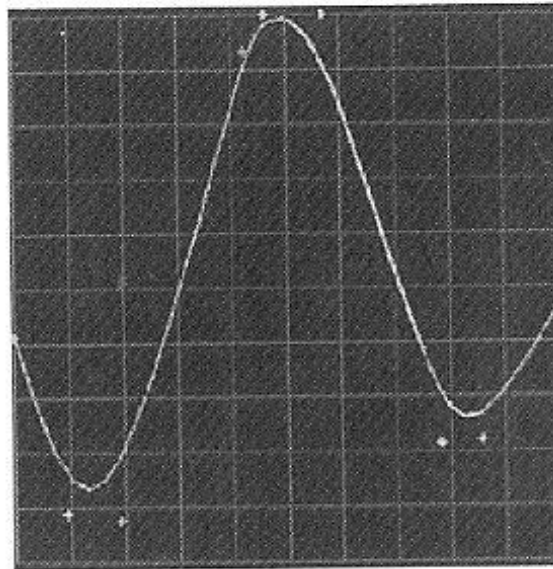
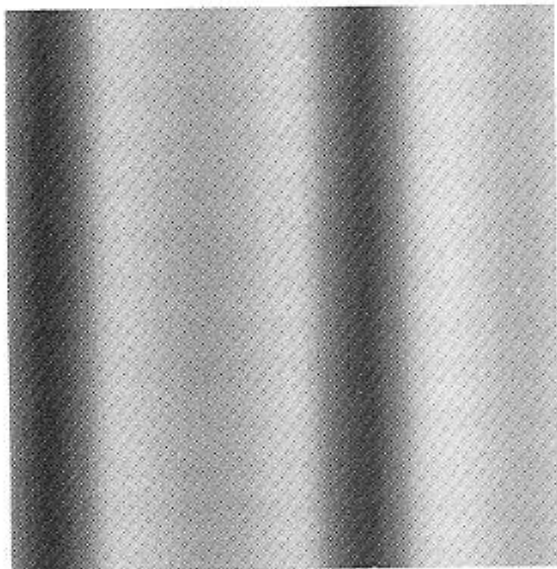
Example: Marble

- **Overall Structure**

- Smoothly alternating layers of different marble colors
- $f_{\text{marble}}(x,y,z) := \text{marble_color}(\sin(x))$
- marble_color : transfer function (see lower left)

- **Realistic Appearance**

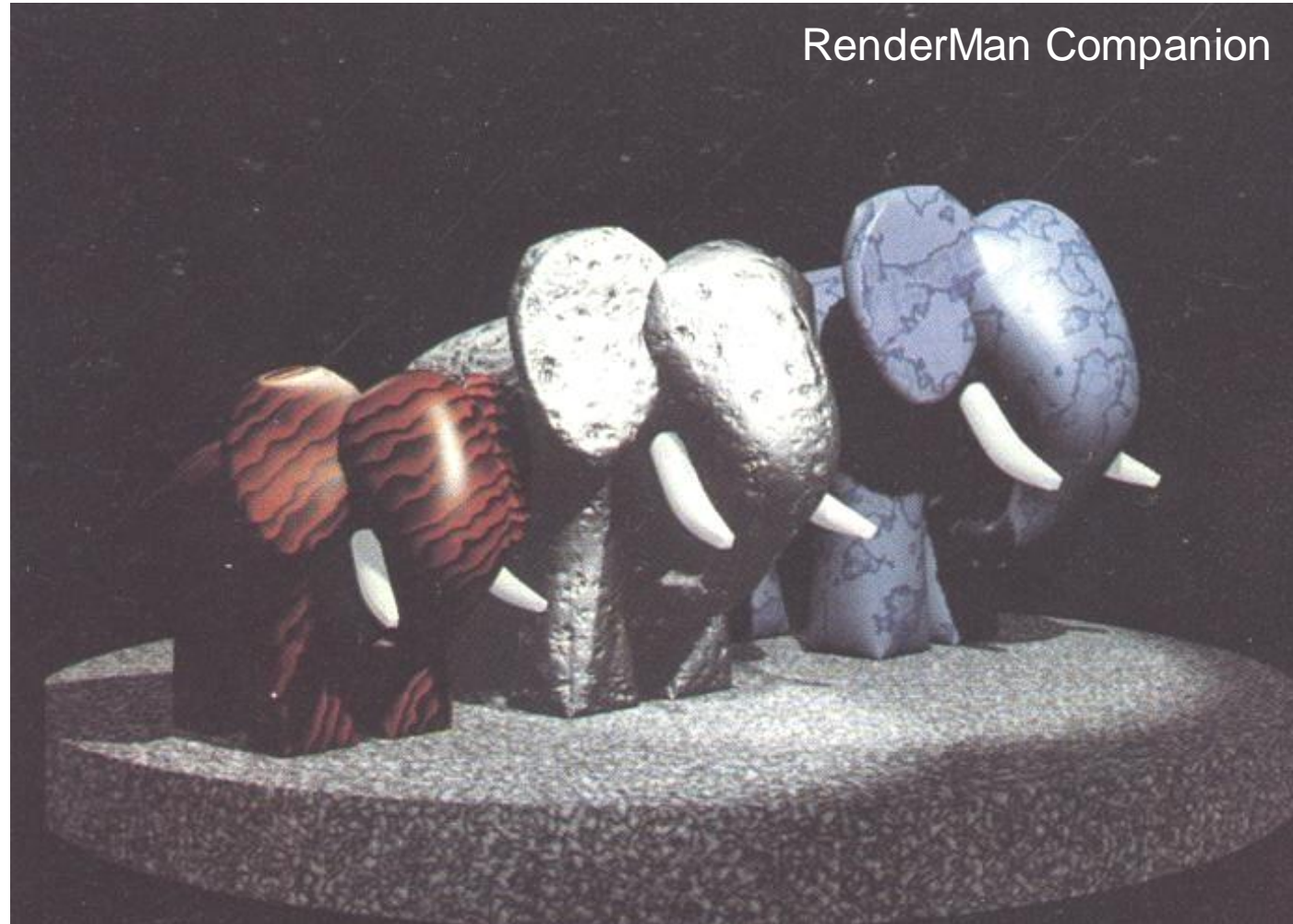
- Simulated turbulence
- $f_{\text{marble}}(x,y,z) := \text{marble_color}(\sin(x + \text{turbulence}(x, y, z)))$



Solid Noise

- **3D Noise Texture**

- Wood
- Erosion
- Marble
- Granite
- ...



Other Applications

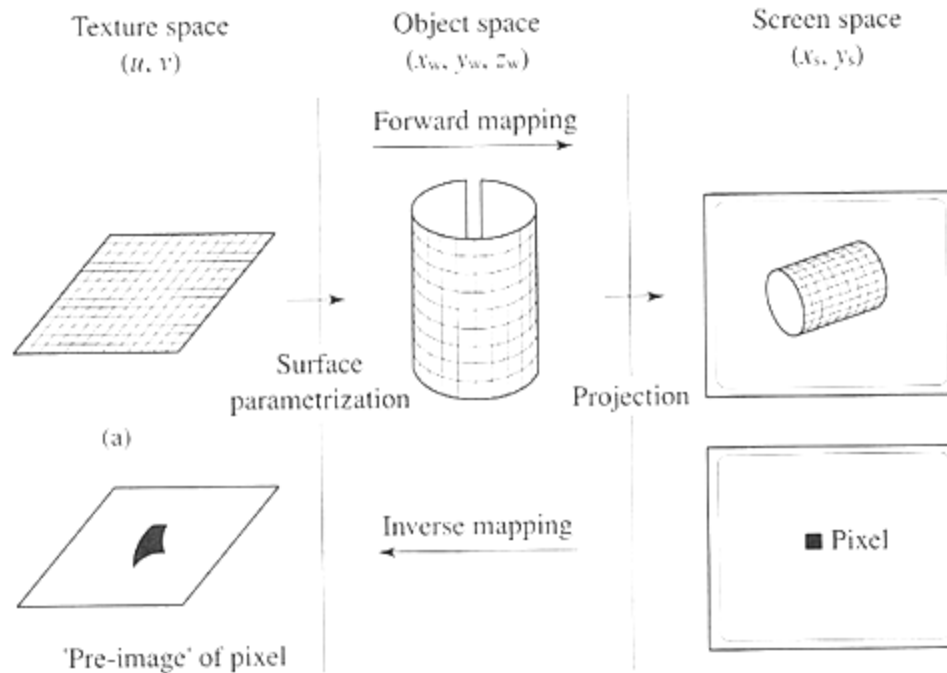
- **Bark**
 - Turbulated saw-tooth function
 - **Clouds**
 - White blobs
 - Turbulated transparency along edge
 - **Animation**
 - Vary procedural texture function's parameters over time
-

Shading Languages

- **Small program fragments (plugins)**
 - Compute certain aspects of the rendering process
 - Executing at innermost loop, must be extremely efficient
 - Executed at each intersection
- **Typical shaders**
 - Material/surface shaders: Compute reflected color
 - Light shaders: Compute illumination from light source at some point
 - Volume shader: Compute interaction in participating medium
 - Displacement shader: Compute changes to the geometry
 - Camera shader: Compute rays for each pixel
- **Shading languages**
 - RenderMan (the mother of all shading languages)
 - HLSL (DX only), GLSL (OpenGL only), CG (Nvidia only)
 - OSL (Modern approach)
 - Currently no portable shading format usable for exchange
 - But Material Definition Language (MDL, Nvidia), shade.js (UdS)
- **More details later**

TEXTURE MAPPING

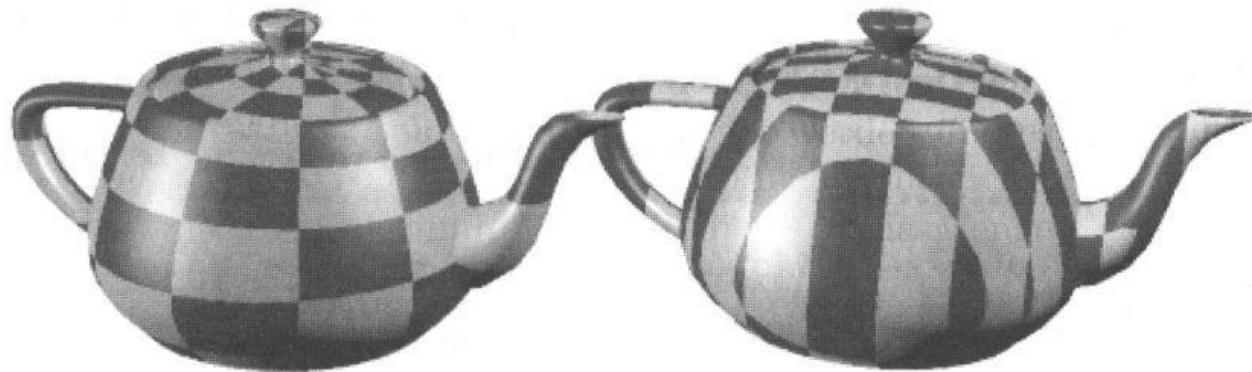
2D Texture Mapping



- **Forward mapping**
 - Object surface parameterization
 - Projective transformation
- **Inverse mapping**
 - Find corresponding pre-image/footprint of each pixel in texture
 - Integrate over pre-image

Surface Parameterization

- To apply textures we need 2D coordinates on surfaces
→ **Parameterization**
- **Some objects have a natural parameterization**
 - Sphere: spherical coordinates $(\varphi, \theta) = (2\pi u, \pi v)$
 - Cylinder: cylindrical coordinates $(\varphi, h) = (2\pi u, H v)$
 - Parametric surfaces (such as B-spline or Bezier surfaces → later)
- **Parameterization is less obvious for**
 - Polygons, implicit surfaces, teapots, ...



Triangle Parameterization

- **Triangle is a planar object**
 - Has implicit parameterization (e.g., barycentric coordinates)
 - But we need more control: Placement of triangle in texture space
- **Assign texture coordinates (u,v) to each vertex (x_o, y_o, z_o)**
- **Apply viewing projection $(x_o, y_o, z_o) \rightarrow (x,y)$ (details later)**
- **Yields full texture transformation (warping) $(u,v) \rightarrow (x,y)$**

$$x = \frac{au + bv + c}{gu + hv + i} \qquad y = \frac{du + ev + f}{gu + hv + i}$$

- In homogeneous coordinates (by embedding (u,v) as $(u,v,1)$)

$$\begin{bmatrix} x' \\ y' \\ w \end{bmatrix} = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} \begin{bmatrix} u' \\ v' \\ q \end{bmatrix}; (x, y) = \left(\frac{x'}{w}, \frac{y'}{w} \right), (u, v) = \left(\frac{u'}{q}, \frac{v'}{q} \right)$$

- Transformation coefficients determined by 3 pairs $(u,v) \rightarrow (x,y)$
 - Three linear equations
 - Invertible if neither set of points is collinear

Triangle Parameterization (2)

- **Given**
$$\begin{bmatrix} x' \\ y' \\ w \end{bmatrix} = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} \begin{bmatrix} u' \\ v' \\ q \end{bmatrix}$$

- **The inverse transform $(x,y) \rightarrow (u,v)$ is**

$$\begin{bmatrix} u' \\ v' \\ q \end{bmatrix} = \begin{bmatrix} ei - fh & ch - bi & bf - ce \\ fg - di & ai - cg & cd - af \\ dh - eg & bg - ah & ae - bd \end{bmatrix} \begin{bmatrix} x' \\ y' \\ w \end{bmatrix}$$

- **Coefficients must be calculated for each triangle**
 - Rasterization
 - Incremental bilinear update of (u',v',q) in screen space
 - Using the partial derivatives of the linear function (i.e. constants)
 - Ray tracing
 - Evaluated at every intersection (via barycentric coordinates)
- **Often (partial) derivatives are needed as well**
 - Explicitly given in matrix (colored for $\partial u/\partial x$, $\partial v/\partial x$, $\partial q/\partial x$)

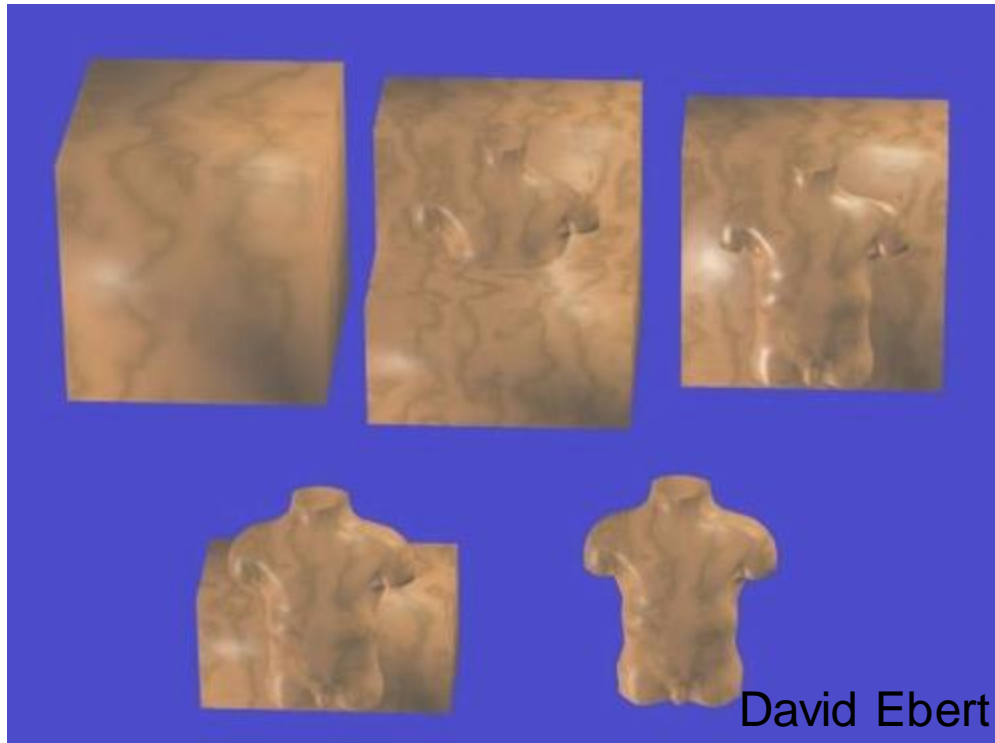
Textures Coordinates

- **Solid Textures**

- 3D world/object (x,y,z) coords \rightarrow 3D (u,v,w) texture coordinates
- Similar to carving object out of material block

- **2D Textures**

- 3D Cartesian (x,y,z) coordinates \rightarrow 2D (u,v) texture coordinates?

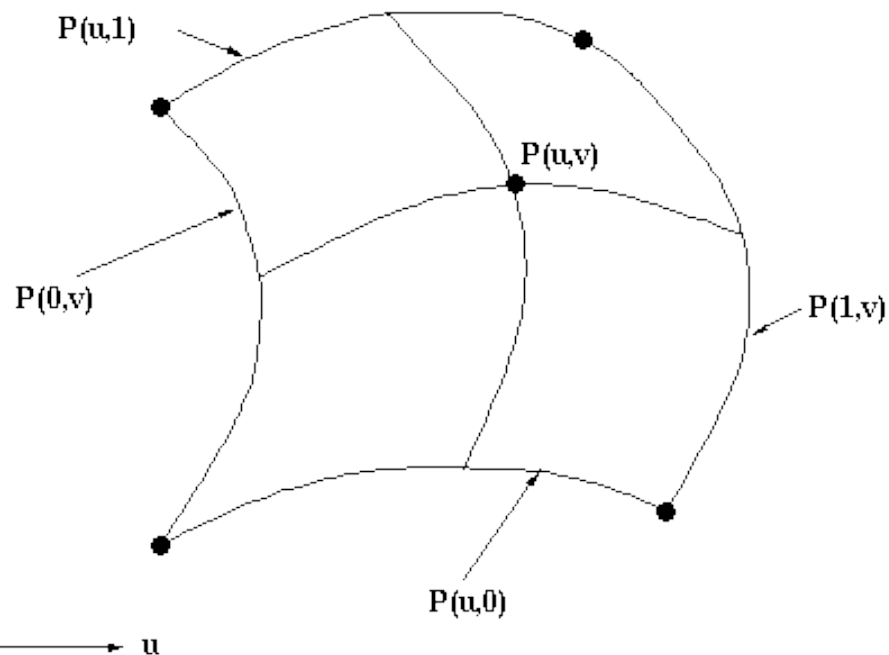


David Ebert

Parametric Surfaces

- **Definition (more detail later)**

- Surface defined by parametric function
 - $(x, y, z) = p(u, v)$
- Input
 - Parametric coordinates: (u, v)
- Output
 - Cartesian coordinates: (x, y, z)



- **Texture Coordinates**

- Directly derived from surface parameterization
- Invert parametric function
 - From world coordinates to parametric coordinates
 - Usually computed implicitly anyway (e.g., in ray tracing)

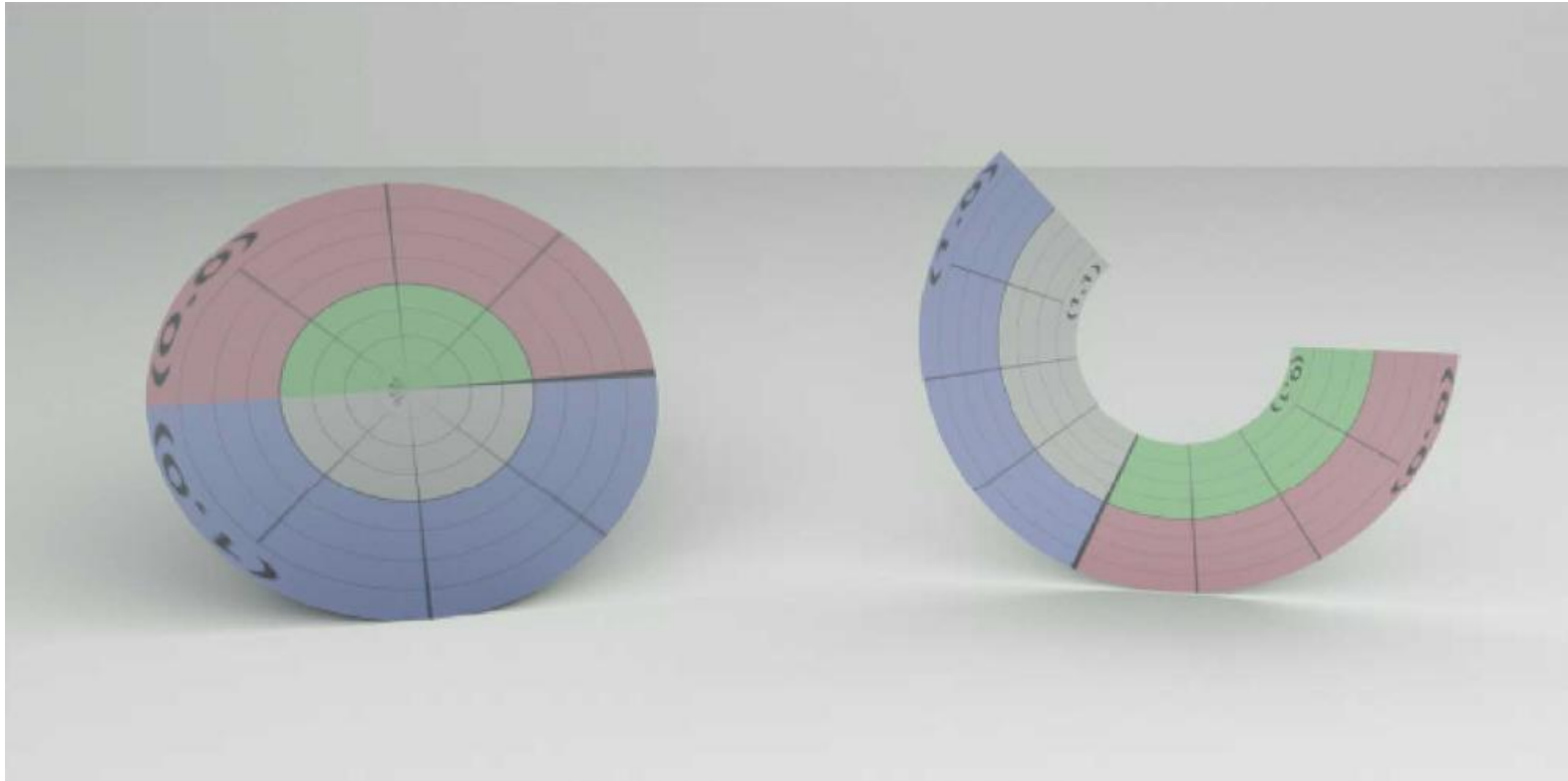
Parametric Surfaces

- **Polar Coordinates**

- $(x, y, 0) = \text{Polar2Cartesian}(r, \varphi)$

- **Disc**

- $p(u, v) = \text{Polar2Cartesian}(R v, 2 \pi u)$ // disc radius R



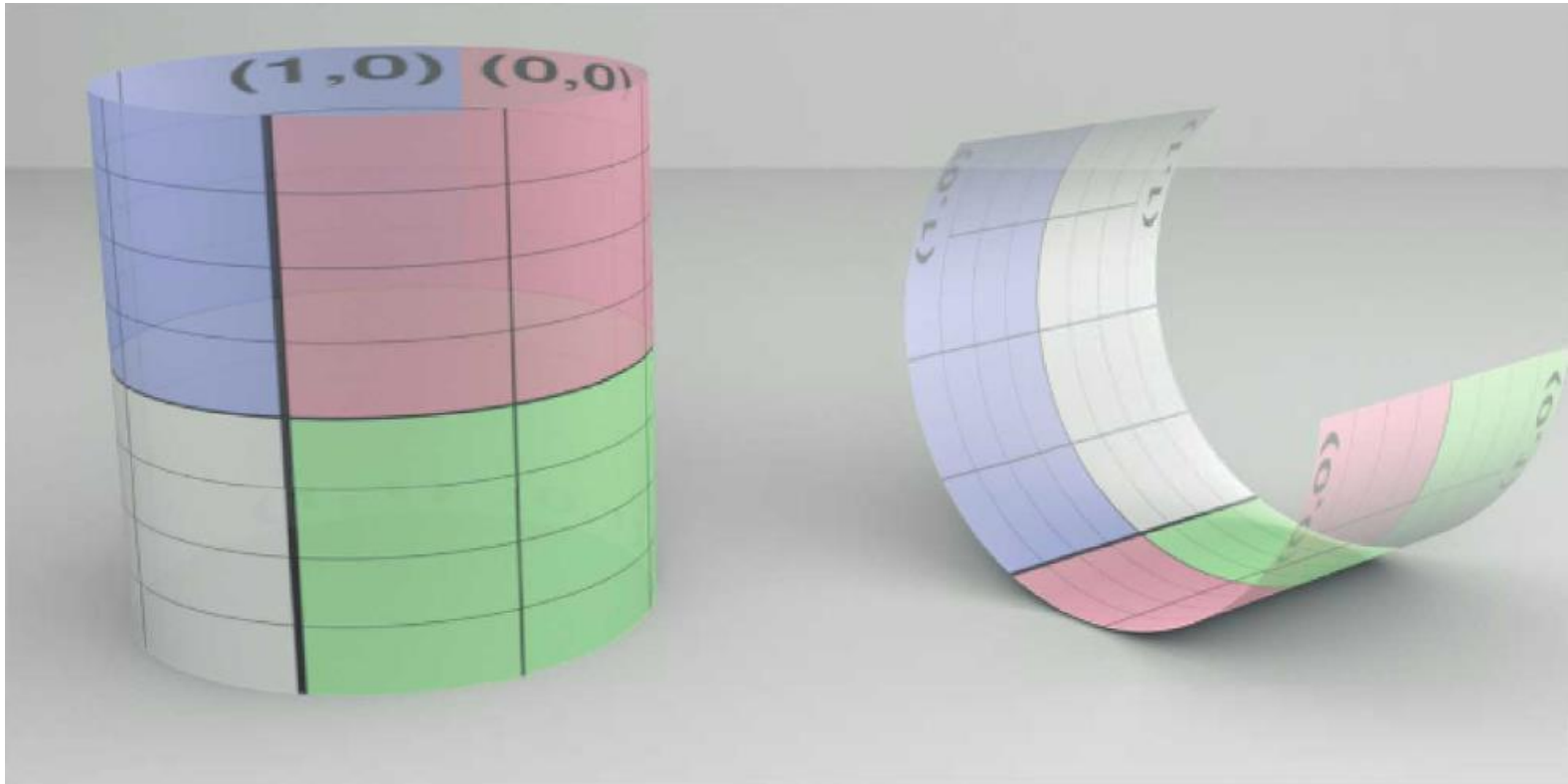
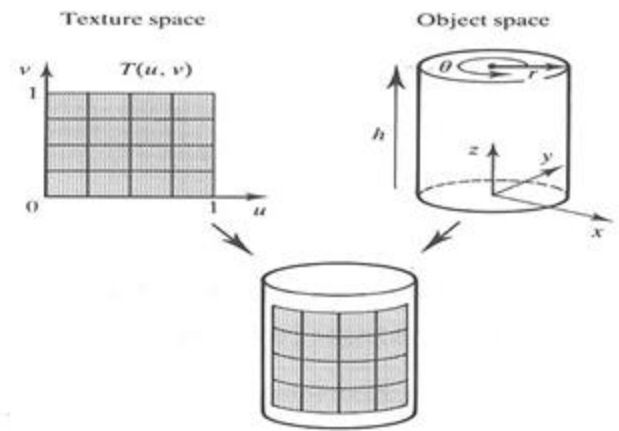
Parametric Surfaces

- **Cylindrical Coordinates**

- $(x, y, z) = \text{Cylindrical2Cartesian}(r, \phi, z)$

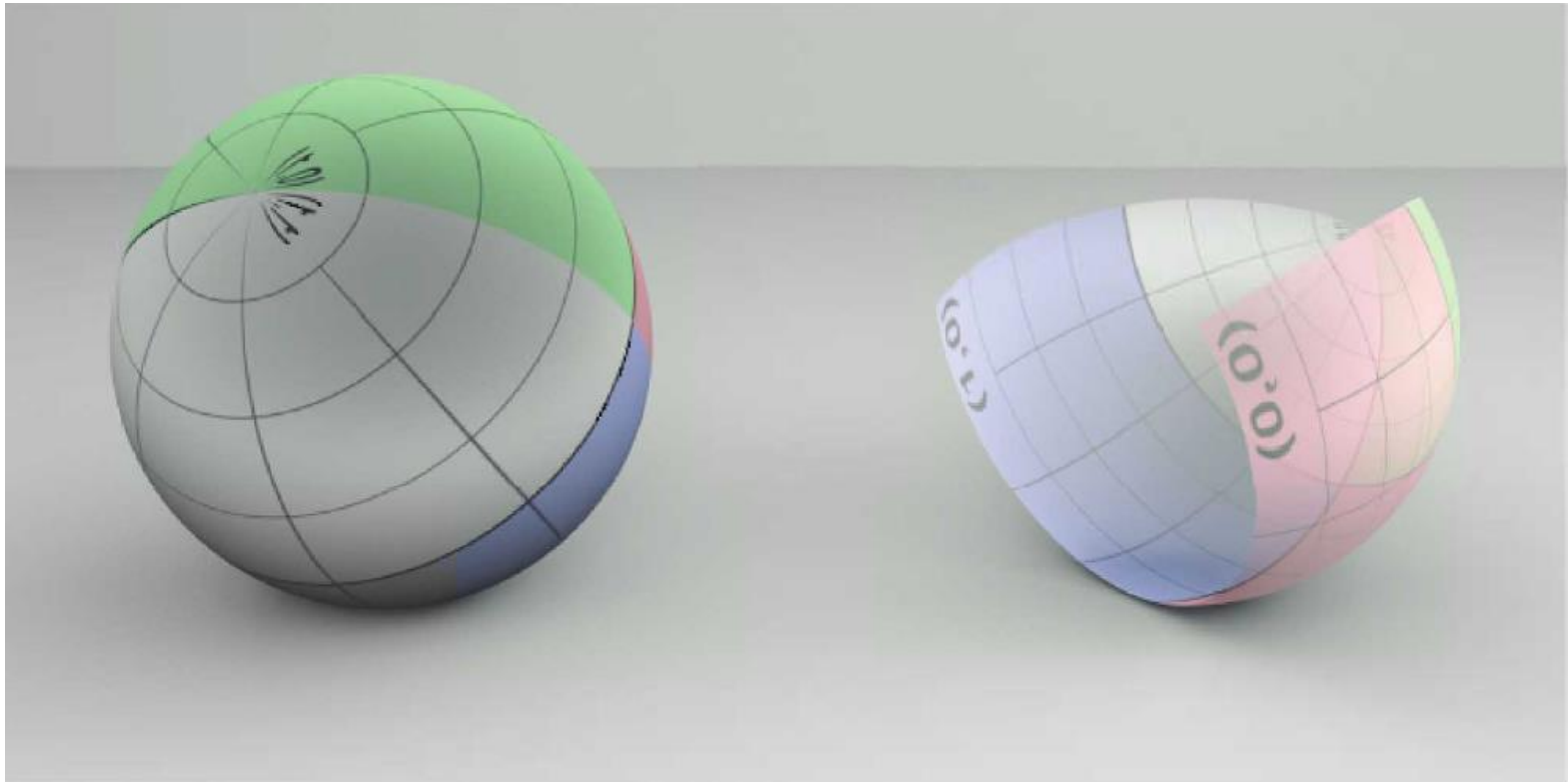
- **Cylinder**

- $p(u, v) = \text{Cylindrical2Cartesian}(r, 2\pi u, H v)$ // cylinder height H



Parametric Surfaces

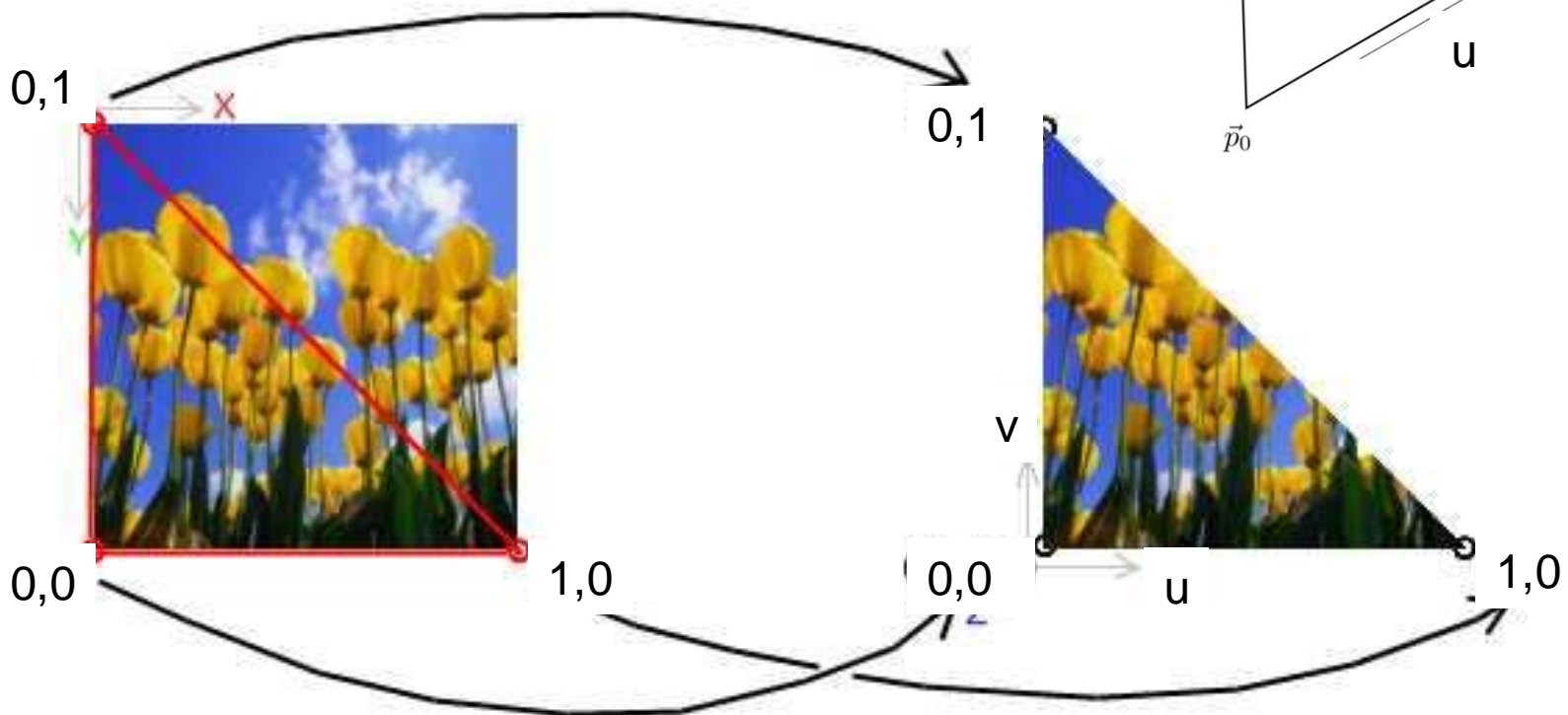
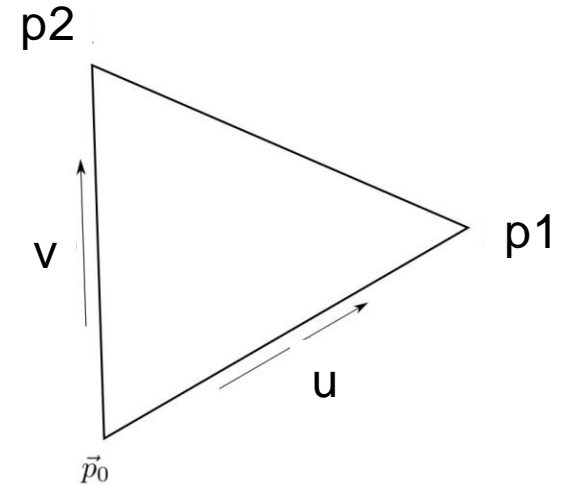
- **Spherical Coordinates**
 - $(x, y, z) = \text{Spherical2Cartesian}(r, \theta, \varphi)$
- **Sphere**
 - $p(u, v) = \text{Spherical2Cartesian}(r, \pi v, 2 \pi u)$



Parametric Surfaces

- **Triangle**

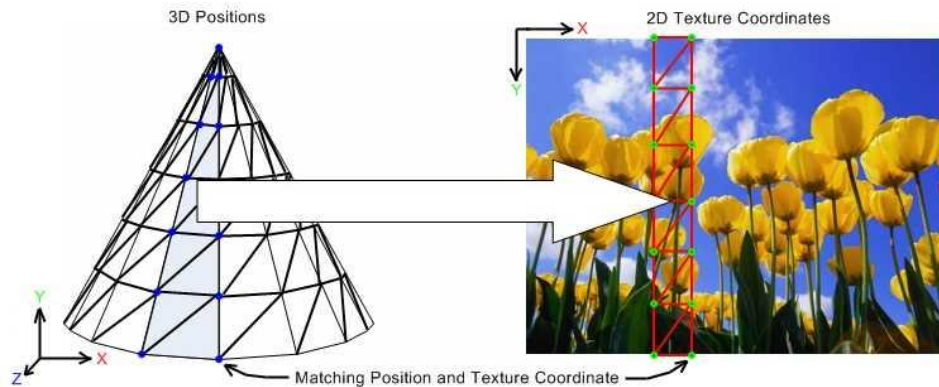
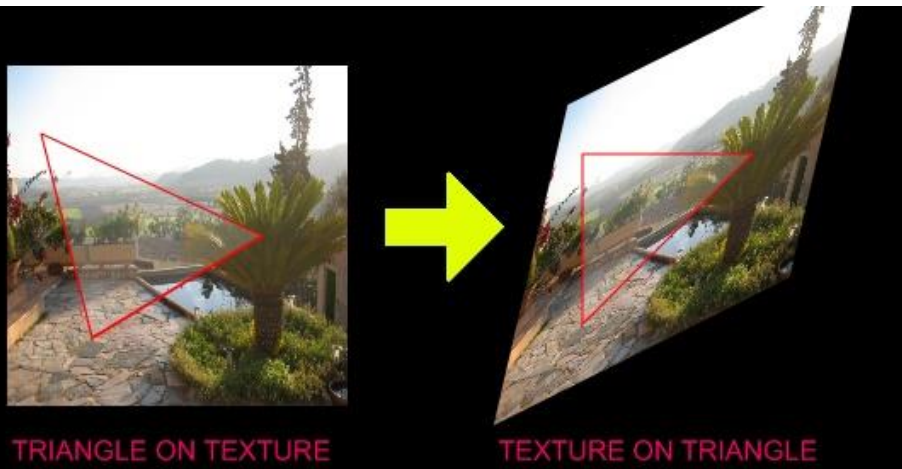
- Use barycentric coordinates directly
- $p(u, v) = (1 - u - v)p_0 + up_1 + v p_2$



Parametric Surfaces

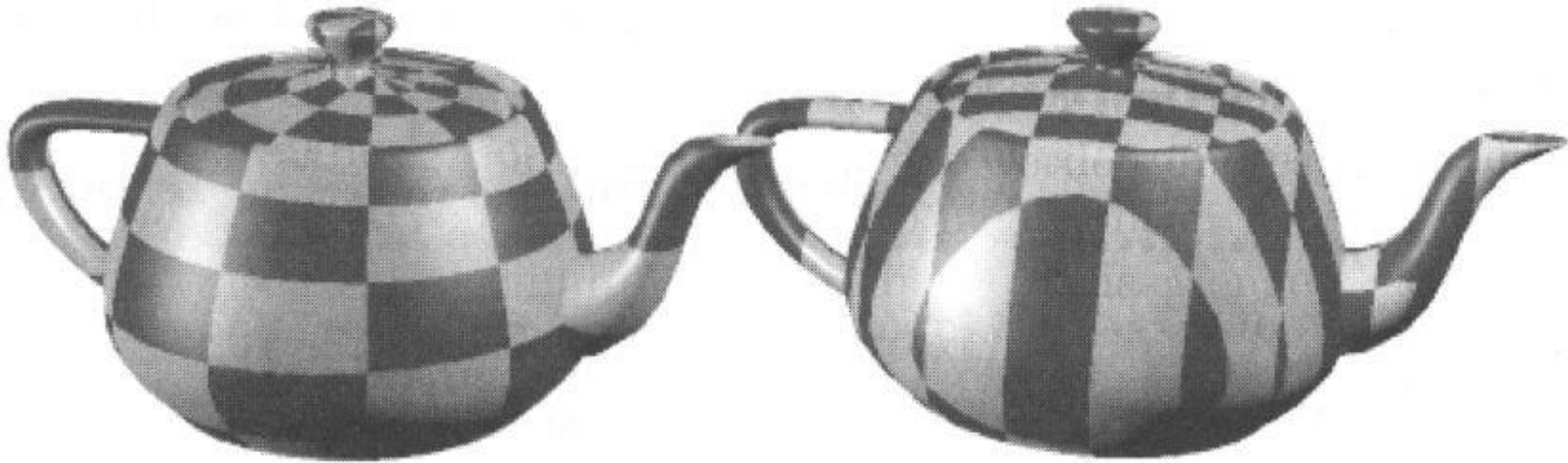
- **Triangle Mesh**

- Associate a predefined texture coordinate to each triangle vertex
 - Interpolate texture coordinates using barycentric coordinates
 - $u = \lambda_0 p_{0u} + \lambda_1 p_{1u} + \lambda_2 p_{2u}$
 - $v = \lambda_0 p_{0v} + \lambda_1 p_{1v} + \lambda_2 p_{2v}$
- Texture mapped onto manifold
 - Single texture shared by many triangles



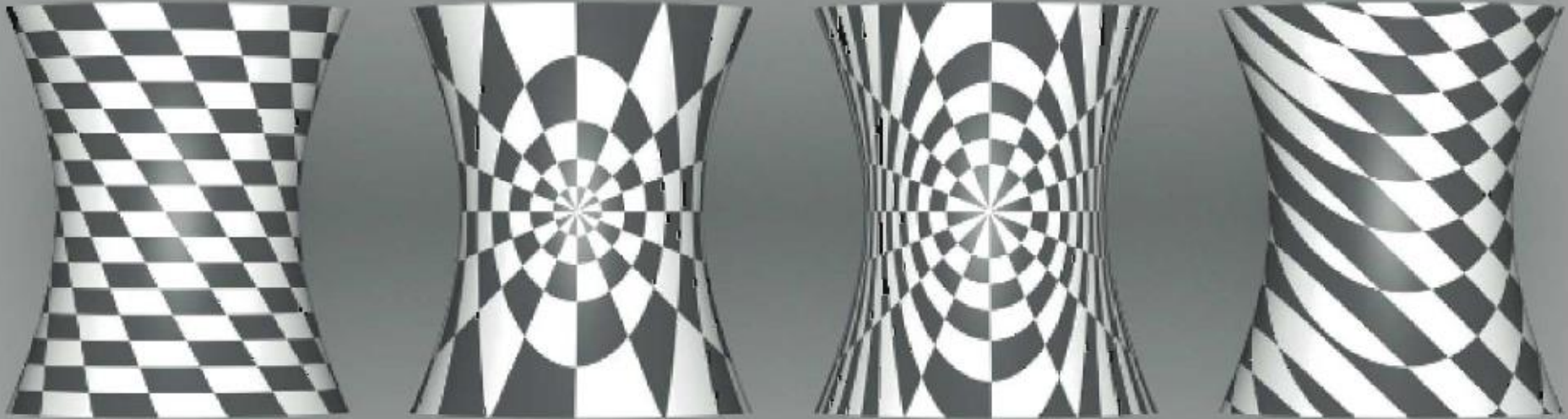
Surface Parameterization

- **Other Surfaces**
 - No intrinsic parameterization??



Intermediate Mapping

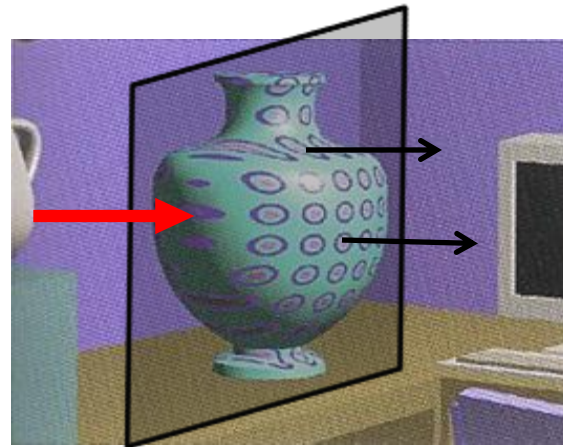
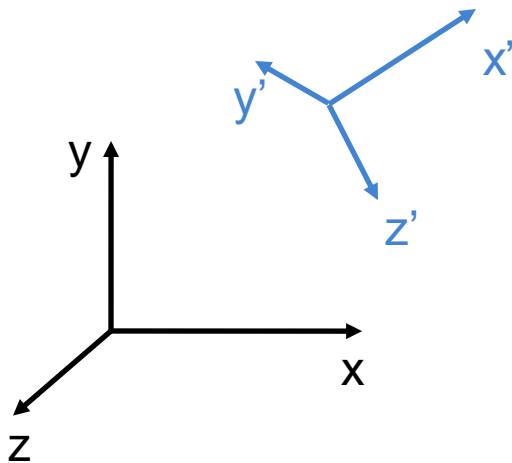
- **Coordinate System Transform**
 - Express Cartesian coordinates into a given coordinate system
- **3D to 2D Projection**
 - Drop one coordinate
 - Compute u and v from remaining 2 coordinates



Intermediate Mapping

- **Planar Mapping**

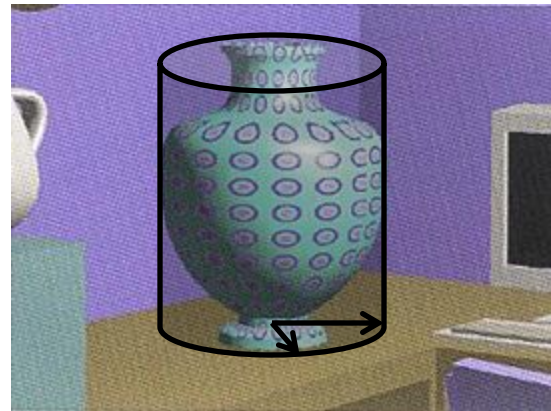
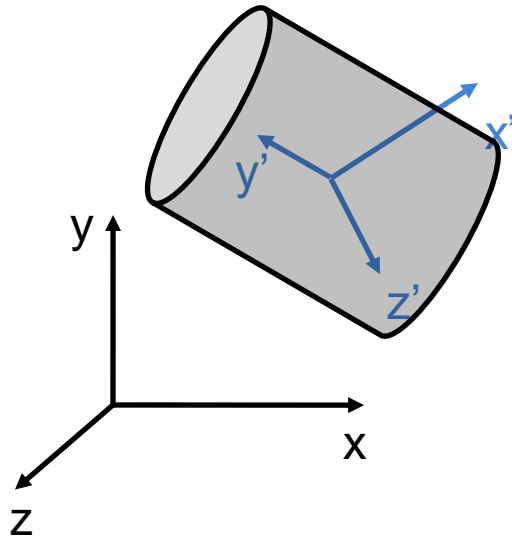
- Map to different Cartesian coordinate system
- $(x', y', z') = \text{AffineTransformation}(x, y, z)$
 - Orthogonal basis: translation + row-vector rotation matrix
 - Non-orthogonal basis: translation + inverse column-vector matrix
- Drop z' , map $u = x'$, map $v = y'$
- E.g.: Issues when surface normal orthogonal to projection axis



Intermediate Mapping

- **Cylindrical Mapping**

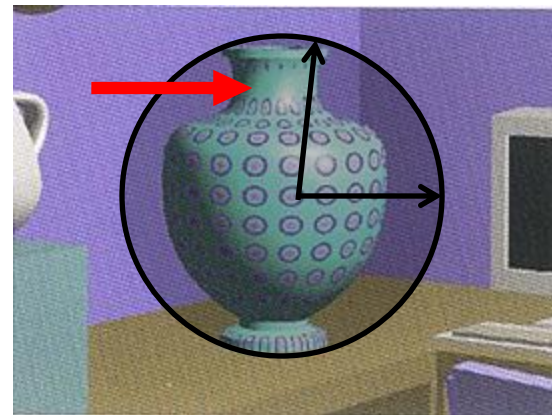
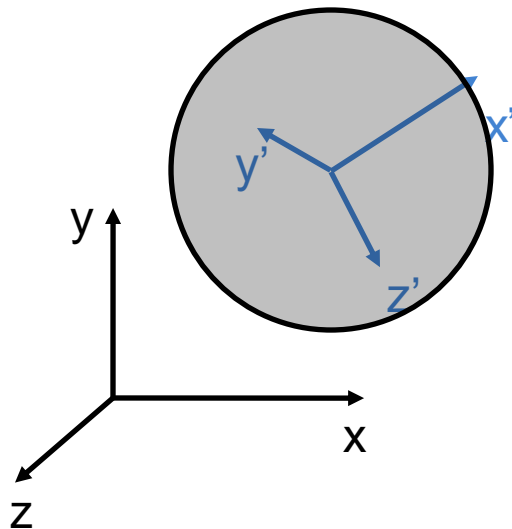
- Map to cylindrical coordinates (possibly after translation/rotation)
- $(r, \phi, z) = \text{Cartesian2Cylindrical}(x, y, z)$
- Drop r , map $u = \phi / 2\pi$, map $v = z / H$
- Extension: add scaling factors: $u = \alpha \phi / 2\pi$
- E.g.: Similar topology gives reasonable mapping



Intermediate Mapping

- **Spherical Mapping**

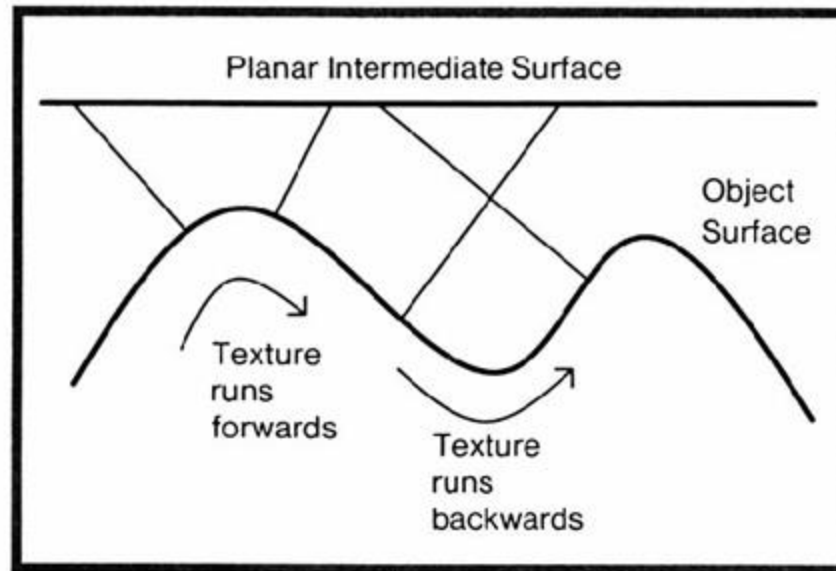
- Map to spherical coordinates (possibly after translation/rotation)
- $(r, \theta, \phi) = \text{Cartesian2Spherical}(x, y, z)$
- Drop r , map $u = \phi / 2\pi$, map $v = \theta / \pi$
- Extension: add scaling factors to both u and v
- E.g.: Issues in concave regions



Two-Stage Mapping: Problems

- **Problems**

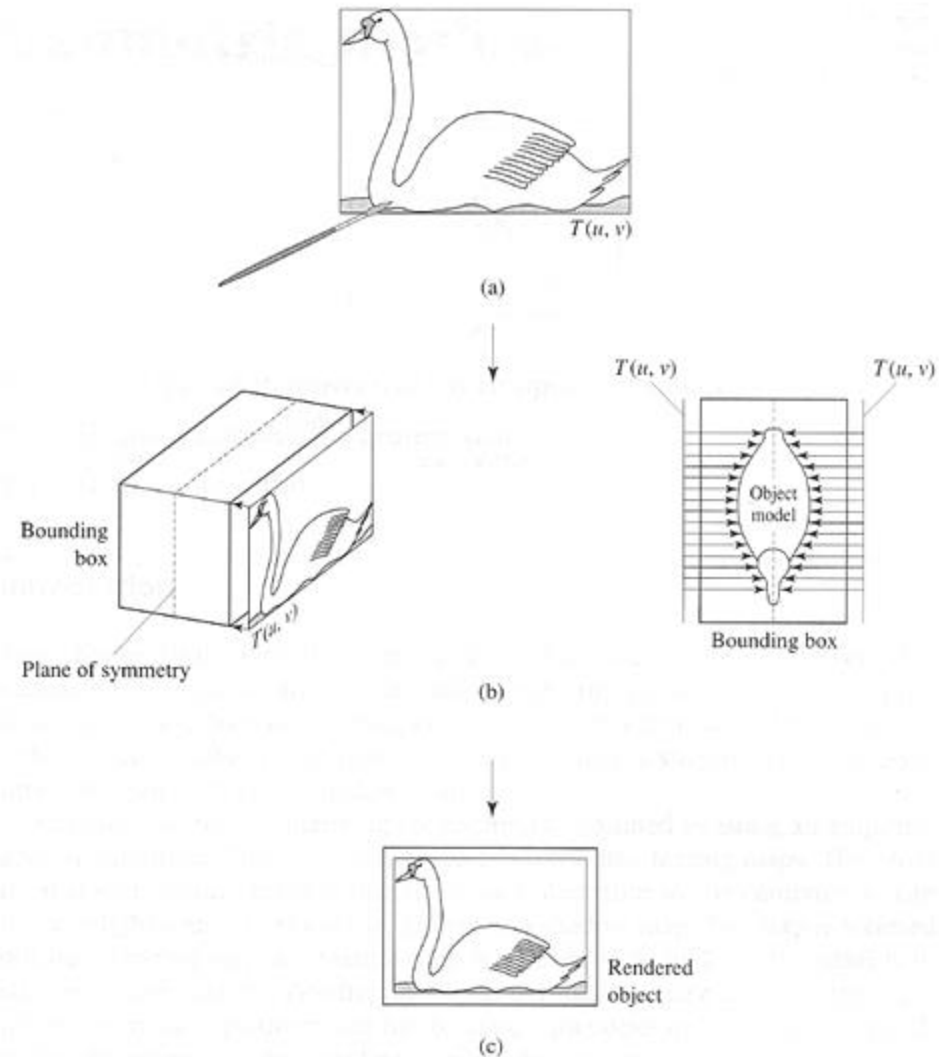
- May introduce undesired texture distortions if the intermediate surface differs too much from the destination surface
- Still often used in practice because of its simplicity



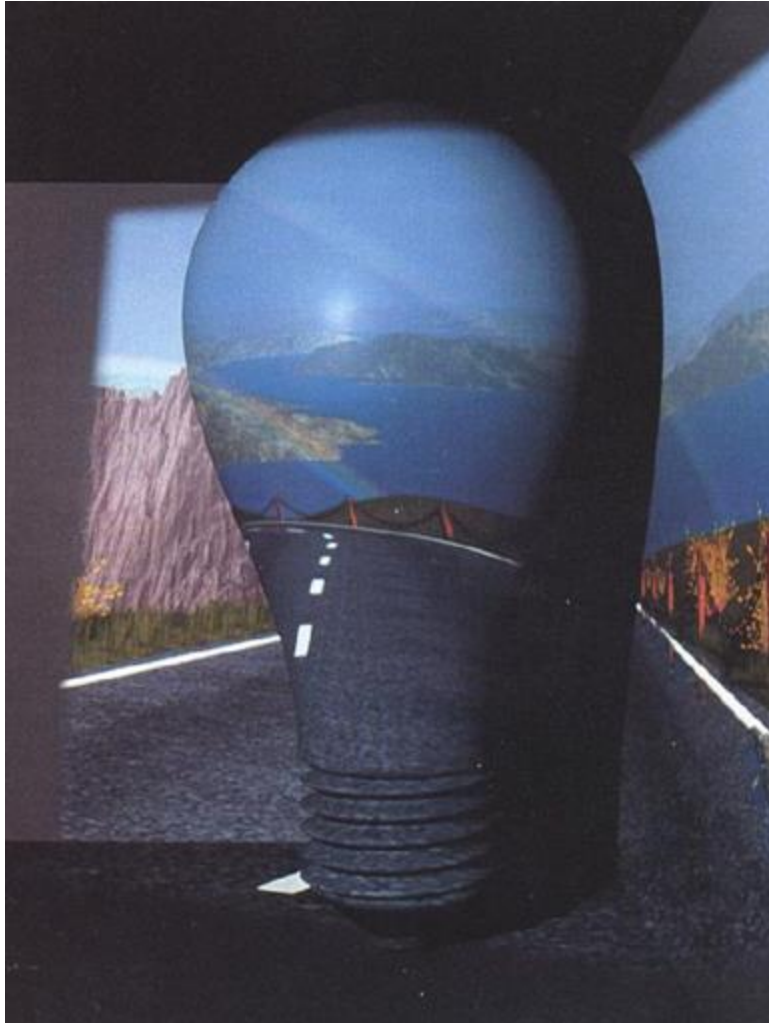
Surface concavities can cause the texture pattern to reverse if the object normal mapping is used.

Projective Textures

- **Project texture onto object surfaces**
 - Slide projector
- **Parallel or perspective projection**
- **Use photographs (or drawings) as textures**
 - Used a lot in film industry!
- **Multiple images**
 - View-dependent texturing (advanced topic)
- **Perspective Mapping**
 - Re-project photo on its 3D environment



Projective Texturing: Examples



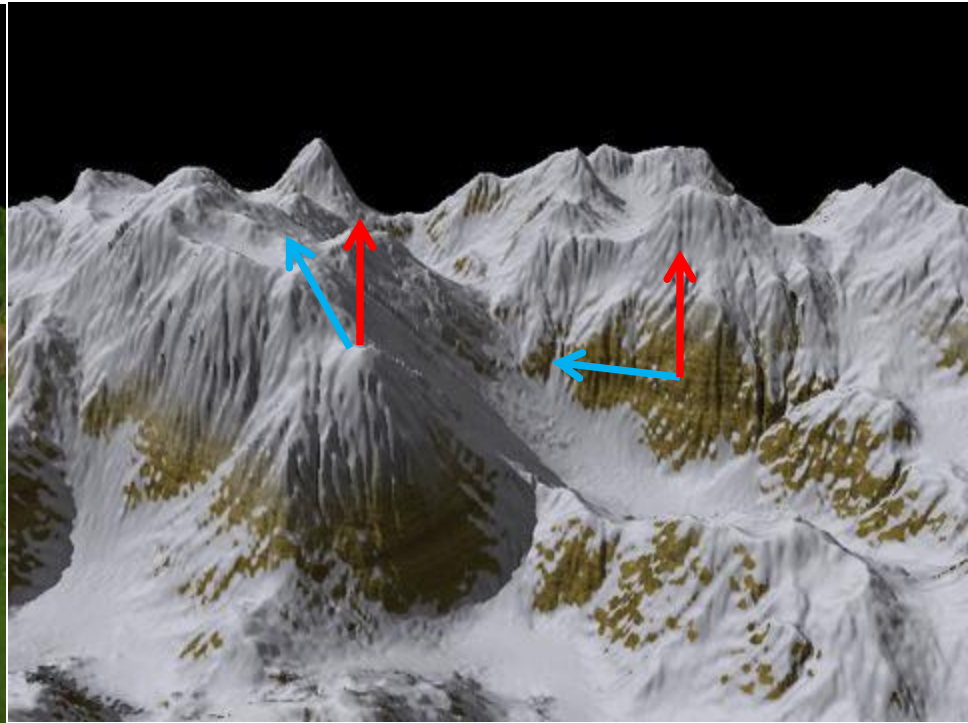
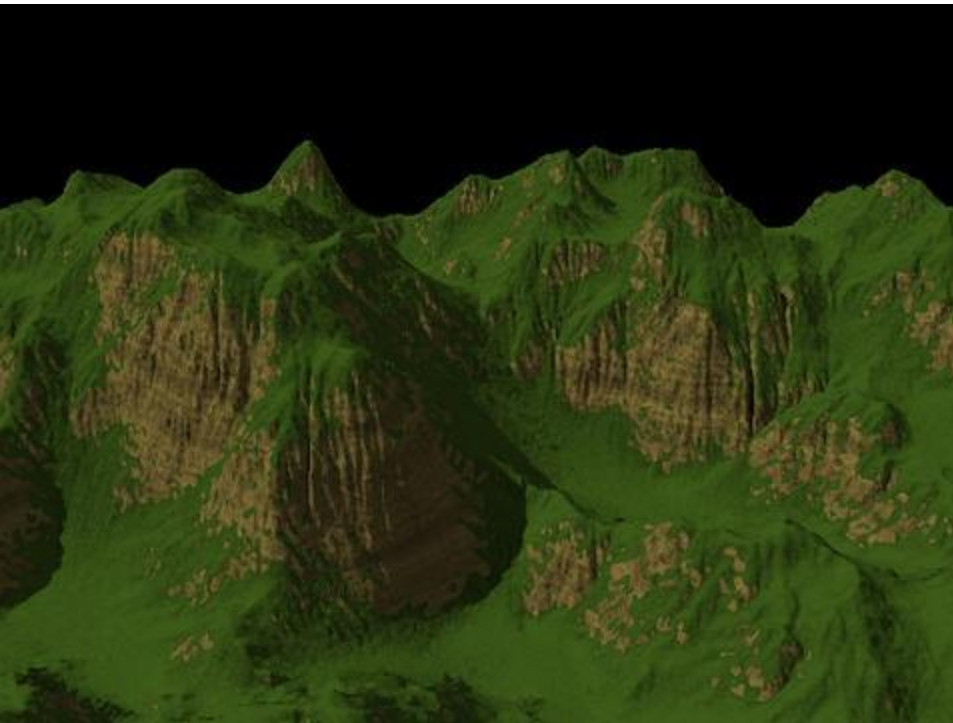
Slope-Based Mapping

- **Definition**

- Depends on **surface normal** and **predefined vector**

- **Example**

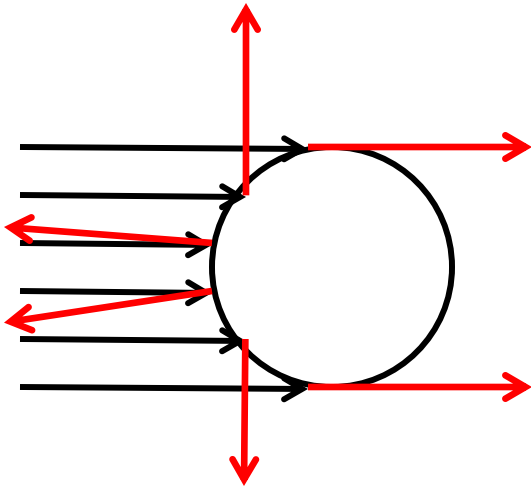
- $\alpha = \mathbf{n} \cdot \mathbf{\omega}$
 - return $\alpha \text{ flatColor} + (1 - \alpha) \text{ slopeColor};$



Environment Map

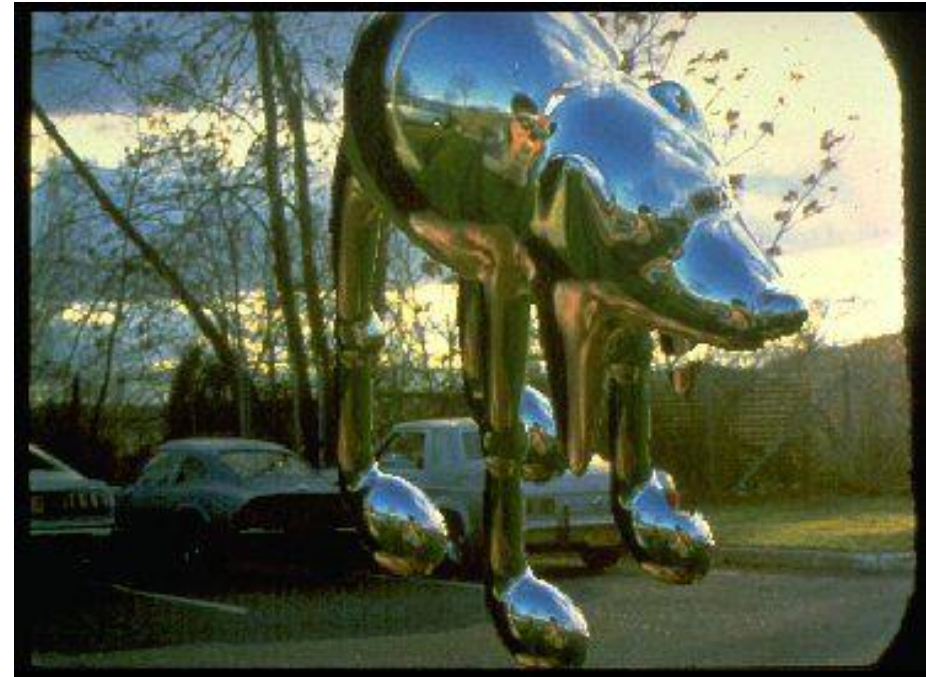
- **Spherical Map**

- Photo of a reflective sphere (gazing ball)
- Photos with a fish-eye camera
 - Only gives hemi-sphere mapping



Environment Map

- **Latitude-Longitude Map**
 - Remapping 2 images of reflective sphere
 - Photo with an environment camera
- **Algorithm**
 - If no intersection found, use ray direction to find background color
 - Cartesian coords of ray dir. \rightarrow spherical coords \rightarrow uv tex coords



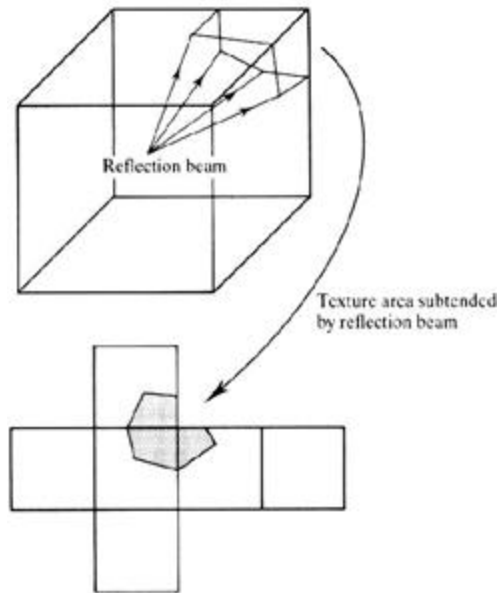
Environment Map

- **Cube Map**

- Remapping 2 images of reflective sphere
- Photos with a perspective camera

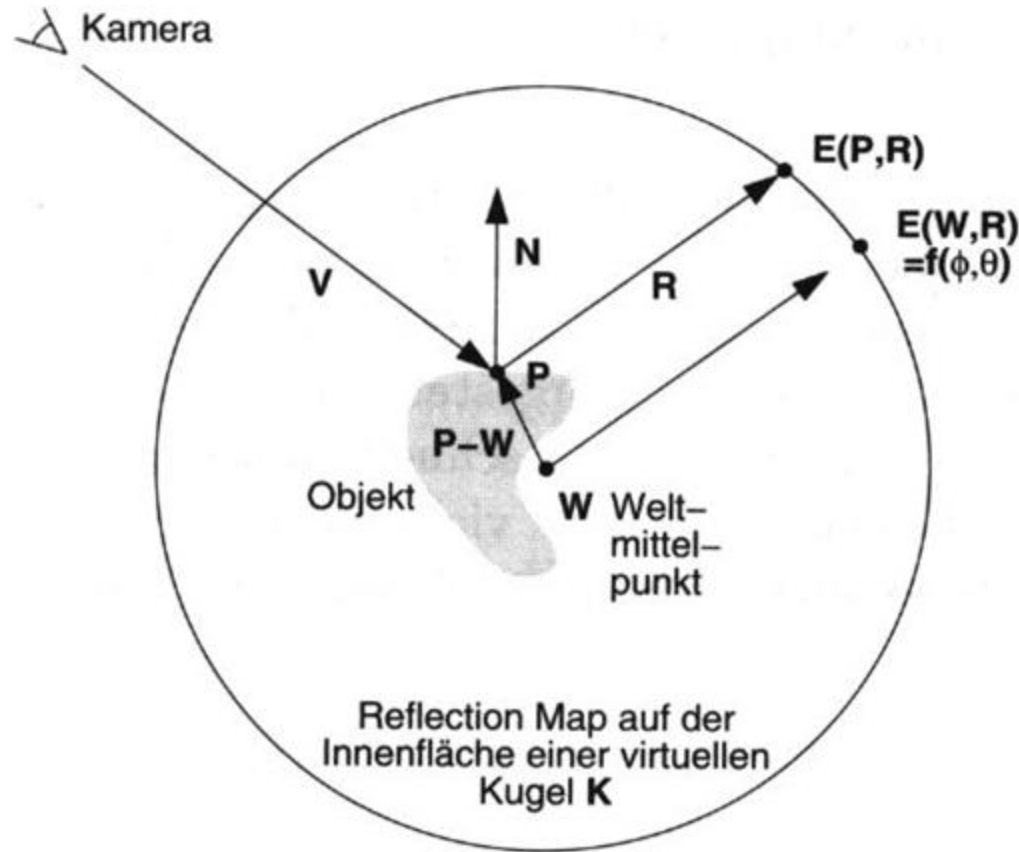
- **Algorithm**

- Find main axis ($-x$, $+x$, $-y$, $+y$, $-z$, $+z$) of ray direction
- Use other 2 coordinates to access corresponding face texture
 - Akin to a 90° projective light



Reflection Map Rendering

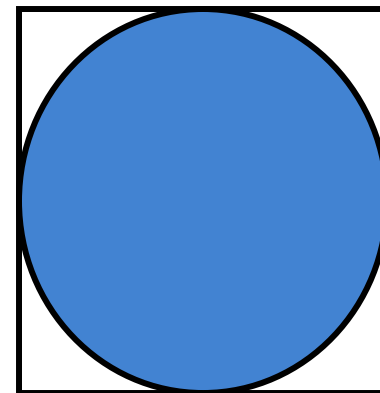
- Spherical parameterization
- O-mapping using reflected view ray intersection



Reflection Map Parameterization

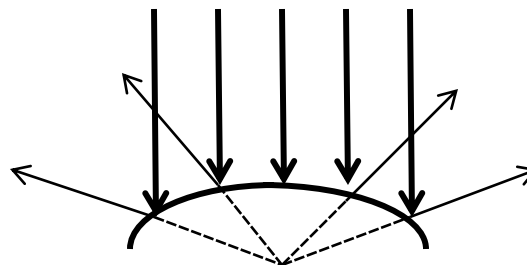
- **Spherical mapping**

- Single image
- Bad utilization of the image area
- Bad scanning on the edge
- Artifacts, if map and image do not have the same viewpoint



- **Double parabolic mapping**

- Yields spherical parameterization
- Subdivide in 2 images (front-facing and back-facing sides)
- Less bias near the periphery
- Arbitrarily reusable
- Supported by OpenGL extensions



Reflection Mapping Example



Terminator II motion picture

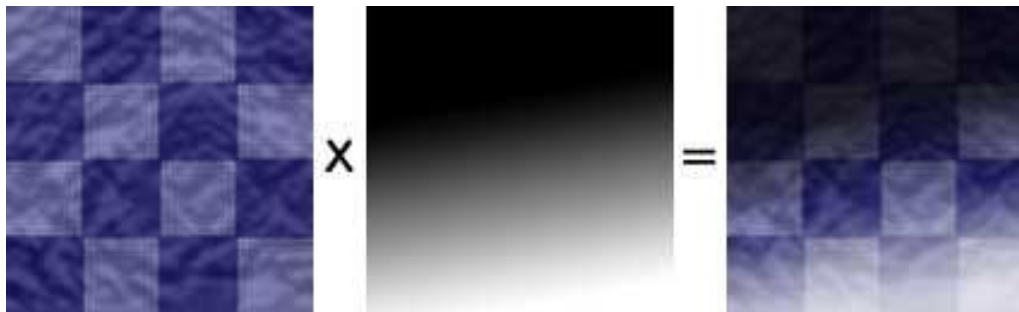
Reflection Mapping Example II

- **Reflection mapping with Phong reflection**
 - Two maps: diffuse & specular
 - Diffuse: index by surface normal
 - Specular: indexed by reflected view vector



Light Maps

- **Light maps (e.g., in Quake)**
 - Pre-calculated illumination (local irradiance)
 - Often very low resolution: smoothly varying
 - Multiplication of irradiance with base texture
 - Diffuse reflectance only
 - Provides surface radiosity
 - View-independent out-going radiance
 - Animated light maps
 - Animated shadows, moving light spots, etc...

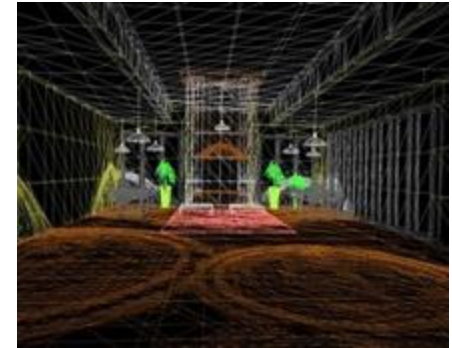


Reflectance

Irradiance

Radiosity

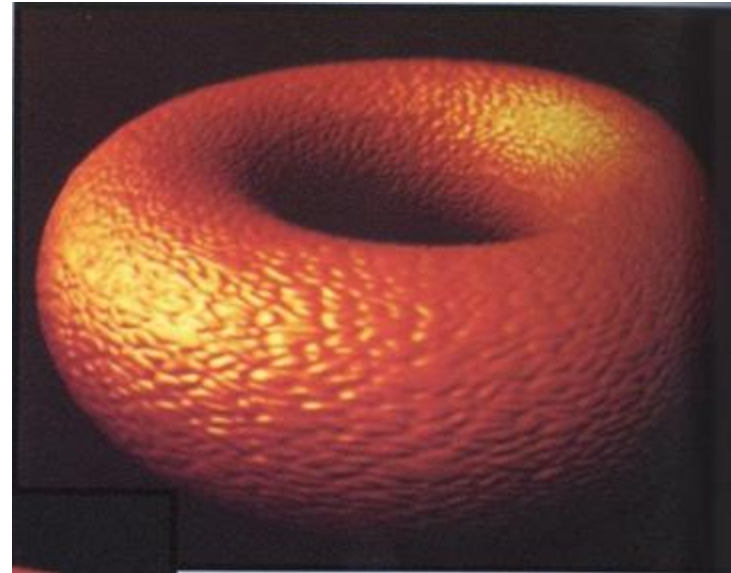
$$B(x) = \rho(x) E(x) = \pi L_o(x)$$



Representing radiosity
in a mesh or texture

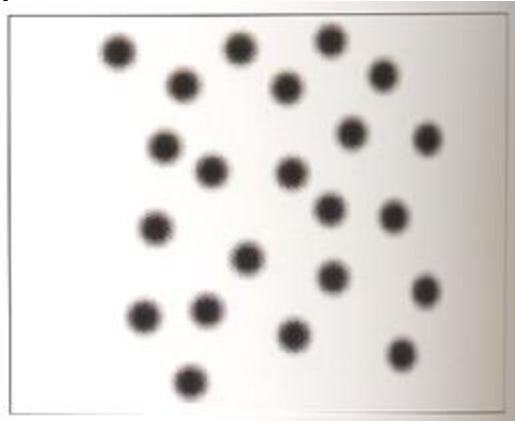
Bump Mapping

- **Modulation of the normal vector**
 - Surface normals changed only
 - Influences shading only
 - No self-shadowing, contour is **not** altered

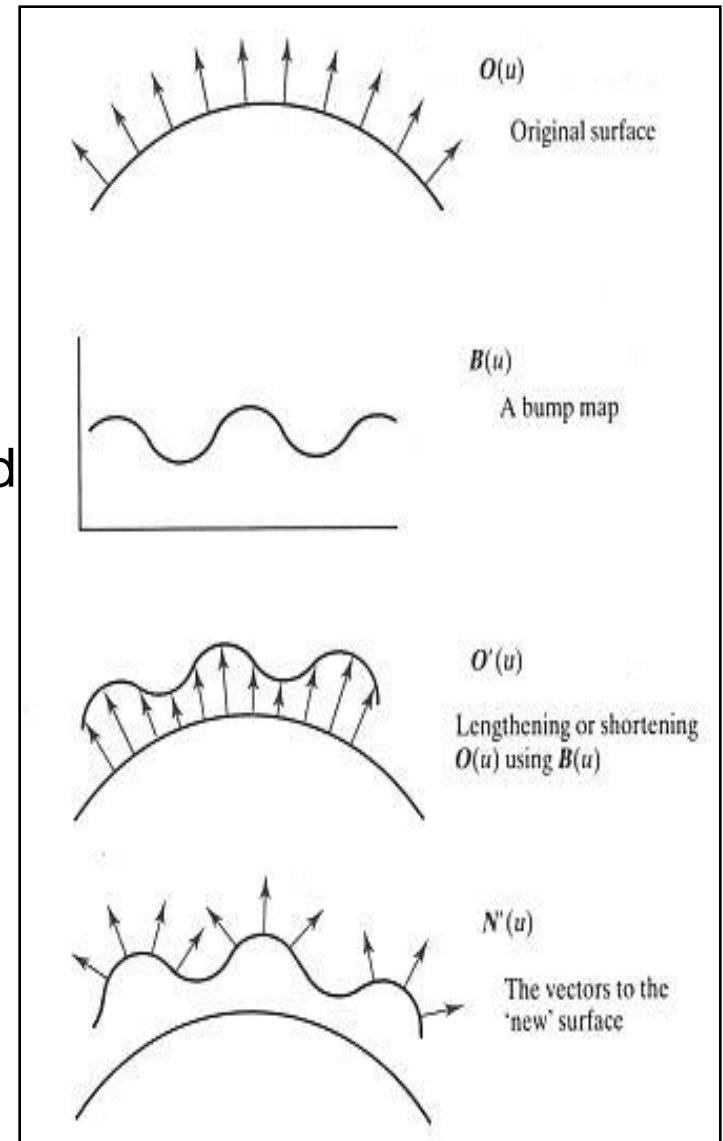


Bump Mapping

- **Original surface:** $O(u, v)$
 - Surface normals are known
- **Bump map:** $B(u, v) \in \mathbb{R}$
 - Surface is offset in normal direction according to bump map intensity
 - New normal directions $N'(u, v)$ are calculated based on virtually displaced surface $O'(u, v)$
 - Original surface is rendered with new normals $N'(u, v)$



Grey-valued texture used for bump height



Bump Mapping

- **Displaced surface:**

$$O'(u, v) = O(u, v) + B(u, v) N(u, v)$$

- **Computing the normal:**

- Normal is cross-product of derivatives:

$$N'(u, v) = O'_u \times O'_v$$

- Where:

$$O'_u = O_u + B_u N + B N_u$$

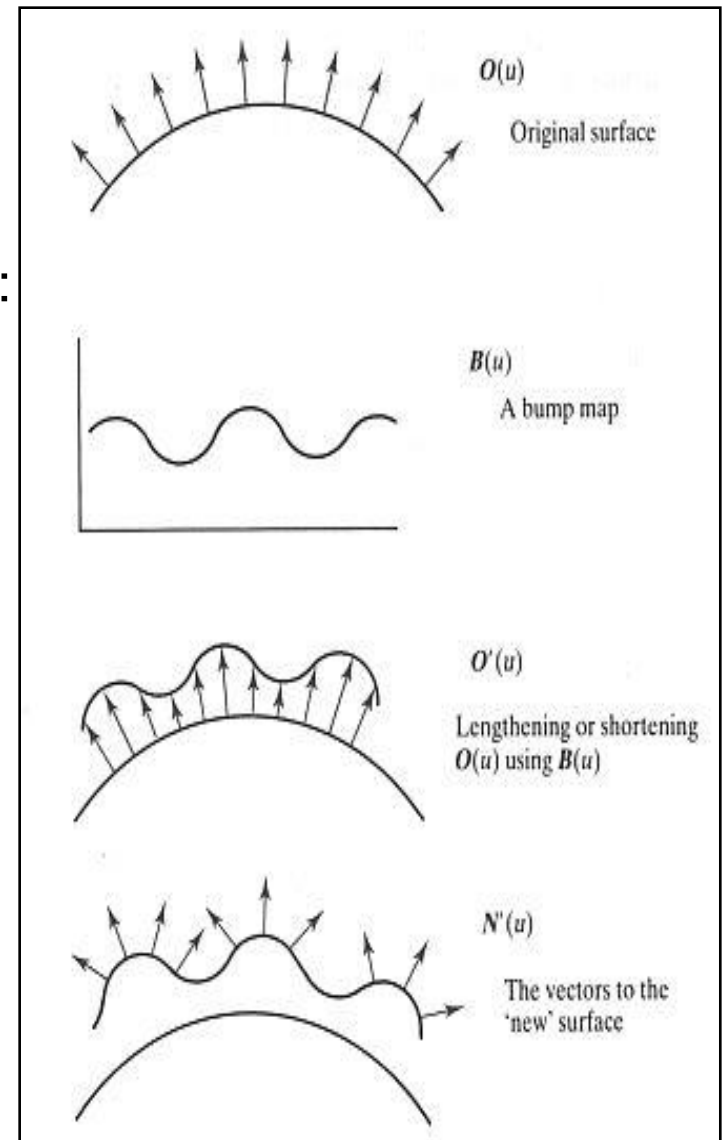
$$O'_v = O_v + B_v N + B N_v$$

- If B is small the **last term** in each equation can be ignored, yielding:

$$\begin{aligned} N'(u, v) &= O_u \times O_v + B_u(N \times O_v) + B_v(O_u \times N) \\ &\quad + B_u B_v(N \times N) \end{aligned}$$

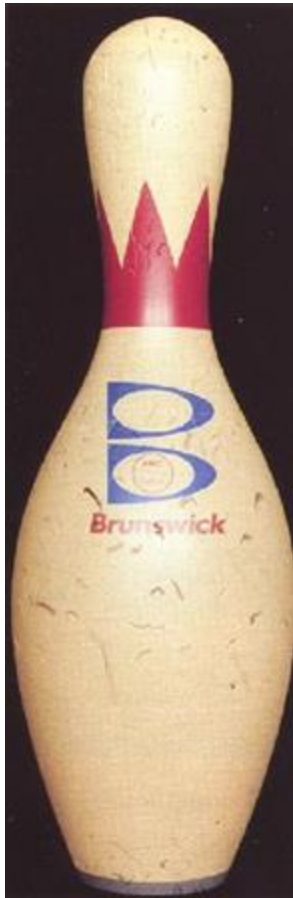
- The first term is the normal to the surface and **the last is zero**, giving:

$$\begin{aligned} D &= B_u(N \times O_v) - B_v(N \times O_u) \\ N' &= N + D \end{aligned}$$



Texture Examples

- **Complex optical effects**
 - Combination of multiple texture effects



RenderMan Companion



Billboards

- **Single textured polygons**
 - Often with opacity texture
 - Rotates, always facing viewer
 - Used for rendering distant objects
 - Best results if approximately radially or spherically symmetric
- **Multiple textured polygons**
 - Azimuthal orientation: different view-points
 - Complex distribution: trunk, branches, ...

