# Computer Graphics

## Texture Filtering

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# Sensors

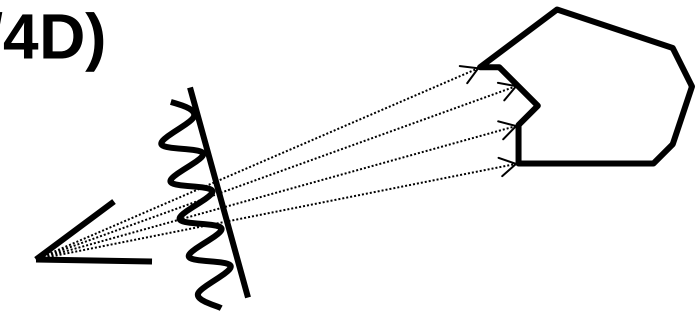
- **Measurement of signal**
  - Conversion of a continuous signal to discrete samples by integrating over the sensor field
    - Weighted with some sensor sensitivity function *P*

$$R(i,j) = \int_{A_{ij}} E(x,y)\, P_{ij}(x,y)\, dx\, dy$$

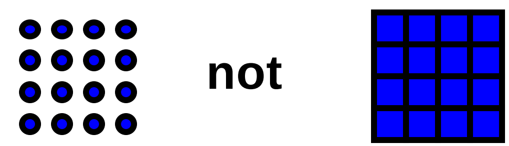
  - Similar to physical processes
    - Different sensitivity of sensor to photons
- **Examples**
  - Photo receptors in the retina
  - CCD or CMOS cells in a digital camera
- **Virtual cameras in computer graphics**
  - Analytic integration is expensive or even impossible
    - Needs to sample and integrate numerically
  - Ray tracing: mathematically ideal point samples
    - Origin of aliasing artifacts !

# The Digital Dilemma

- **Nature: continuous signal (2D/3D/4D)**
  - Defined at every point



- **Acquisition: sampling**
  - Rays, pixels/texels, spectral values, frames, ... (aliasing !)
- **Representation: discrete data**
  - Discrete points, discretized values

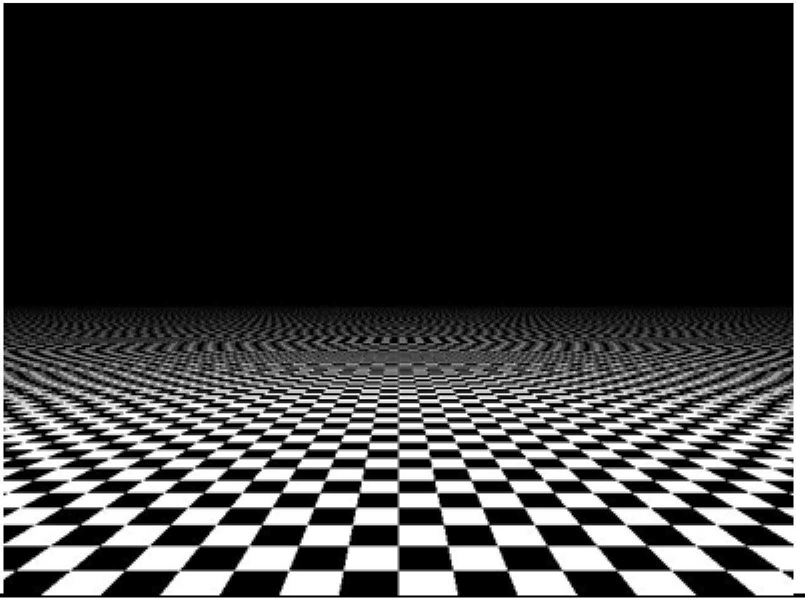


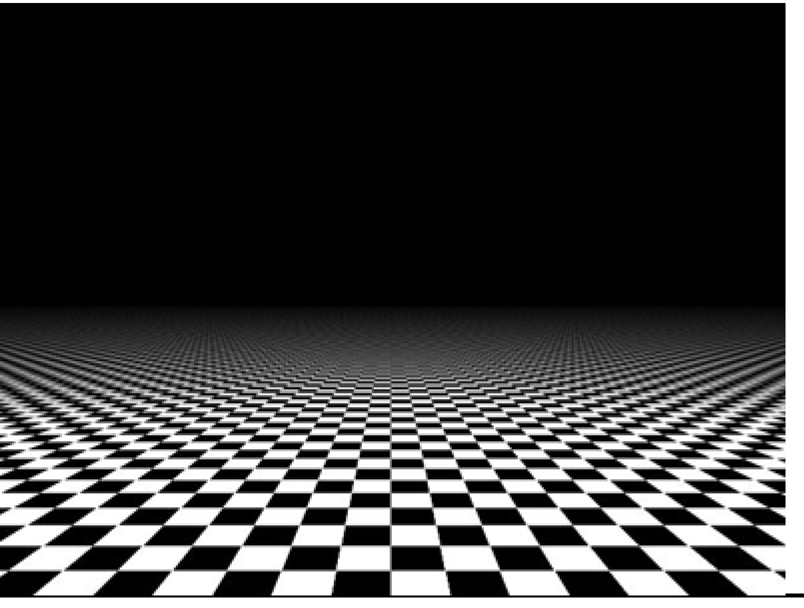

Pixels are usually point sampled

- **Reconstruction: filtering**
  - Recreate continuous signal
- **Display and perception (on some mostly unknown device!)**
  - Hopefully similar to the original signal, no artifacts

# Aliasing Example

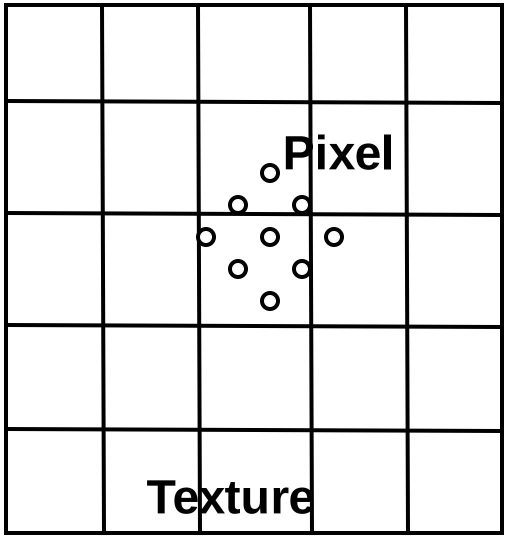
- **Ray tracing**
  - Textured plane with one ray for each pixel (say, at pixel center)
    - No texture filtering: equivalent to modeling with b/w tiles
  - Checkerboard period becomes smaller than two pixels
    - At the Nyquist sampling limit
  - Hits textured plane at only one point per pixel
    - Can be either black or white – essentially by "chance"
    - Can have correlations at certain locations

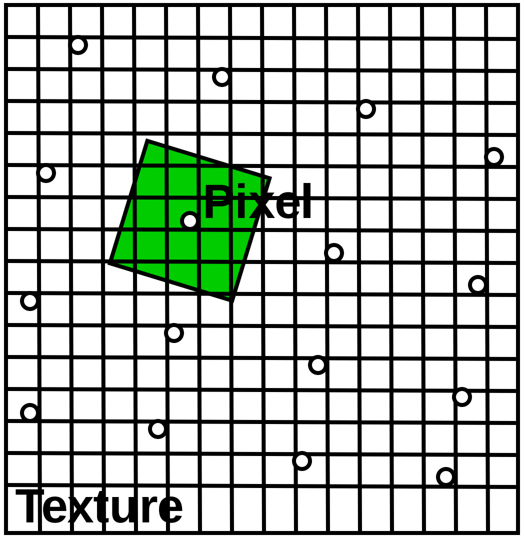




# Filtering

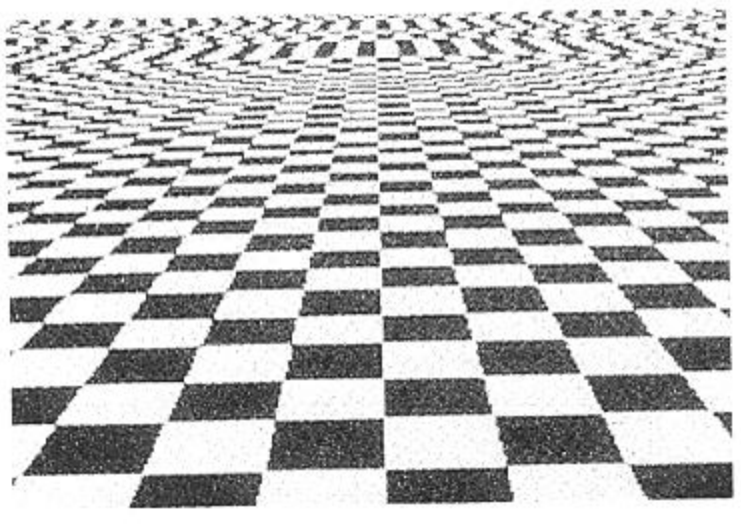
- **Magnification (Zoom-in)**
  - Map few texels onto many pixels
  - Reconstruction filter:
    - Nearest neighbor interpolation:
      - Take the nearest texel
    - Bilinear interpolation:
      - Interpolation between 4 nearest texels
      - Need fractional accuracy of coordinates
    - Higher order interpolation
- **Minification (Zoom-out)**
  - Map many texels to one pixel
    - Aliasing: Reconstructing high-frequency signals with low-frequency sampling
  - Antialising (low-pass filtering)
    - Averaging over (many) texels associated with the given pixel
    - Computationally expensive

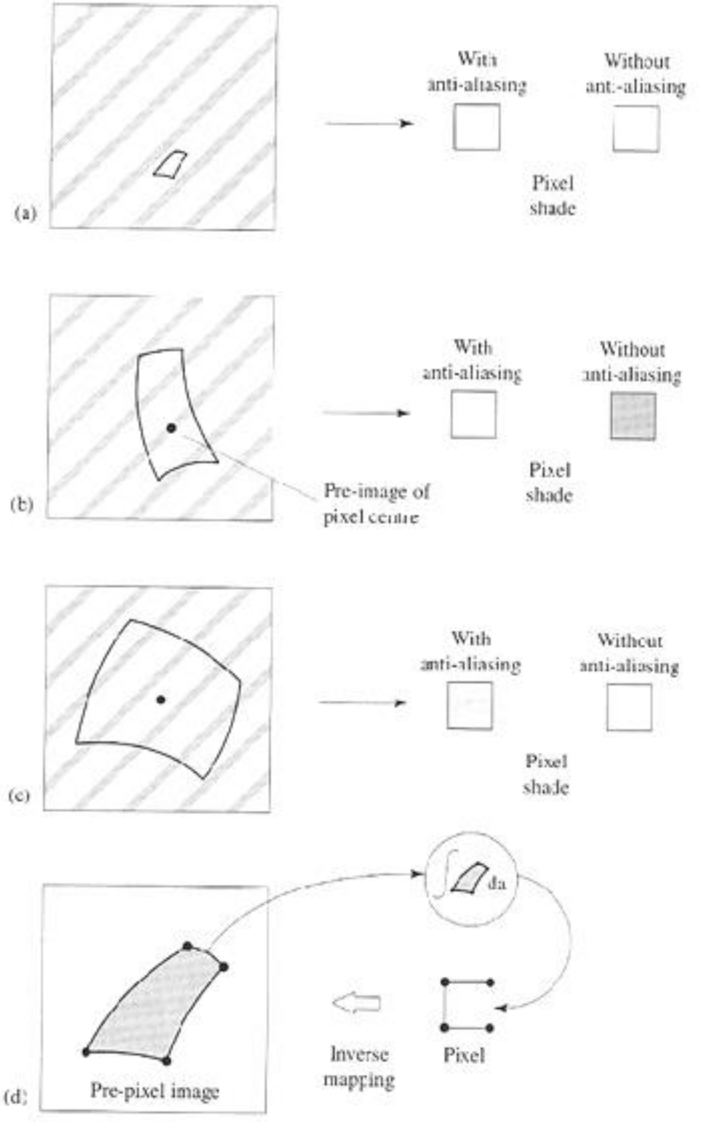


# Aliasing Artifacts

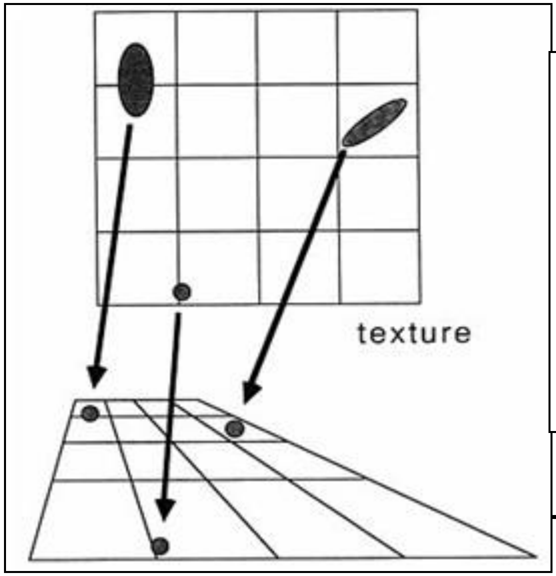
- **Aliasing**
  - Texture insufficiently sampled
  - Incorrect pixel values
  - “Randomly” changing pixels when moving
- **Integration of Pre-Image**
  - Integration over pixel footprint in texture space

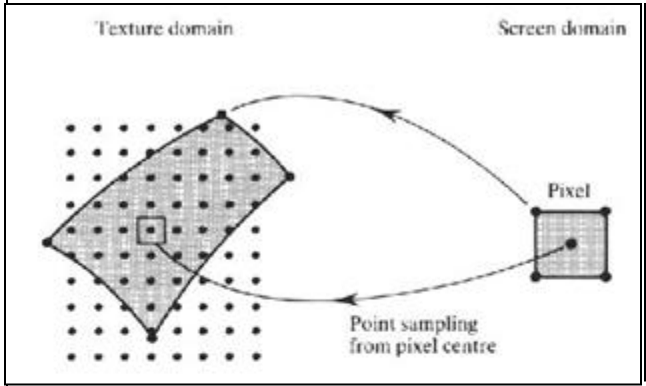


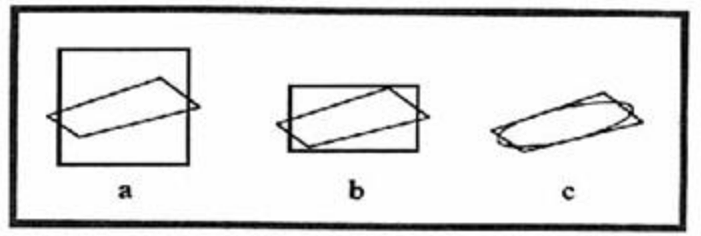


# Pixel Pre-Image in Texture Space

- **Circular pixel footprints have elliptic pre-images on planar surfaces**
- **Square screen pixels form quadrilaterals**
  - On curved surface shape can be arbitrary (non-connected, etc…)
- **Possible approximation by quadrilateral or parallelogram**
  - Or taking multiple samples within a pixel



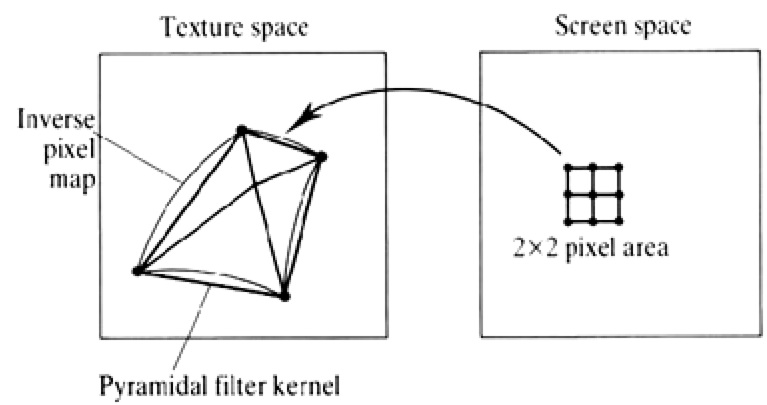

Approximating a quadrilateral texture area with (a) a square, (b) a rectangle, and (c) an ellipse. Too small an area causes aliasing; too large an area causes blurring.

# Space-Variant Filtering

- **Space-variant filtering**
  - Mapping from texture space $(u,v)$ to screen space $(x,y)$ not affine
  - Filtering changes with position
- **Space-variant filtering methods**
  - Direct convolution
    - Numerically compute the integral
  - Pre-filtering
    - Precompute the integral for certain regions $\Rightarrow$ more efficient
    - Approximate actual footprint with precomputed regions

# Direct Convolution

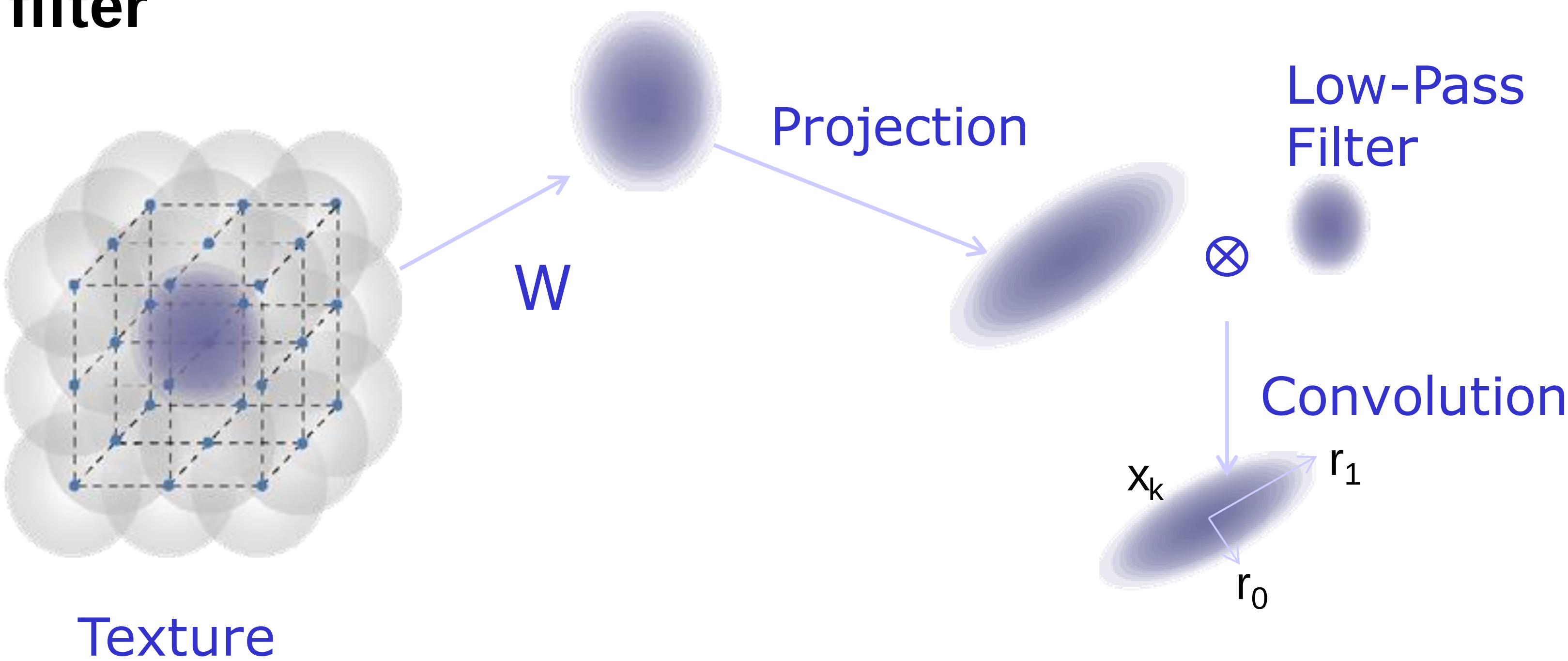
- **Convolution in texture space**
  - Texels weighted according to distance from pixel center (e.g. pyramidal filter kernel)
    - Essentially a low-pass filter




- **Convolution in image space**
  - Center the filter function on the pixel (in image space) and find its bounding rectangle.
  - Transform the rectangle to the texture space, where it is a quadrilateral whose sides are assumed to be straight.
  - Find a bounding rectangle for this quadrilateral.
  - Map all pixels inside the texture space rectangle to screen space.
  - Form a weighted average of the mapped texels (e.g. using a two-dimensional lookup table indexed by each sample's location within the pixel).

# EWA Filtering

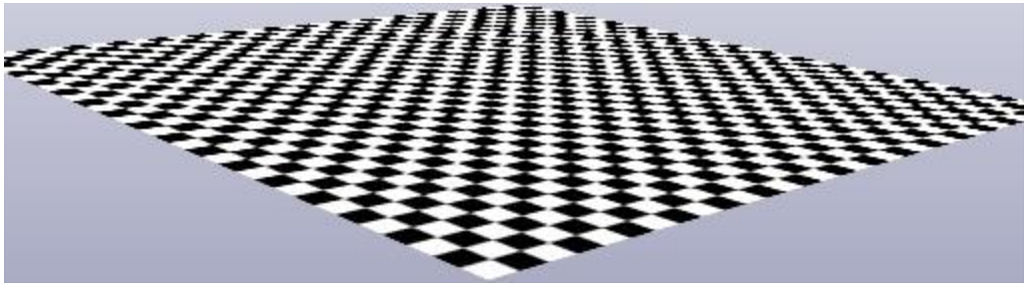
- **EWA: Elliptical Weighted Average**
- **Compensate aliasing artifacts caused by perspective projection**
- **EWA Filter = low-pass filter ⊗ warped reconstruction filter**




EWA texture resampling filter $\rho_k$

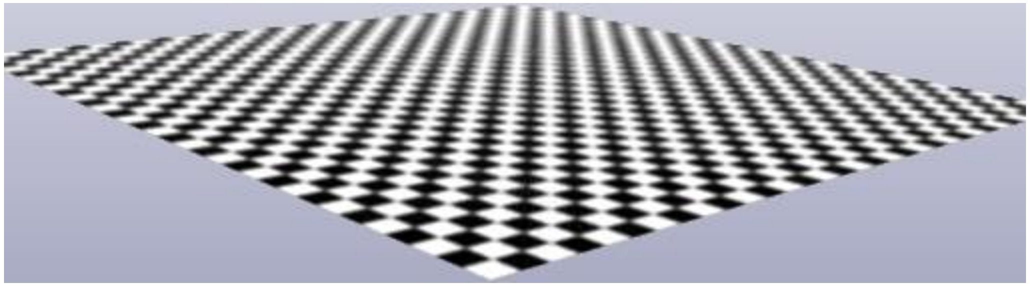
# EWA Filtering

- **Four step algorithm:**
  1. Calculate the ellipse
  2. Choose low-pass filter
  3. Scan conversion in the ellipse
  4. Determine the color of the pixel



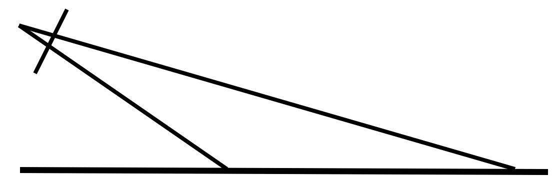
Without EWA filtering

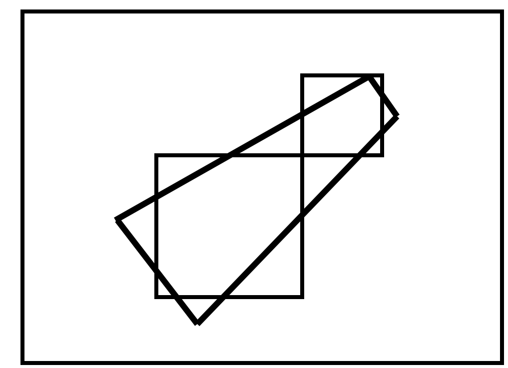
With EWA filtering



# Footprint Assembly

- **Footprint assembly: Approximation of pixel integral**
  - Good for space variant filtering
    - E.g. inclined view of terrain
  - Approximation of the pixel area by rectangular texel-regions
  - More footprints → better accuracy





- **In practice**
  - Often fixed number of area samples
  - Done by sampling multiple locations within a pixel (e.g. 2x2), each with smaller footprint

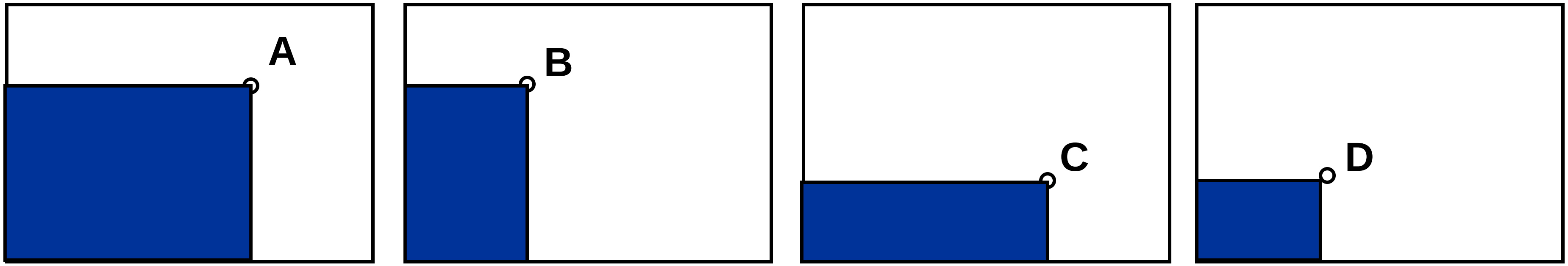
  ➔ Anisotropic (Texture) Filtering (AF)
    - GPUs allow selection of #samples (e.g. 4x, 8x, etc.)
    - Each sample has its own footprint area/extent
    - Each gets independently projected and filtered

# Pre-Filtering

- **Direct convolution methods are slow**
  - A pixel pre-image can be arbitrarily large
    - Along silhouettes
    - At the horizon of a textured plane
  - Can require averaging over thousands of texels
  - Texture filtering cost grows in proportion to projected texture area
- **Speed-up**
  - The texture can be prefiltered before rendering
    - Only a few samples are accessed for each screen sample
  - Two data structures are commonly used for prefiltering:
    - Integrated arrays (summed area tables - SAT)
    - Image pyramids (MIP-maps)

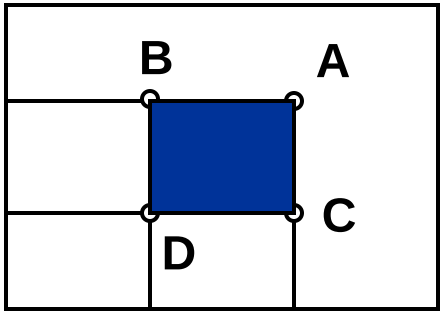
# Summed Area Tables (SAT)

- **Per texel, store sum from *(0, 0)* to *(u, v)***




- **Evaluation of 2D integrals over AA-boxes in constant time!**

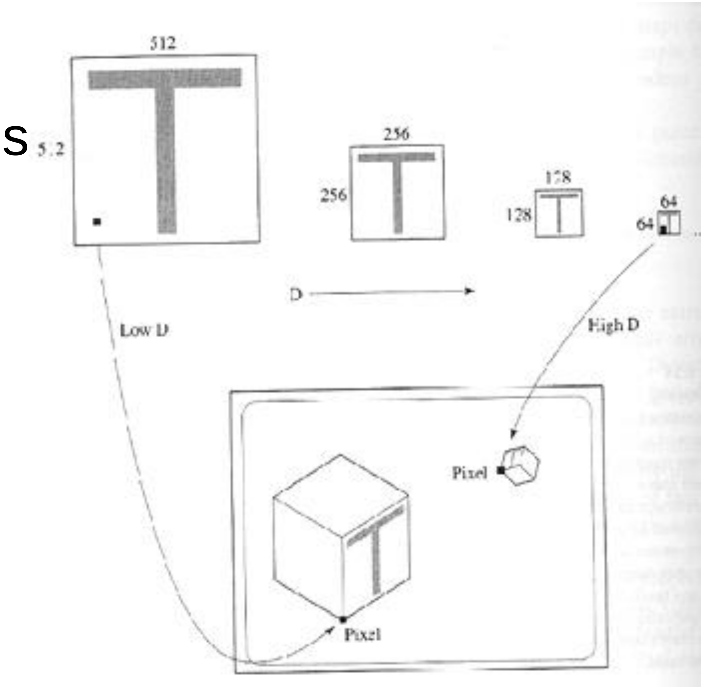
$$\int_{Bx}^{Ax}\int_{Cy}^{Ay} \mathrm{I}(\mathrm{x},\mathrm{y})\,dx\,dy = \mathrm{A}-\mathrm{B}-\mathrm{C}+\mathrm{D}$$




- **Needs many bits per texel (sum over million of pixels!)**

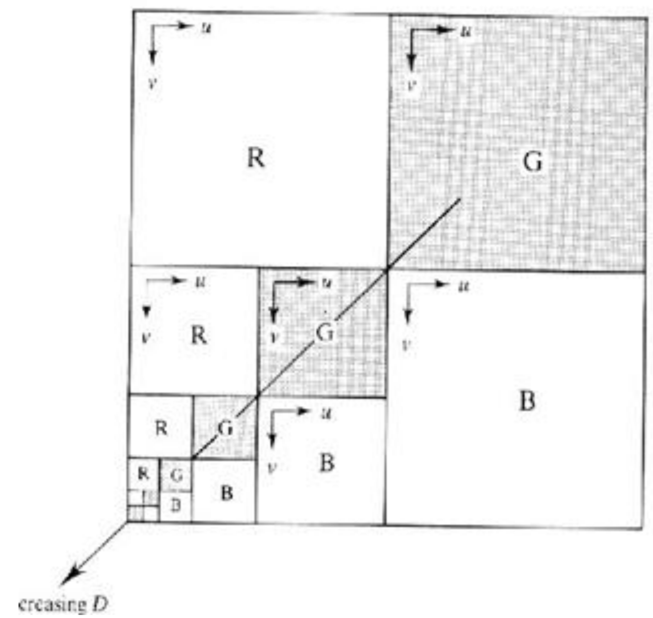
# MIP-Mapping

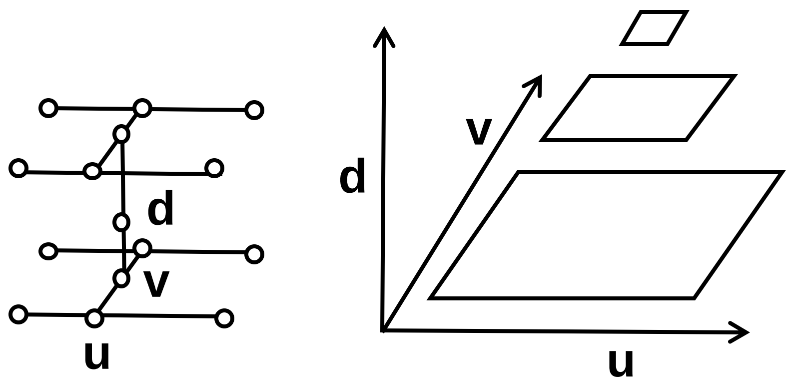
- **Texture available in multiple resolutions**
  - Pre-processing step that filters textures in each step
  - Discrete number of texture sizes (powers of 2)
- **Rendering**
  - Select appropriate texture resolution level *n* (per pixel !!!)
    - s.t.: texel size(*n*) < extent of pixel footprint < texel size(*n+1*)
  - Needs derivative of texture coordinates
  - Can be computed from differences between pixels (divided differences)
    - → Quad rendering (2x2 pixels)



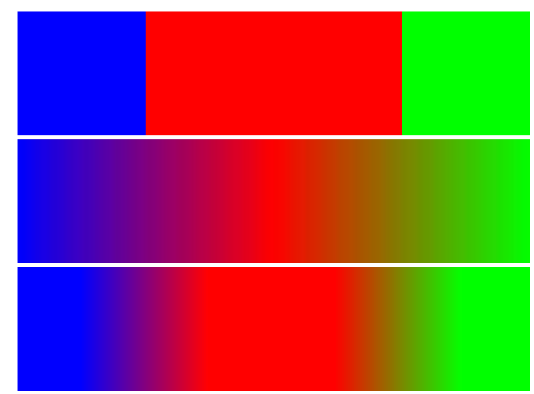
# MIP-Mapping (2)

- **Multum In Parvo (MIP): much in little**
- **Hierarchical resolution pyramid**
  - Repeated filtering over texture by 2x
- **Rectangular arrangement (RGB)**
- **Reconstruction**
  - Tri-linear interpolation of 8 nearest texels
    - Bilinear interpolation in levels *n* and *n+1*
    - Linear interpolation between the two levels



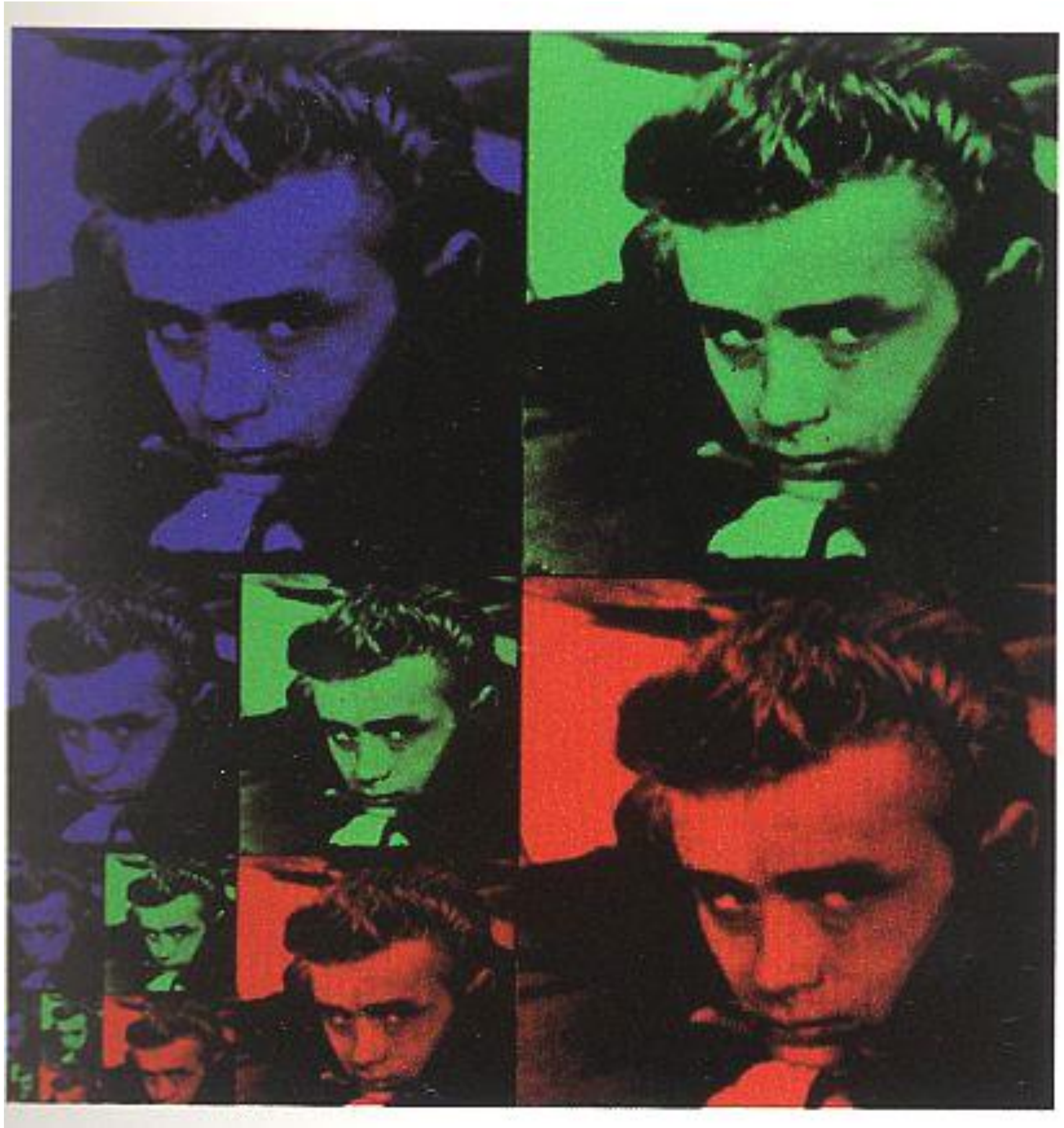




  - “Brilinear”: Trilinear only near transitions
    - Avoid reading 8 texels, most of the time



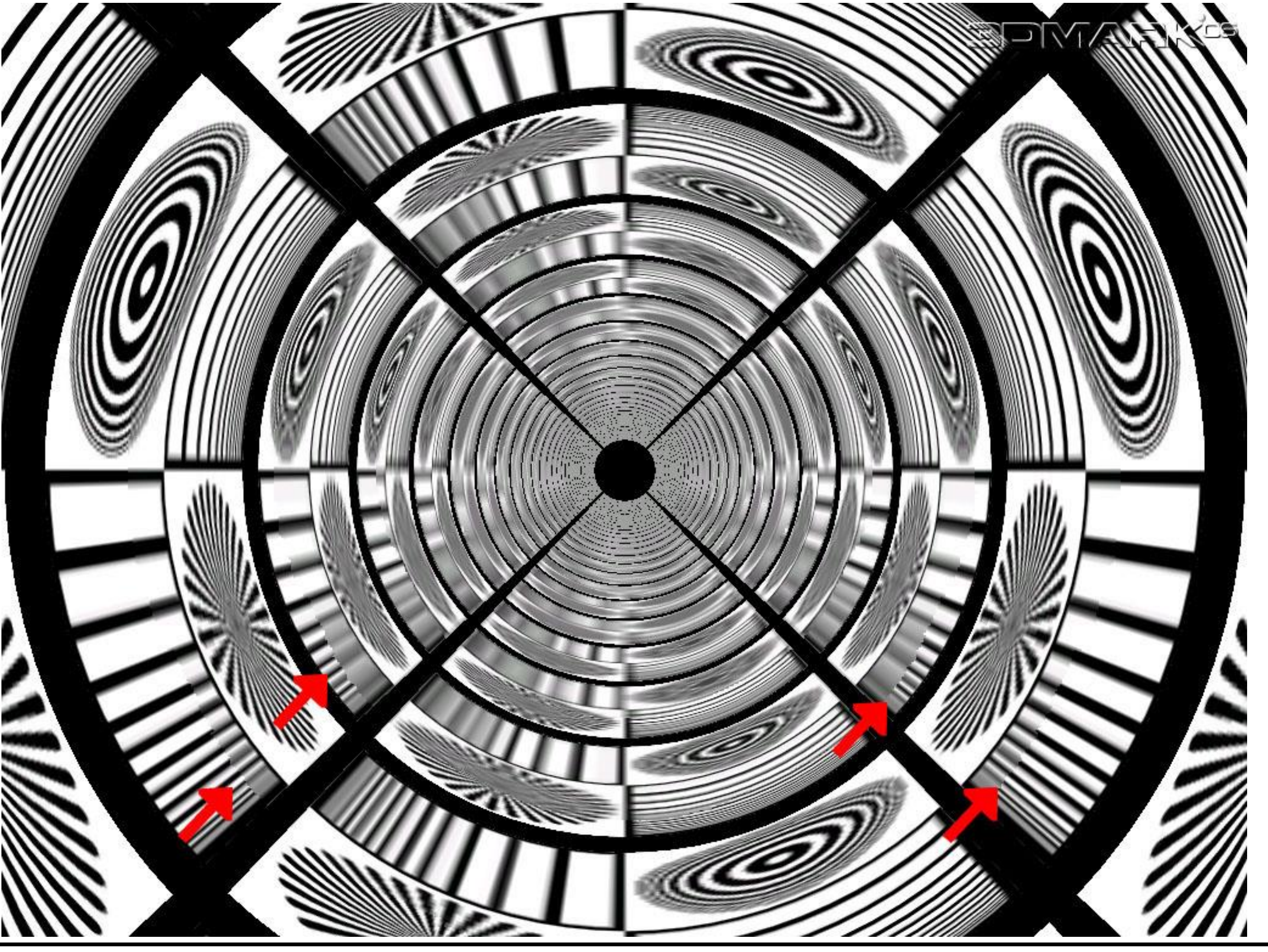
Reducing the domain for linear interpolation improves performance

# MIP-Map Example



# Hardware Texture Filtering

- **Bilinear filtering (in std. textured tunnel benchmark)**
  - Clearly visible transition between MIP-map levels



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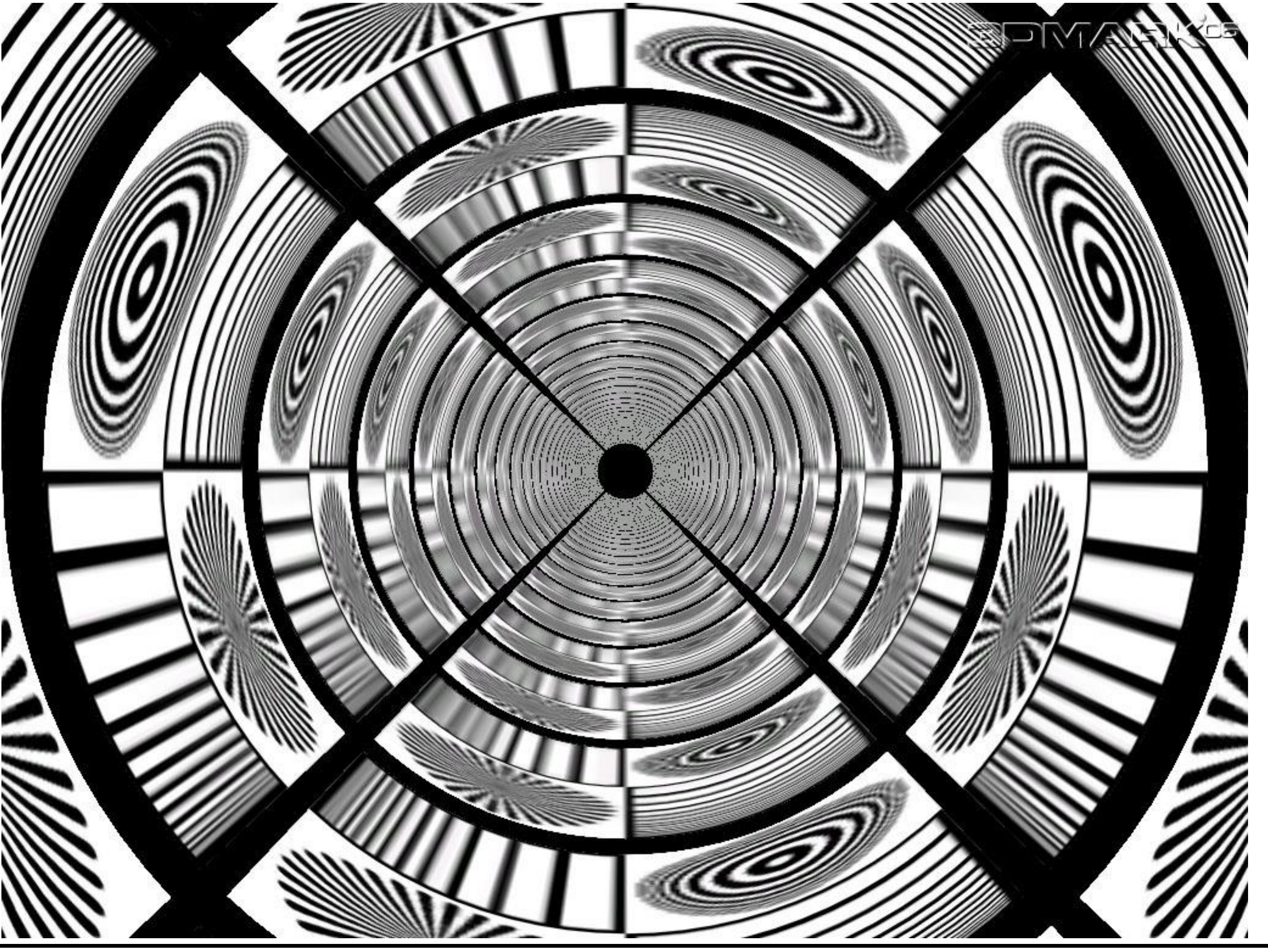
# Hardware Texture Filtering

- **Trilinear filtering**
  - Hides the transitions between MIP-map levels



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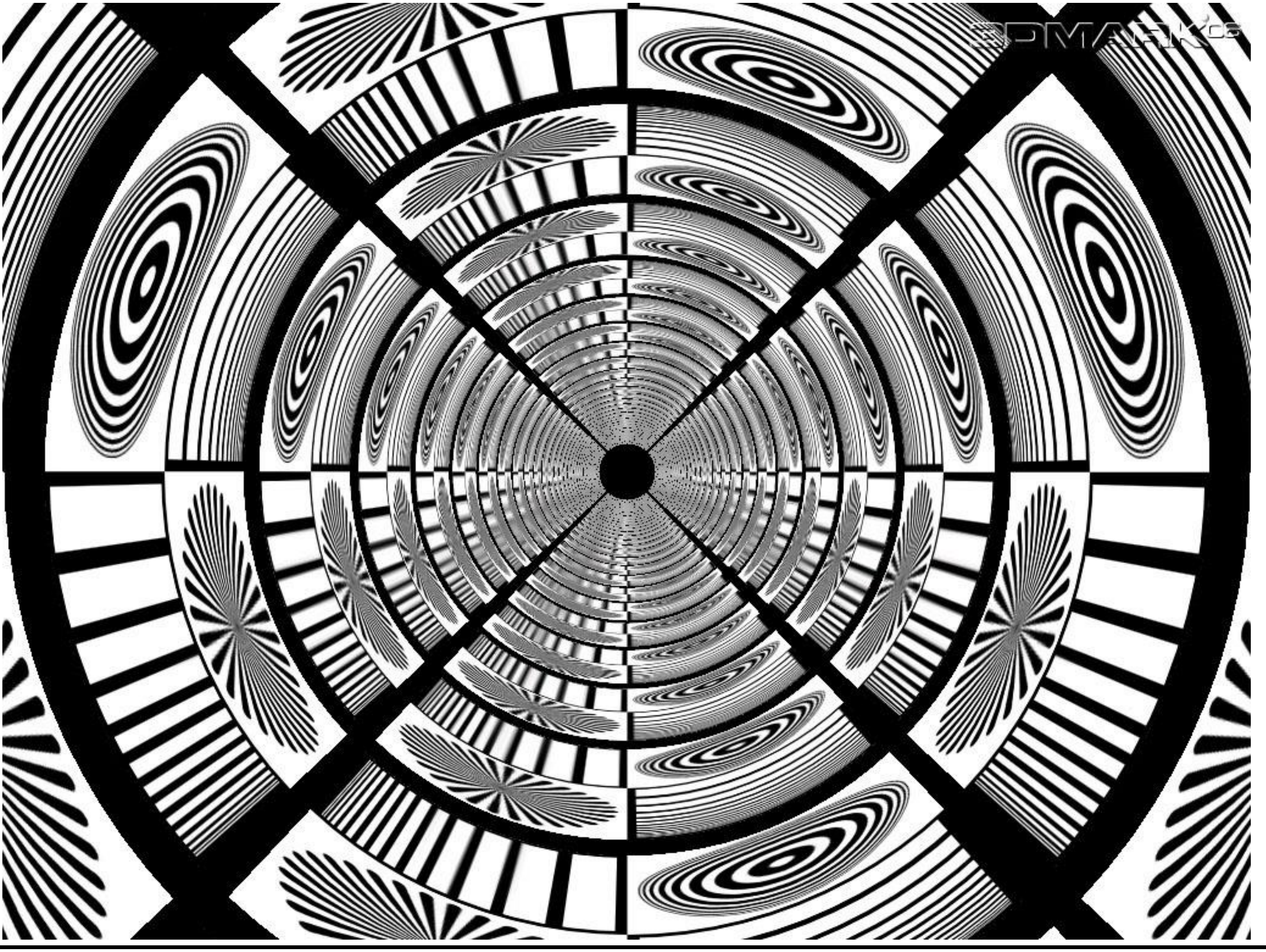
# Hardware Texture Filtering

- **Anisotropic filtering (8x)**
  - Makes the textures much sharper along azimuthal coordinate

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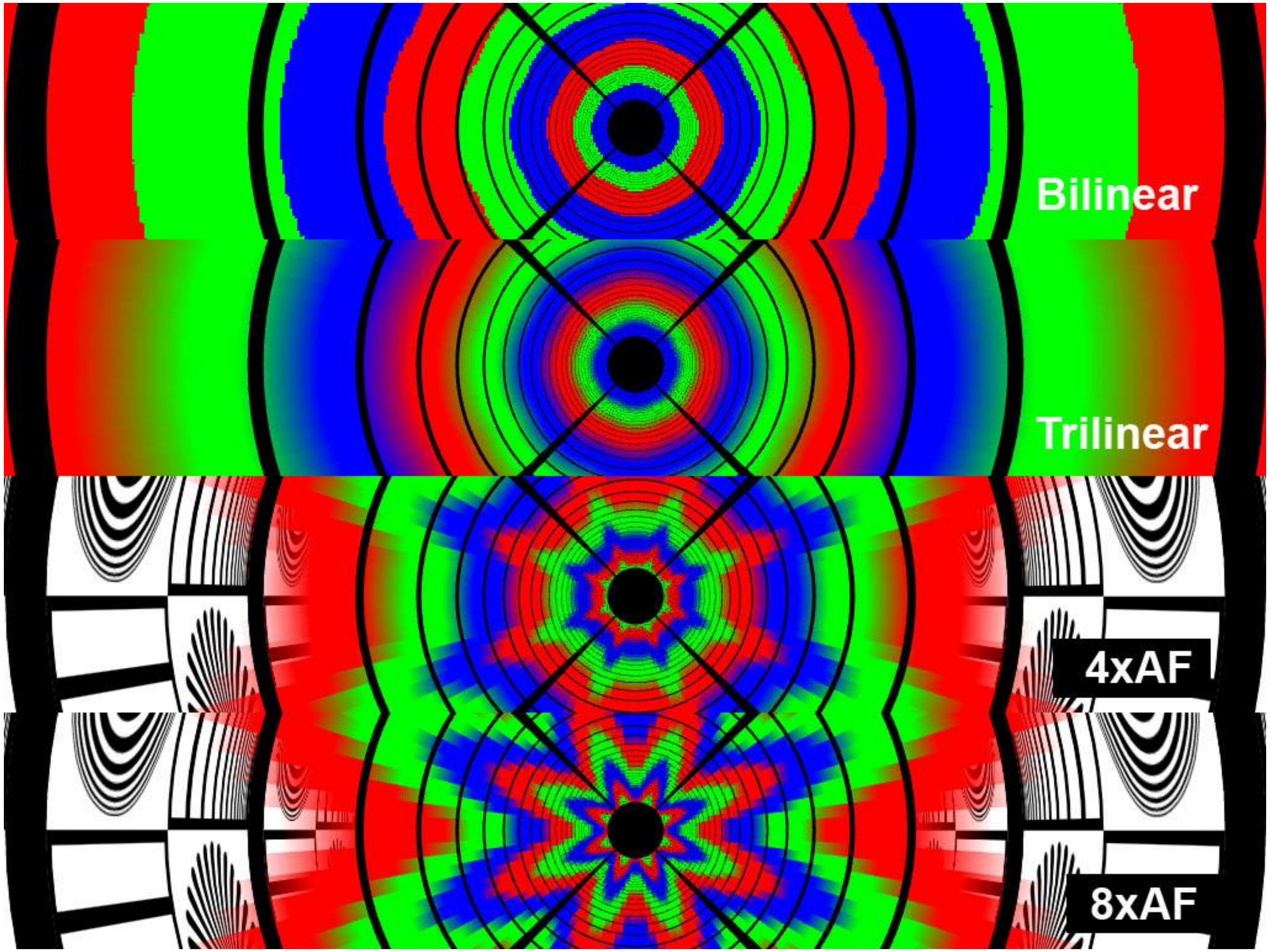
# Hardware Texture Filtering

- **Bilinear vs. trilinear vs. anisotropic filtering**
  - Using colored MIP-map levels

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# Texture Caching in Hardware

- **All GPUs have small texture caches**
  - Designed for local effects (streaming cache)
    - No effects between frames, or so!
- **Mipmapping ensures ~1:1 ratio**
  - From pixel to texels
  - Both horizontally & vertically
- **Pixels rendered in small 2D groups**
  - Basic block is 2x2 „quad“
    - Used to compute „derivatives“
    - Using divided differences (left/right, up/down)
  - Lots of local coherence
- **Bi-/tri-linear filtering needs adjacent texels (up to 8 for trilinear)**
  - Most often just 1-2 new texel per pixel not in (local) cache