

Computer Graphics

- Material Models -

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Material Samples

- **How do materials reflect light?**
 - At the same point or in neighborhood (subsurface scattering)



COMPOSITES



GLASS



LEATHER



CONCRETE



LIQUID



GRANITE



MESH



METAL



PAINT



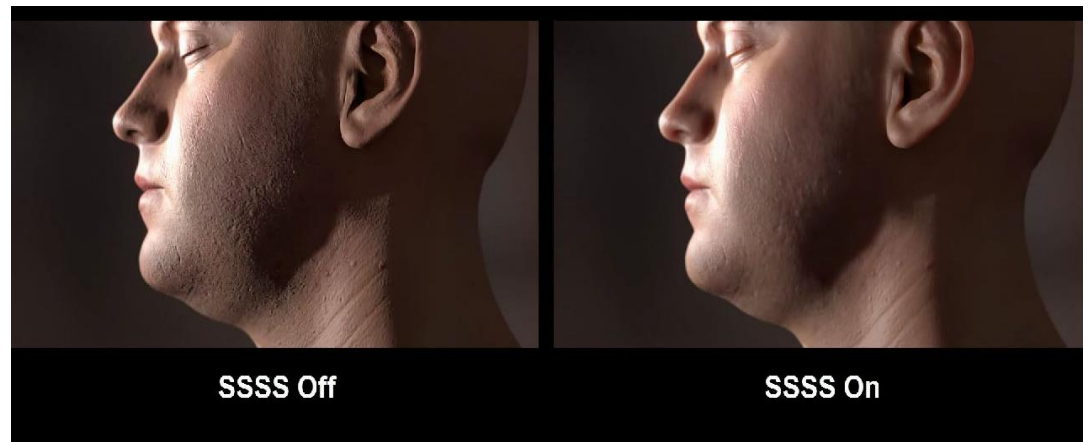
BRICK



WOOD

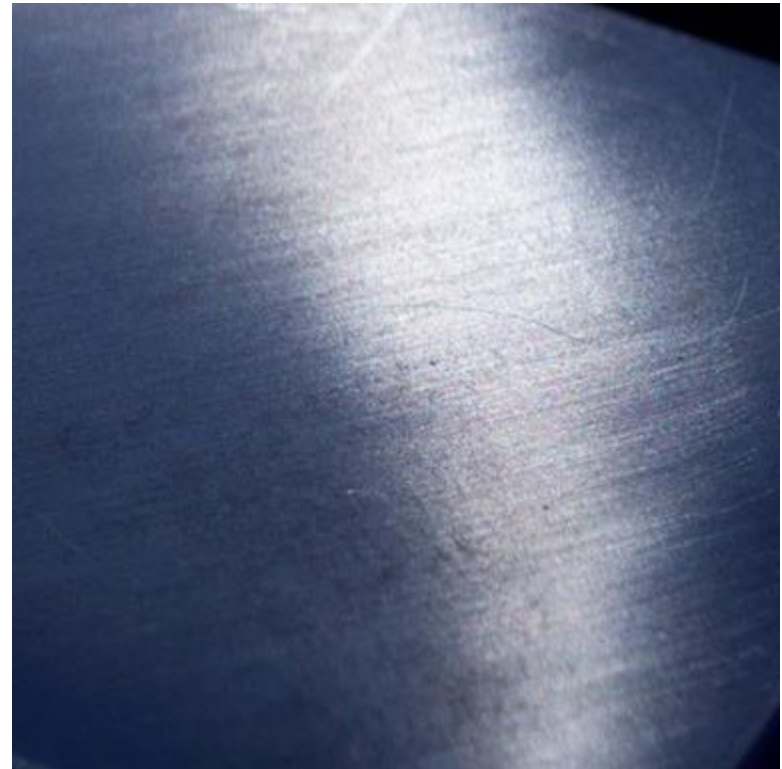
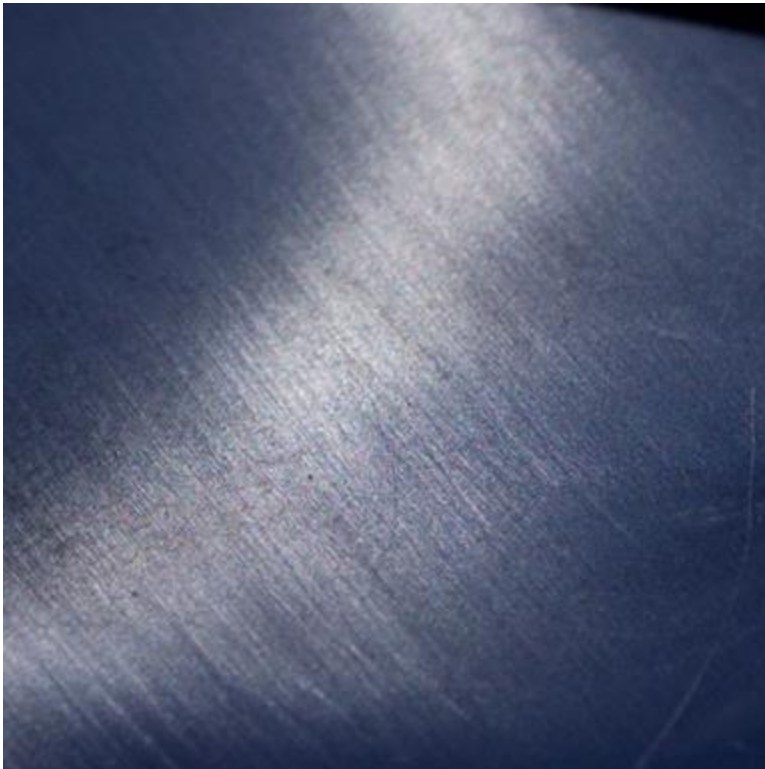


PAINT



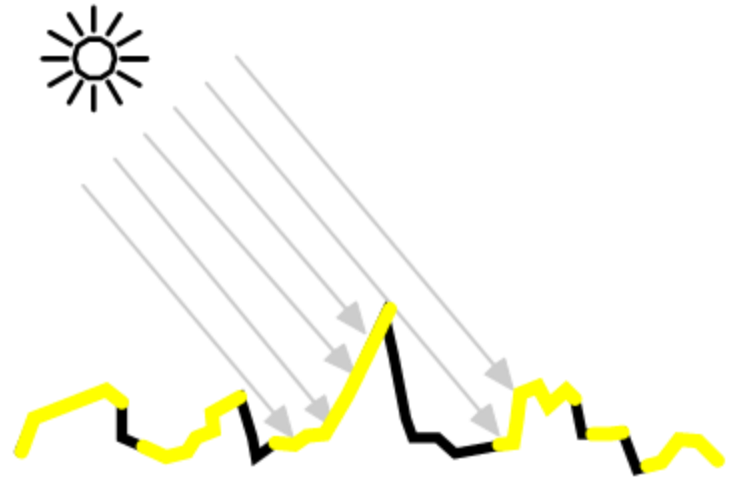
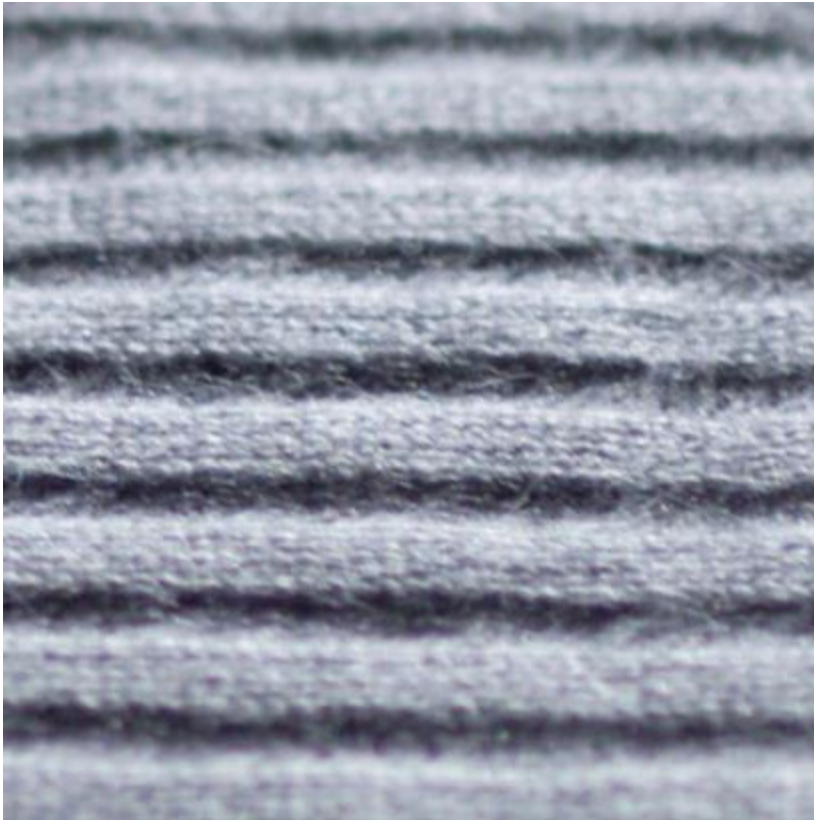
Material Samples

- **Anisotropic surfaces**



Material Samples

- **Complex surface meso-structure**



Material Samples

- Lots of details: Fibers

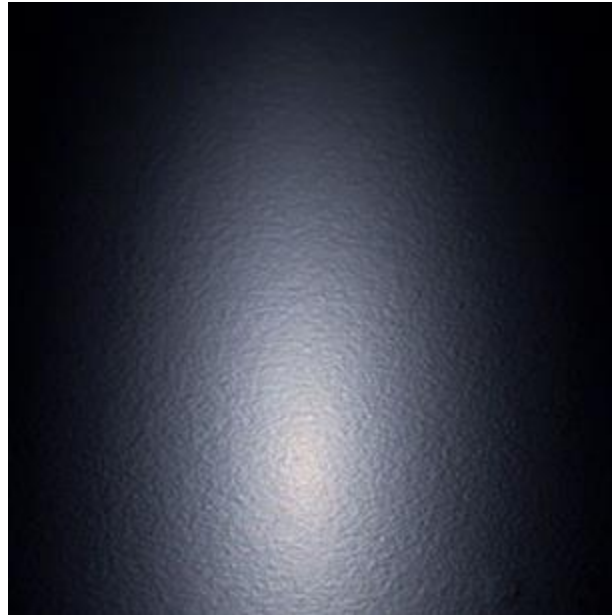


Material Samples

- Typical material types: Photos of samples with light source at exactly the same position



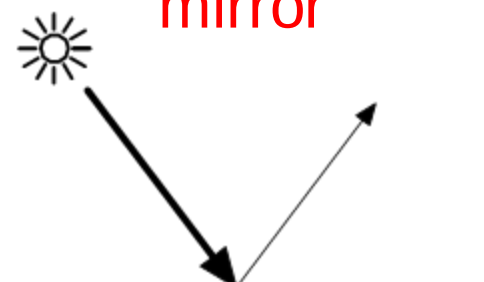
diffuse



glossy



mirror



How to describe materials?

- **Surface roughness**

- Cause of different reflection properties (often in combination):
 - Perfectly smooth: Mirror reflection
 - Slightly rough: Glossy highlights, approx. in direction of reflection
 - Very rough: Diffuse reflection, light reflected many times in material, loses directionality
 - Combination of the above

- **Geometry**

- Macro structure: Described as explicit geometry (e.g. triangles)
- Micro structure: Captured in scattering function (BRDF)
- Meso structure: Difficult to handle: integrate into BRDF (offline simulation), use geometry and simulate (online)

- **Representation of reflection properties**

- Bidirectional reflection distribution function (BRDF)
 - For reflections at a single point (approx.)
- More complex scattering functions (e.g. subsurface scattering)

- **Goal: Relightable representation of appearance**

Rendering Equation

- Reflection equation

$$L_o(x, \omega_o) = \int_{\Omega_+} f_r(\omega_i, x, \omega_o) L_i(x, \omega_i) \cos \theta_i d\omega_i$$

- **BRDF Definition**

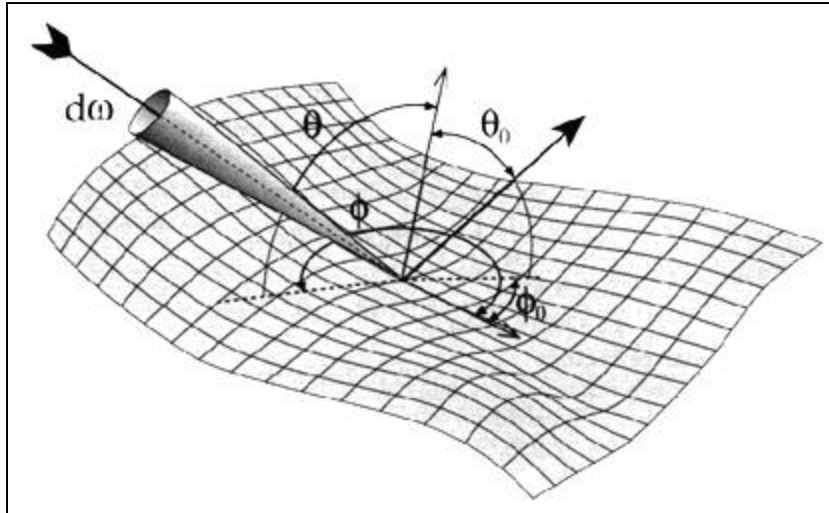
– Ratio of *reflected radiance* to *incident irradiance*

$$f_r(\omega_i, x, \omega_o) = \frac{dL_o(x, \omega_o)}{dE_i(x, \omega_i)} \quad \text{Units: } \left[\frac{1}{\text{sr}} \right]$$

BRDF

- **BRDF describes surface reflection**
 - for light incident from direction $\omega_i = (\theta_i, \varphi_i)$
 - observed from direction $\omega_o = (\theta_o, \varphi_o)$
- **Bidirectional**
 - Depends on 2 directions ω_i, ω_o and position x (a 6-D function)

$$f_r(\omega_i, x, \omega_o) = \frac{dL_o(x, \omega_o)}{dE_i(x, \omega_i)} = \frac{dL_o(x, \omega_o)}{L_i(x, \omega_i) \cos \theta_i d\omega_i}$$

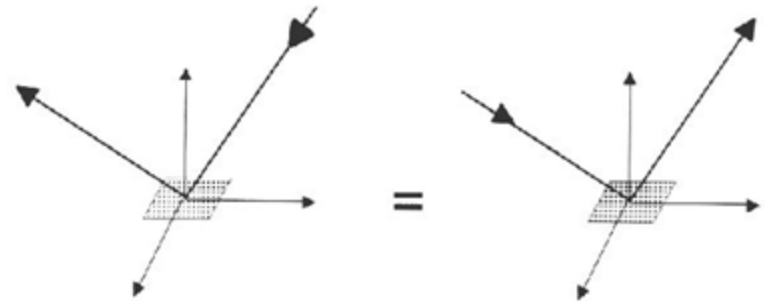


BRDF Properties

- **Helmholtz reciprocity principle**

- BRDF remains unchanged if incident and reflected directions are interchanged
- Due to physical principle of time reversal

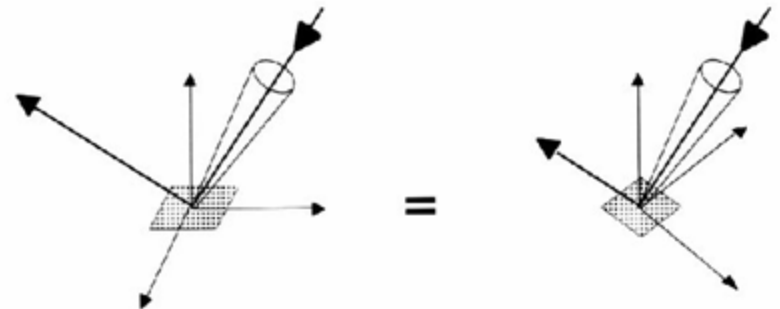
$$f_r(\omega_i, \omega_o) = f_r(\omega_o, \omega_i)$$



- **No surface structure: Isotropic BRDF**

- Reflectivity independent of rotation around surface normal
- BRDF has only 3 instead of 4 directional degrees of freedom

$$f_r(x, \theta_i, \theta_o, \varphi_o - \varphi_i)$$



BRDF Properties

- **Characteristics**

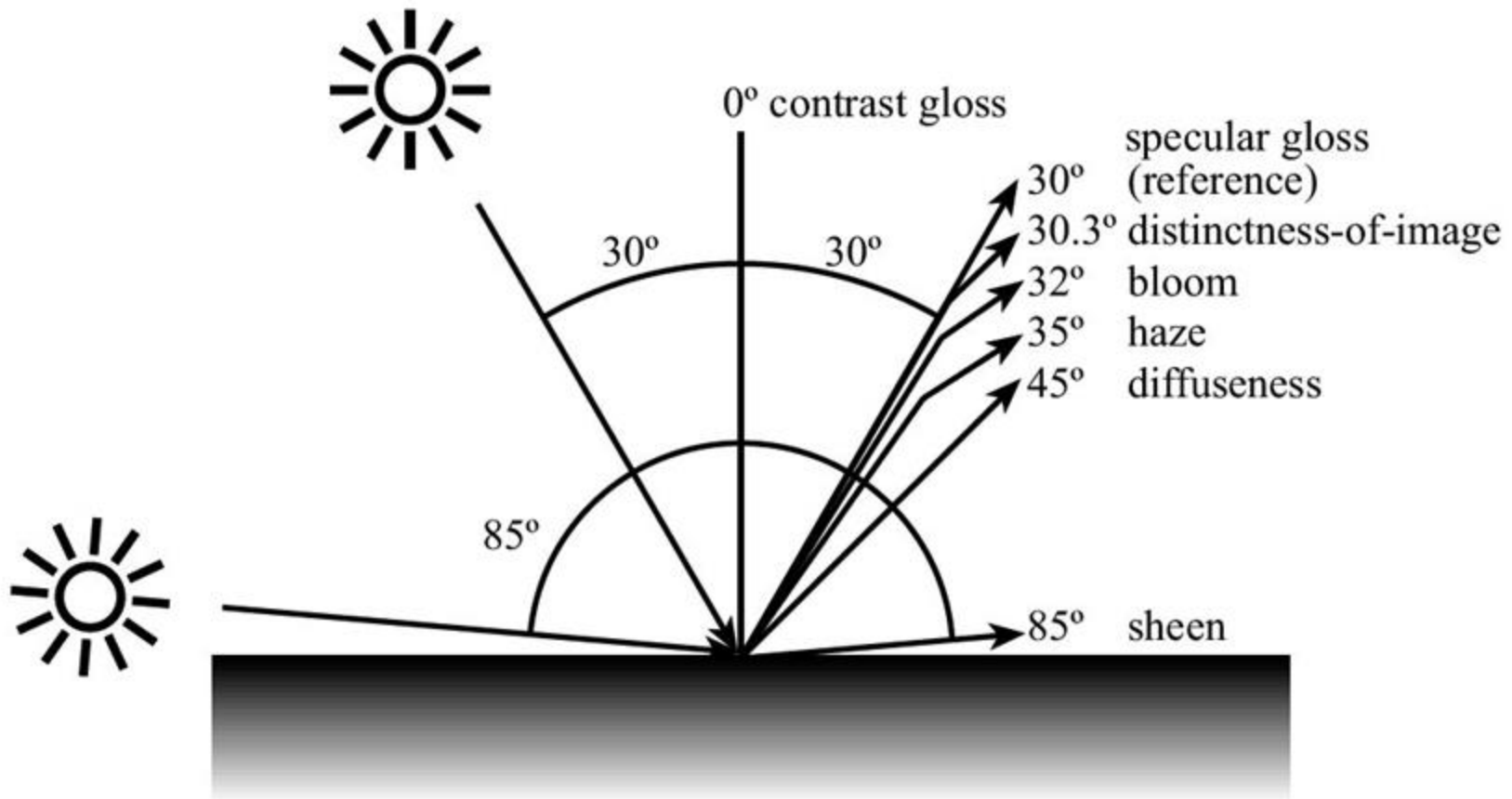
- BRDF units
 - Inverse steradian: sr^{-1} (not really intuitive)
- Range of values: distribution function is positive, **can be infinite**
 - From 0 (no reflection in that direction, perfectly black)
 - to ∞ (perfect reflection into exactly one direction, δ -function, mirror)
- Energy conservation law
 - Absorption physically unavoidable and assuming no self-emission
 - Integral of f_r over **outgoing** directions integrates to less than one
 - For any incoming direction

$$\int_{\Omega_+} f_r(\omega_i, x, \omega_o) \cos \theta_o d\omega_o \leq 1, \quad \forall \omega_i$$

- **Reflection only at the point of entry ($x_i = x_o$)**
 - Ignoring subsurface scattering (SSS)

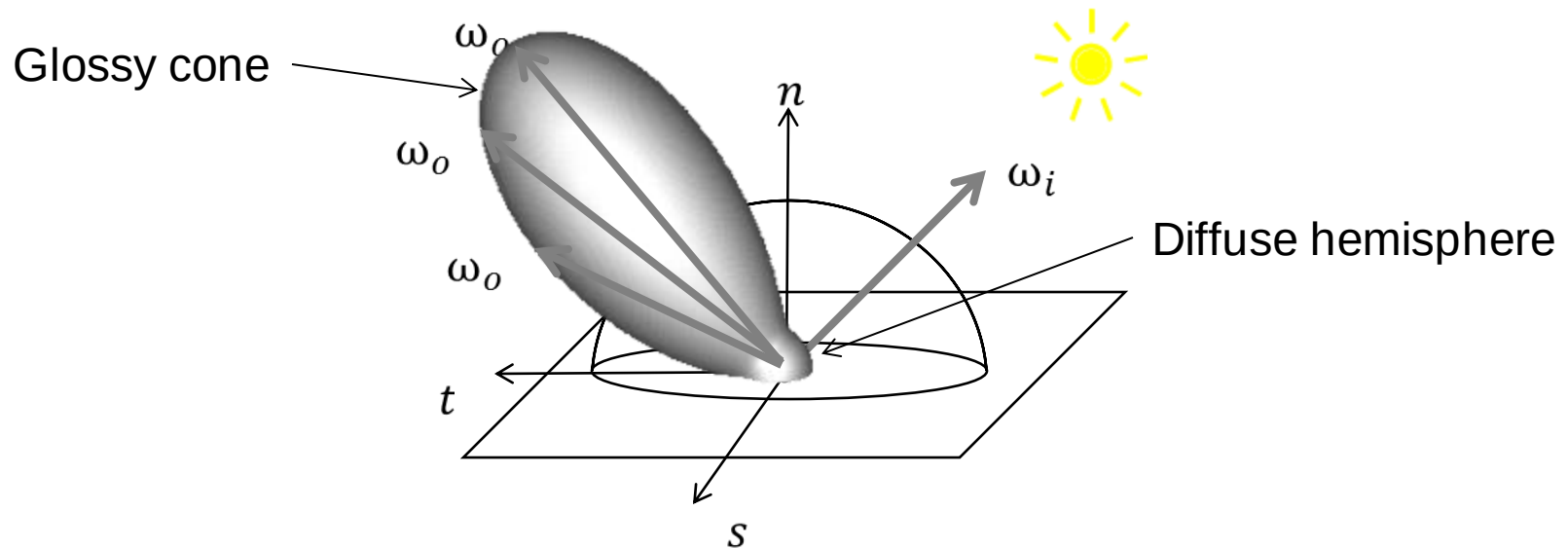
Standardized Gloss Model

- **Industry often uses only a subset of BRDF values**
 - Reflection only measured at discrete set of angles in plane of incidence (not typically used in graphics)



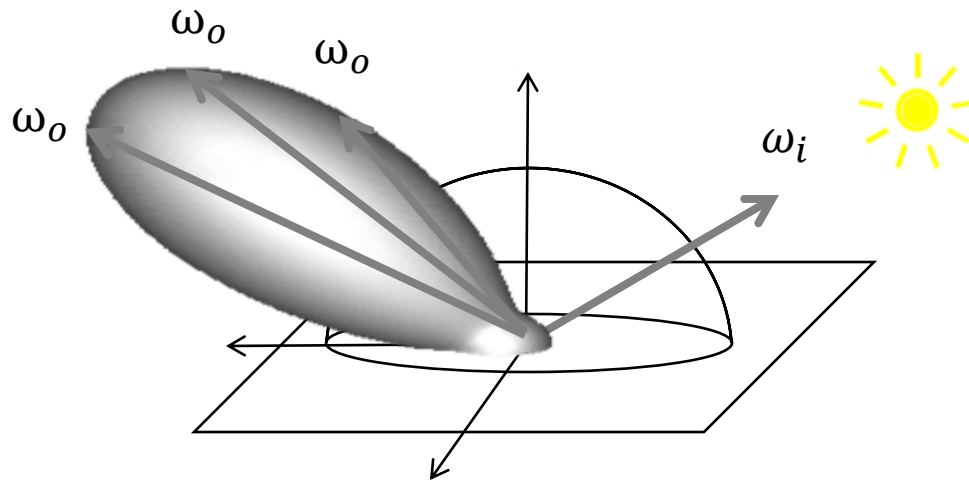
Reflection on an Opaque Surface

- **BRDF is often shown as a slice of the 6D function**
 - Given point x and given incident direction ω_i
 - Show 3D polar plot (intensity as length of vector from origin)
 - Often consists of some mostly diffuse component (here small)
 - and a somewhat glossy component (here rather large)



Reflection on an Opaque Surface

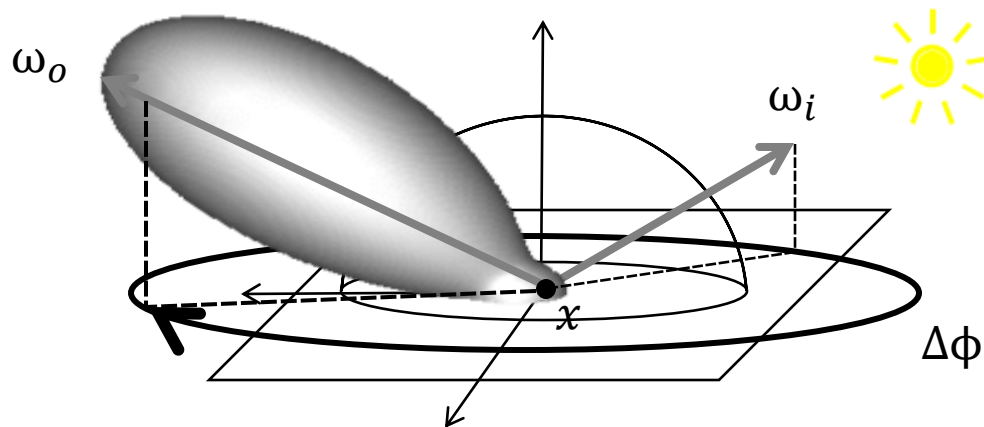
- **BRDF slice varies with incident direction**
 - and possibly location



Homog. & Isotropic BRDF – 3D

- **Invariant with respect to rotation about the normal**
 - Homogeneous and isotropic across surface
 - Only depends on azimuth difference to incoming angle

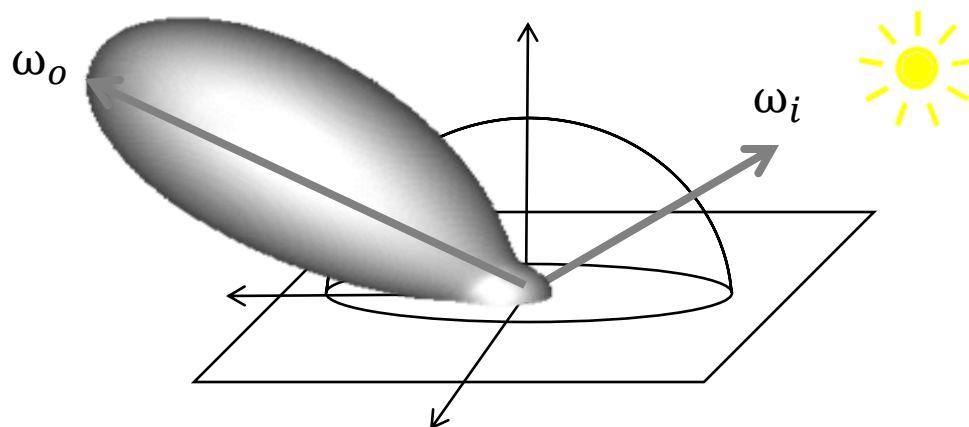
$$f_r((\theta_i, \varphi_i) \rightarrow (\theta_o, \varphi_o)) \Rightarrow$$
$$f_r(\theta_i \rightarrow \theta_o, (\varphi_i - \varphi_o)) = f_r(\theta_i \rightarrow \theta_o, \Delta\varphi)$$



Homogeneous BRDF – 4D

- **Homogeneous bidirectional reflectance distribution function**
 - Ratio of reflected radiance to incident irradiance
 - Independent of position

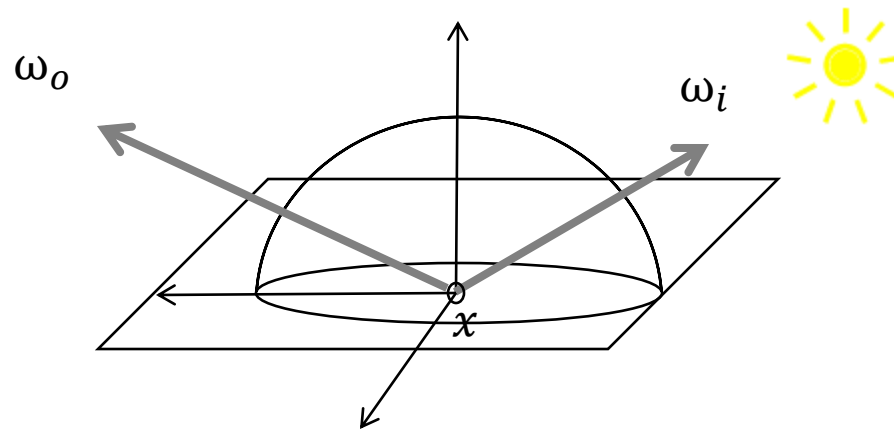
$$f_r(\omega_i \rightarrow \omega_o) = \frac{dL_o(\omega_o)}{dE_i(\omega_i)}$$



Spatially Varying BRDF – 6D

- **Heterogeneous materials (standard model for BRDF)**
 - Dependent on position, and two directions
 - Reflection at the point of incidence

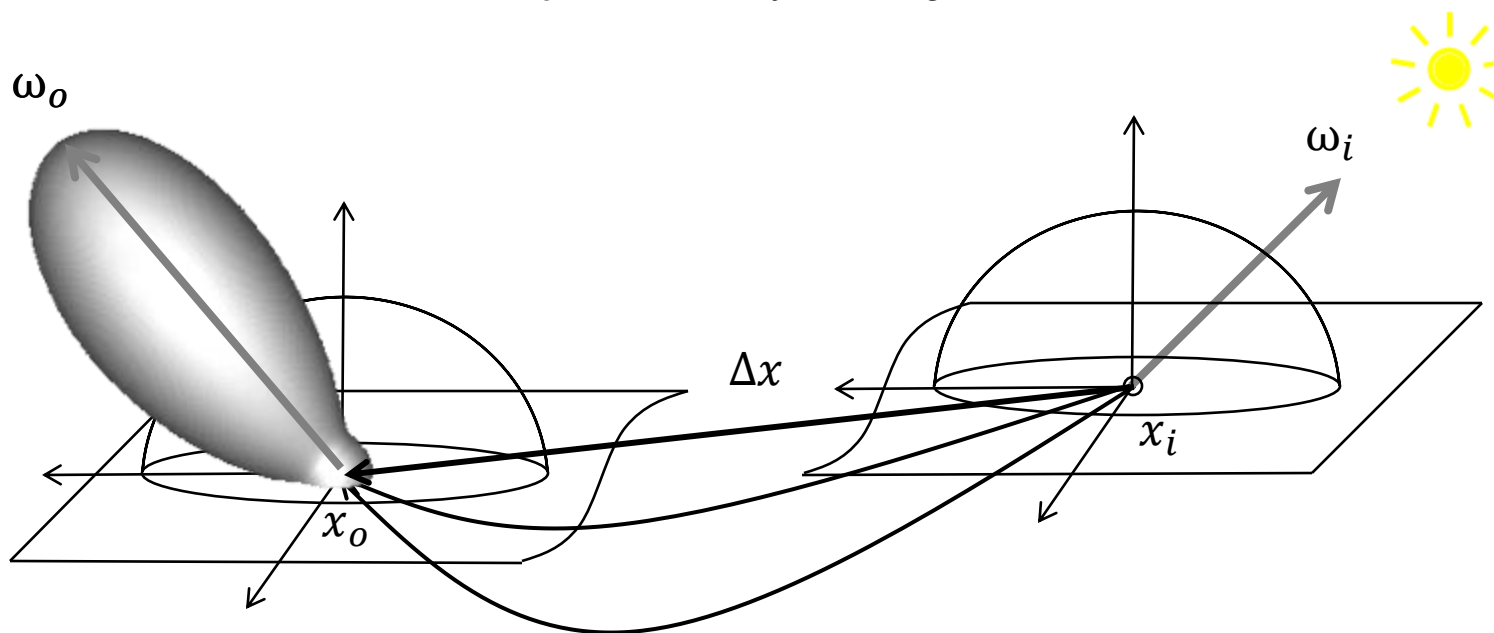
$$f_r(x, \omega_i \rightarrow \omega_o)$$



Homogeneous BSSRDF – 6D

- **Homogeneous bidirectional scattering surface reflectance distribution function**
 - Assumes a homogeneous and flat surface
 - Only depends on the difference vector to the outgoing point

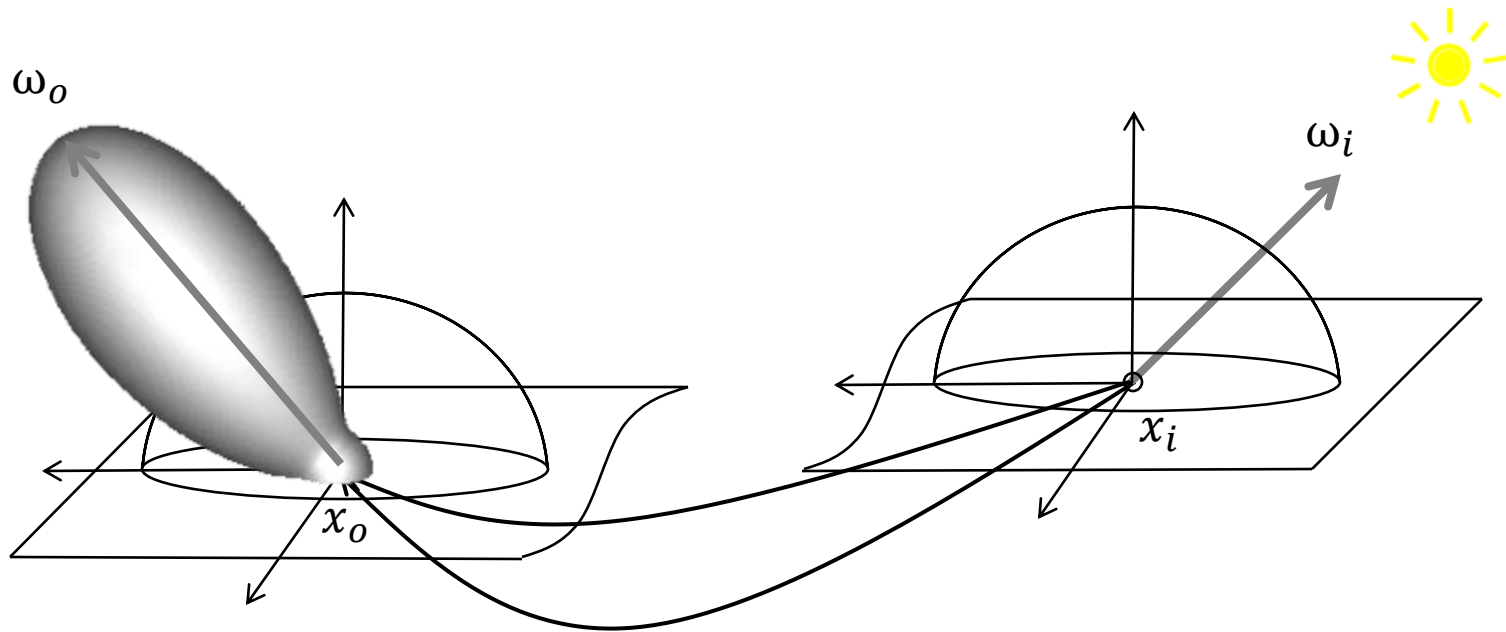
$$f_r(\Delta x, \omega_i \rightarrow \omega_o)$$



BSSRDF – 8D

- Bidirectional scattering surface reflectance distribution function

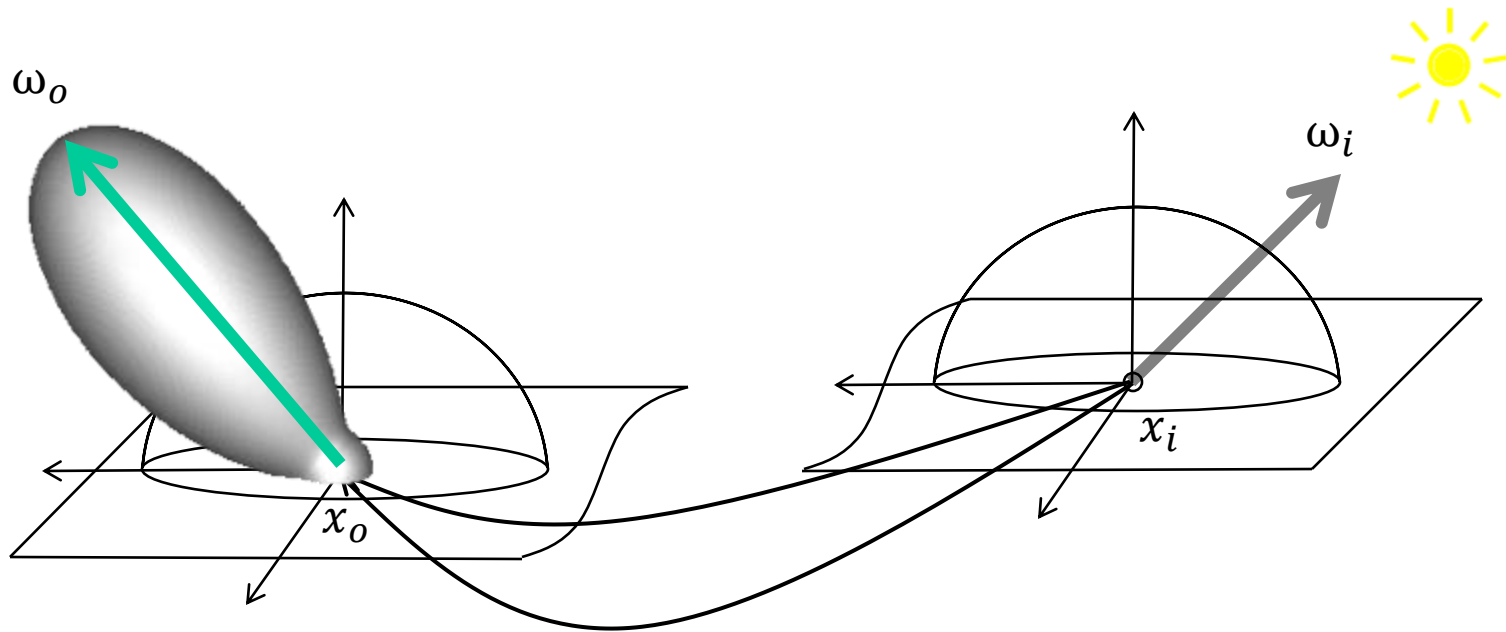
$$f_r((x_i, \omega_i) \rightarrow (x_o, \omega_o))$$



Generalization – 9D

- **Generalizations**
 - Add wavelength dependence

$$f_r(\lambda, (x_i, \omega_i) \rightarrow (x_o, \omega_o))$$

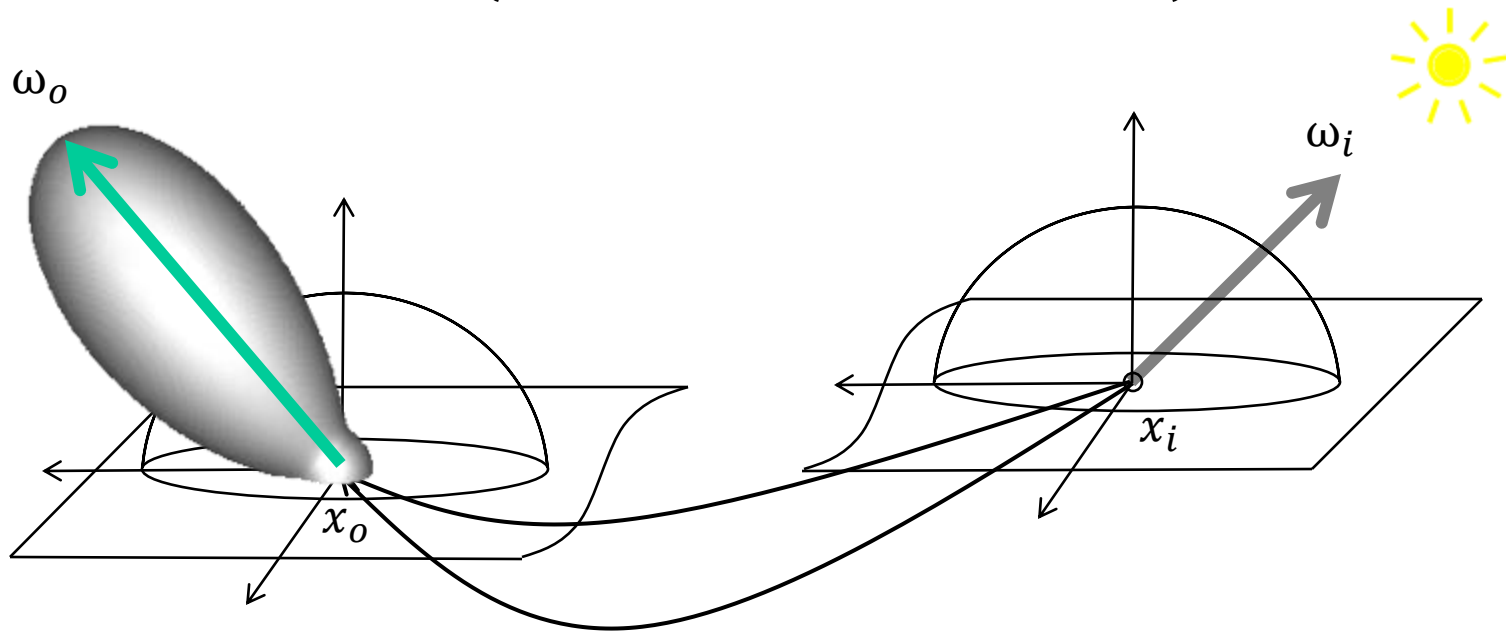


Generalization – 10D

- **Generalizations**

- Add wavelength dependence
- Add **fluorescence**
 - Change to longer wavelength during scattering

$$f_r((x_i, \omega_i, \lambda_i) \rightarrow (x_o, \omega_o, \lambda_o))$$

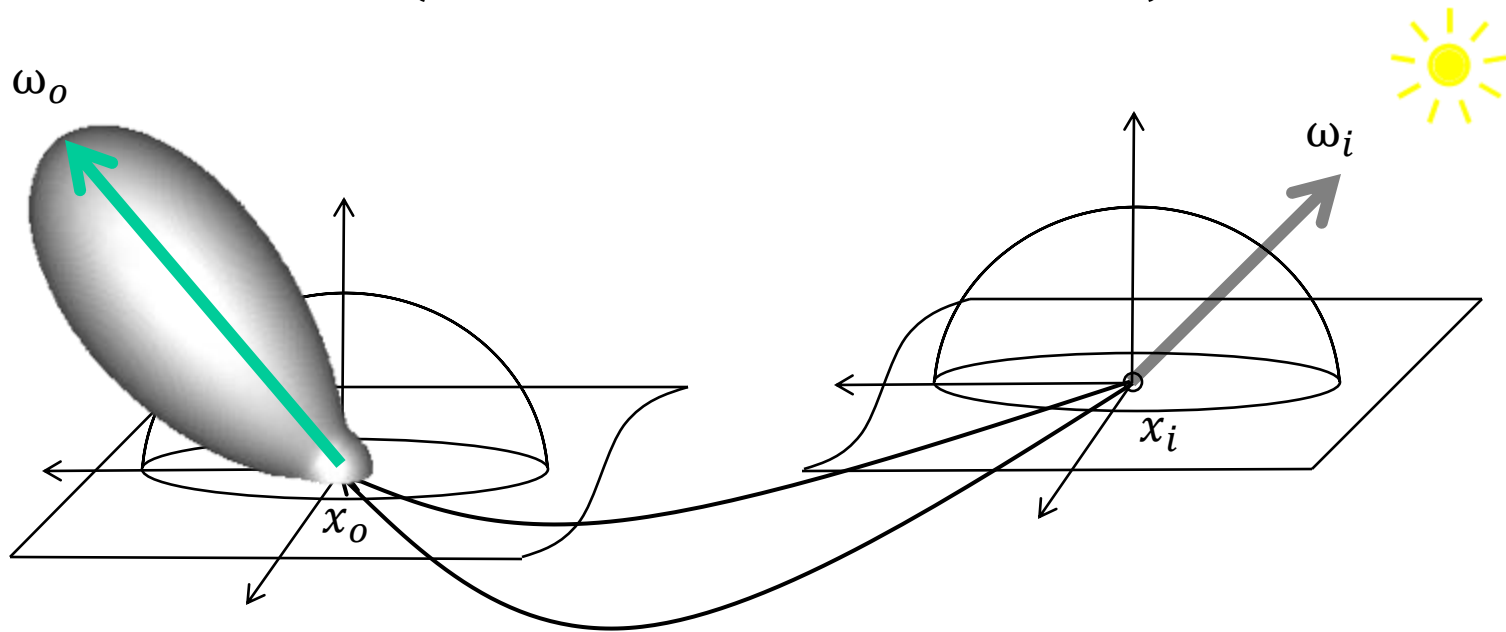


Generalization – 11D

- **Generalizations**

- Add wavelength dependence
- Add fluorescence (change to longer wavelength for reflection)
- Time varying surface characteristics

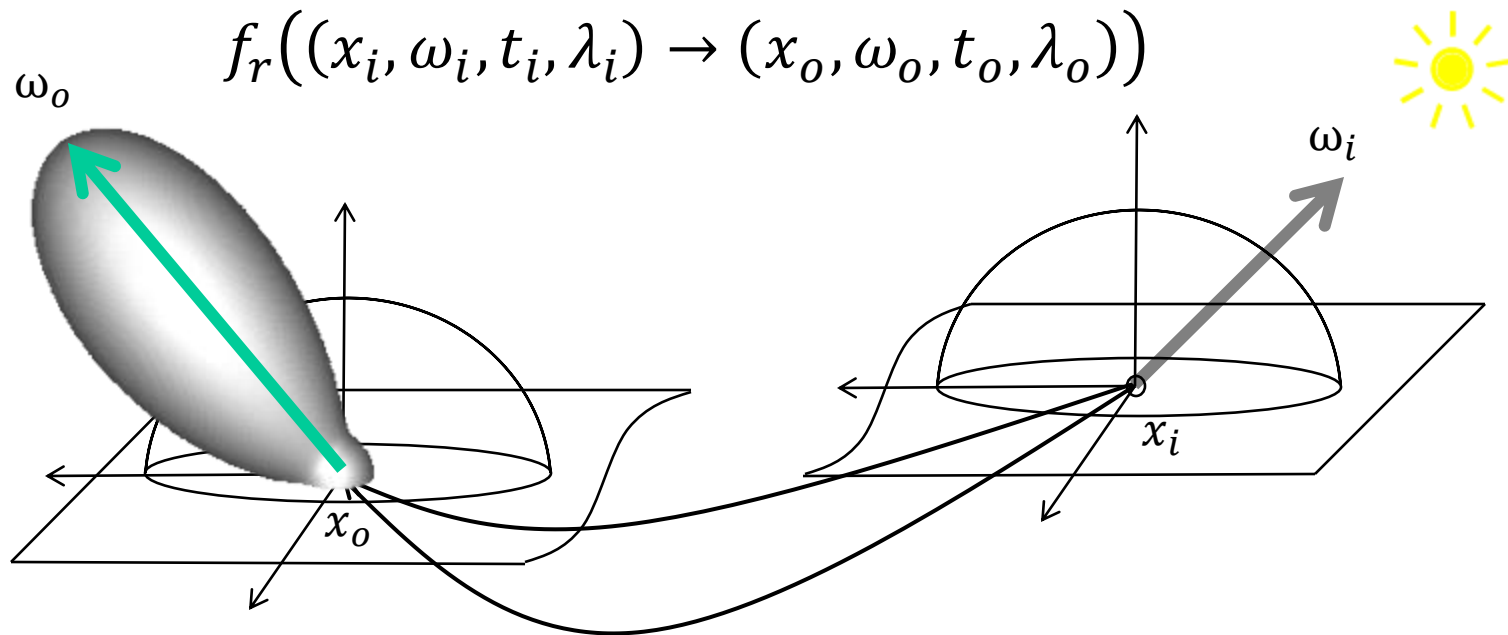
$$f_r(t, (x_i, \omega_i, \lambda_i) \rightarrow (x_o, \omega_o, \lambda_o))$$



Generalization – 12D

- **Generalizations**

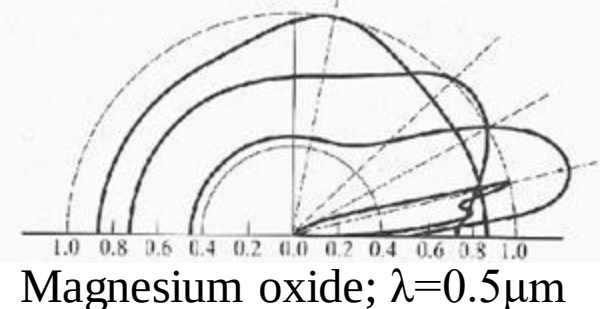
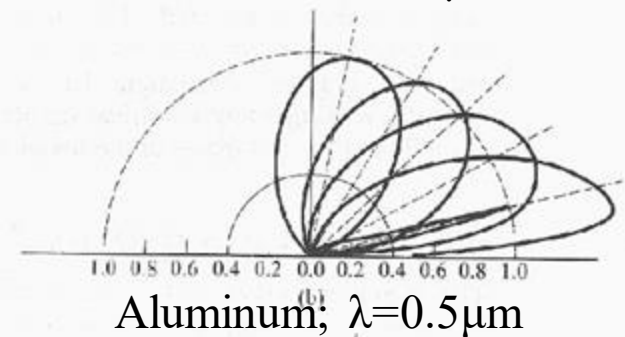
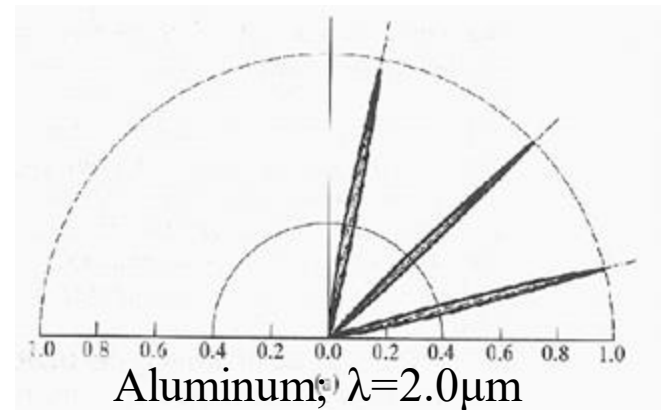
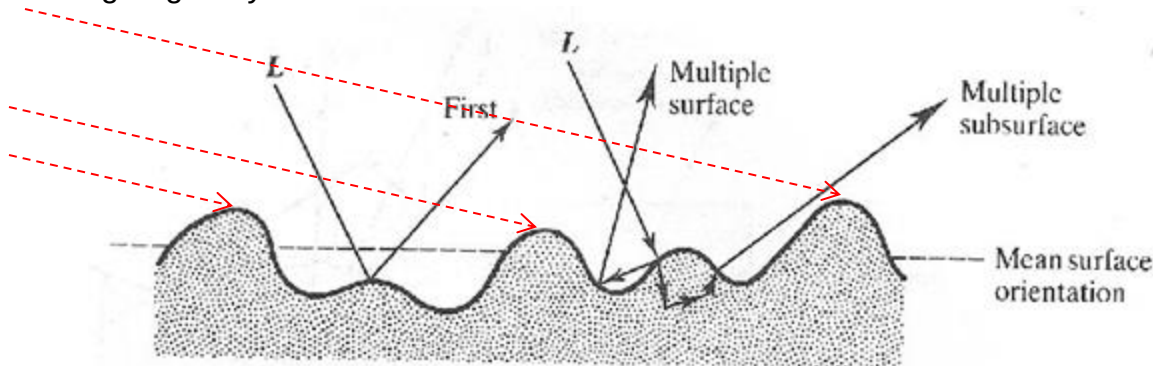
- Add wavelength dependence
- Add fluorescence (change to longer wavelength for reflection)
- Time varying surface characteristics
- **Phosphorescence**
 - Temporal storage of light



Reflectance

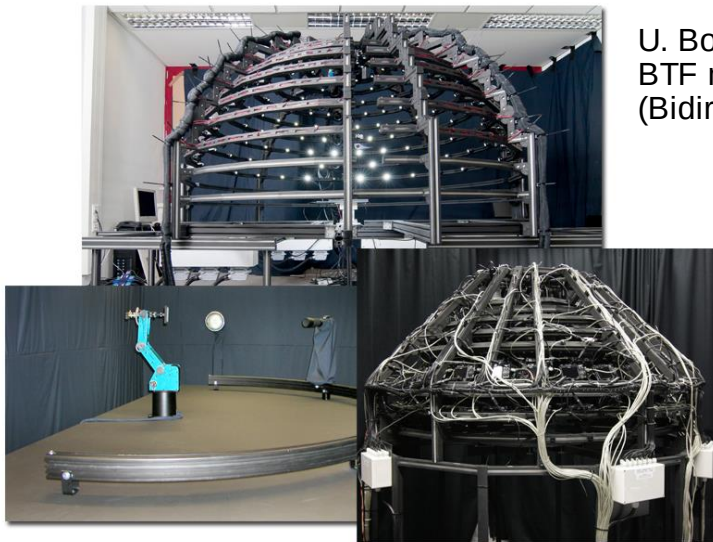
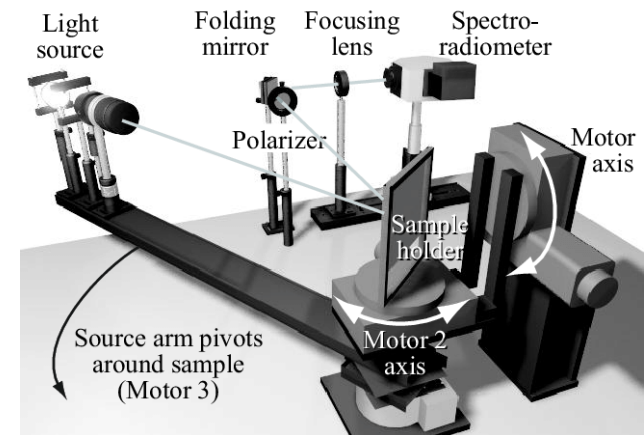
- **Reflectance may vary with**
 - Illumination angle
 - Viewing angle
 - Wavelength
 - (Polarization, ...)
- **Variations due to**
 - Absorption
 - Surface micro-geometry
 - Index of refraction / dielectric constant
 - Scattering in material (e.g. paint)

Grazing angle rays



BRDF Measurement

- **Gonio-Reflectometer**
- **BRDF measurement**
 - Point light source position (θ_i, φ_i)
 - Light detector position (θ_o, φ_o)
- **4 directional degrees of freedom**
- **BRDF representation (large!!!)**
 - m (in) * n (out) directional samples
 - Additional position (e.g. image $\rightarrow$ 6D)



U. Bonn,
BTF measurement,
(Bidir Texture Func.)



Stanford
light gantry

Rendering from Measured BRDF

- **Linearity, superposition principle**

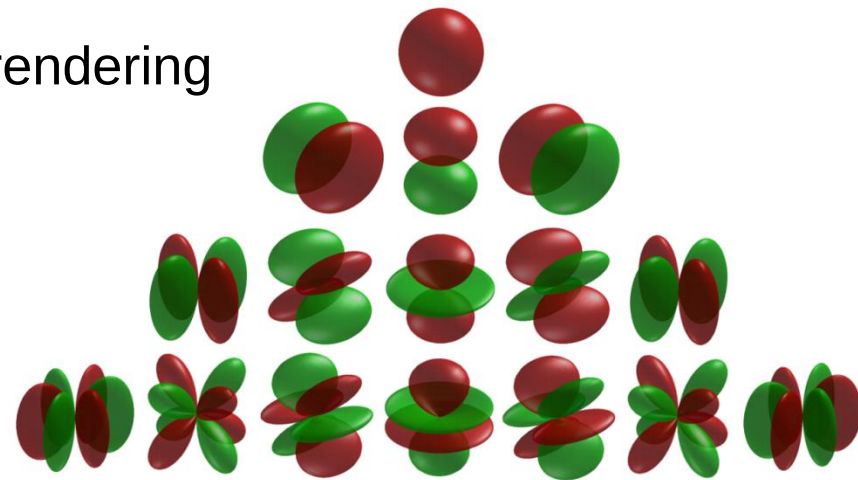
- Continuous illumin.: integrating light distribution against BRDF
- Sampled illumination: superimposing many point light sources

- **Interpolation**

- Look-up of BRDF values during rendering
- Sampled BRDF must be filtered

- **BRDF Modeling**

- Fitting of parameterized BRDF models to measured data
 - Continuous, analytic function
 - No interpolation
 - Typically fast evaluation



Spherical Harmonics

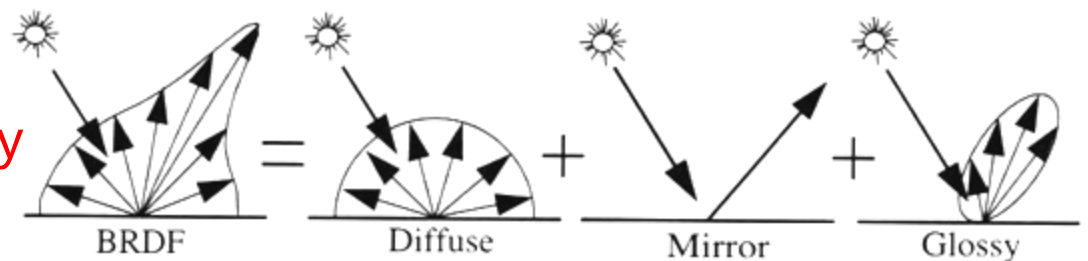
Red is positive, green negative [Wikipedia]

- **Representation in a basis**

- Often: Spherical harmonics (ortho-normal basis on sphere)
 - Or BTFs (bidirectional texture function)
- Mathematically elegant filtering, illumination-BRDF integration

BRDF Modeling

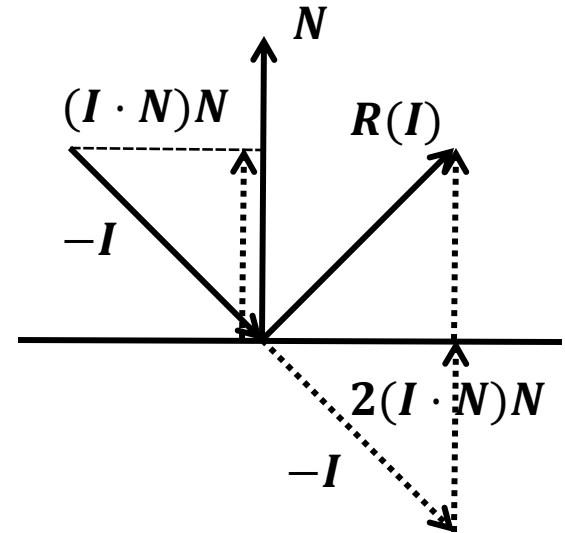
- **Phenomenological approach (not physically correct)**
 - Description of visual surface appearance
 - Composition of different terms:
- **Ideal diffuse reflection +**
 - Lambert's law, interactions within material
 - Matte surfaces
- **Ideal specular/mirror reflection +**
 - Reflection law
 - Mirror surfaces
- **Glossy reflection**
 - “Directional diffuse”, reflection on surface that is somewhat rough
 - Shiny surface
 - Glossy highlights
 - Sometimes incorrectly called “specular”



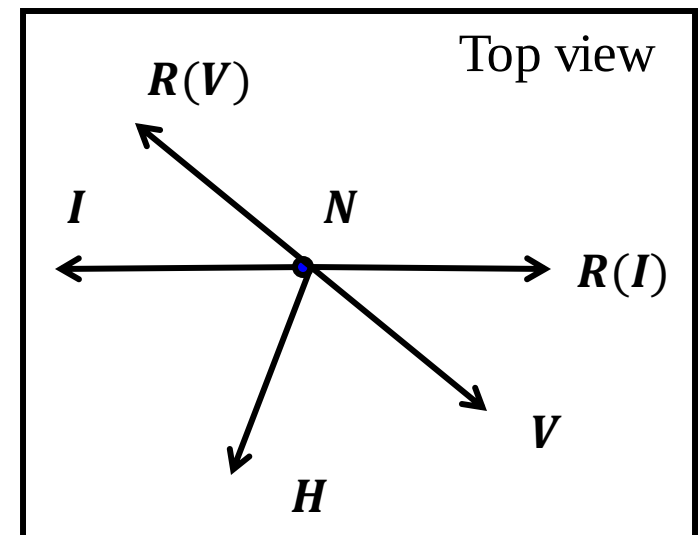
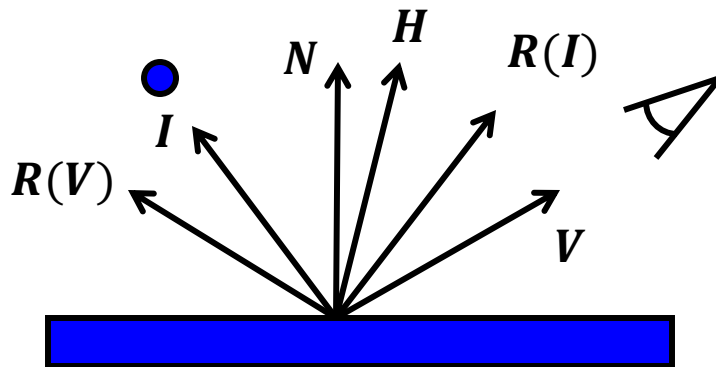
Reflection Geometry

- **Direction vectors (normalize):**

- N : Surface normal
- I : Light source direction vector
- V : Viewpoint direction vector
- $R(I)$: Reflection vector
 - $R(I) = -I + 2(I \cdot N)N$
- H : Halfway vector
 - $H = (I + V) / |I + V|$



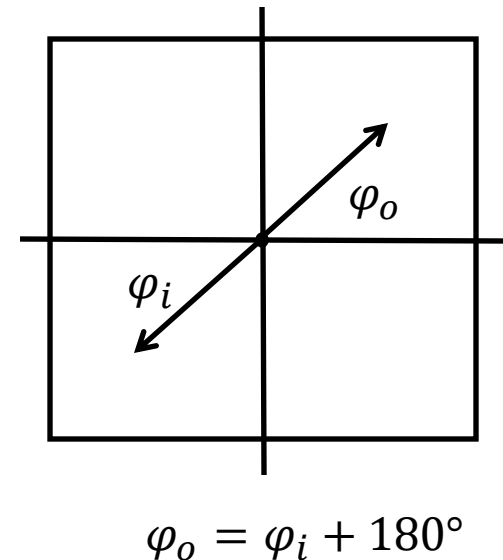
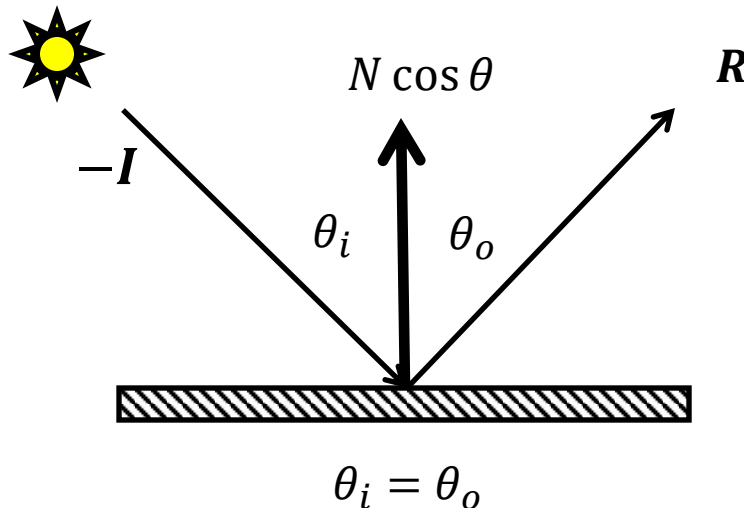
- **Tangential surface: local plane**



Ideal Specular (Mirror) Reflection

- Angle of reflectance equal to angle of incidence
- Reflected vector in a plane with incident ray and surface normal vector

$$R + I = 2 \cos \theta N = 2(I \cdot N)N \Rightarrow$$
$$R(I) = -I + 2(I \cdot N)N$$



Mirror BRDF

- **Dirac Delta function $\delta(x)$**

- $\delta(x)$: zero everywhere except at $x = 0$
- Unit integral iff domain contains $x = 0$ (else zero)

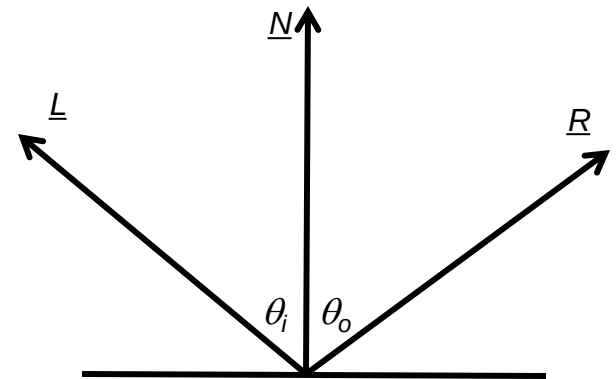
$$f_{r,m}(\omega_i, x, \omega_o) = \rho_s(\theta_i) \frac{\delta(\cos\theta_i - \cos\theta_o)}{\cos\theta_i} \delta(\varphi_i - \varphi_o \pm \pi)$$

$$L_o(x, \omega_o) = \int_{\Omega_+} f_{r,m}(\omega_i, x, \omega_o) L_i(x, \omega_i) \cos\theta_i d\omega_i = \rho_s(\theta_o) L_i(x, \theta_o, \varphi_o \pm \pi)$$

- **Specular reflectance ρ_s**

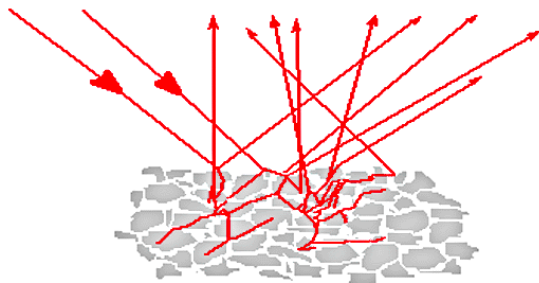
- Ratio of reflected radiance in specular direction and incoming radiance
- Dimensionless quantity between 0 and 1

$$\rho_s(x, \theta_i) = \frac{L_o(x, \theta_o)}{L_i(x, \theta_o)}$$

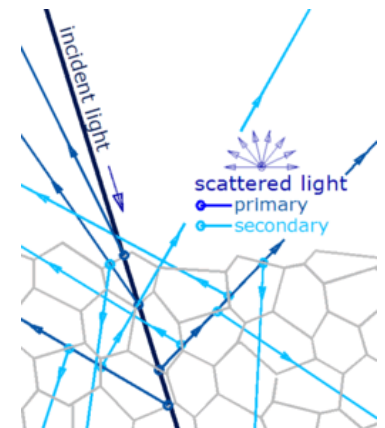
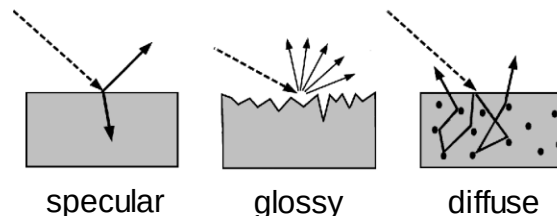
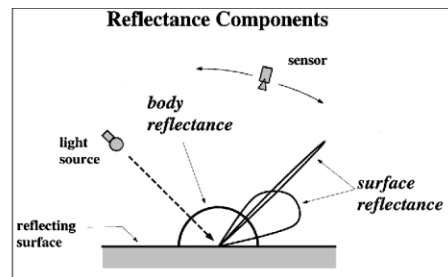


“Diffuse” Reflection

- **Theoretical explanation**
 - Multiple scattering within the material (at very short range)
- **Experimental realization**
 - Pressed magnesium oxide powder (or foam/snow)
 - Random mixture of tiny, highly reflective surfaces
 - Almost never valid at grazing angles of incidence
 - Paint manufacturers attempt to create ideal diffuse paints



Highly reflective particles
(e.g. magnesium oxide, plaster
paper fibers)



Highly reflective/refractive
foam-like materials

Diffuse Reflection Model

- Light equally likely to be reflected in any output direction (independent of input direction, idealized)

- **Constant BRDF**

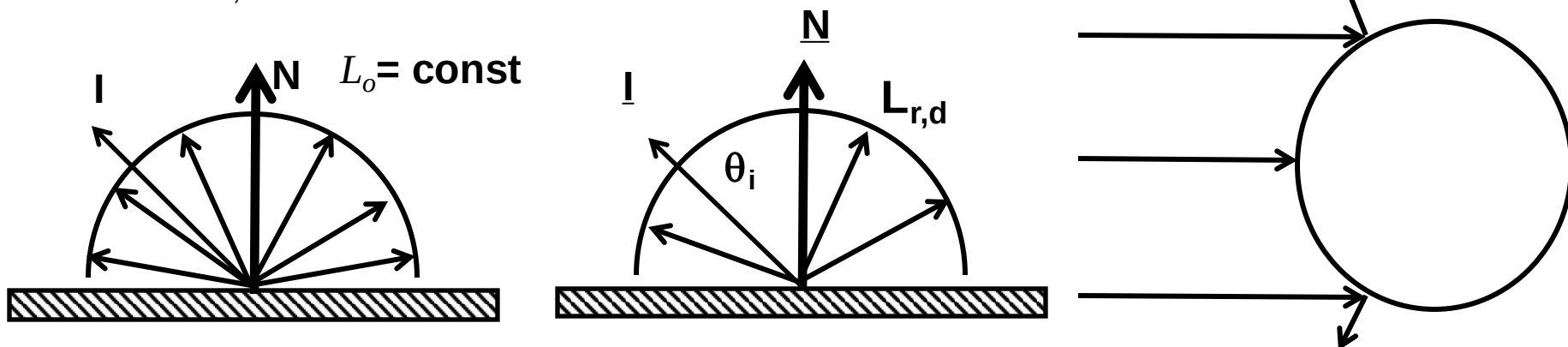
$$f_{r,d}(\omega_i, x, \omega_o) = k_d = \text{const} = \rho_d / \pi [\text{sr}] \quad \text{with } \rho_d \in [0,1]$$

$$L_o(x, \omega_o) = k_d \int_{\Omega_+} L_i(x, \omega_i) \cos \theta_i d\omega_i = k_d E = \frac{\rho_d}{\pi [\text{sr}]} E$$

– ρ_d : diffuse reflection coefficient, material property [1/sr]

- **For each point light source**

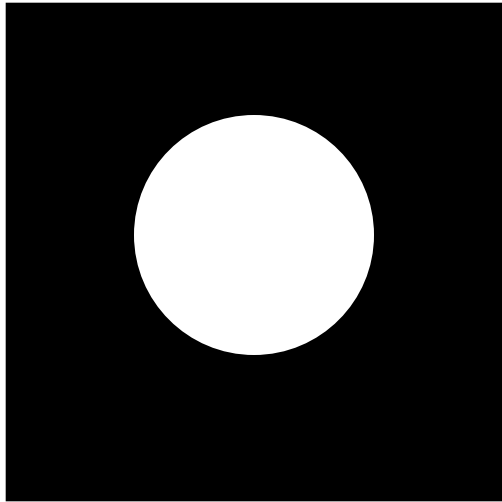
– $L_{r,d} = k_d L_i \cos \theta_i = k_d L_i (\underline{l} \cdot \underline{N})$



Lambertian Objects

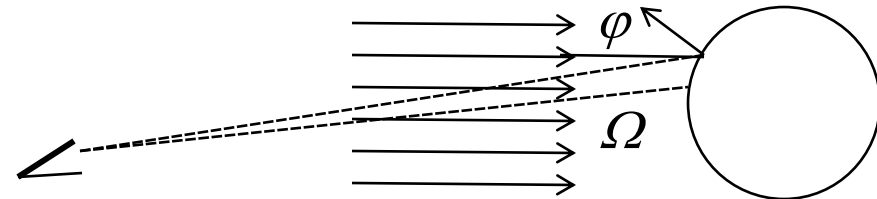
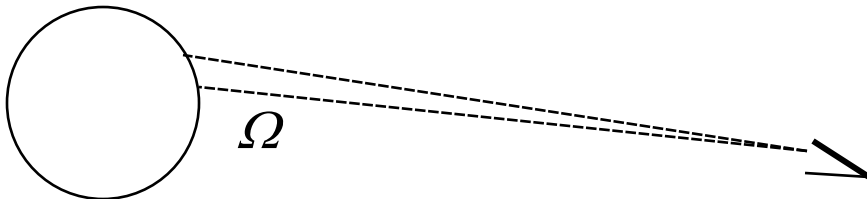
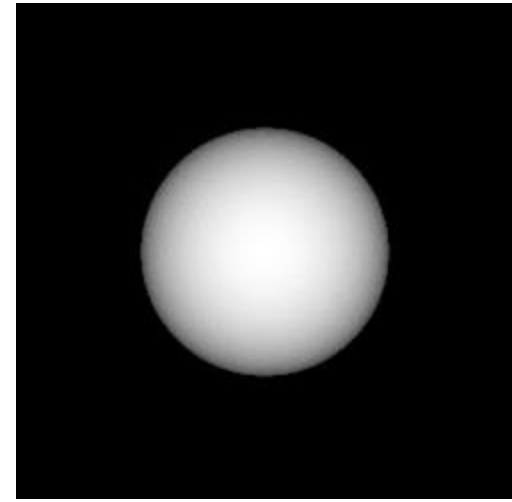
Self-luminous
spherical Lambertian light source

$$\Phi_0 \propto L_0 \cdot \Omega$$



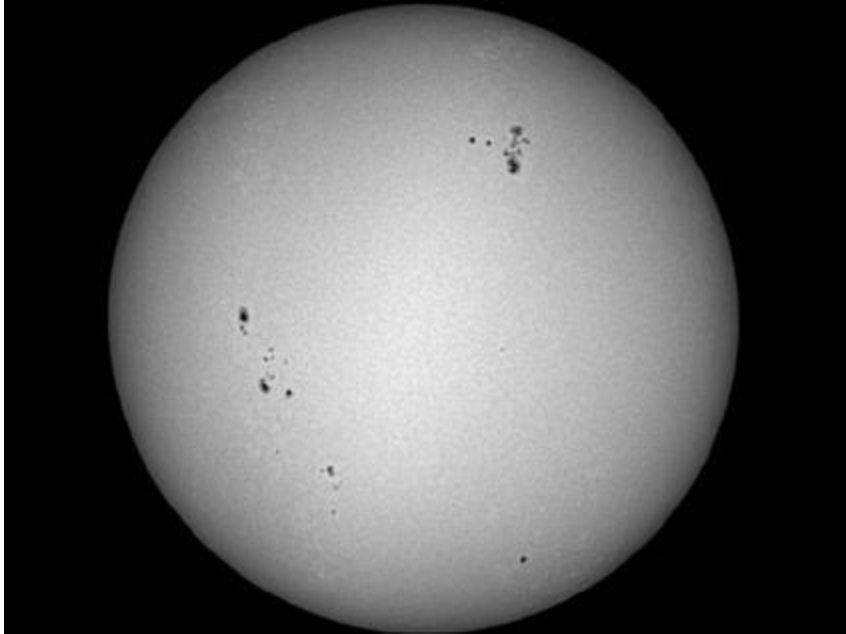
Eye-light illuminated
spherical Lambertian reflector

$$\Phi_1 \propto L_i \cdot \cos \theta \cdot \Omega$$



Lambertian Objects (?)

The Sun



- Some absorption in photosphere
- Path length through photosphere longer from the Sun's rim

The Moon

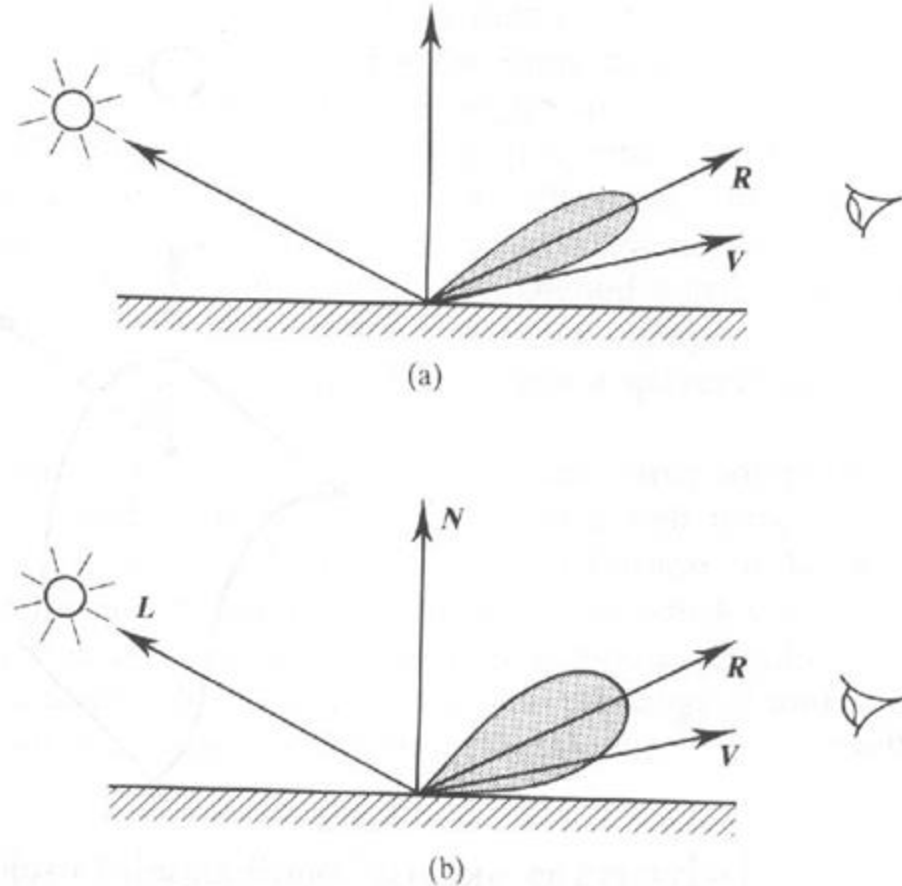


- Surface covered with fine dust
- Dust visible best from slanted viewing angle

⇒ Neither the Sun nor the Moon are Lambertian

Glossy Reflection

- Due to surface roughness
- Empirical models (phenomenological)
 - Phong
 - Blinn-Phong
- Physically-based models
 - Blinn
 - Cook & Torrance
- Sometimes incorrectly called “specular”



Phong Glossy Reflection Model

- Simple experimental description: Cosine power lobe

$$f_r(\omega_i, x, \omega_o) = k_s (R(I) \cdot V)^{k_e} / I \cdot N$$

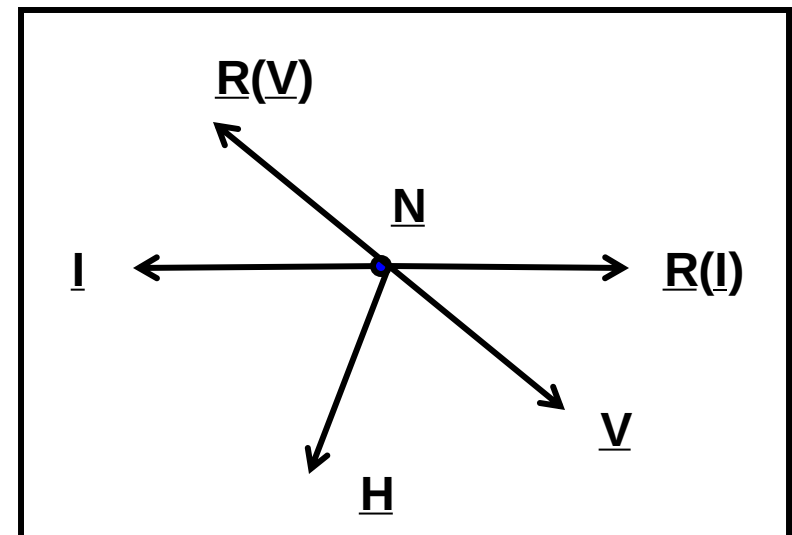
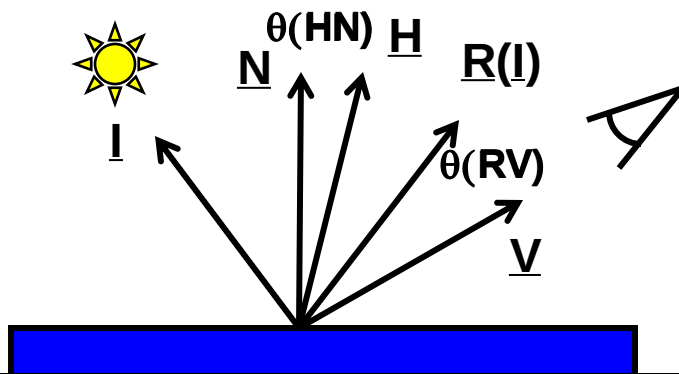
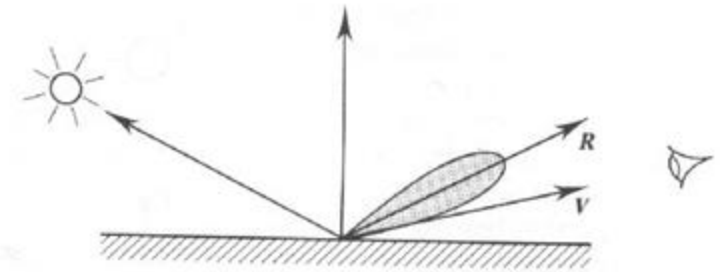
- Take angle to reflection direction to some
 - $L_{r,s} = L_i k_s \cos^{k_e} \theta_{RV}$

- **Issues**

- Not energy conserving/reciprocal
- Plastic-like appearance

- **Dot product & power**

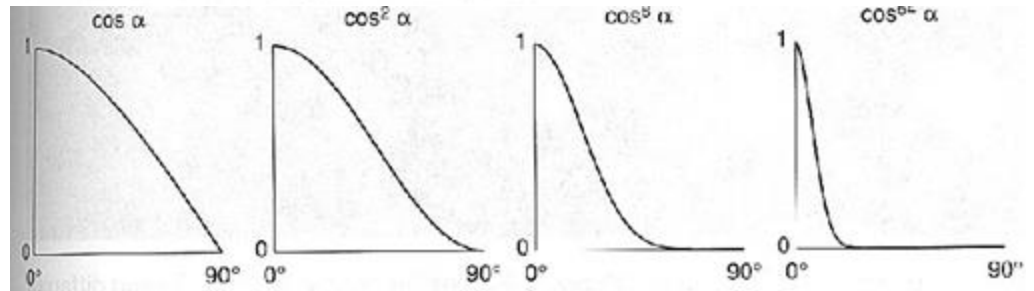
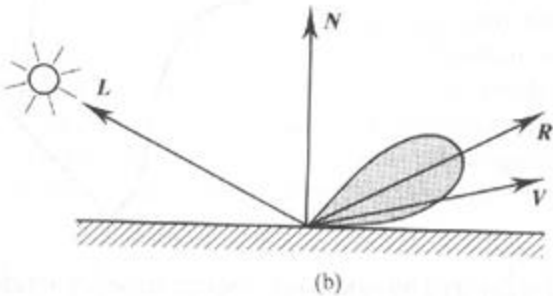
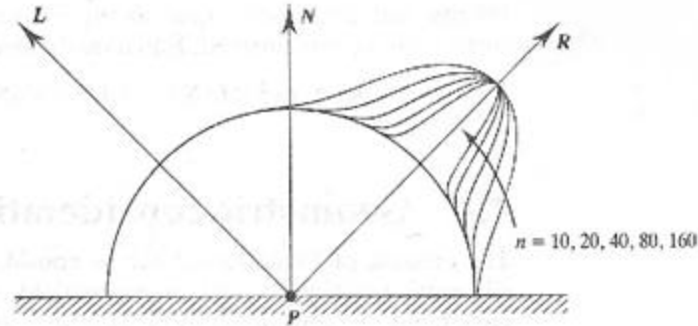
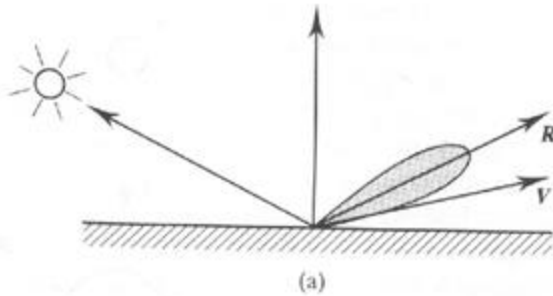
- Still widely used in CG



Phong Exponent k_e

$$f_r(\omega_i, x, \omega_o) = k_s(R(I) \cdot V)^{k_e} / I \cdot N$$

- **Determines size of highlight**



- **Beware: Non-zero contribution into the material !!!**
 - Cosine is non-zero between -90 and 90 degrees

Blinn-Phong Glossy Reflection

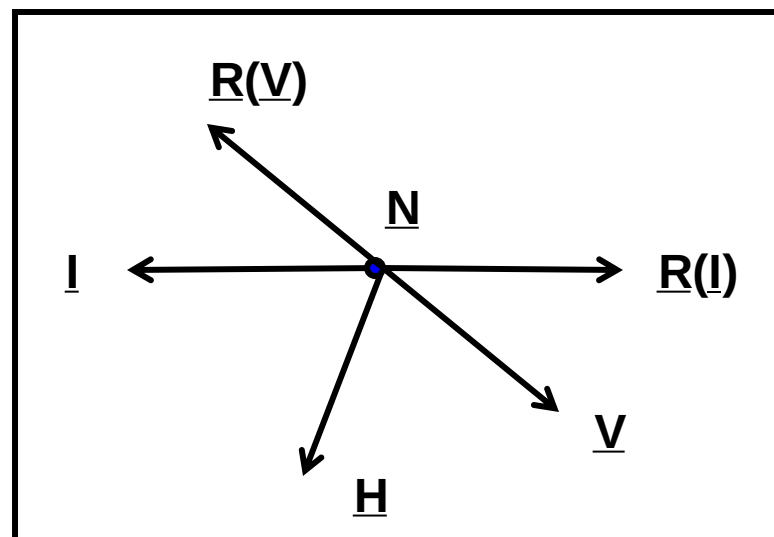
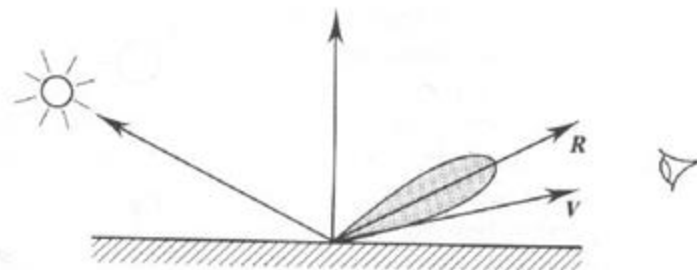
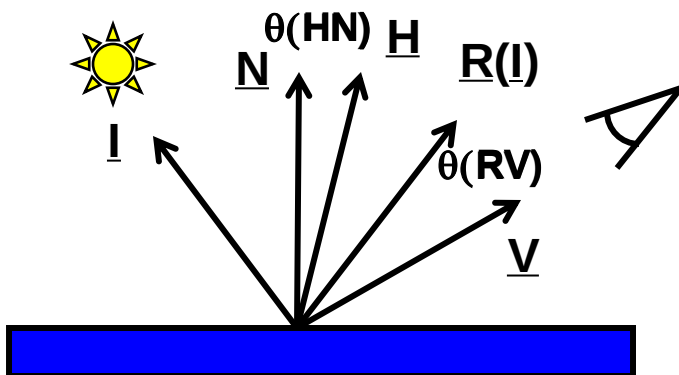
- Same idea: Cosine power lobe

$$f_r(\omega_i, x, \omega_o) = k_s (H \cdot N)^{k_e} / I \cdot N$$

$$- L_{r,s} = L_i k_s \cos^{k_e} \theta_{HN}$$

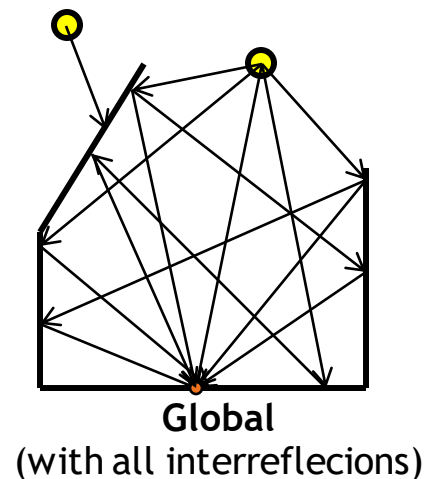
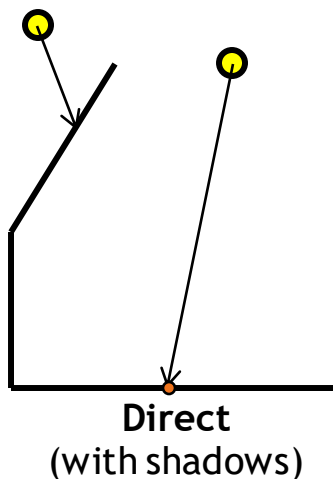
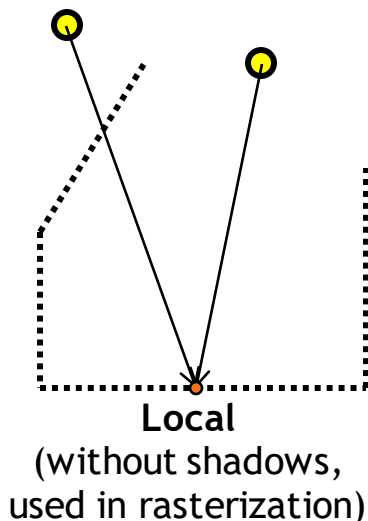
- Dot product & power

- $\theta_{RV} \rightarrow \theta_{HN}$
- Special case: Light source, viewer far away
 - I, R constant: H constant
 - θ_{HN} less expensive to compute



Different Types of Illumination

- Three types of illumination computations in CG



- Ambient Illumination**

- Global illumination is costly to compute
- Indirect illumination (through interreflections) is typically smooth
 - ➔ Approximate via a constant term $L_{i,a}$ (incoming ambient illum.)
- Has no incoming direction, provide ambient reflection term k_a
 - Often chosen to be the same as the diffuse term $k_a = k_d$

$$L_o(x, \omega_o) = k_a L_{i,a}$$

Full Phong Reflection Model

- Phong illumination model for *multiple* point light sources

$$L_r = k_a L_{i,a} + k_d \sum_l L_l (I_l \cdot N) + k_s \sum_l L_l (R(I_l) \cdot V)^{k_e} \text{ (Phong)}$$

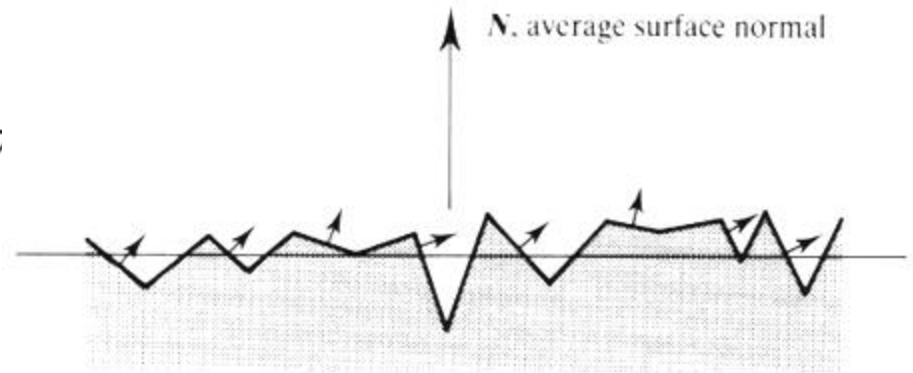
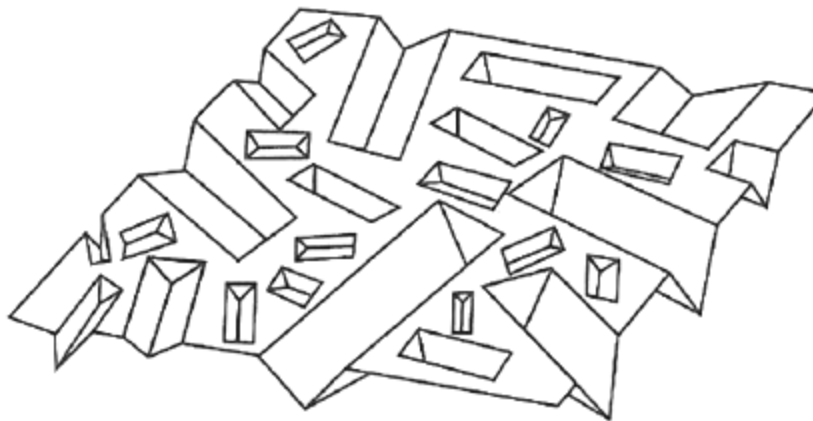
$$L_r = k_a L_{i,a} + k_d \sum_l L_l (I_l \cdot N) + k_s \sum_l L_l (H_l \cdot N)^{k_e} \text{ (Blinn)}$$

- Diffuse reflection (contribution only depends on incoming cosine)
 - Ambient and glossy reflection (Phong or Blinn-Phong)
- **Typically: Color of specular reflection k_s is white**
 - Often separate specular and diffuse color (common extension, OGL)
- **Empirical reflection model!**
 - Contradicts physics
 - Purely local illumination
 - Only direct light from the light sources + constant ambient term
- **Optimization: Lights & viewer assumed to be far away**



Microfacet BRDF Model

- **Physically-Inspired Models**
 - Isotropic microfacet collection
 - Microfacets assumed as perfectly smooth reflectors
- **BRDF**
 - Distribution of microfacets
 - Often probabilistic distribution of orientation or V-groove assumption
 - Planar reflection properties
 - Self-masking, shadowing

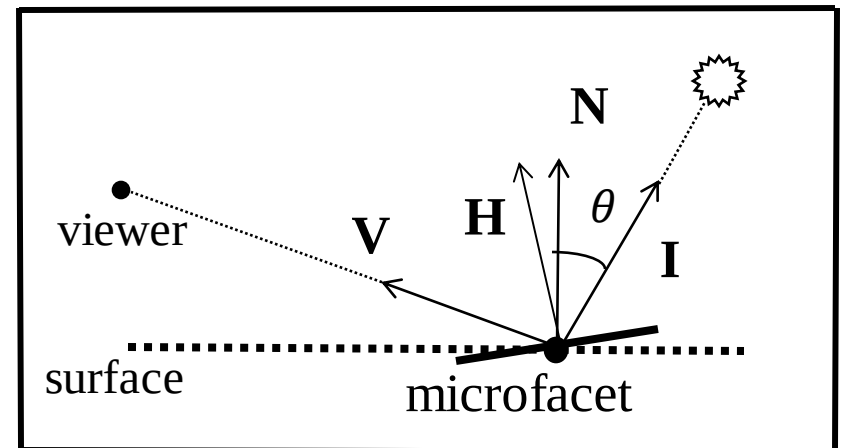


Ward Reflection Model

- **BRDF**

$$f_r = \frac{\rho_d}{\pi} + \frac{\rho_s}{\sqrt{(I \cdot N)(V \cdot N)}} \frac{\exp\left(-\frac{\tan^2 \angle H, N}{\sigma^2}\right)}{4\pi\sigma^2}$$

- σ standard deviation (RMS) of surface slope
 - Simple expansion to anisotropic model (σ_x, σ_y)
 - Empirical, not physics-based
- **Inspired by notion of reflecting microfacets**
 - Convincing results
 - Good match to measured data



Cook-Torrance Reflection Model

- **Cook-Torrance reflectance model**

- Is based on the *microfacet* model
- BRDF is defined as the sum of a diffuse and a glossy component:

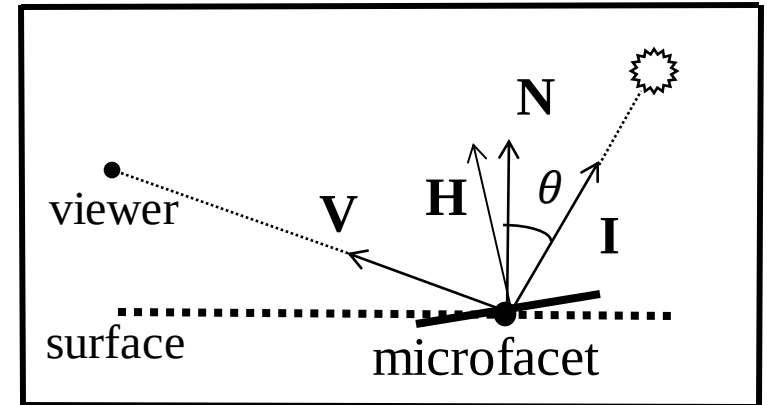
$$f_r = \kappa_d \rho_d + \kappa_g \rho_g; \quad \rho_d + \rho_g \leq 1$$

where ρ_g and ρ_d are the glossy and diffuse coefficients.

- Derivation of the glossy component κ_g is based on a physically derived theoretical reflectance model
- (The original paper talks about “specular” instead of “glossy” as the glossy reflection originates from averaging the specular reflections of many microfacets)

Cook-Torrance Specular Term

$$\kappa_s = \frac{F_\lambda DG}{\pi(N \cdot V)(N \cdot I)}$$



- **D : Distribution function of microfacet orientations**
- **G : Geometrical attenuation factor**
 - represents self-masking and shadowing effects of microfacets
- **F_λ : Fresnel term**
 - computed by Fresnel equation
 - Fraction of specularly reflected light for each planar microfacet
- **$N \cdot V$: Proportional to visible surface area**
- **$N \cdot I$: Proportional to illuminated surface area**

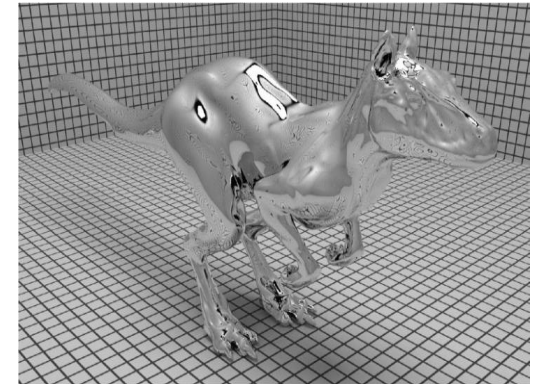
Electric Conductors (e.g. Metals)

- Assume ideally smooth surface
- Perfect specular reflection of light, rest is absorbed
- Reflectance is defined by Fresnel formula based on:
 - Index of refraction η
 - Absorption coefficient κ
 - Both wavelength dependent
- Given for parallel and perpendicular polarized light

Object	η	k
Gold	0.370	2.820
Silver	0.177	3.638
Copper	0.617	2.63
Steel	2.485	3.433

$$r_{\parallel}^2 = \frac{(\eta^2 + k^2) \cos^2 \theta_i - 2\eta \cos \theta_i + 1}{(\eta^2 + k^2) \cos^2 \theta_i + 2\eta \cos \theta_i + 1}$$

$$r_{\perp}^2 = \frac{(\eta^2 + k^2) - 2\eta \cos \theta_i + \cos^2 \theta_i}{(\eta^2 + k^2) + 2\eta \cos \theta_i + \cos^2 \theta_i}$$



- θ_i, θ_t : Angle between ray & plane, incident & transmitted

- For unpolarized light:

$$F_r = \frac{1}{2}(r_{\parallel}^2 + r_{\perp}^2)$$

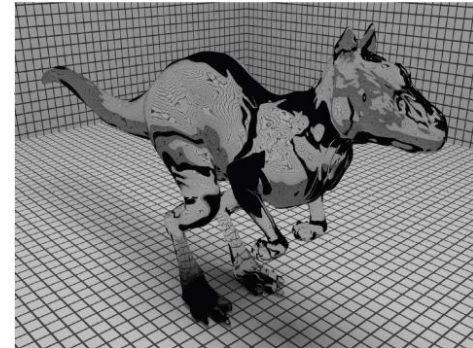
Dielectrics (e.g. Glass)

- Assume ideally smooth surface
- Non-reflected light is perfectly transmitted: $1 - F_r$
 - They do not conduct electricity
- Fresnel formula depends on:
 - Refr. index: speed of light in vacuum vs. medium
 - Refractive index in incident medium $\eta_i = c_0 / c_i$
 - Refractive index in transmitted medium $\eta_t = c_0 / c_t$
- Given for parallel and perpendicular polarized light

$$r_{\parallel} = \frac{\eta_t \cos \theta_i - \eta_i \cos \theta_t}{\eta_t \cos \theta_i + \eta_i \cos \theta_t}$$

$$r_{\perp} = \frac{\eta_i \cos \theta_i - \eta_t \cos \theta_t}{\eta_i \cos \theta_i + \eta_t \cos \theta_t},$$

Medium	Index of refraction η
Vacuum	1.0
Air at sea level	1.00029
Ice	1.31
Water (20° C)	1.333
Fused quartz	1.46
Glass	1.5–1.6
Sapphire	1.77
Diamond	2.42



- For unpolarized light:

$$F_r = \frac{1}{2}(r_{\parallel}^2 + r_{\perp}^2)$$

Microfacet Distribution Functions

- **Isotropic Distributions** $D(\omega) \Rightarrow D(\alpha)$ $\alpha = \angle N, H$
 - α : angle to average normal of surface
 - m : average slope of the microfacets

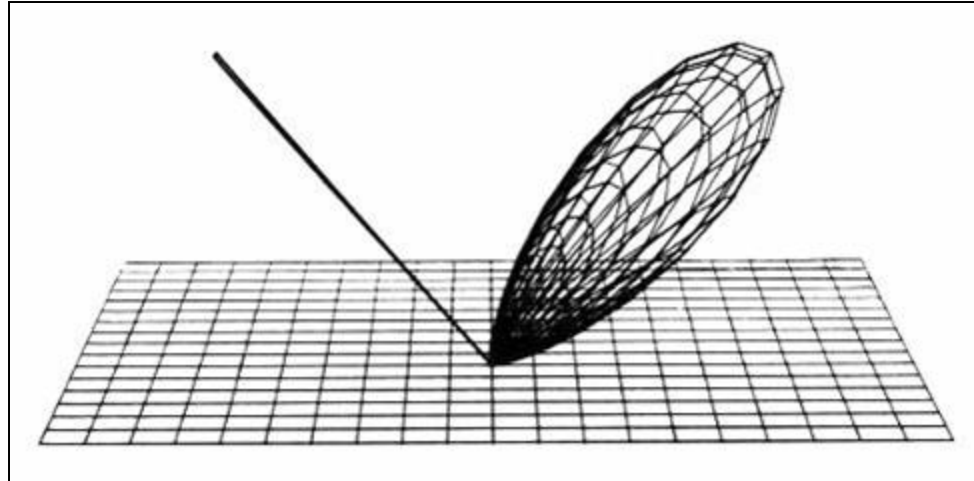
- **Blinn:**
$$D(\alpha) = \cos^{\frac{\ln 2}{\cos m}} \alpha$$

- **Torrance-Sparrow**
 - Gaussian
$$D(\alpha) = e^{-\left(\frac{\alpha}{m}\right)^2}$$

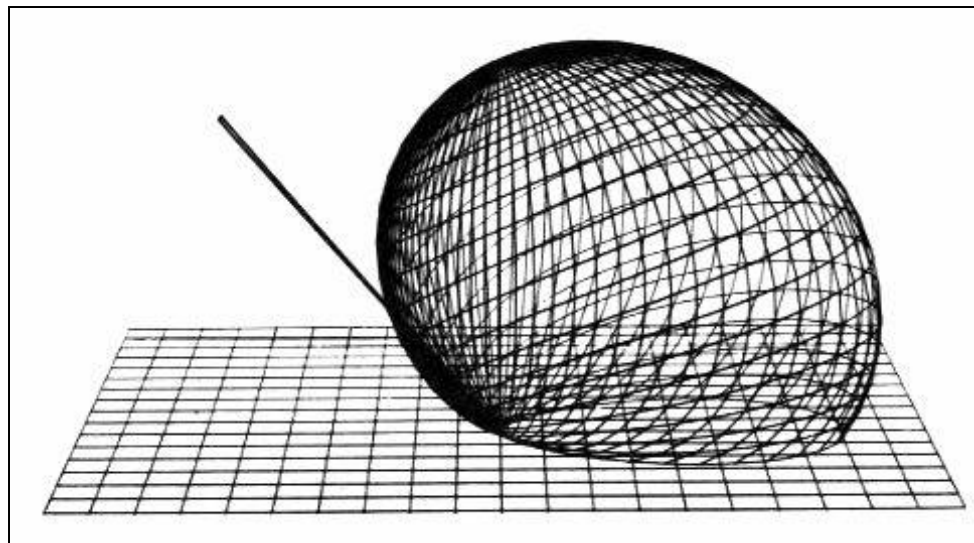
- **Beckmann**
 - Used by Cook-Torrance
$$D(\alpha) = \frac{1}{\pi m^2 \cos^4 \alpha} e^{-\left(\frac{\tan \alpha}{m}\right)^2}$$

Beckman Microfacet Distribution

$m=0.2$



$m=0.6$



Geometric Attenuation Factor

- V-shaped grooves
- Fully illuminated and visible

$$G = 1$$

- Partial masking of reflected light

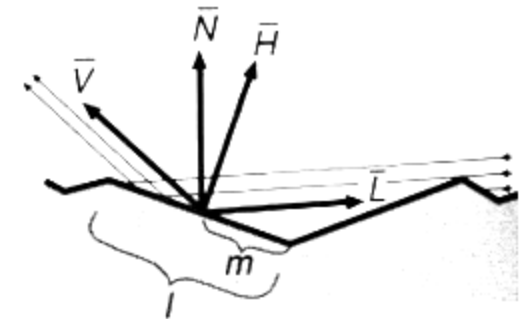
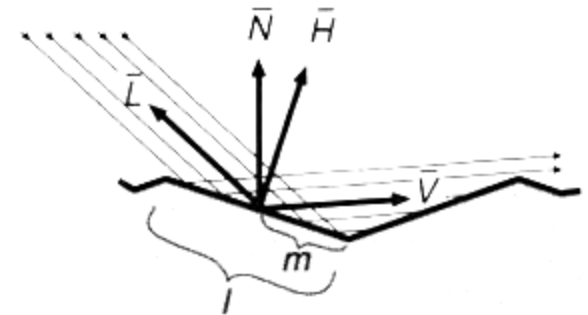
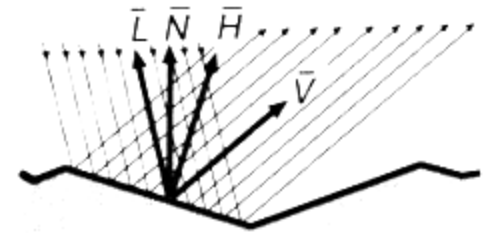
$$G = \frac{2(\underline{N} \cdot \underline{H})(\underline{N} \cdot \underline{V})}{(\underline{V} \cdot \underline{H})}$$

- Partial shadowing of incident light

$$G = \frac{2(\underline{N} \cdot \underline{H})(\underline{N} \cdot \underline{I})}{(\underline{V} \cdot \underline{H})}$$

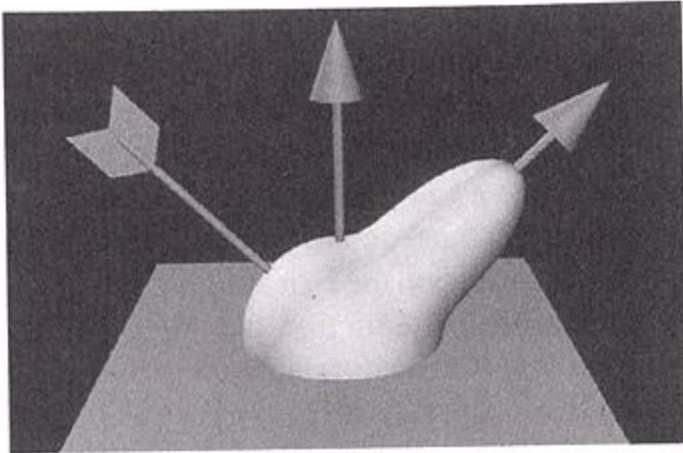
- Final

$$G = \min \left\{ 1, \frac{2(\underline{N} \cdot \underline{H})(\underline{N} \cdot \underline{V})}{(\underline{V} \cdot \underline{H})}, \frac{2(\underline{N} \cdot \underline{H})(\underline{N} \cdot \underline{I})}{(\underline{V} \cdot \underline{H})} \right\}$$

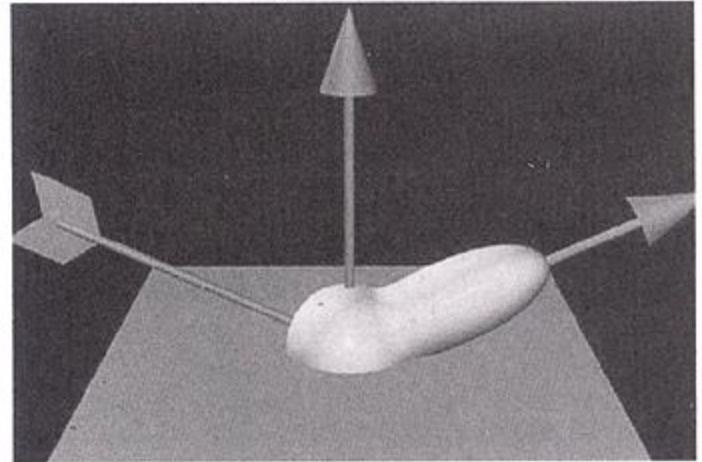


Comparison Phong vs. Torrance

Phong:

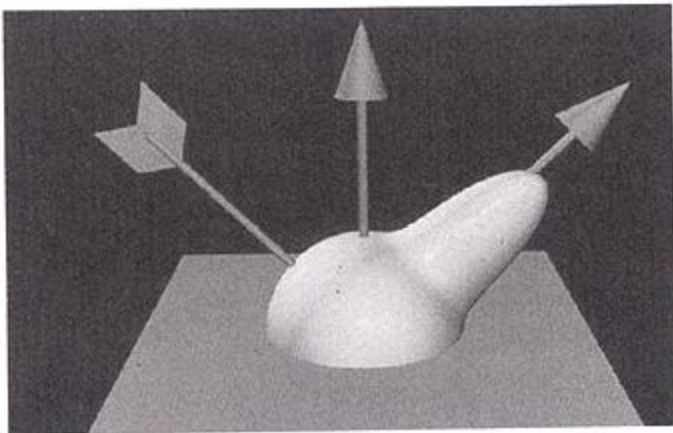


(a)

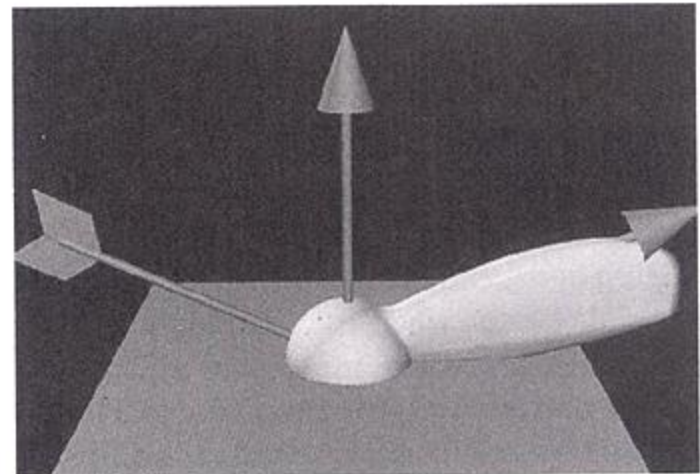


(b)

Torrance:



(c)



(d)

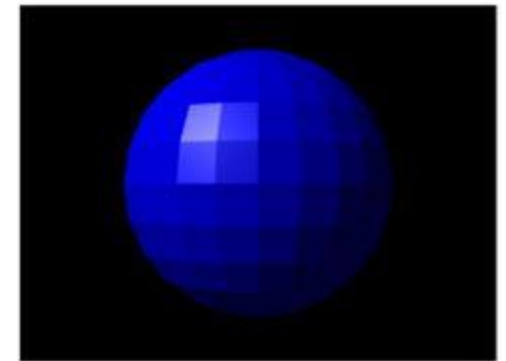
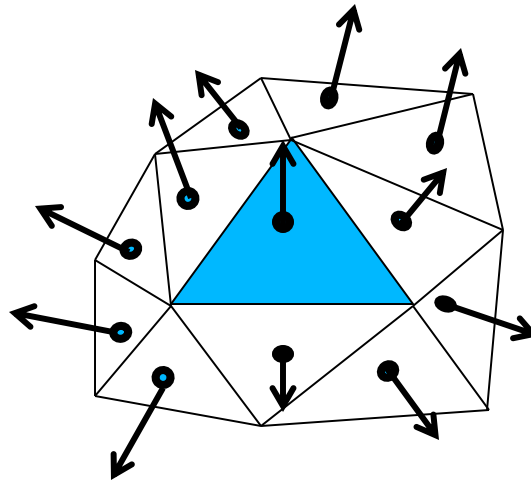
SHADING

What is Shading?

- **Shading**
 - Computation of reflected light (radiance) at every pixel
 - In ray tracing typically computed at every hit point
 - In rasterization computed per triangle, vertex, pixel, or sample
- **What is required for shading**
 - Position of shaded point
 - Position of viewpoint
 - Position of light source and its description/parameters
 - Surface normal / local coordinate frame at shaded point
 - Reflectance model (BRDF)

Flat Shading Model

- **Most simple: Constant Shading**
 - Fixed color per polygon/triangle
- **Shading Model: Flat Shading**
 - Single per-surface normal
 - Single color per polygon
 - Evaluated at one of the vertices (→ OpenGL) or at center



[wikipedia]

Gouraud Shading Model

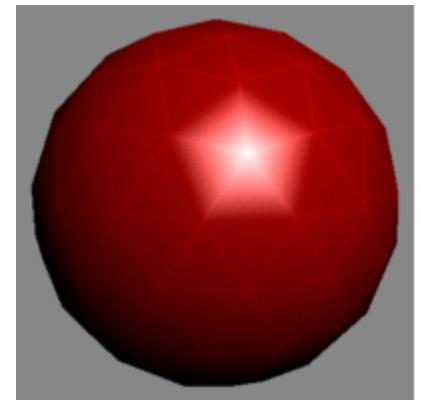
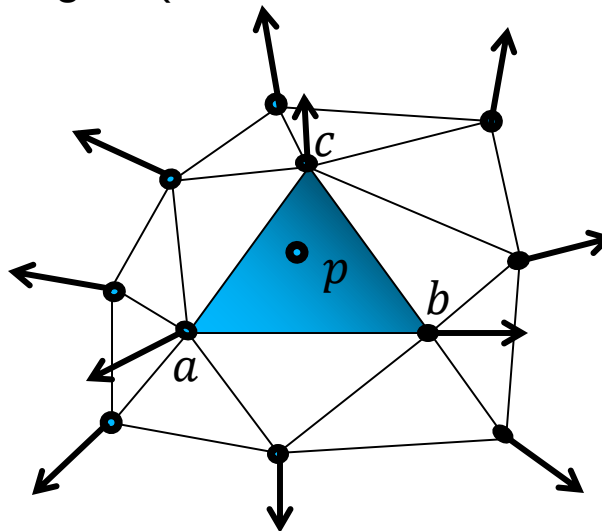
- **Shading Model: Gouraud Shading**

- Computed only at vertices (with per-vertex normal)
 - Normal can be computed from adjacent triangle normals
- Linear interpolation of the shaded colors
 - Computed at all vertices and interpolated
- Often results in shading artifacts along edges
 - **Mach Banding** (i.e. discontinuous 1st derivative)
 - Flickering of highlights (when one of the normal generates strong reflection)

$$L_x \sim f_r(\omega_o, n_x, \omega_i) L_i \cos \theta_i$$

$$L_p = \lambda_1 L_a + \lambda_2 L_b + \lambda_3 L_c$$

- **Barycentric interpolation** within triangle



[wikipedia]

Phong Shading Model

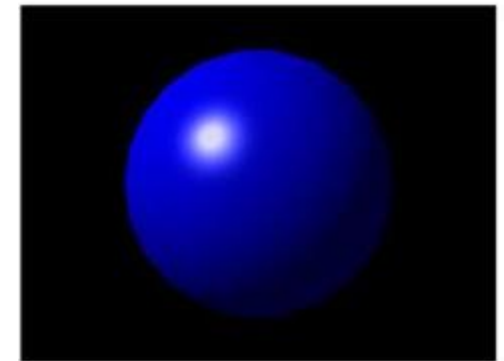
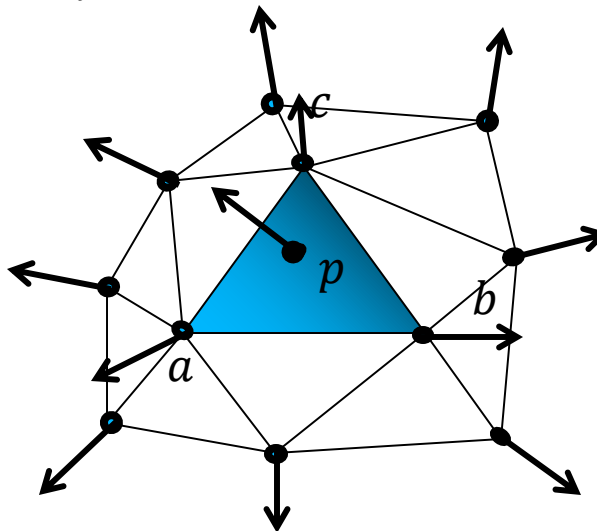
- **Shading Model: Phong Shading**

- Linear interpolation of the surface normal
- Shading is evaluated at every point separately
- Smoother but still off due to hit point offset from apparent surface

$$n_p = \frac{\lambda_1 n_1 + \lambda_2 n_2 + \lambda_3 n_3}{\| \lambda_1 n_1 + \lambda_2 n_2 + \lambda_3 n_3 \|}$$

$$L_p \sim f_r(\omega_o, n_p, \omega_i) L_i \cos \theta_i$$

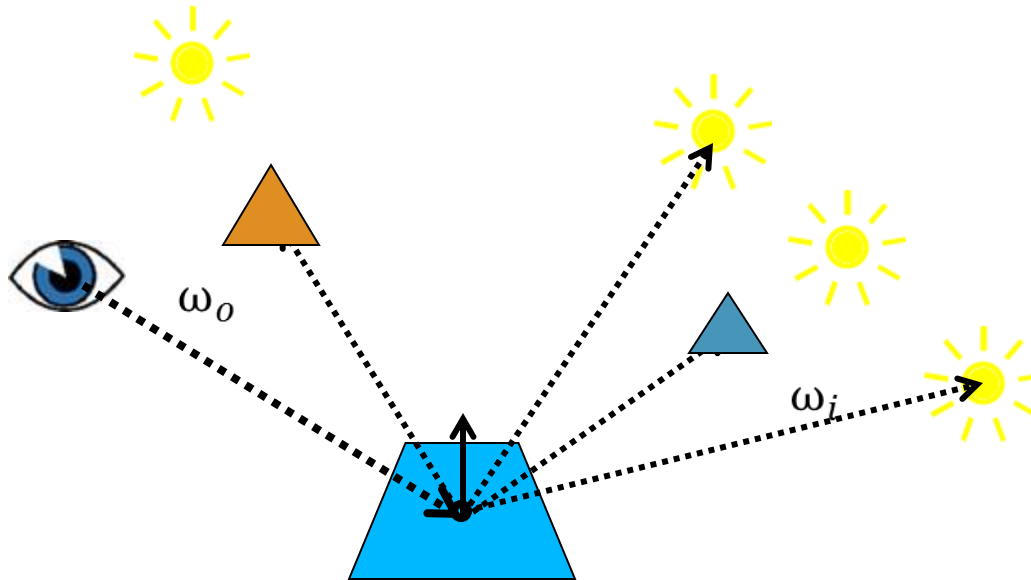
- Barycentric interpolation of normal within the triangle
- With subsequent renormalization



[wikipedia]

Occlusion / Shadows

- **The point on the surface might be in shadow**
 - Rasterization (OpenGL):
 - Not easily done
 - Can use shadow map or shadow volumes (→ later)
 - Ray tracing
 - Simply trace ray to light source and test for occlusion



Area Light sources

- **Typically approximated by sampling**
 - Replacing area with some point light sources
 - Often randomly sampled

