

Computer Graphics

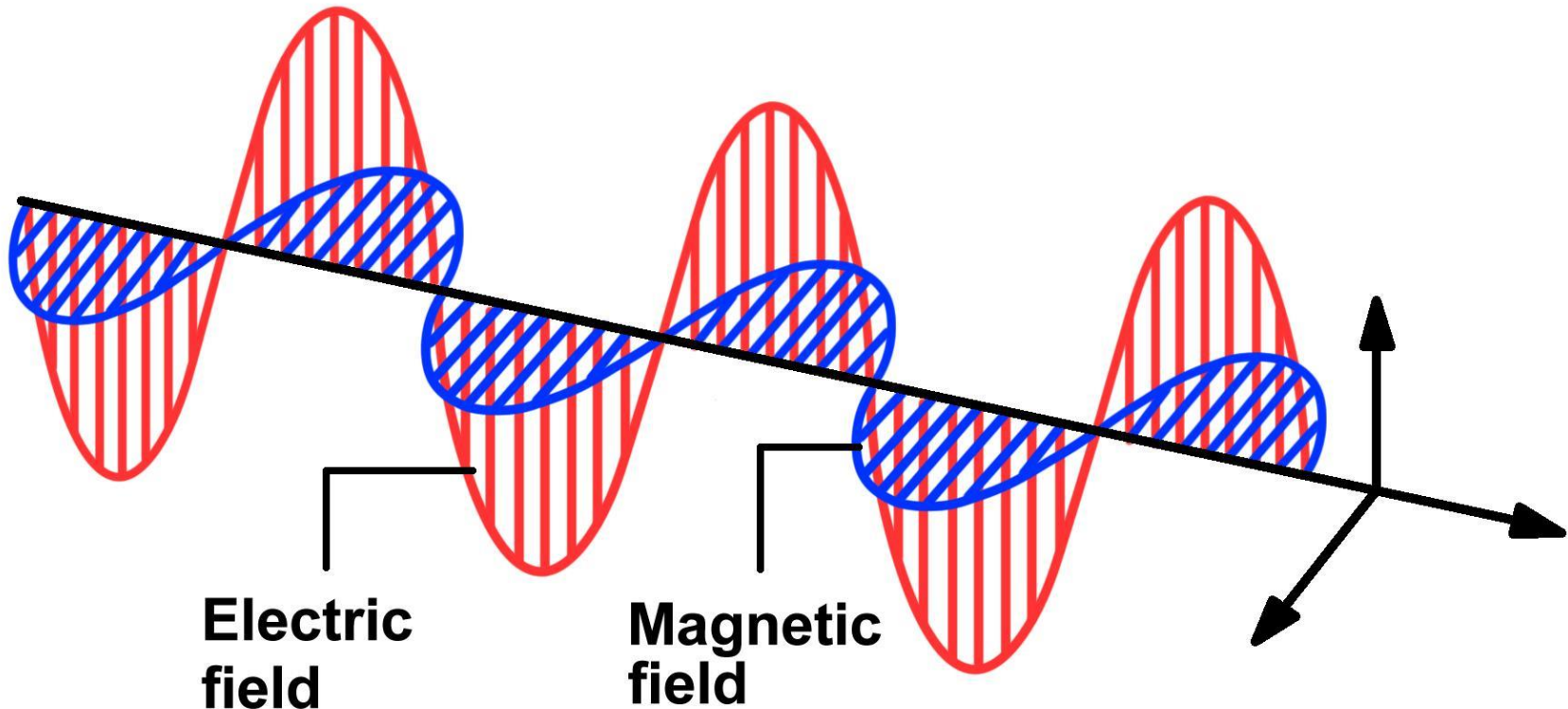
- Light Transport -

Philipp Slusallek

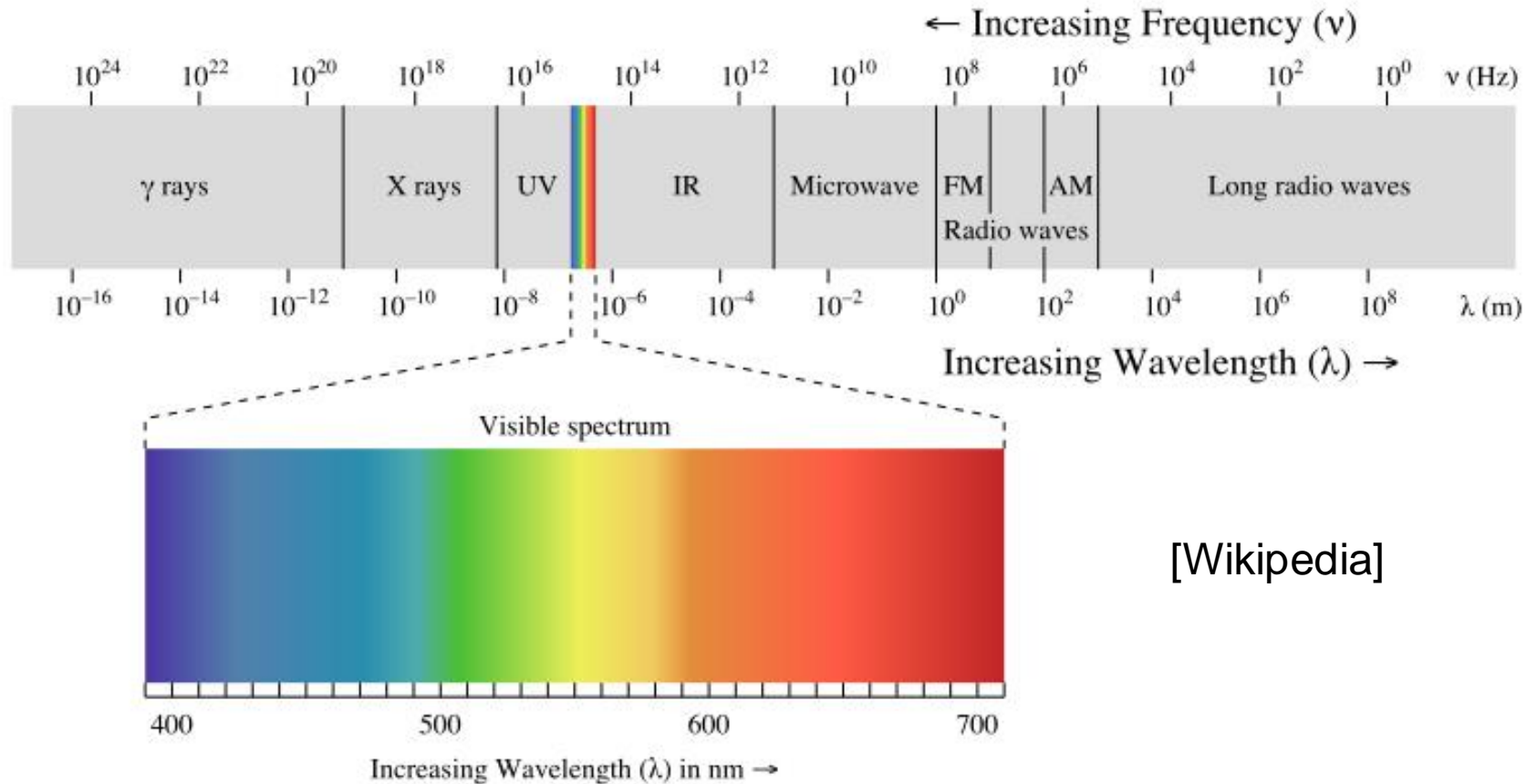
LIGHT

What is Light ?

- **Electro-magnetic wave propagating at speed of light**



What is Light ?



[Wikipedia]

What is Light ?

- **Ray**
 - Linear propagation
 - Geometrical optics / ray optics
- **Vector**
 - Polarization
 - **Jones Calculus**: matrix representation,
 - Has been used in graphics with extended ray model
- **Wave**
 - Diffraction, interference
 - **Maxwell equations**: propagation of light
 - Partial simulation possible using extended ray model, e.g. radar
- **Particle**
 - Light comes in discrete energy quanta: photons
 - **Quantum theory**: interaction of light with matter
- **Field**
 - Electromagnetic force: exchange of virtual photons
 - **Quantum Electrodynamics (QED)**: interaction between particles

What is Light ?

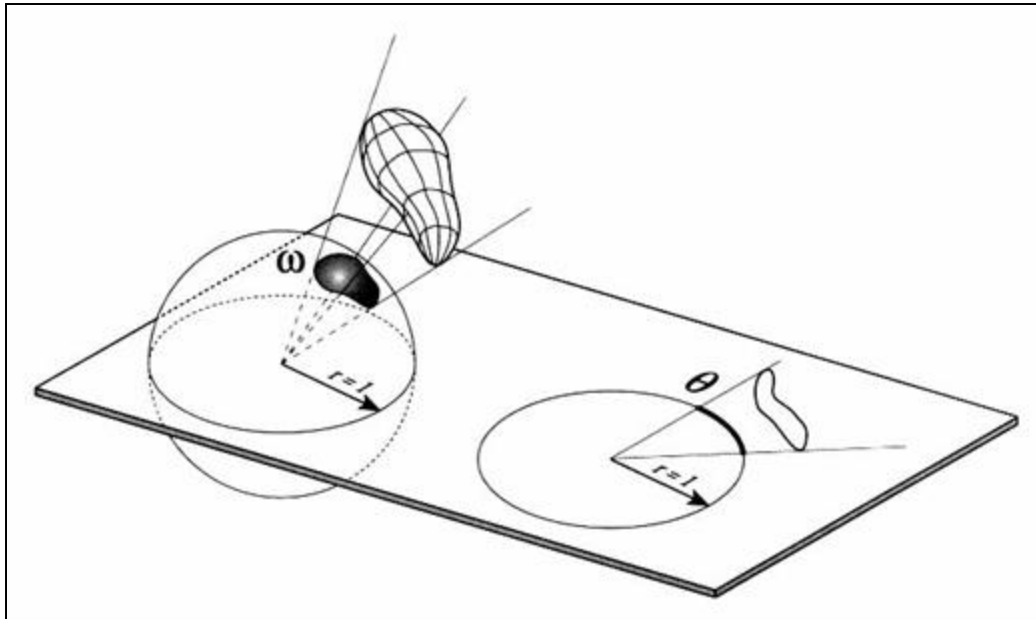
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Light in Computer Graphics

- **Based on human visual perception**
 - Focused on macroscopic geometry (→ Reflection Models)
 - Only tristimulus color model (e.g. RGB, → Human Visual System)
 - Psycho-physics: tone mapping, compression, ... (→ RIS course)
- **Ray optic assumptions**
 - Macroscopic objects
 - Incoherent light
 - Light: scalar, real-valued quantity
 - Linear propagation
 - Superposition principle: light contributions add up, do not interact
 - No attenuation in free space
- **Limitations**
 - No microscopic structures ($\approx \lambda$), no volumetric effects (for now)
 - No polarization, no coherent light (e.g. laser)
 - No diffraction, interference, dispersion, etc. ...

Angle and Solid Angle

- The **angle** θ (in radians) subtended by a curve in the plane is the length of the corresponding arc on the unit circle: $l = \theta r = 1$
- The **solid angle** Ω , $d\omega$ subtended by an object is the surface area of its projection onto the unit sphere
 - Units for solid angle: steradian [sr] (dimensionless, $\leq 4\pi$)



Solid Angle in Spherical Coords

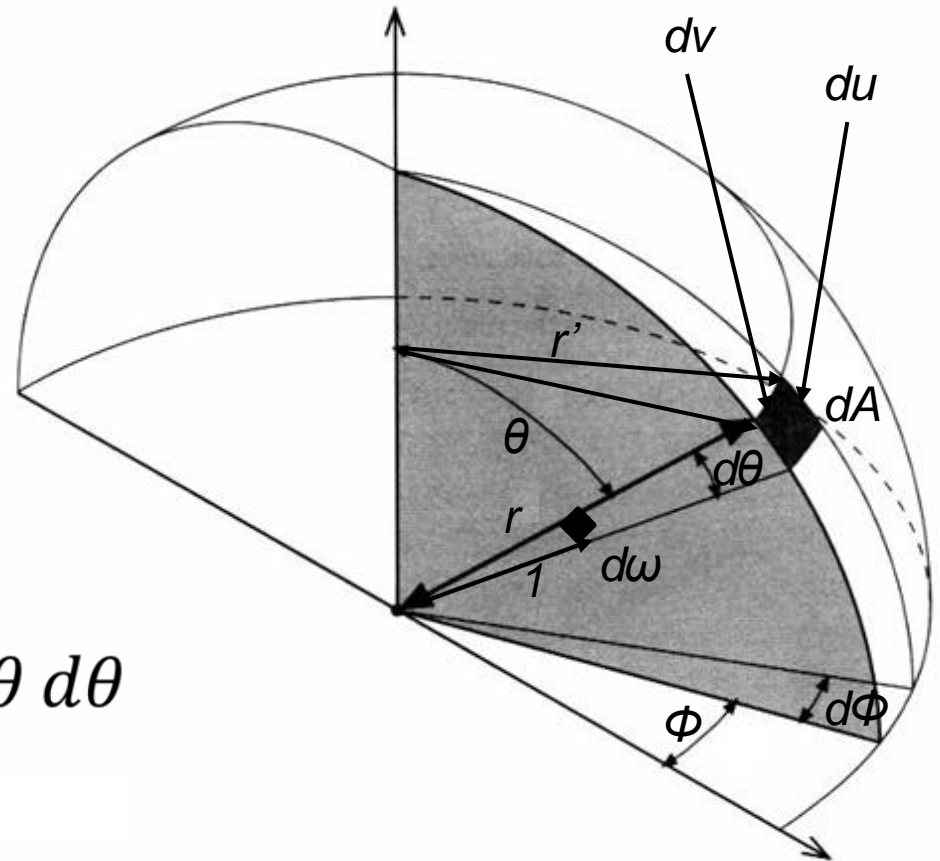
- **Infinitesimally small solid angle $d\omega$**

- $du = r d\theta$
- $dv = r' d\Phi = r \sin \theta d\Phi$
- $dA = du dv = r^2 \sin \theta d\theta d\Phi$
- $d\omega = dA/r^2 = \sin \theta d\theta d\Phi$

- **Finite solid angle**

- Integration of area, e.g.

$$\Omega = \int_{\phi_0}^{\phi_1} d\phi \int_{\theta_0(\phi)}^{\theta_1(\phi)} \sin \theta d\theta$$



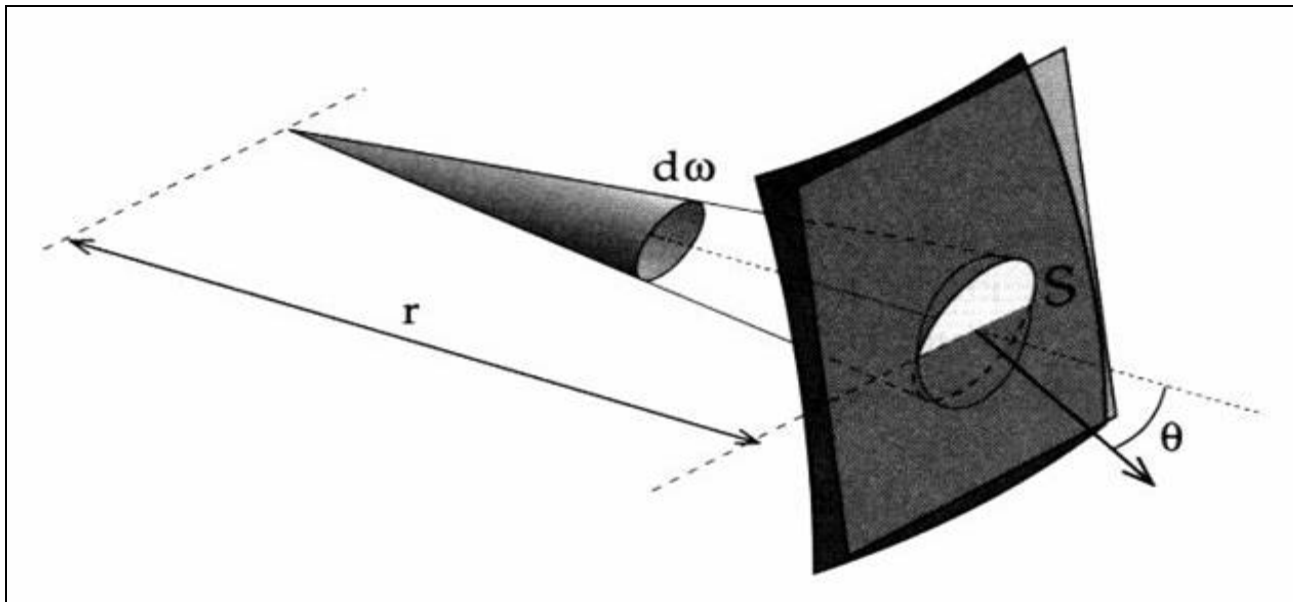
Solid Angle for a Surface

- The solid angle subtended by a small surface patch S with area dA is obtained (i) by projecting it orthogonal to the vector r from the origin:

$$dA \cos \theta$$

and (ii) dividing by the squared distance to the origin: $d\omega = \frac{dA \cos \theta}{r^2}$

$$\Omega = \iint_S \frac{\vec{r} \cdot \vec{n}}{r^3} dA$$



Radiometry

- **Definition:**

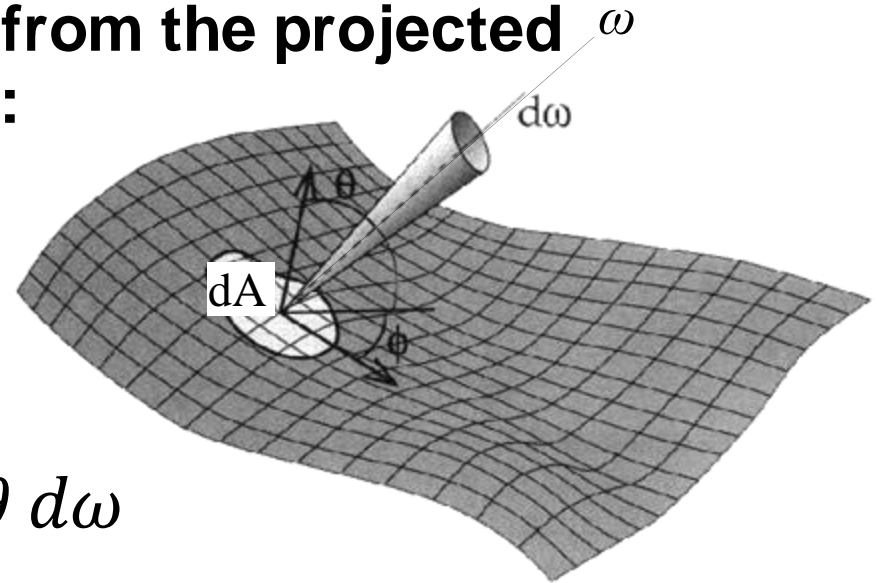
- Radiometry is the science of measuring radiant energy transfer. Radiometric quantities have physical meaning and can be directly measured using proper equipment such as spectral photometers.

- **Radiometric Quantities**

- | | | | |
|-----------------|----------------------------|--------|---------------------------------------|
| – Energy | [J] | Q | (#Photons x Energy = $n \cdot h\nu$) |
| – Radiant power | [watt = J/s] | Φ | (Total Flux) |
| – Intensity | [watt/sr] | I | (Flux from a point per s.angle) |
| – Irradiance | [watt/m ²] | E | (Incoming flux per area) |
| – Radiosity | [watt/m ²] | B | (Outgoing flux per area) |
| – Radiance | [watt/(m ² sr)] | L | (Flux per area & proj. s. angle) |

Radiometric Quantities: Radiance

- Radiance is used to describe radiant energy transfer
- Radiance L is defined as
 - The power (flux) traveling through areas dA around some point x
 - In a specified **direction** $\omega = (\theta, \varphi)$
 - Per unit area **perpendicular** to the direction of travel
 - Per unit solid angle
- Thus, the differential power $d^2\Phi$ radiated through the **differential solid angle** $d\omega$, from the projected differential area $dA \cos \theta$ is:



$$d^2\Phi = L(x, \omega) dA(x) \cos \theta d\omega$$

Radiometric Quantities: Irradiance

- **Irradiance E** is defined as the **total power per unit area** (flux density) incident onto a surface. To obtain the total flux incident to dA , the **incoming** radiance L_i is integrated over the upper hemisphere Ω_+ above the surface:

$$E \equiv \frac{d\Phi}{dA}$$
$$d\Phi = \left[\int_{\Omega_+} L_i(x, \omega) \cos \theta d\omega \right] dA$$
$$E(x) = \int_{\Omega_+} L_i(x, \omega) \cos \theta d\omega = \int_0^{2\pi} \int_0^{\frac{\pi}{2}} L_i(x, \omega) \cos \theta \sin \theta d\theta d\phi$$

Radiometric Quantities: Radiosity

- **Radiosity B** is defined as the **total power per unit area** (flux density) **exitant from** a surface. To obtain the total flux incident to dA , the **outgoing** radiance L_o is integrated over the upper hemisphere Ω_+ above the surface:

$$B \equiv \frac{d\Phi}{dA}$$
$$d\Phi = \left[\int_{\Omega_+} L_o(x, \omega) \cos \theta d\omega \right] dA$$
$$B(x) = \int_{\Omega_+} L_o(x, \omega) \cos \theta d\omega = \int_0^{2\pi} \int_0^{\frac{\pi}{2}} L_o(x, \omega) \cos \theta \sin \theta d\theta d\phi$$

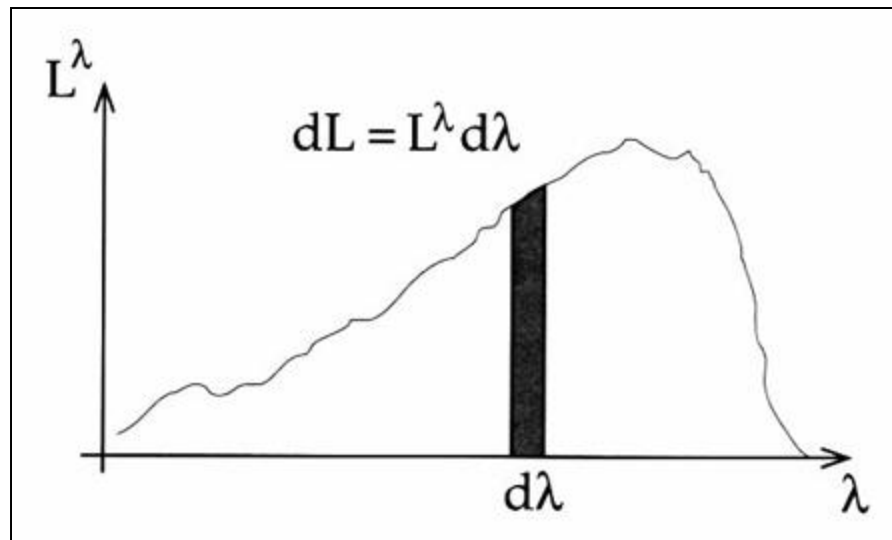
Spectral Properties

- **Wavelength**

- Light is composed of electromagnetic waves
- These waves have different frequencies (and wavelengths)
- Most transfer quantities are continuous functions of wavelength

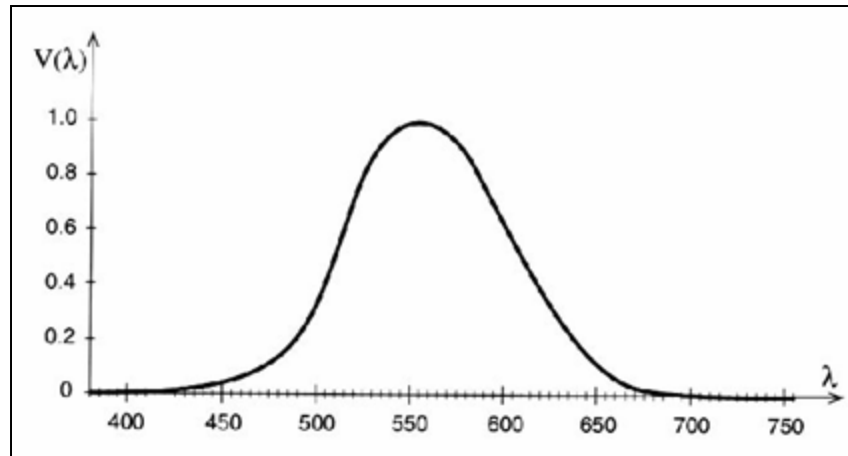
- **In graphics**

- Each measurement $L(x, \omega)$ is for a discrete band of wavelength only
 - Often **R**(ed, long), **G**(reen, medium), **B**(lue, short) (but see later)



Photometry

- The human eye is sensitive to a limited range of wavelengths
 - Roughly from 380 nm to 780 nm
- Our visual system responds differently to different wavelengths
 - Can be characterized by the *Luminous Efficiency Function $V(\lambda)$*
 - Represents the average human spectral response
 - Separate curves exist for light and dark adaptation of the eye
- Photometric quantities are derived from radiometric quantities by *integrating* them against this function



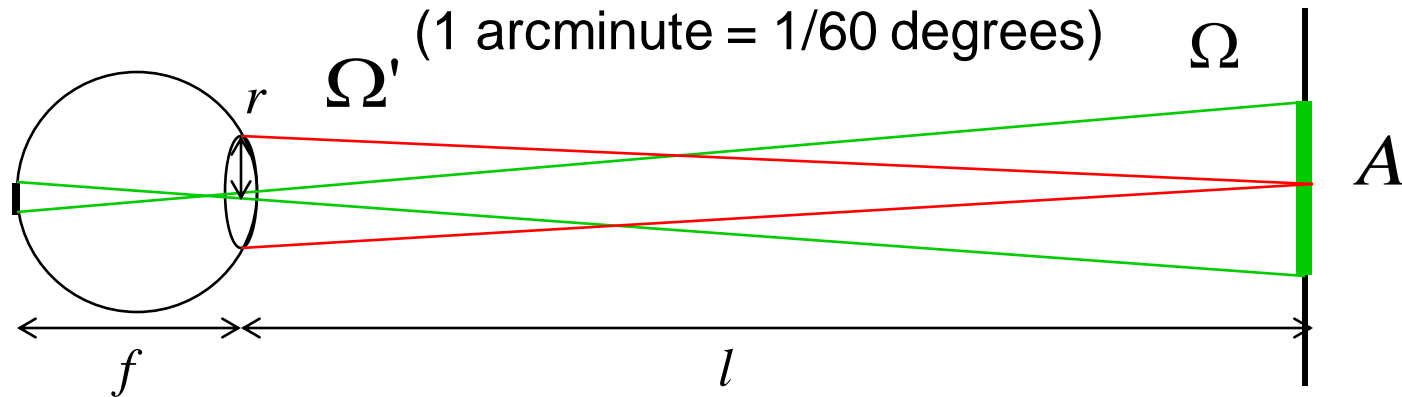
Radiometry vs. Photometry

Physics-based quantities

Perception-based quantities

Radiometry		→	Photometry	
W	Radiant power	→	Luminous power	Lumens (lm)
W/m ²	Radiosity	→	Luminosity	Lux (lm/m ²)
	Irradiance		Illuminance	
W/m ² /sr	Radiance	→	Luminance	cd/m ² (lm/m ² /sr)

Perception of Light



photons / second = **flux** = **energy / time** = **power** (Φ)

rod sensitive to flux

Solid angle of a rod = **resolution** ($\approx 1 \text{ arcminute}^2$)

Ω

projected rod size = **area A**

$$A \approx l^2 \cdot \Omega$$

angular extent of pupil aperture ($r \leq 4 \text{ mm}$) = **solid angle**

$$\Omega' \approx \pi \cdot r^2 / l^2$$

flux proportional to area and solid angle

$$\Phi = L A \Omega'$$

radiance = flux per unit area per unit solid angle

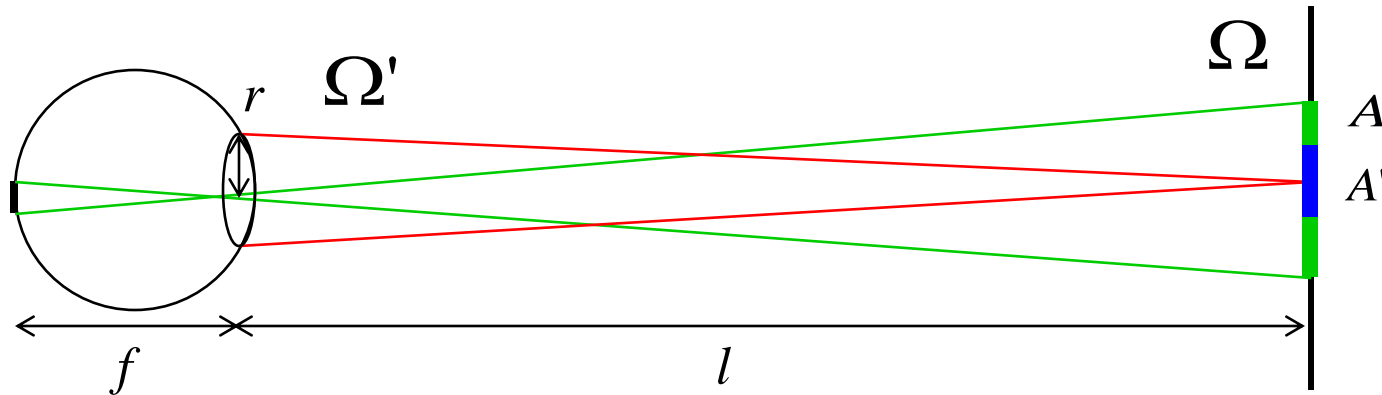
$$L = \frac{\Phi}{\Omega' \cdot A}$$

The eye detects radiance

As l increases:

$$\Phi_0 = L \cdot l^2 \cdot \Omega \cdot \pi \frac{r^2}{l^2} = L \cdot \text{const}$$

Brightness Perception

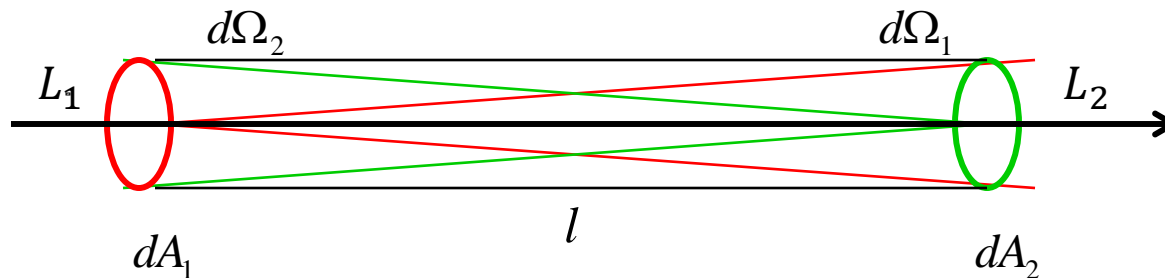


- $A' > A$: area of sun covers more than one rod:
photon flux per rod stays constant
- $A' < A$: photon flux per rod decreases

Where does the Sun turn into a star ?

- Depends on apparent Sun disc size on retina
- Photon flux per rod stays the same on Mercury, Earth or Neptune
- Photon flux per rod decreases when $\Omega' < 1 \text{ arcminute}^2$ (~ beyond Neptune)

Radiance in Space



Flux leaving surface 1 must be equal to flux arriving on surface 2

$$L_1 d\Omega_1 dA_1 = L_2 d\Omega_2 dA_2$$

From geometry follows $d\Omega_1 = \frac{dA_2}{l^2}$ $d\Omega_2 = \frac{dA_1}{l^2}$

Ray throughput T : $T = d\Omega_1 \cdot dA_1 = d\Omega_2 \cdot dA_2 = \frac{dA_1 \cdot dA_2}{l^2}$

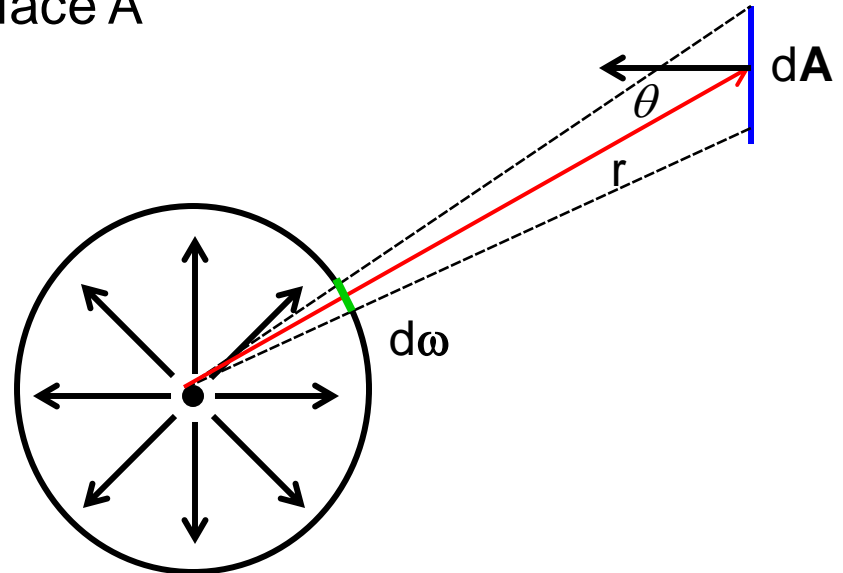
$$L_1 = L_2$$

The **radiance** in the direction of a light ray **remains constant** as it propagates along the ray

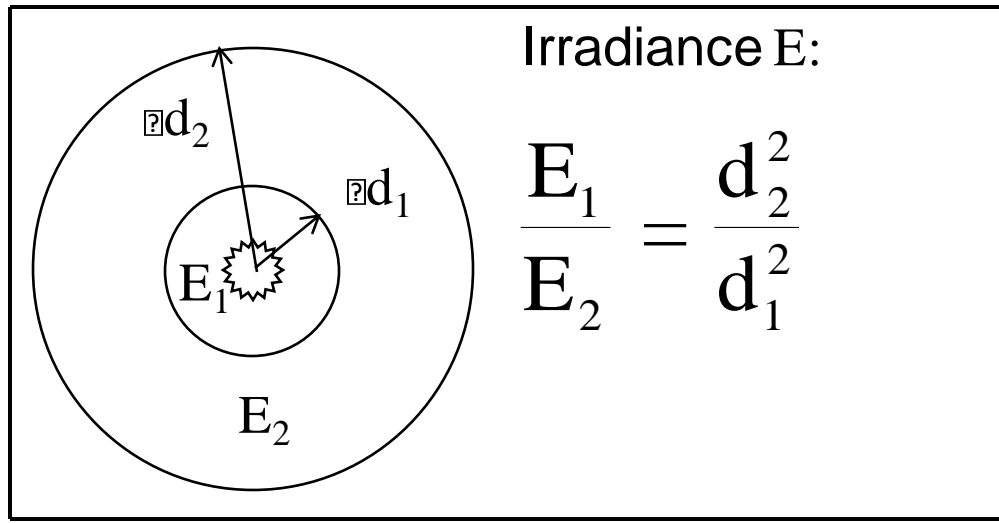
Point Light Source

- **Point light with *isotropic* (same in all dir.) radiance**
 - Power (total flux) of a point light source
 - Φ_g = Power of the light source [watt]
 - Intensity of a light source (radiance cannot be defined, no area)
 - $I = \Phi_g / 4\pi$ [watt/sr]
 - Irradiance on a sphere with radius r around light source:
 - $E_r = \Phi_g / (4 \pi r^2)$ [watt/m²]
 - Irradiance on some other surface A

$$\begin{aligned} E(x) &= \frac{d\Phi_g}{dA} = \frac{d\Phi_g}{d\omega} \frac{d\omega}{dA} = I \frac{d\omega}{dA} \\ &= \frac{\Phi_g}{4\pi} \cdot \frac{dA \cos \theta}{r^2 dA} \\ &= \frac{\Phi_g}{4\pi} \cdot \frac{\cos \theta}{r^2} = \frac{\Phi_g}{4\pi r^2} \cdot \cos \theta \end{aligned}$$



Inverse Square Law

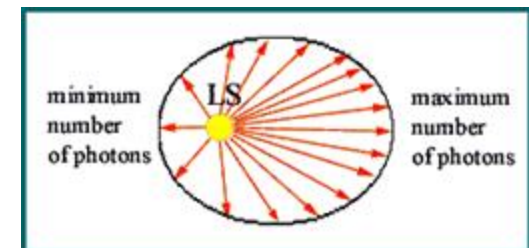
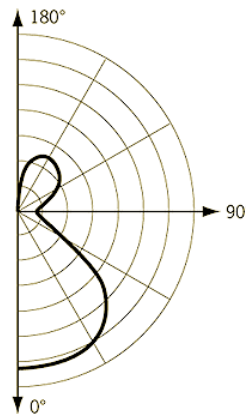
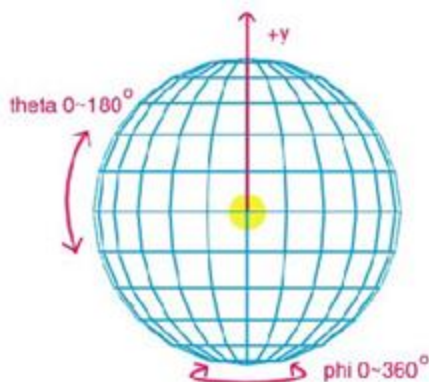
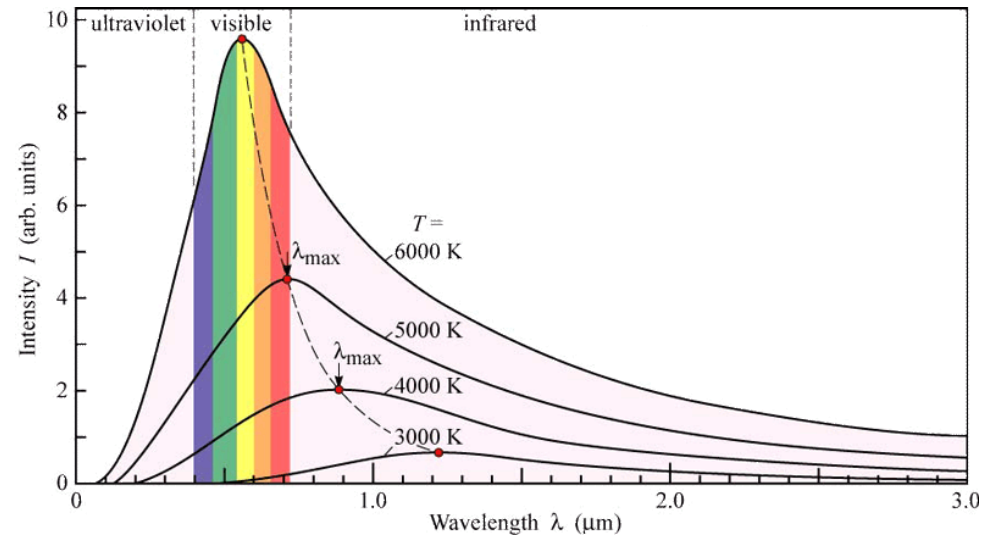


- **Irradiance E : power per m^2**
 - Illuminating quantity
- **Distance-dependent**
 - Double distance from emitter: area of sphere is four times bigger
- **Irradiance falls off with inverse of squared distance**
 - Only for point light sources (!)

Light Source Specifications

- **Power (total flux)**
 - Emitted energy / time
- **Active emission size**
 - Point, line, area, volume
- **Spectral distribution**
 - Thermal, line spectrum
- **Directional distribution**
 - Goniometric diagram

Black body radiation (see later)



Light Source Classification

Radiation characteristics

- **Directional light**
 - Spot-lights
 - Projectors
 - Distant sources
- **Diffuse emitters**
 - Torchieres
 - Frosted glass lamps
- **Ambient light**
 - “Photons everywhere”

Emitting area

- **Volume**
 - Neon advertisements
 - Sodium vapor lamps
 - Fire
 - **Area**
 - CRT/LCD display
 - (Overcast) sky
 - **Line**
 - Clear light bulb, filament
 - **“Point”**
 - Xenon lamp
 - Arc lamp
 - Laser diode
-

Sky Light

- **Sun**
 - Point source (approx.)
 - White light (by def.)
- **Sky**
 - Area source
 - Scattering: blue
- **Horizon**
 - Brighter
 - Haze: whitish
- **Overcast sky**
 - Multiple scattering in clouds
 - Uniform grey
- **Several sky models are available**



Courtesy Lynch & Livingston

LIGHT TRANSPORT

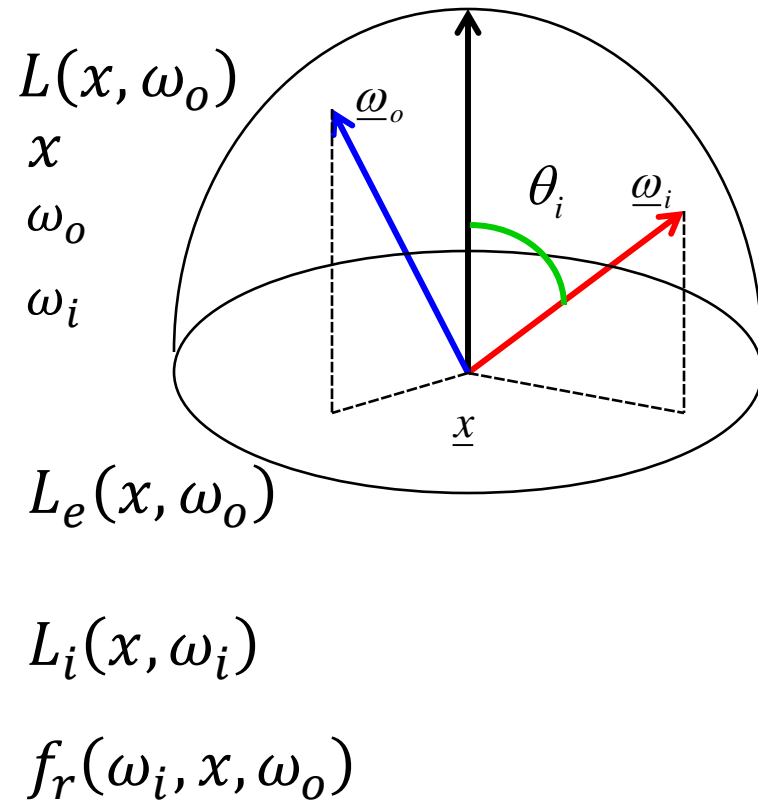
Light Transport in a Scene

- **Scene**
 - Lights (emitters)
 - Object surfaces (partially absorbing)
- **Illuminated object surfaces become emitters, too!**
 - Radiosity = Irradiance minus absorbed photons flux density
 - Radiosity: photons per second per m^2 leaving surface
 - Irradiance: photons per second per m^2 incident on surface
 - But also need to look at directional distribution
- **Light bounces between all mutually visible surfaces**
- **Invariance of radiance in free space**
 - No absorption in-between objects
- **Dynamic energy equilibrium in a scene**
 - Emitted photons = absorbed photons (+ escaping photons)
 - **Global Illumination, discussed in RIS lecture**

Surface Radiance

$$L(x, \omega_o) = L_e(x, \omega_o) + \int_{\Omega_+} f_r(\omega_i, x, \omega_o) L_i(x, \omega_i) \cos \theta_i d\omega_i$$

- **Visible surface radiance**
 - Surface position
 - Outgoing direction
- **Incoming illumination direction**
- **Emission**
- **Reflected light**
 - Incoming radiance from all directions
 - Direction-dependent reflectance
(**BRDF: bidirectional reflectance distribution function**)



Rendering Equation

Typ. Exam
Question!

- **Most important equation for graphics**
 - Expresses energy equilibrium in scene

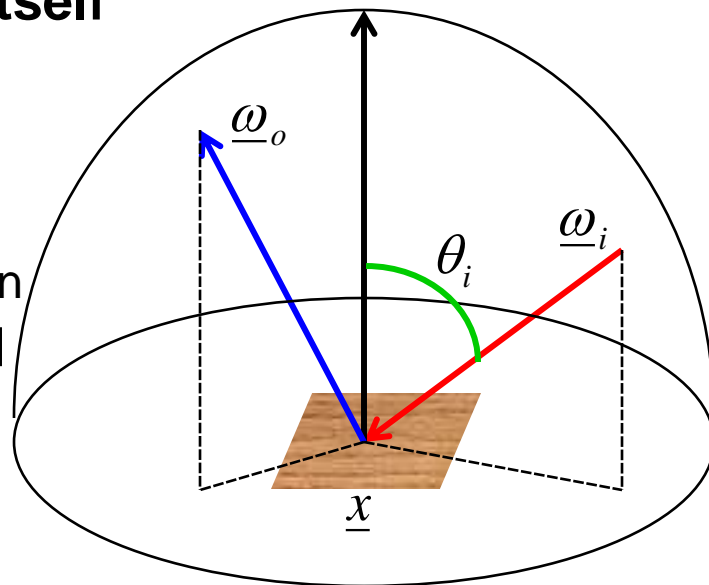
$$L(x, \omega_o) = L_e(x, \omega_o) + \int_{\Omega_+} f_r(\omega_i, x, \omega_o) L_i(x, \omega_i) \cos \theta_i d\omega_i$$

total radiance = emitted + reflected radiance

- **First term: Emission from the surface itself**
 - Non-zero only for light sources

- **Second term: reflected radiance**
 - Integral over all possible incoming directions of radiance times angle-dependent surface reflection function

- **Fredholm integral equation of 2nd kind**
 - **Difficulty: Unknown radiance appears both on the left-hand side and inside the integral**
 - Numerical methods necessary to compute approximate solution



RE: Integrating over Surfaces

- **Outgoing illumination at a point**

$$L(x, \omega_o) = L_e(x, \omega_o) + L_r(x, \omega_o)$$

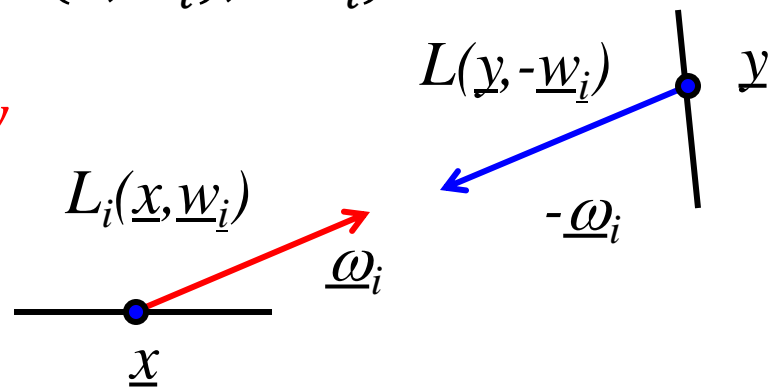
$$L(x, \omega_o) = L_e(x, \omega_o) + \int_{\Omega_+} f_r(\omega_i, x, \omega_o) L_i(x, \omega_i) \cos \theta_i d\omega_i$$

- **Linking with other surface points**

- Incoming radiance at x is outgoing radiance at y

$$L_i(x, \omega_i) = L(y, -\omega_i) = L(RT(x, \omega_i), -\omega_i)$$

- **Ray-Tracing operator:** $RT(x, \omega_i) = y$



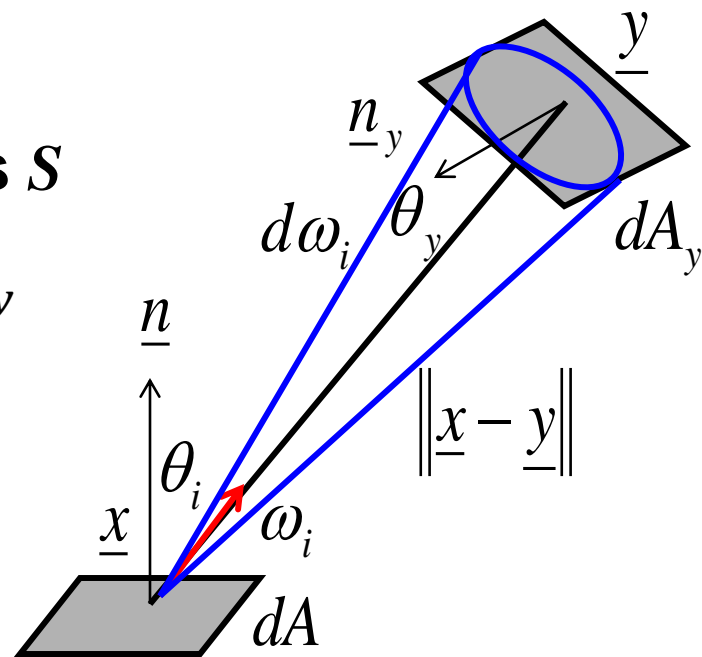
Integrating over Surfaces

- **Outgoing illumination at a point**

$$L(\underline{x}, \omega_o) = L_e(\underline{x}, \omega_o) + \int_{\Omega_+} f_r(\omega_i, \underline{x}, \omega_o) L_i(\underline{x}, \omega_i) \cos \theta_i d\omega_i$$

- **Re-parameterization over surfaces S**

$$d\omega_i = \frac{\cos \theta_y}{\|\underline{x} - \underline{y}\|^2} dA_y$$



$$\begin{aligned} L(\underline{x}, \omega_o) \\ = L_e(\underline{x}, \omega_o) \end{aligned}$$

$$+ \int_{\underline{y} \in S} f_r(\omega(\underline{x}, \underline{y}), \underline{x}, \omega_o) L_i(\underline{x}, \omega(\underline{x}, \underline{y})) V(\underline{x}, \underline{y}) \frac{\cos \theta_i \cos \theta_y}{\|\underline{x} - \underline{y}\|^2} dA_y$$

Integrating over Surfaces

$$\begin{aligned} L(x, \omega_o) \\ &= L_e(x, \omega_o) \\ &+ \int_{y \in S} f_r(\omega(x, y), x, \omega_o) L_i(x, \omega(x, y)) V(x, y) \frac{\cos \theta_i \cos \theta_y}{\|x - y\|^2} dA_y \end{aligned}$$

- **Geometry term:** $G(x, y) = V(x, y) \frac{\cos \theta_i \cos \theta_y}{\|x - y\|^2}$
- **Visibility term:** $V(x, y) = \begin{cases} 1, & \text{if visible} \\ 0, & \text{otherwise} \end{cases}$
- **Integration over all surfaces:** $\int_{y \in S} \cdots dA_y$
$$L(x, \omega_o) = L_e(x, \omega_o) + \int_{y \in S} f_r(\omega(x, y), x, \omega_o) L_i(x, \omega(x, y)) G(x, y) dA_y$$

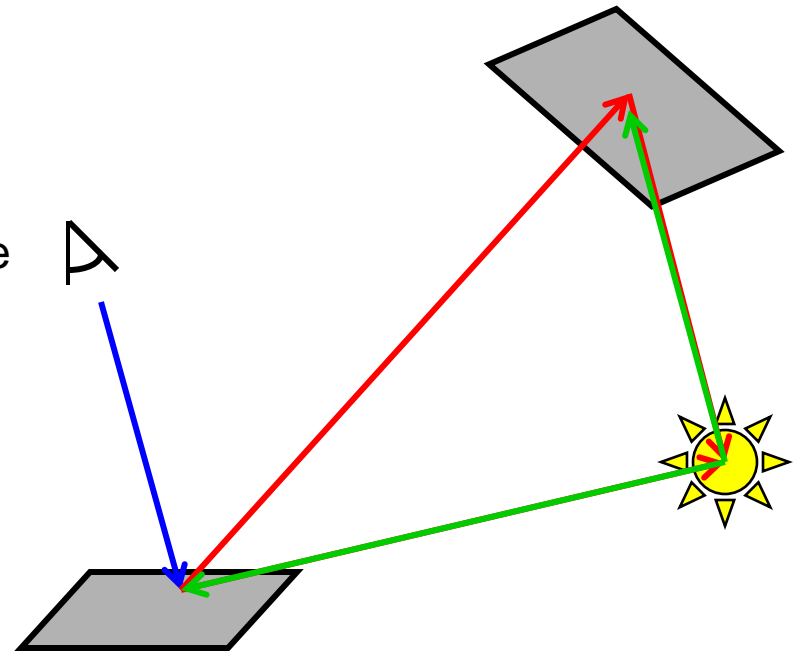
Rendering Equation: Approximations

- **Approximations based only on empirical foundations**
 - An example: polygon rendering in OpenGL (→ later)
- **Using RGB instead of full spectrum**
 - Follows roughly the eye's sensitivity (L , f_r are 3D vectors for RGB)
- **Sampling hemisphere only at discrete directions**
 - Simplifies integration to a summation
- **Reflection function model (BRDF, see later)**
 - Approximation by parameterized functions
 - **Diffuse**: light reflected uniformly in all directions
 - **Specular**: perfect reflection/refraction direction
 - **Glossy**: mirror reflection, but from a rough surface
 - And mixture thereof

Ray Tracing

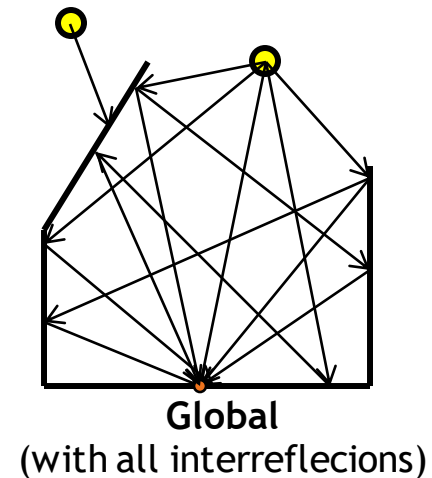
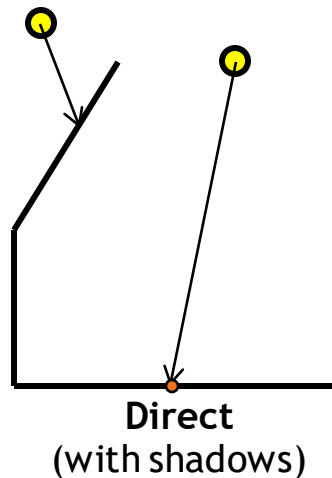
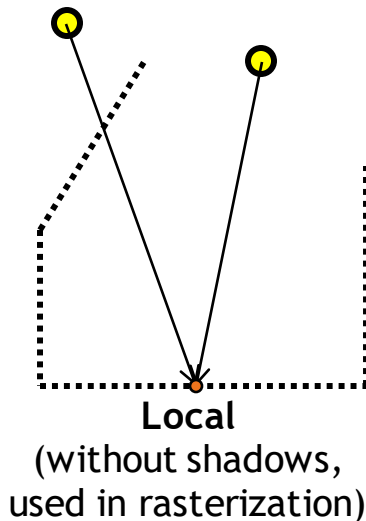
$$L(x, \omega_o) = L_e(x, \omega_o) + \int_{\Omega_+} f_r(\omega_i, x, \omega_o) L_i(x, \omega_i) \cos \theta_i d\omega_i$$

- **Simple ray tracing**
 - Illumination from discrete point light sources only – **direct illumination only**
 - Integral → sum of contributions from each light
 - **No global illumination**
 - Evaluates angle-dependent reflectance function (BRDF) – **shading process**
- **Advanced ray tracing techniques**
 - Recursive ray tracing
 - Multiple reflections/refractions (e.g. for specular surfaces)
 - Ray tracing for global illumination
 - Stochastic sampling (Monte Carlo methods) → RIS course



Different Types of Illumination

- Three types of illumination computations in CG



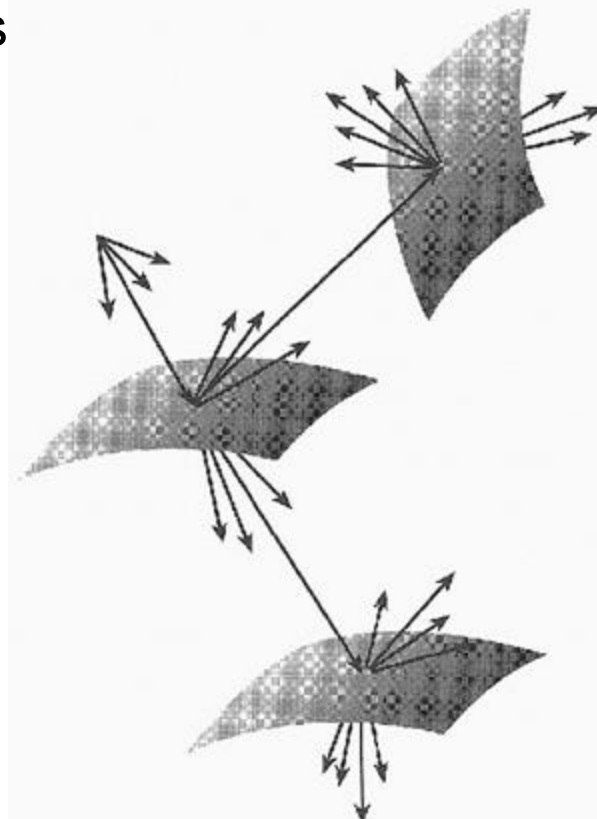
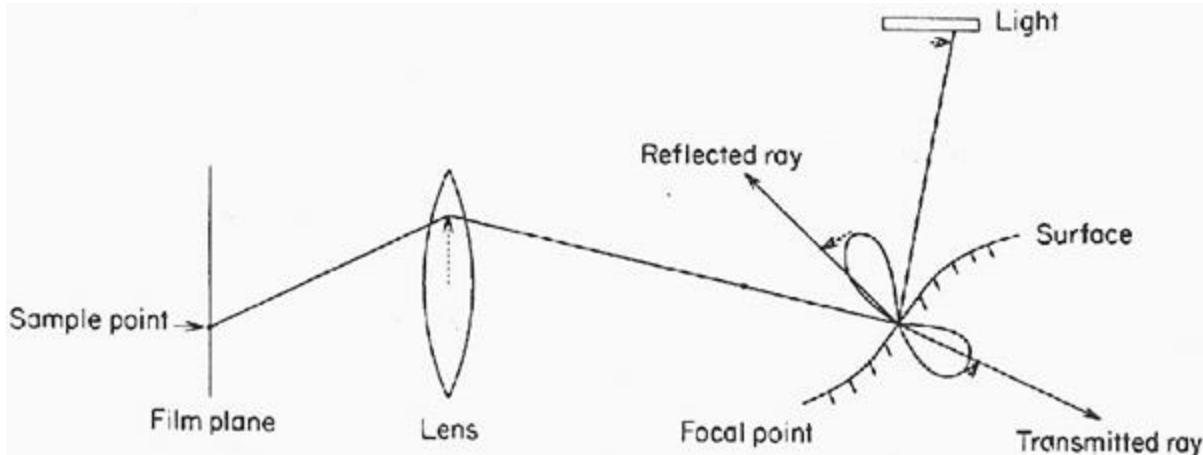
- Ambient Illumination**

- Global illumination is costly to compute
- Indirect illumination (through interreflections) is typically smooth
 - ➔ Approximate via a constant term $L_{i,a}$ (incoming ambient illum.)
- Has no incoming direction, provide ambient reflection term k_a
 - Often chosen to be the same as the diffuse term $k_a = k_d$

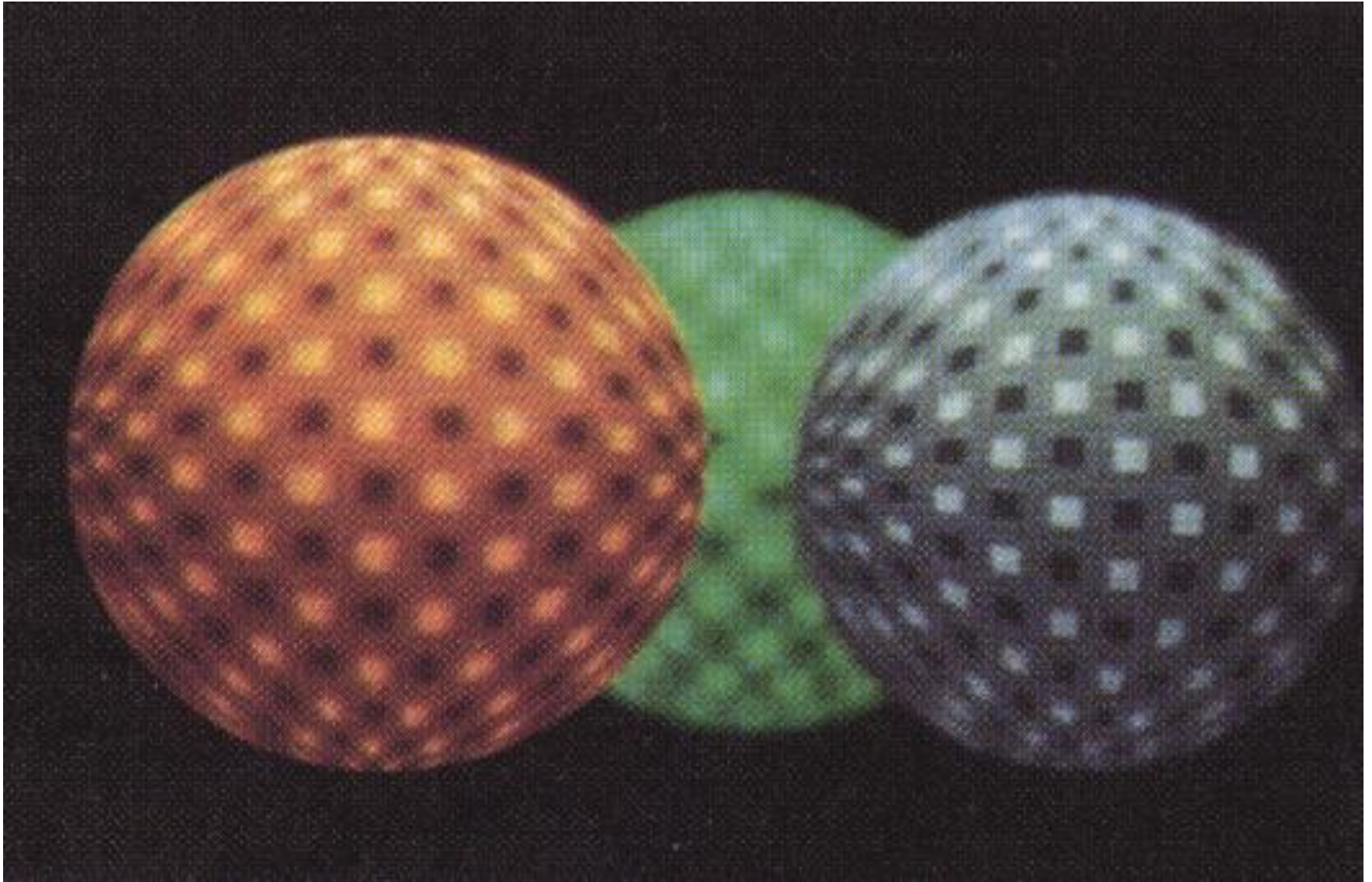
$$L_o(x, \omega_o) = k_a L_{i,a}$$

Distribution Ray Tracing

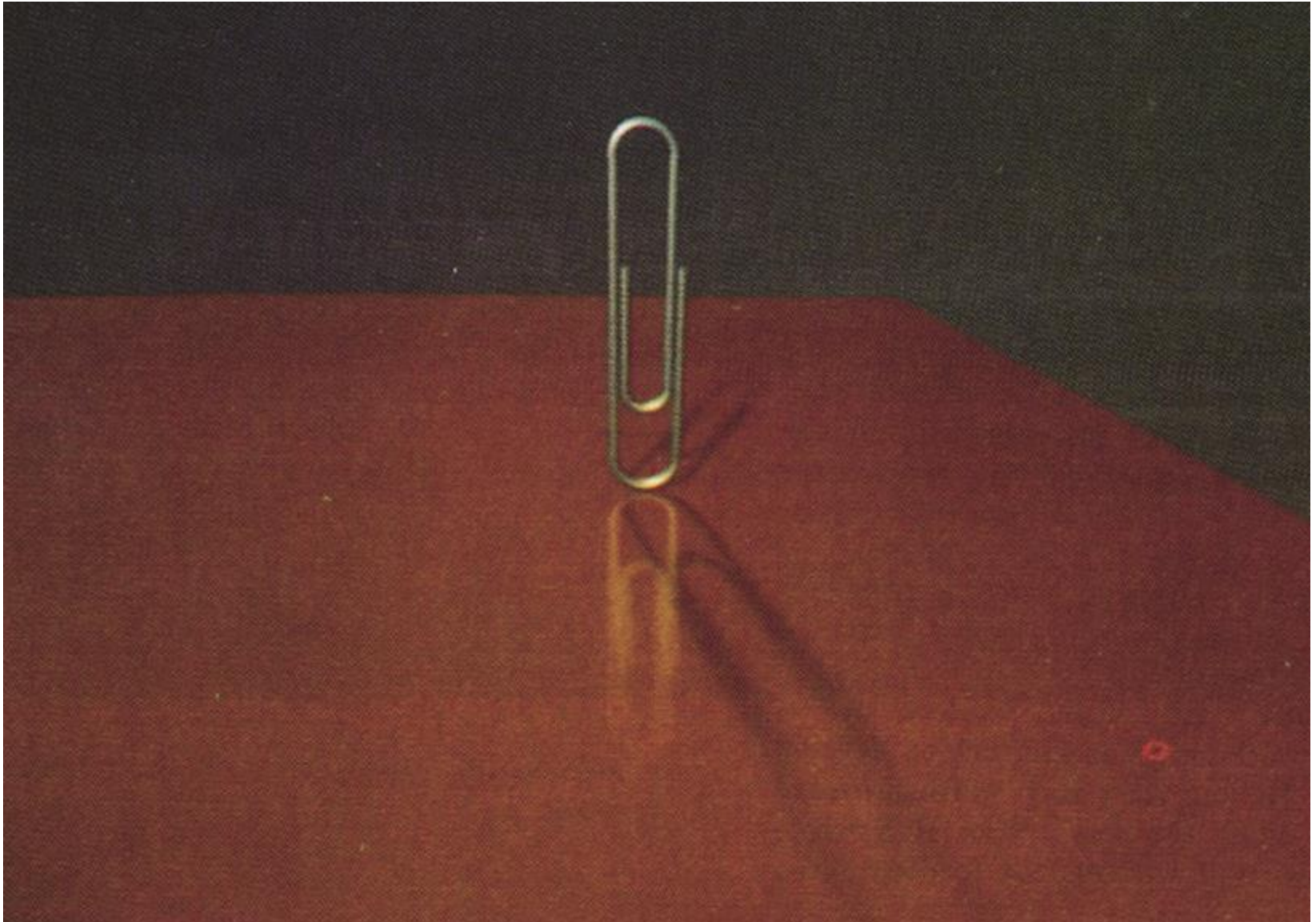
- Formerly called *Distributed* Ray Tracing [Cook`84]
- **Stochastic Sampling of**
 - Pixel: Antialiasing
 - Lens: Depth-of-field
 - BRDF: Sampling of hemisphere & lobes
 - Lights: Smooth shadows from area light sources
 - Time: Motion blur
- **Covered in detail in RIS course**



Depth of Field



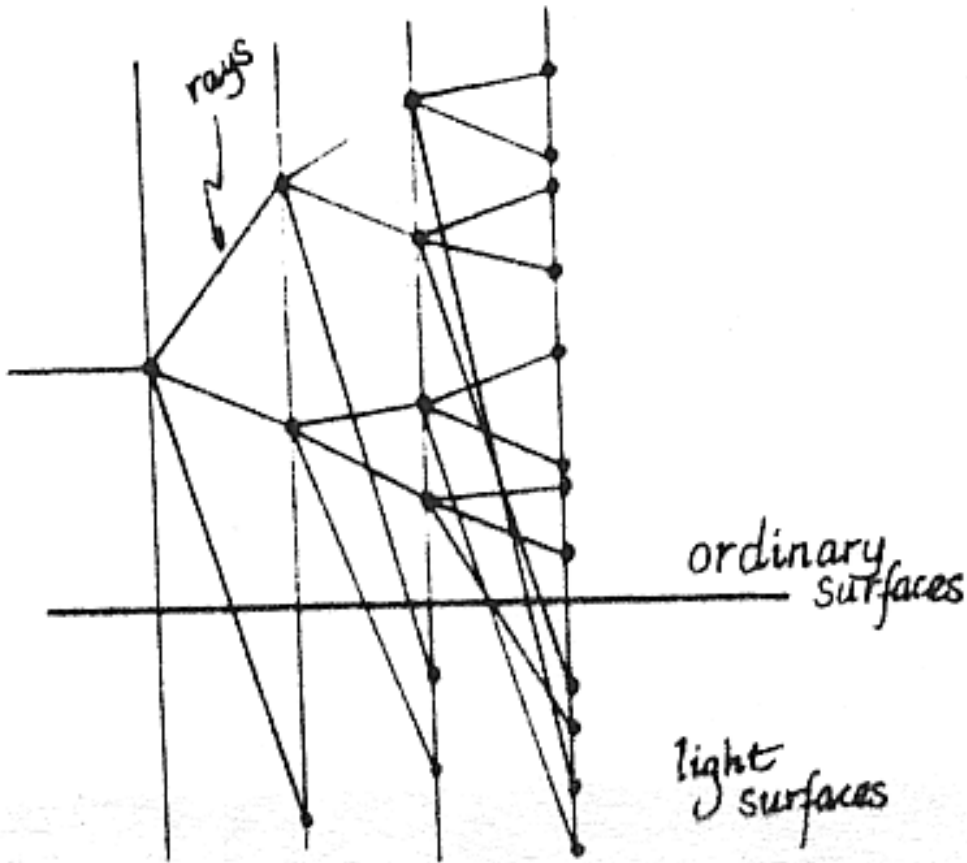
Glossy Reflection



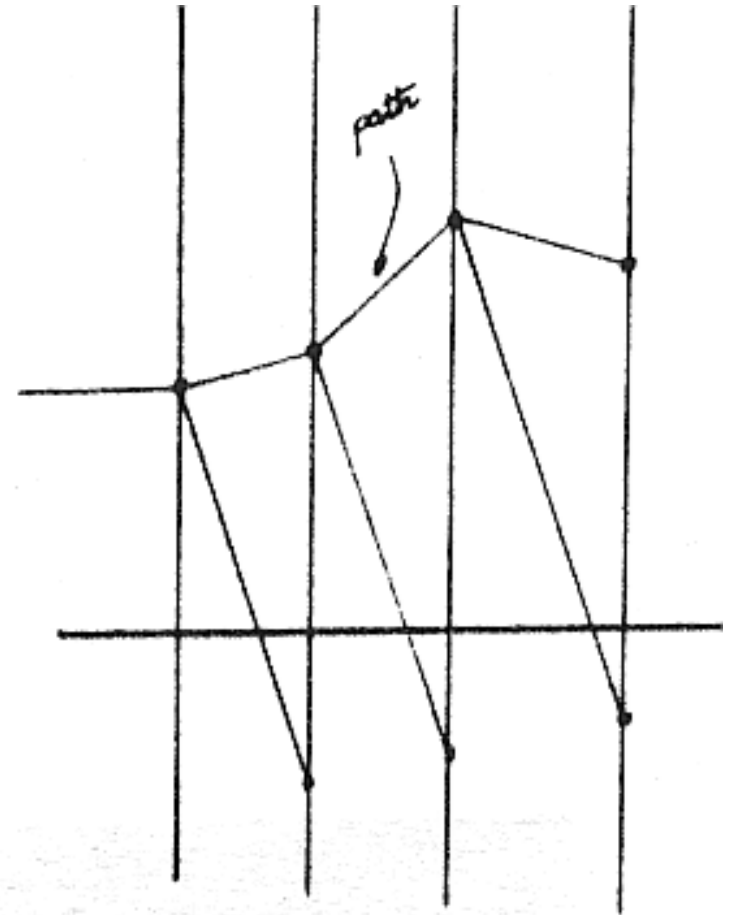
Motion Blur



Comparison to Path Tracing



Distribution Ray Tracing

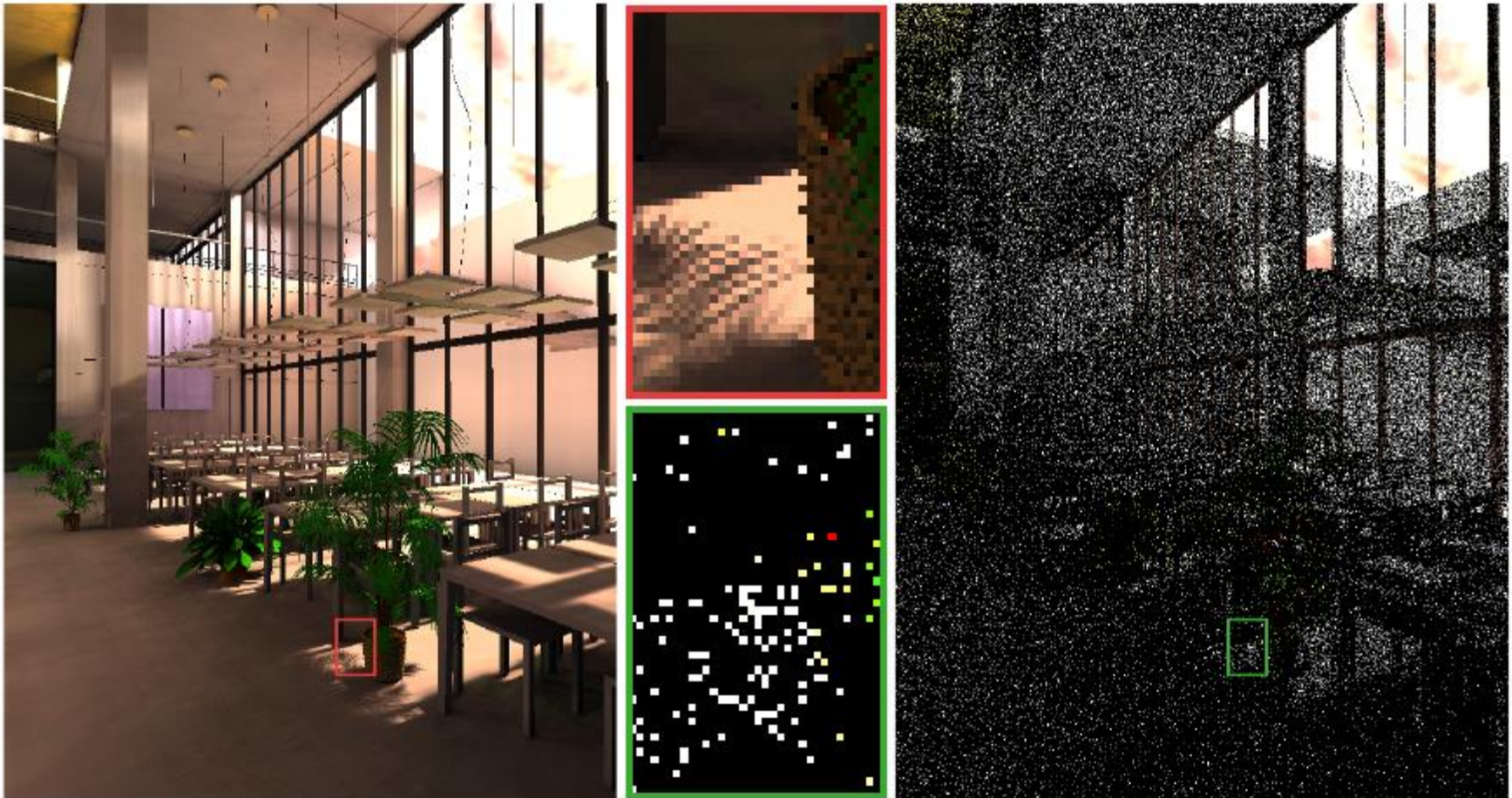


Path Tracing

(figure by Kajiya)

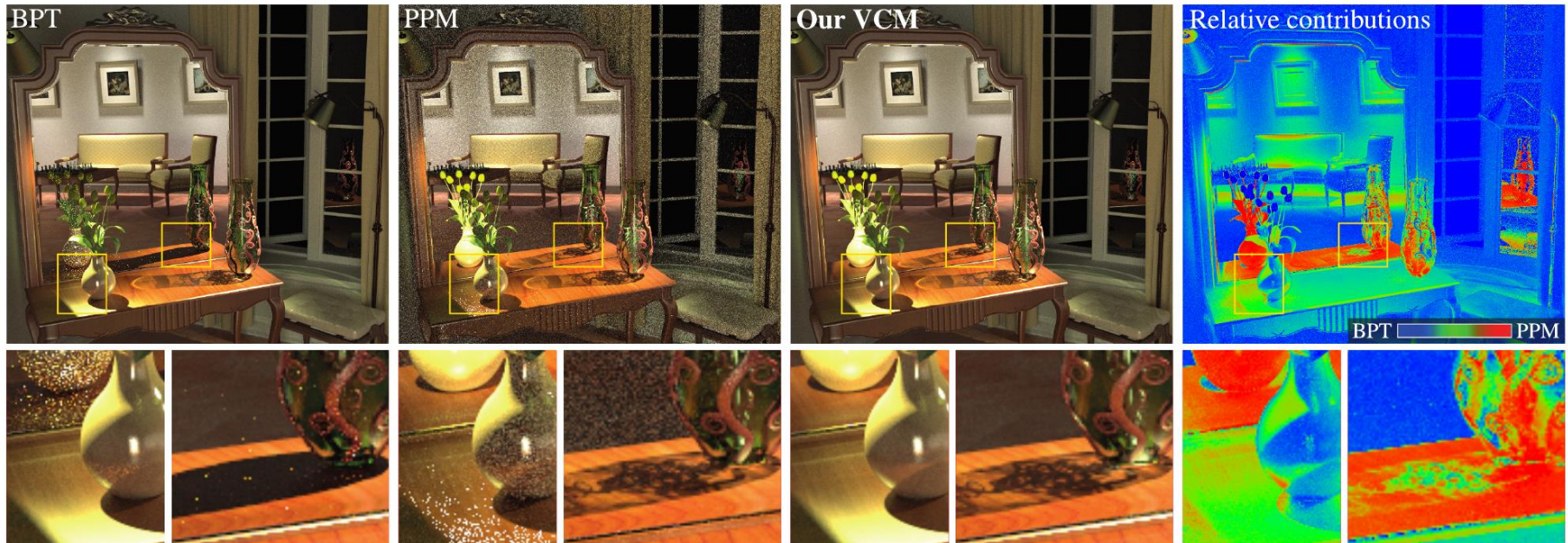
Recent Advances in Lighting Sim.

- **Importance Caching for Complex Illumination**
 - By Iliyan Georgiev et al., Eurographics 2012



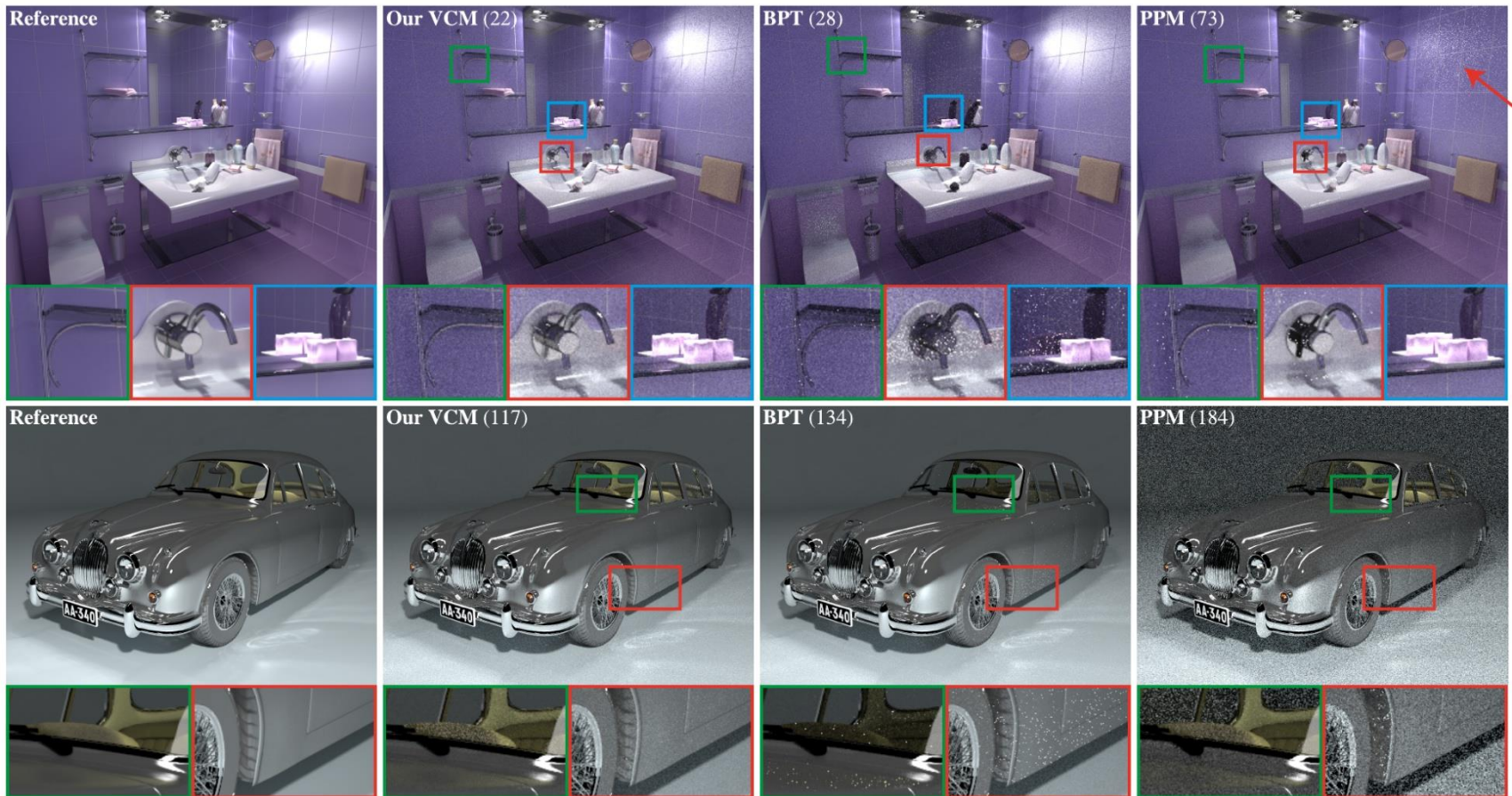
Recent Advances in Lighting Sim.

- **Light Transport Simulation with Vertex Connection and Merging (VCM)**
 - By Iliyan Georgiev et al., Siggraph 2012



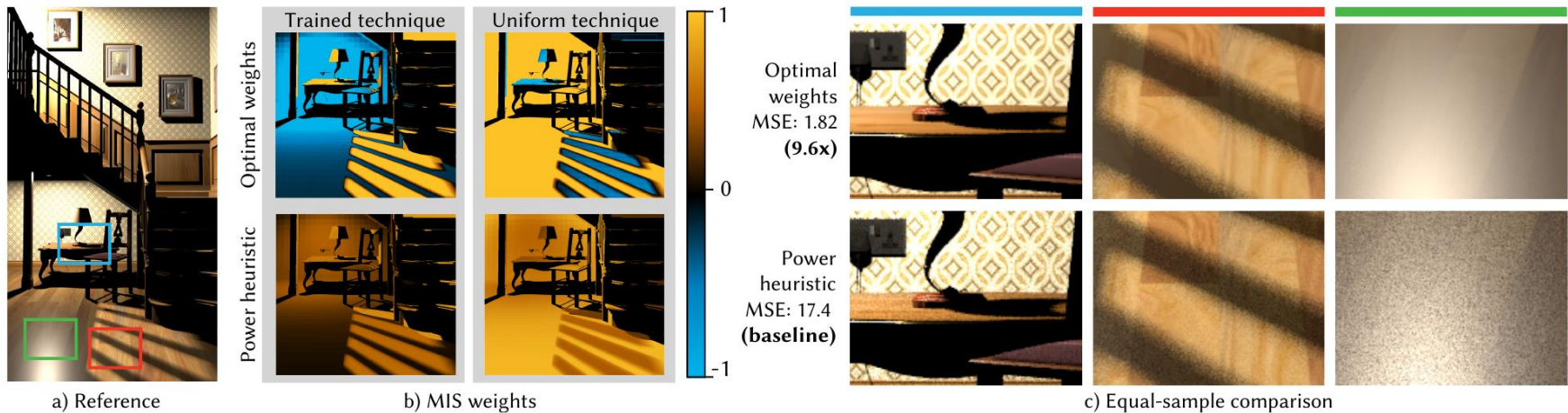
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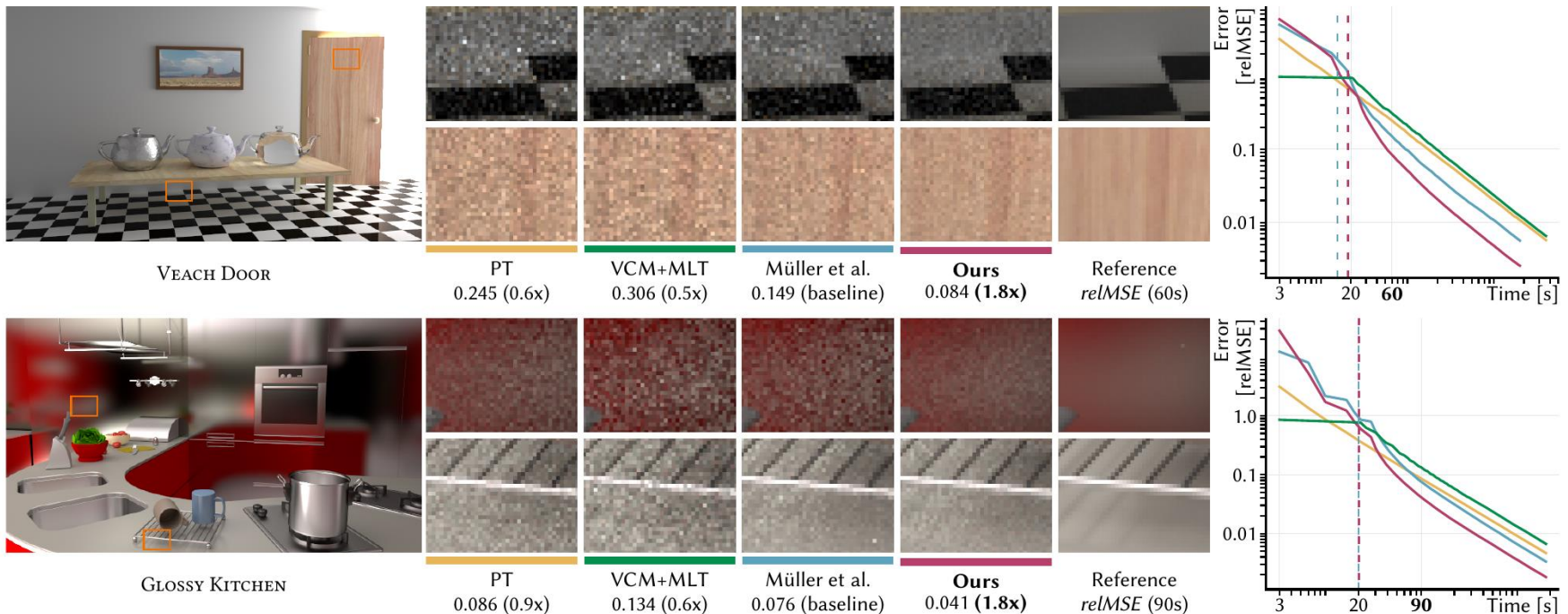
Recent Advances in Lighting Sim.

- **Optimal Multiple Importance Sampling**
 - By Pascal Grittmann, Jaroslav Krivanek, et al., Siggraph 2019



Recent Advances in Lighting Sim.

- **Variance-Aware Path Guiding**
 - By Alexander Rath, Pascal Grittmann, et al., Siggraph 2020



Wrap Up

- **Physical Quantities in Rendering**
 - Radiance
 - Radiosity
 - Irradiance
 - Intensity
- **Light Perception**
- **Light Source Definition**
- **Rendering Equation**
 - Key equation in graphics (!)
 - Integral equation
 - Describes global balance of radiance in a scene