Computer Graphics

- Introduction to Ray Tracing -

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Rendering Algorithms

• **Rendering**
  – Definition: Given a 3D scene description as input and a camera, generate a 2D image as a view from the camera of the 3D scene

• **Algorithms**
  – **Ray Tracing**
    • Declarative scene description
    • Physically-based simulation of light transport
    • Throughout the scene from light sources to the camera
  – **Rasterization**
    • Traditional procedural/imperative drawing of scene content
      – One triangle at a time (conceptually)
    • See later in the course!
Scene Description

• **Surface Geometry**
  – 3D geometry of objects in a scene
  – Geometric primitives – triangles, polygons, spheres, …

• **Surface Appearance**
  – Color, texture, absorption, reflection, refraction, subsurface scattering
  – Types of materials: Diffuse, glossy, mirror, glass, …

• **Illumination**
  – Position and emission characteristics of light sources
  – Note: Light also reflects off of surfaces!
    • Secondary/indirect/global illumination
  – Assumption: air/empty space is totally transparent
    • Simplification that excludes scattering effects in participating media or volumes, e.g. smoke, solid object (CT scan), …
    • See later in course

• **Camera**
  – View point, viewing direction, field of view, resolution, …
OVERVIEW OF RAY-TRACING
Light Transport (1)
Light Transport (2)

• **Light Distribution in a Scene**
  – Dynamic equilibrium: As much light is absorbed as is emitted

• **Forward Light Transport**
  – Shoot photons from the light sources into scene
  – Scatter at surfaces and record when a detector is hit
    • Photons that hit the camera produce the final image
    • Most photons will not reach the camera!
  – **Particle or Light Tracing**

• **Backward Light Transport**
  – Start at the detector (camera)
  – Trace only paths that might transport light towards camera
    • May be hard to find and connect to light sources
  – **Ray Tracing**
Ray Tracing Is…

• **Fundamental rendering algorithm**

• **Automatic, simple and intuitive**
  – Easy to understand and implement
  – Delivers “correct“ images by default

• **Powerful and efficient**
  – Many optical global effects
    • Shadows, reflections, refractions, …
  – Efficient real-time implementation in SW and HW
  – Can work in parallel and distributed environments
  – Logarithmic scalability with scene size: $O(\log n)$ vs. $O(n)$
  – Output sensitive and demand-driven approach

• **Concept of light rays is not new**
  – Empedocles (492-432 BC), Renaissance (Dürer, 1525), …
  – Used in lens design, geometric optics, neutron transport, …
Fundamental Ray Tracing Steps

- **Generation of primary rays**
  - Rays from viewpoint along viewing directions into 3D scene
  - (At least) one ray per picture element (pixel) in image plane

- **Ray casting**
  - Traversal of spatial index structures
    - To avoid unnecessary intersection computations
  - Ray-primitive intersection → hit point

- **Shading the hit point**
  - Compute light towards camera → pixel color
    - Light power (really “radiance”) travelling along primary ray
    - Needed for computation
      - Local reflection/scattering properties: material color, texture, …
      - Local illumination at intersection point
        - Compute through *recursive tracing of rays*
        - Can be hard to determine correctly
Ray Tracing Pipeline (1)

Ray Generation

Ray Traversal

Intersection

Shading

Pixel Color
Ray Tracing Pipeline (2)
Ray Tracing Pipeline (3)
Ray Tracing Pipeline (4)
Ray Tracing Pipeline (5)
Ray Tracing Pipeline (6)
Ray Tracing Pipeline (7)
Ray Tracing Pipeline (8)
Ray Tracing Pipeline (9)

- Ray Generation
- Ray Traversal
- Intersection
- Shading
- Pixel Color
Recursive Ray Tracing

- Searching recursively for paths to light sources
  - Interaction of light & material at intersections
  - Trace rays to light sources
  - Recursively trace new ray paths in reflection & refraction directions
Ray Tracing Algorithm

- **Trace(ray)**
  - Search the next intersection point \((hit, material)\)
  - Return Shade(ray, hit, material)
- **Shade(ray, hit, material)**
  - For each light source
    - if ShadowTrace(ray to light source, distance to light)
      - Calculate reflected radiance at hit
      - Adding radiance to the reflected radiance
    - If mirroring material
      - Calculate radiance in reflected direction: Trace(R(ray, hit))
      - Adding mirroring part to the reflected radiance
    - Same for transmission
    - If emissive (i.e. light source), add emitted light
    - Return reflected radiance
- **ShadowTrace(ray, dist)**
  - Return false, if intersection with distance \(< dist\) has been found
  - Can be changed to handle transparent objects as well
    - But not with refraction – WHY?
Ray Tracing Algorithm

• **Trace(ray)**
  - Search the next intersection point (hit, material)
  - Return Shade(ray, hit, material)

• **Shade(ray, hit, material)**
  - For each light source
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  - Same for transmission
  - If emissive (i.e. light source), add emitted light
  - Return reflected radiance

• **ShadowTrace(ray, dist)**
  - Return false, if intersection with distance < dist has been found
  - Can be changed to handle transparent objects as well
    - But not with refraction – WHY?
Shading (Material)

- **Intersection point determines primary ray’s “color”**
  - Diffuse object: isotropic reflection at hit point of illumination
    - No variation with viewing angle: diffuse (or Lambertian)
  - Perfect reflection/refraction (mirror, glass)
    - Only one outgoing direction each → Trace secondary ray path(s)
- **Non-Lambertian Reflectance**
  - Appearance depends on illumination and viewing direction
  - Local *Bi-directional Reflectance Distribution Function* (BRDF)

- **Illumination**
  - Point/directional light sources
  - Area light sources
    - Approximate with multiple samples / shadow rays
  - Indirect/global illumination
    - See Realistic Image Synthesis (RIS) course in next semester

- **More details later**
Common Approximations

• **Usually RGB color model**
  – Instead of full spectrum

• **Light only from finite # of point lights**
  – Instead of full indirect light

• **Approximate material reflectance properties**
  – Ambient: constant, non-directional background light
  – Diffuse: light reflected uniformly in all directions
  – Specular: perfect reflection, refraction

• **Reflection models are often empirical**
  – Often using Phong/Blinn shading model (or variation thereof)
  – But physically-based models are available as well
Ray Tracing Features

• **Incorporates into a single framework**
  – Hidden surface removal
    • Front to back traversal
    • Early termination once first hit point is found
  – Shadow computation
    • Shadow rays are traced between a point on a surface & light sources
  – Exact simulation of some light paths
    • Reflection (reflected rays at a mirror surface)
    • Refraction (refracted rays at a transparent surface, Snell’s law)

• **Limitations**
  – Many reflections or refractions
    • Exponential increase in number of rays
  – Indirect illumination requires many rays to sample all incoming directions
    • Easily gets inefficient for full global illumination computations
  – Solved with Path Tracing (→ later)
Ray Tracing Can…

- **Produce Realistic Images**
  - By simulating light transport
What is Possible?

- **Models Physics of Global Light Transport**
  - Dependable, physically-correct visualization
VW Visualization Center
Realistic Visualization: CAD
Realistic Visualization: VR/AR
Lighting Simulation
What is Possible?

- **Huge Models**
  - Logarithmic scaling in scene size

12.5 Million Triangles

~1 Billion Triangles
Outdoor Environments

• $90 \times 10^{12}$ (trillion) triangles
Boeing 777

Boeing 777: ~350 million individual polygons, ~30 GB on disk
Volume Visualization

- Iso-surface rendering
Games? (in 2006)
Games!

Nvidia RTX (Turing)
(up to 10 Grays/s)
Ray Tracing in CG

- **In the Past (until end of 90ies)**
  - Only used as an off-line technique
  - Was computationally far too demanding (minutes to hours per frame)
  - Believed to not be suitable for a HW implementation

- **More Recently**
  - Interactive ray tracing on supercomputers [Parker, U. Utah‘98]
  - Interactive ray tracing on PCs [Wald‘01]
  - Distributed Real-time ray tracing on PC clusters [Wald‘01]
  - RPU: First full HW implementation [Siggraph 2005]
  - Commercial tools: Embree (Intel/CPU), OptiX (Nvidia/GPU)
  - Complete film industry has switched to ray tracing (Monte-Carlo)

- **Own conference**
  - Symposium on Interactive RT, now High-Performance Graphics (HPG)

- **Ray tracing systems**
  - Research: PBRT (offline, physically-based, based on book, OSS), Mitsuba-2 renderer (EPFL), Rodent (SB), ...
  - Products: Blender (OSS), V-Ray (Chaos Group), Arnold & VRED (Autodesk), Corona (Render Legion), MentalRay/iRay (MI), ...
Ray Casting Outside CG

- **Tracing/Casting a ray**
  - Special type of query
    - "Is there a primitive along a ray"
    - "How far is the closest primitive"

- **Other uses than rendering**
  - Visibility computation
  - Radar simulation
  - Sound waves tracing
  - Volume computation
  - Collision detection
  - ...
RAY-PRIMITIVE INTERSECTIONS
Basic Math - Ray

- **Ray parameterization**
  - \( r(t) = \vec{o} + t\vec{d}, \quad t \in \mathbb{R}; \ \vec{o}, \vec{d} \in \mathbb{R}^3: \) origin and direction

- **Ray**
  - All points on the graph of \( r(t), \) with \( t \in \mathbb{R}_{0+} \)
Pinhole Camera Model

// For given image resolution {resx, resy}
// Loop over pixel raster coordinates [0, res-1]
for(prcx = 0; prcx < resx; prcx++)
    for(prcy = 0; prcy < resy; prcy++)
    {
        // Normalized device coordinates [0, 1]
        ndcx = (prcx + 0.5) / resx;
        ndcy = (prcy + 0.5) / resy;
        // Screen space coordinates [-1, 1]
        sscx = ndcx * 2 - 1;
        sscy = ndcy * 2 - 1;
        // Generate direction through pixel center
        d = f + sscx \cdot x + sscy \cdot y;
        d = d / |d|; // May normalize here
        // Trace ray and assign color to pixel
        color = trace_ray(o, d);
        write_pixel(prcx, prcy, color);
    }
Basic Math - Sphere

- **Sphere** $S$
  - $\vec{c} \in \mathbb{R}^3, r \in \mathbb{R}$: center and radius
  - $\forall \vec{p} \in \mathbb{R}^3: \vec{p} \in S \iff (\vec{p} - \vec{c}) \cdot (\vec{p} - \vec{c}) - r^2 = 0$
  - The distance between points on the sphere and its center equals the radius
Ray-Sphere Intersection

• Given
  – Ray: \( r(t) = \vec{o} + t\vec{d}, \quad t \in \mathbb{R}; \vec{o}, \vec{d} \in \mathbb{R}^3 \)
  – Sphere: \( \vec{c} \in \mathbb{R}^3, r \in \mathbb{R} \):
    • \( \forall \vec{p} \in \mathbb{R}^3: \vec{p} \in S \iff (\vec{p} - \vec{c}) \cdot (\vec{p} - \vec{c}) - r^2 = 0 \)

• Find closest intersection point
  – Algebraic approach: substitute ray equation
    • \( (\vec{p} - \vec{c}) \cdot (\vec{p} - \vec{c}) - r^2 = 0 \) with \( \vec{p} = \vec{o} + t\vec{d} \)
    • \( t^2 \vec{d} \cdot \vec{d} + 2t \vec{d} \cdot (\vec{o} - \vec{c}) + (\vec{o} - \vec{c}) \cdot (\vec{o} - \vec{c}) - r^2 = 0 \)
    • Solve for \( t \)
Ray-Sphere Intersection (2)

• Given
  - Ray: \( r(t) = \vec{o} + t\vec{d} \), \( t \in \mathbb{R}; \vec{o}, \vec{d} \in \mathbb{R}^3 \)
  - Sphere: \( \vec{c} \in \mathbb{R}^3, r \in \mathbb{R} \):
    - \( \forall \vec{p} \in \mathbb{R}^3: \vec{p} \in S \iff (\vec{p} - \vec{c}) \cdot (\vec{p} - \vec{c}) - r^2 = 0 \)

• Find closest intersection point
  - Geometric approach
    - Ray and center span a plane
    - Solve in 2D
    - Compute \( |\vec{b} - \vec{o}|, |\vec{b} - \vec{c}| \)
      - Such that \( \angle obc = 90^\circ \)
    - Intersection(s) if \( |\vec{b} - \vec{c}| \leq r \)

  - Be aware of floating point issues if \( o \) is far from sphere
Basic Math - Plane

- **Plane** $P$
  - $\vec{n}, \vec{a} \in \mathbb{R}^3$: normal and point in $P$ (Hesse normal form for plane)
  - $\forall \vec{p} \in \mathbb{R}^3$: $\vec{p} \in P \iff (\vec{p} - \vec{a}) \cdot \vec{n} = 0$
  - The difference vector between any two points on the plane is either 0 or orthogonal to the plane’s normal
Ray-Plane Intersection

- **Given**
  - Ray: \( r(t) = \vec{o} + t\vec{d} \), \( t \in \mathbb{R}; \vec{o}, \vec{d} \in \mathbb{R}^3 \)
  - Plane: \( \vec{n}, \vec{a} \in \mathbb{R}^3 \): normal and point in \( P \)

- **Compute intersection point**
  - Plane equation: \( \vec{p} \in P \iff (\vec{p} - \vec{a}) \cdot \vec{n} = 0 \)
  \[ \iff \vec{p} \cdot \vec{n} - D = 0, \text{ with } D = \vec{a} \cdot \vec{n} \]
  - Substitute ray parameterization: \( (\vec{o} + t\vec{d}) \cdot \vec{n} - D = 0 \)
  - Solve for \( t \)
    - What are possible solutions?
Ray-Plane Intersection

- **Given**
  - Ray: \( r(t) = \vec{o} + t\vec{d} \), \( t \in \mathbb{R} \); \( \vec{o}, \vec{d} \in \mathbb{R}^3 \)
  - Plane: \( \vec{n}, \vec{a} \in \mathbb{R}^3 \): normal and point in \( P \)

- **Compute intersection point**
  - Plane equation: \( \vec{p} \in P \iff (\vec{p} - \vec{a}) \cdot \vec{n} = 0 \)
    \[ \iff \vec{p} \cdot \vec{n} - D = 0, \text{ with } D = \vec{a} \cdot \vec{n} \]
  - Substitute ray parameterization: \( (\vec{o} + t\vec{d}) \cdot \vec{n} - D = 0 \)
  - Solve for \( t \)
    - 0, 1, or infinitely many solutions
    - One intersection or parallel to/in plane
Ray-Disc Intersection

- Intersect ray with plane
- Discard intersection if $||p - a|| > r$
Basic Math - Triangle

- **Triangle** $T$
  - $\vec{a}, \vec{b}, \vec{c} \in \mathbb{R}^3$: vertices
  - Affine combinations of $\vec{a}, \vec{b}, \vec{c} \rightarrow$ points in the plane
    - Non-negative coefficients that sum up to 1 $\rightarrow$ points in the triangle
  - $\forall \vec{p} \in \mathbb{R}^3: \vec{p} \in T \Leftrightarrow \exists \lambda_{1,2,3} \in \mathbb{R}_{0+}, \lambda_1 + \lambda_2 + \lambda_3 = 1$ and
    \[
    \vec{p} = \lambda_1 \vec{a} + \lambda_2 \vec{b} + \lambda_3 \vec{c}
    \]

- **Barycentric coordinates** $\lambda_{1,2,3}$
  - $\lambda_1 = S_{pbc}/S_{abc}$, etc.
  - $S$: signed area of triangles, based on CLW/CCW orientation
Barycentric Coordinates (BCs)

• **Triangle** \(T\)
  - \(\vec{a}, \vec{b}, \vec{c} \in \mathbb{R}^3\): vertices
  - \(\lambda_{1,2,3}\): Barycentric coordinates
  - \(\lambda_1 + \lambda_2 + \lambda_3 = 1\)
  - \(\lambda_1 = S_{pbc}/S_{abc}\), etc.

• **Easy geometric interpretation**
Triangle Intersection: Plane-Based

• **Compute intersection with triangle plane**
  – Plane equation easily computable from vertices

• **Compute barycentric coordinates**
  – Signed areas of subtriangles
  – Can be done in 2D, after “projection” onto major plane, depending on largest component of normal vector

• **Test for positive BCs**

• **Issues:**
  – Edges of neighboring triangles might not be identical
  – Due to inaccuracies of floats
  – Need a better method!
Triangle Intersection: Edge-Based

- 3D linear function across triangle (3D edge functions)
  - Ray: $\vec{o} + t \vec{d}$, $t \in \mathbb{R}; \vec{o}, \vec{d} \in \mathbb{R}^3$
  - Triangle: $\vec{a}, \vec{b}, \vec{c} \in \mathbb{R}^3$
Triangle Intersection: Edge-Based

- **3D linear function across triangle (3D edge functions)**
  - Ray: $\vec{o} + t \vec{d}$, \( t \in \mathbb{R}; \vec{o}, \vec{d} \in \mathbb{R}^3 \)
  - Triangle: $\vec{a}, \vec{b}, \vec{c} \in \mathbb{R}^3$
  - $\vec{n}_{ab} = (\vec{b} - \vec{o}) \times (\vec{a} - \vec{o})$
  - $|\vec{n}_{ab}|$ is the signed area of OAB (2x)
Triangle Intersection: Edge-Based

- **3D linear function across triangle (3D edge functions)**
  - Ray: \( \mathbf{\hat{o}} + t\mathbf{\hat{d}}, \quad t \in \mathbb{R}; \mathbf{\hat{o}}, \mathbf{\hat{d}} \in \mathbb{R}^3 \)
  - Triangle: \( \mathbf{\hat{a}}, \mathbf{\hat{b}}, \mathbf{\hat{c}} \in \mathbb{R}^3 \)
  - \( \mathbf{n}_{ab} = (\mathbf{\hat{b}} - \mathbf{\hat{o}}) \times (\mathbf{\hat{a}} - \mathbf{\hat{o}}) \)
  - \( |\mathbf{n}_{ab}| \) is the signed area of OAB (2x)
  - \( \lambda_3^*(t) = \mathbf{n}_{ab} \cdot t\mathbf{\hat{d}} \)
    - Volume of OBAP (6x)
    - For \( t = t_{hit} \)
Triangle Intersection: Edge-Based

- **3D linear function across triangle (3D edge functions)**
  - Ray: $\vec{o} + t\vec{d}$, \(t \in \mathbb{R}; \vec{o}, \vec{d} \in \mathbb{R}^3\)
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  - $\vec{n}_{ab} = (\vec{b} - \vec{o}) \times (\vec{a} - \vec{o})$
  - $|\vec{n}_{ab}|$ is the signed area of OAB (2x)
  - $\lambda_3^*(t) = \vec{n}_{ab} \cdot t\vec{d}$
    - Volume of OBAP (6x)
    - For $t = t_{hit}$
  - $\lambda_{1,2}^*(t) = \vec{n}_{bc,ac} \cdot t\vec{d}$
  - Normalize
    - $\lambda_i = \frac{\lambda_i^*(t)}{\lambda_1^*(t) + \lambda_2^*(t) + \lambda_3^*(t)}$, $i = 1,2,3$
    - Length of $t\vec{d}$ cancels out
Triangle Intersection: Edge-Based

- **3D linear function across triangle (3D edge functions)**
  - Ray: \( \vec{o} + t \vec{d} \), \( t \in \mathbb{R}; \vec{o}, \vec{d} \in \mathbb{R}^3 \)
  - Triangle: \( \vec{a}, \vec{b}, \vec{c} \in \mathbb{R}^3 \)
  - \( \overrightarrow{n_{ab}} = (\vec{b} - \vec{o}) \times (\vec{a} - \vec{o}) \)
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    - Volume of OBAP (6x)
    - For \( t = t_{hit} \)
  - \( \lambda_{1,2}^*(t) = \overrightarrow{n_{bc,ac}} \cdot t \vec{d} \)
  - Normalize
    - \( \lambda_i = \frac{\lambda_i^*(t)}{\lambda_1^*(t) + \lambda_2^*(t) + \lambda_3^*(t)}, i = 1, 2, 3 \)

- **Hit if all BCs positive:**
  - Compute \( \vec{p} = \lambda_1 \vec{a} + \lambda_2 \vec{b} + \lambda_3 \vec{c} \)
Quadrics

- **Implicit**
  - \( f(x, y, z) = v \)

- **Ray equation**
  - \( x = x_0 + t x_d \)
  - \( y = y_0 + t y_d \)
  - \( z = z_0 + t z_d \)

- **Solve for \( t \)**
Axis Aligned Bounding Box

- **Given**
  - Ray: $\vec{o} + t \vec{d}$, $t \in \mathbb{R}; \vec{o}, \vec{d} \in \mathbb{R}^3$
  - Axis aligned bounding box (AABB): $\overrightarrow{p_{min}}, \overrightarrow{p_{max}} \in \mathbb{R}^3$
Ray-Box Intersection

• Given
  – Ray: $\vec{\omega} + t \vec{d}$, $t \in \mathbb{R}$; $\vec{\omega}, \vec{d} \in \mathbb{R}^3$
  – Axis aligned bounding box (AABB): $\overrightarrow{p_{min}}, \overrightarrow{p_{max}} \in \mathbb{R}^3$

• “Slabs test” for ray-box intersection
  – Ray enters the box in all dimensions before exiting in any
  – $\max(\{t_{i}^{near} | i = x, y, z\}) < \min(\{t_{i}^{far} | i = x, y, z\})$
History of Intersection Algorithms

- **Ray-geometry intersection algorithms**
  - Polygons: [Appel ’68]
  - Quadrics, CSG: [Goldstein & Nagel ’71]
  - Recursive Ray Tracing: [Whitted ’79]
  - Tori: [Roth ’82]
  - Bicubic patches: [Whitted ’80, Kajiya ’82]
  - Algebraic surfaces: [Hanrahan ’82]
  - Swept surfaces: [Kajiya ’83, van Wijk ’84]
  - Fractals: [Kajiya ’83]
  - Deformations: [Barr ’86]
  - NURBS: [Stürzlinger ’98]
  - Subdivision surfaces: [Kobbelt et al ’98]
Precision Problems

- E.g., Cause of „surface acne“

Due to precision problems the calculated intersection is beneath the surface

2. When a shadow ray starts from this point, it hits the sphere surface, and is in shadow

Problem in surface intersection.