Computer Graphics

- Introduction to Ray Tracing -

Philipp Slusallek

Rendering Algorithms

- Rendering
 - Definition: Given a 3D scene description as input and a camera, generate a 2D image as a view from the camera of the 3D scene

Algorithms

- Ray Tracing
 - Declarative scene description
 - Physically-based simulation of light transport
- Rasterization
 - Traditional procedural/imperative drawing of a scene content
 - See later in the course!

Scene

Surface Geometry

- 3D geometry of objects in a scene
- Geometric primitives triangles, polygons, spheres, ...

Surface Appearance

- Color, texture, absorption, reflection, refraction, subsurface scattering
- Diffuse, glossy, mirror, glass, ...

Illumination

- Position and emission characteristics of light sources
- Note: Light is also reflected off of surfaces!
 - Secondary/indirect/global illumination
- Assumption: air/empty space is totally transparent
 - Simplification that excludes scattering effects in participating media volumes
 - Later in course: Volume objects, e.g. smoke, solid object (CT scan), ...

Camera

- View point, viewing direction, field of view, resolution, ...

OVERVIEW OF RAY-TRACING

Ray Tracing Can...

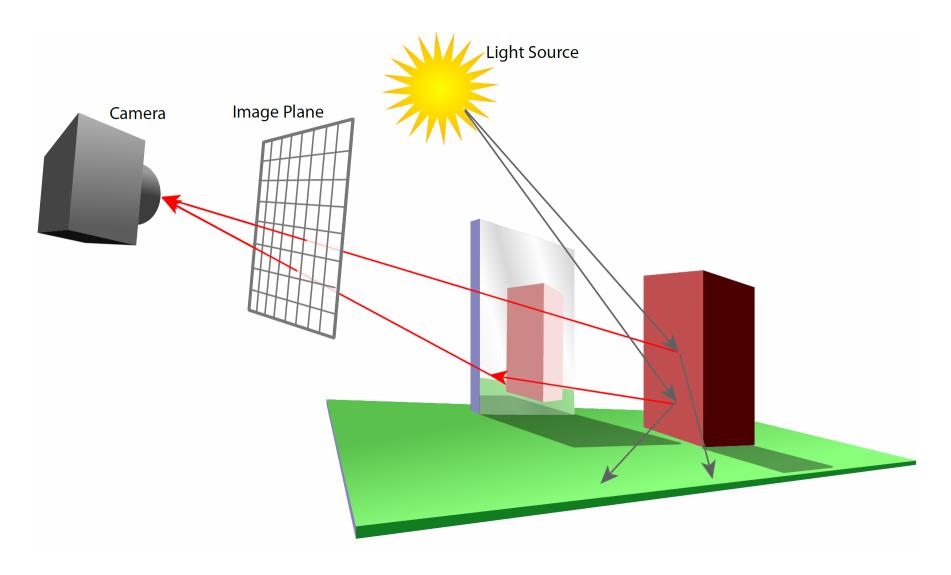
Produce Realistic Images

- By simulating light transport





Light Transport (1)



Light Transport (2)

- Light Distribution in a Scene
 - Dynamic equilibrium
- Forward Light Transport
 - Shoot photons from the light sources into scene
 - Scatter at surfaces and record when a detector is hit
 - Photons that hit the camera produce the final image
 - Most photons will not reach the camera
 - Particle or Light Tracing

Backward Light Transport

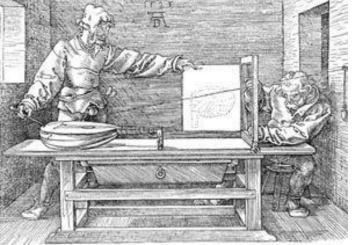
- Start at the detector (camera)
- Trace only paths that might transport light towards camera
 - May be hard to connect to light sources
- Ray Tracing

Ray Tracing Is...

- Fundamental rendering algorithm
- Automatic, simple and intuitive
 - Easy to understand and implement
 - Delivers "correct" images by default

Powerful and efficient

- Many optical global effects
- Shadows, reflections, refractions, ...
- Efficient real-time implementation in SW and HW
- Can work in parallel and distributed environments
- Logarithmic scalability with scene size: O(log n) vs. O(n)
- Output sensitive and demand driven approach
- Concept of light rays is not new
 - Empedocles (492-432 BC), Renaissance (Dürer, 1525), ...
 - Used in lens design, geometric optics, neutron transport, ...



Perspective Machine, Albrecht Dürer

Fundamental Ray Tracing Steps

Generation of primary rays

- Rays from viewpoint along viewing directions into 3D scene
- (At least) one ray per picture element (pixel)

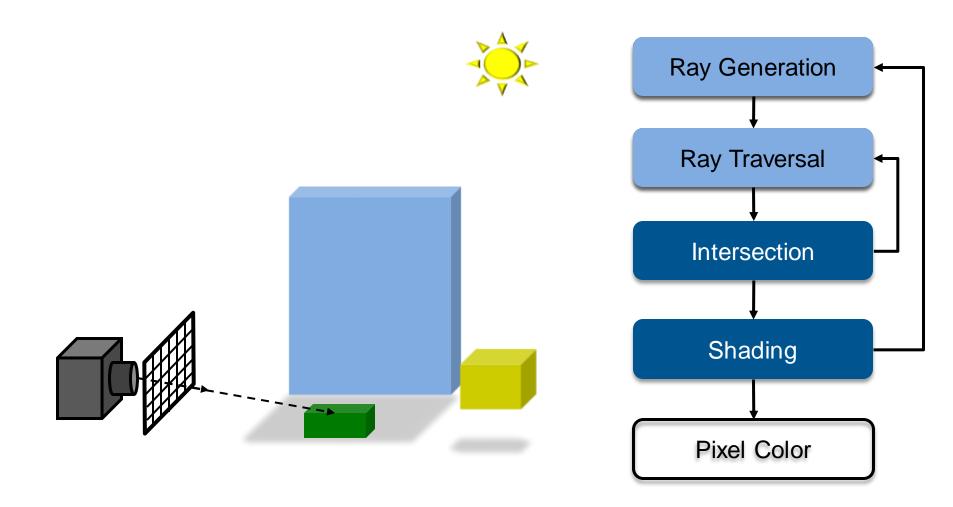
Ray casting

- Traversal of spatial index structures
 - To avoid unnecessary intersection computations
- Ray-primitive intersection \rightarrow hit point

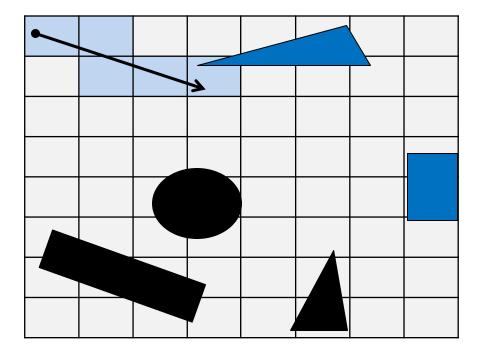
Shading the hit point

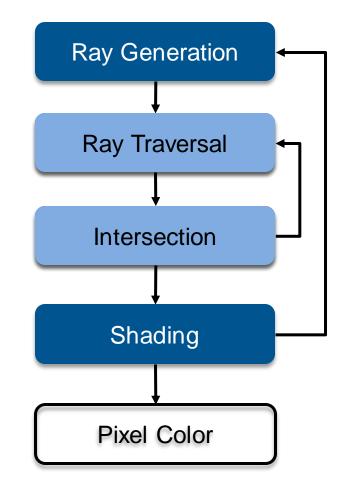
- Compute light towards camera \rightarrow pixel color
 - Light power (really "radiance") travelling along primary ray
- Needed for computation
 - Local reflection/scattering properties: material color, texture, ...
 - Local illumination at intersection point
 - Compute through recursive tracing of rays
 - Can be hard to determine correctly

Ray Tracing Pipeline (1)

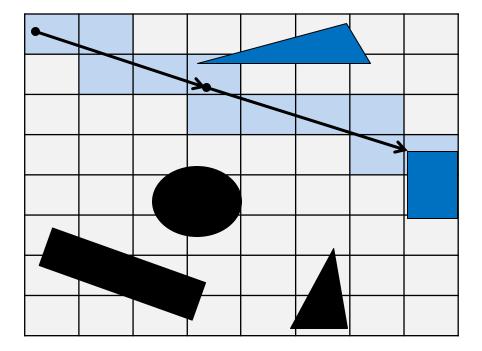


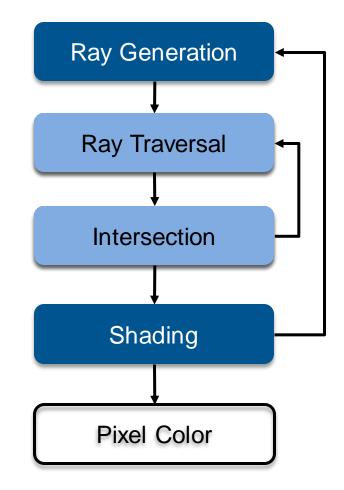
Ray Tracing Pipeline (2)



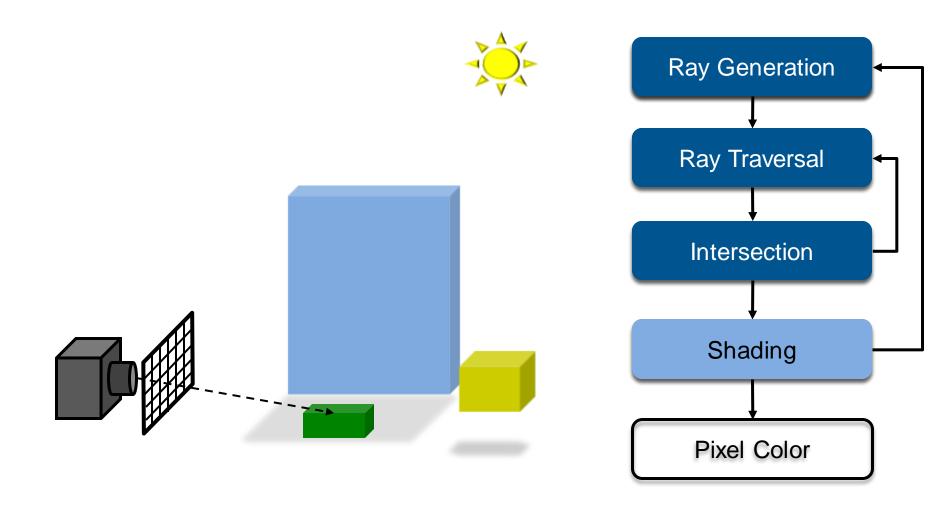


Ray Tracing Pipeline (3)

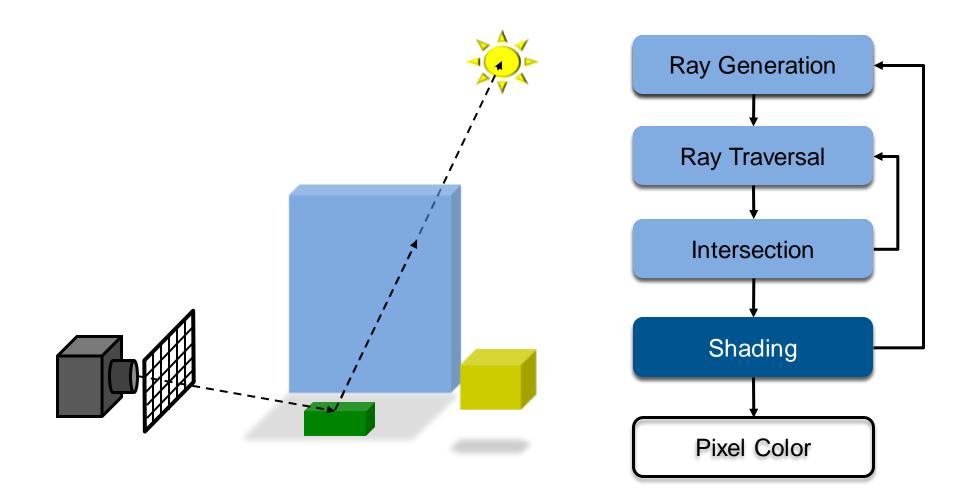




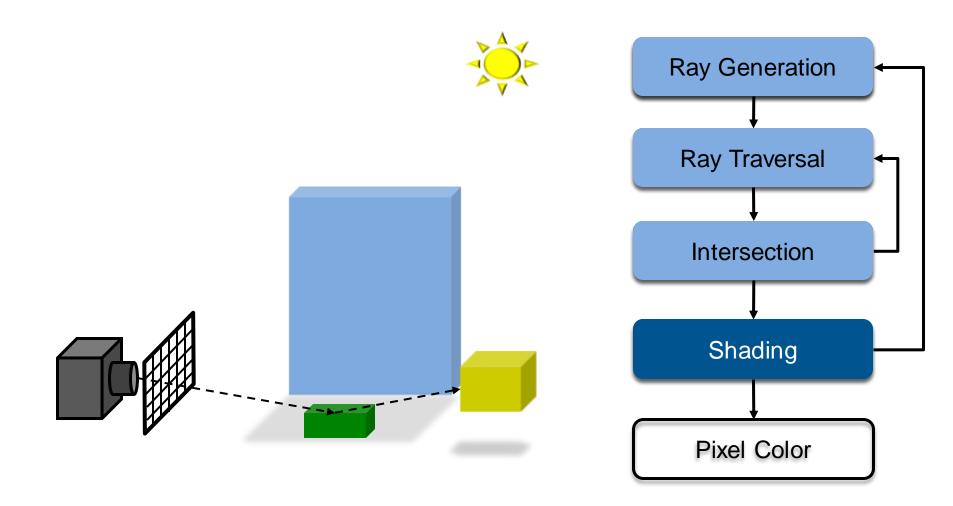
Ray Tracing Pipeline (4)



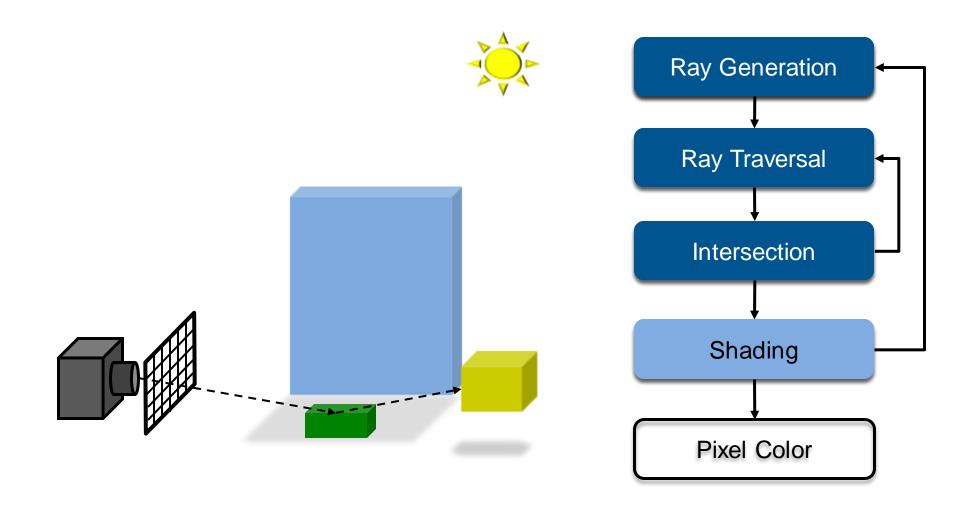
Ray Tracing Pipeline (5)



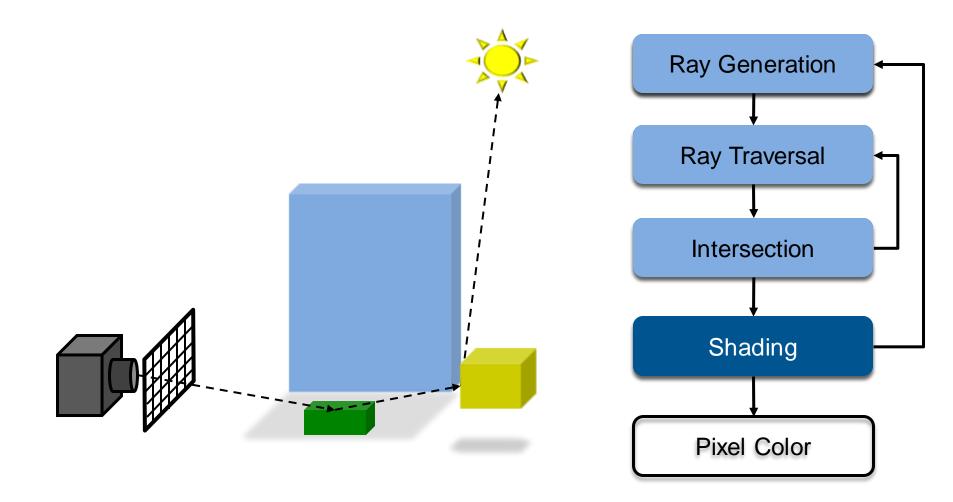
Ray Tracing Pipeline (6)



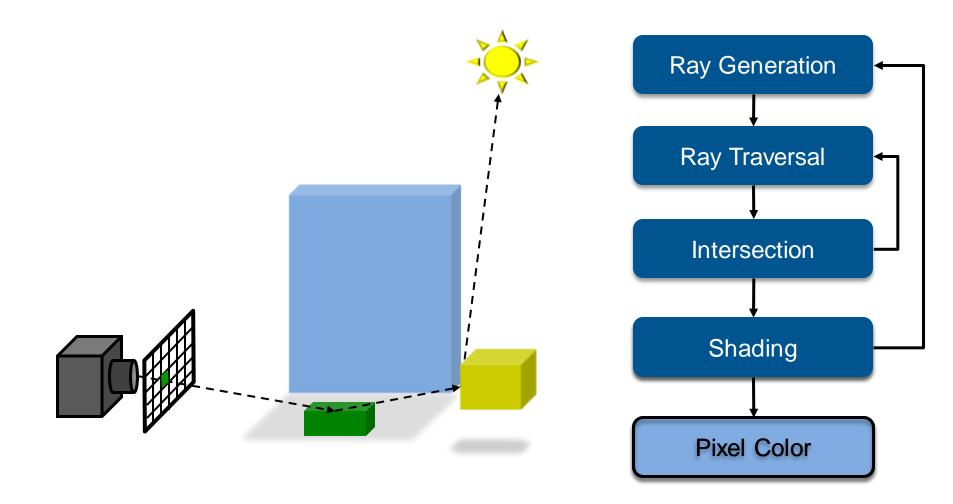
Ray Tracing Pipeline (7)



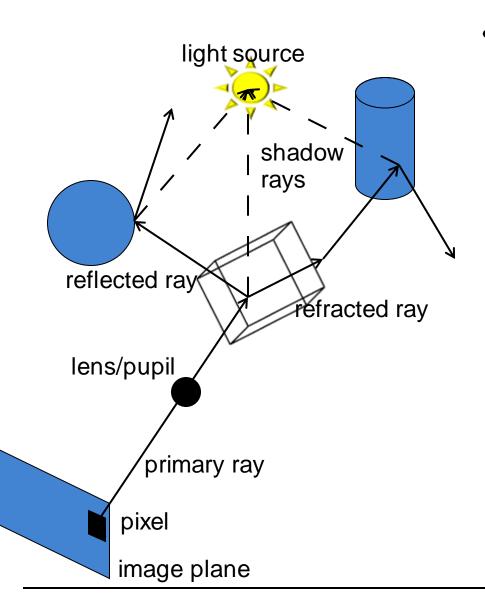
Ray Tracing Pipeline (8)



Ray Tracing Pipeline (9)

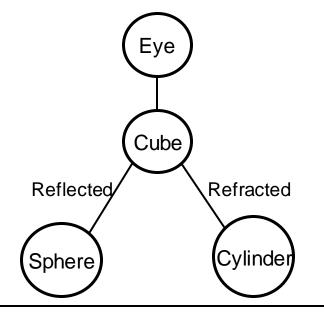


Recursive Ray Tracing



Searching recursively for paths to light sources

- Interaction of light & material at intersections
- Trace rays to light sources
- Recursively trace new ray paths in reflection & refraction directions



Ray Tracing Algorithm

- Trace(ray)
 - Search the next intersection point (hit, material)
 - Return Shade(ray, hit, material)

Shade(ray, hit, material)

- For each light source
 - if ShadowTrace(ray to light source, distance to light)
 - Calculate reflected radiance at hit
 - Adding radiance to the reflected radiance
- If mirroring material
 - Calculate radiance in reflected direction: Trace(R(ray, hit))
 - Adding mirroring part to the reflected radiance
- Same for transmission
- Return reflected radiance
- ShadowTrace(ray, dist)
 - Return false, if intersection with distance < dist has been found
 - Can be changed to handle transparent objects as well
 - But not with refraction WHY?

Ray Tracing Algorithm

- Trace(ray)
 - Search the next intersection point (hit, material)
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Shade(ray, hit, material)

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ShadowTrace(ray, dist)

- Return false, if intersection with distance < dist has been found
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Shading

Intersection point determines primary ray's "color"

- Diffuse object: isotropic reflection at hit point of illumination
 - No variation with viewing angle: diffuse (Lambertian)
- Perfect reflection/refraction (mirror, glass)
 - Only one outgoing direction each \rightarrow Trace secondary ray path(s)
- Non-Lambertian Reflectance
 - Appearance depends on illumination and viewing direction
 - Local Bi-directional Reflectance Distribution Function (BRDF)

Illumination

- Point/directional light sources
- Area light sources
 - Approximate with multiple samples / shadow rays
- Indirect/global illumination
 - See Realistic Image Synthesis (RIS) course in next semester
- More details later

Common Approximations

- Usually RGB color model
 - Instead of full spectrum
- Light only from finite # of point lights
 - Instead of full indirect light
- Approximate material reflectance properties
 - Ambient: constant, non-directional background light
 - Diffuse: light reflected uniformly in all directions
 - Specular: perfect reflection, refraction
- Reflection models are often empirical
 - Often using Phong/Blinn shading model (or variation therof)
 - But physically-based models are available as well

Ray Tracing Features

• Incorporates into a single framework

- Hidden surface removal
 - Front to back traversal
 - Early termination once first hit point is found
- Shadow computation
 - Shadow rays/ shadow feelers are traced between a point on a surface and a light sources
- Exact simulation of some light paths
 - Reflection (reflected rays at a mirror surface)
 - Refraction (refracted rays at a transparent surface, Snell's law)

Limitations

- Many reflections (exponential increase in number of rays)
- Indirect illumination requires many rays to sample all incoming directions
 - Easily gets inefficient for full global illumination computations
- Solved with Path Tracing (\rightarrow later)

Ray Tracing Can...

Produce Realistic Images

- By simulating light transport





What is Possible?

Models Physics of Global Light Transport

- Dependable, physically-correct visualization



VW Visualization Center



Realistic Visualization: CAD



Realistic Visualization: VR/AR





Lighting Simulation

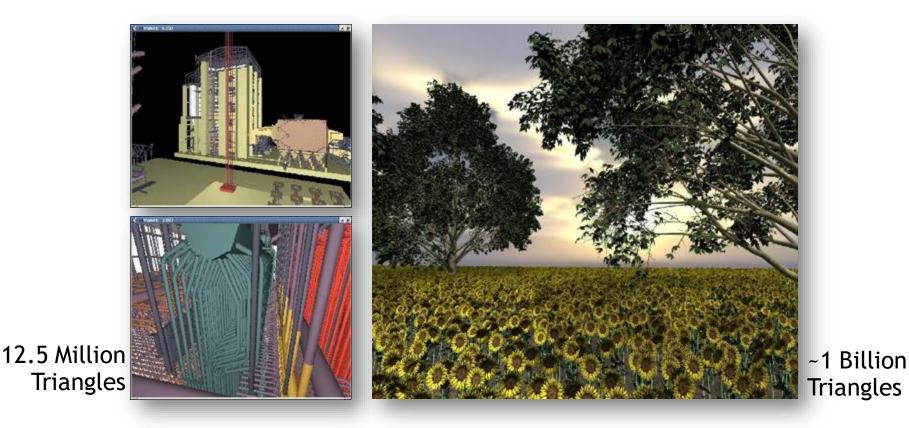




What is Possible?

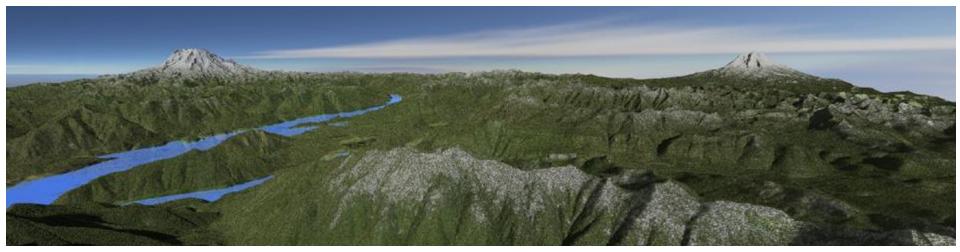
Huge Models

- Logarithmic scaling in scene size



Outdoor Environments

• 90 x 10^12 (trillion) triangles



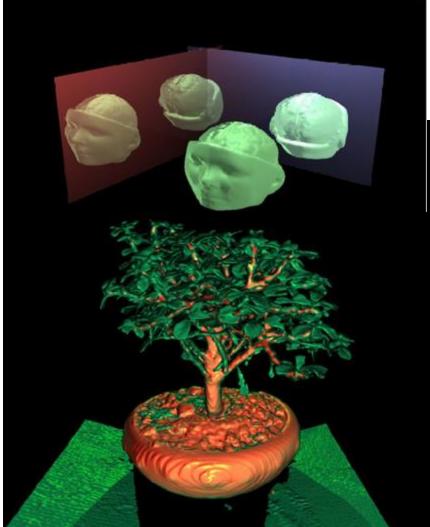




Boeing 777: ~350 million individual polygons, ~30 GB on disk

Volume Visualization

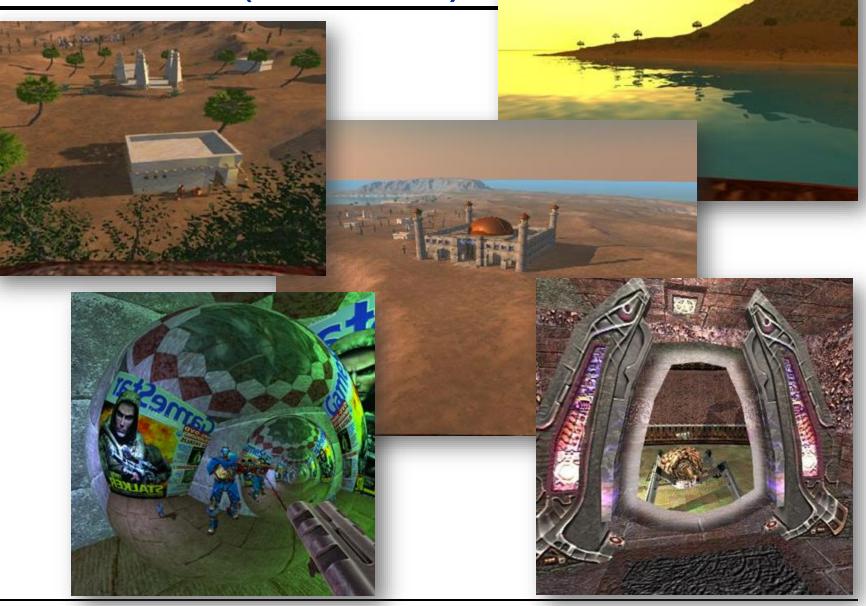
• Iso-surface rendering



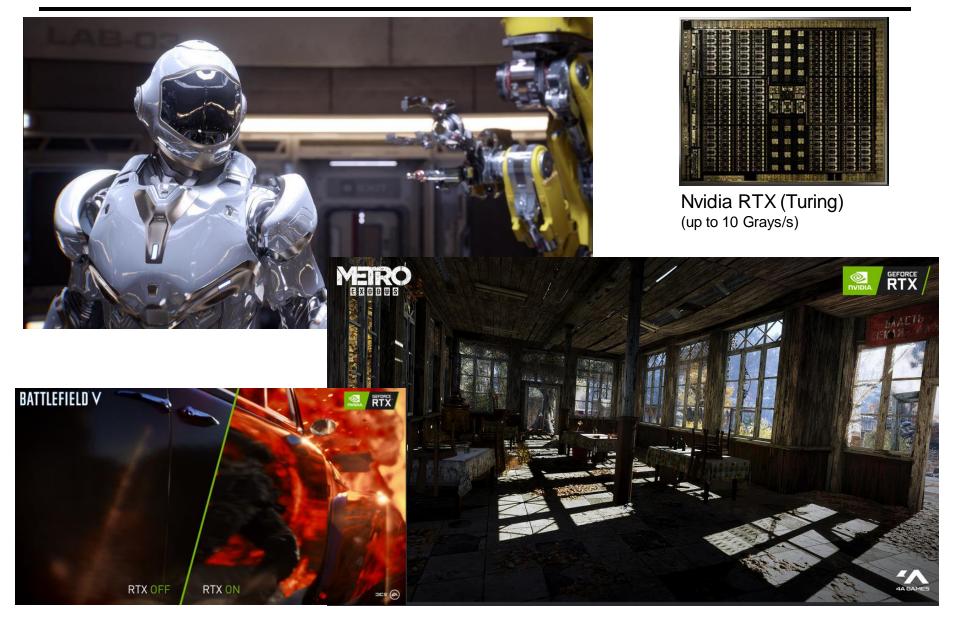




Games? (in 2006)



Games!



Ray Tracing in CG

In the Past

- Only used as an off-line technique
- Was computationally far too demanding (minutes to hours per frame)
- Believed to not be suitable for a HW implementation

More Recently

- Interactive ray tracing on supercomputers [Parker, U. Utah'98]
- Interactive ray tracing on PCs [Wald'01]
- Distributed Real-time ray tracing on PC clusters [Wald'01]
- RPU: First full HW implementation [Siggraph 2005]
- Commercial tools: Embree/OSPRey (Intel/CPU), OptiX (Nvidia/GPU)
- Complete film industry has switched to ray tracing (Monte-Carlo)

Own conference

– Symposium on Interactive RT, now High-Performance Graphics (HPG)

Ray tracing systems

- Research: PBRT (offline, physically-based, based on book, OSS), Mitsuba renderer (EPFL), Rodent (SB), ...
- Commercial: V-Ray (Chaos Group), Corona (Render Legion), VRED (Autodesk), MentalRay/iRay (MI), ...

Ray Casting Outside CG

Tracing/Casting a ray

- Special type of query
 - "Is there a primitive along a ray"
 - "How far is the closest primitive"

Other uses than rendering

- Volume computation
- Sound waves tracing
- Collision detection

— ...

RAY-PRIMITIVE INTERSECTIONS

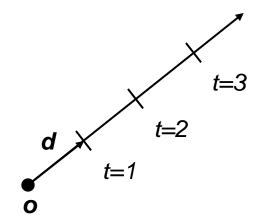
Basic Math - Ray

Ray parameterization

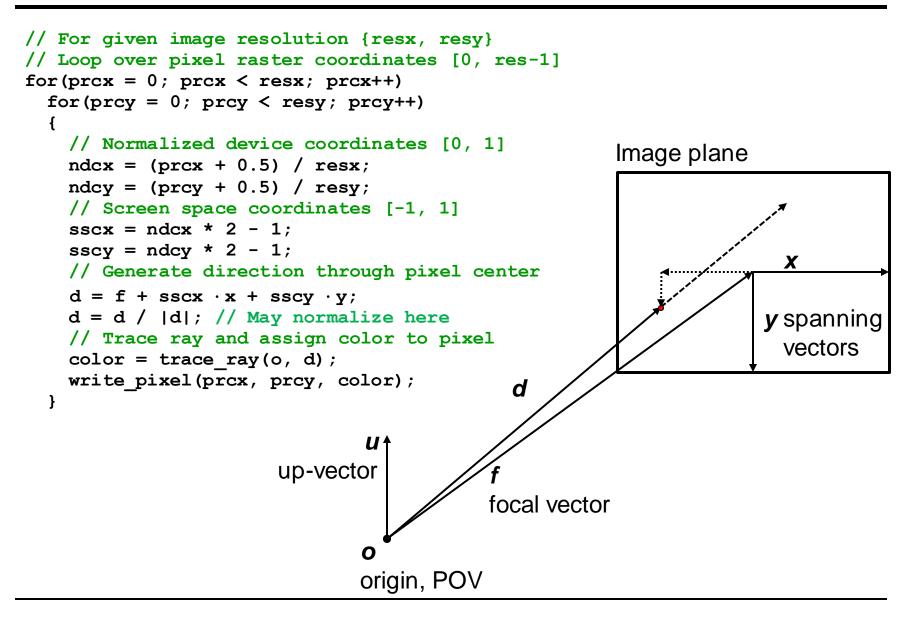
 $-r(t) = \vec{o} + t\vec{d}$, $t \in \mathbb{R}; \vec{o}, \vec{d} \in \mathbb{R}^3$: origin and direction

Ray

- All points on the graph of r(t), with $t \in \mathbb{R}_{0+1}$

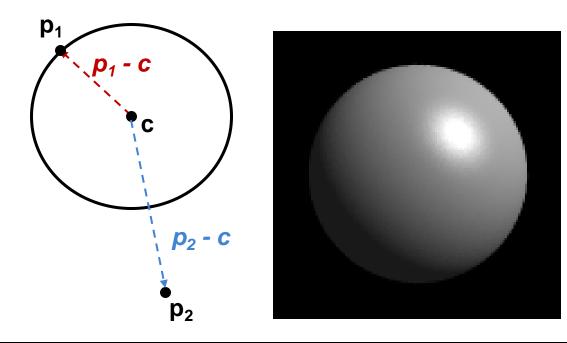


Pinhole Camera Model



Basic Math - Sphere

- Sphere S
 - $-\vec{c} \in \mathbb{R}^3$, $r \in \mathbb{R}$: center and radius
 - $\ \forall \vec{p} \in \mathbb{R}^3 \colon \vec{p} \in S \Leftrightarrow (\vec{p} \vec{c}) \cdot (\vec{p} \vec{c}) r^2 = 0$
 - The distance between points on the sphere and its center equals the radius



Ray-Sphere Intersection

- Given
 - $\operatorname{Ray:} r(t) = \vec{o} + t\vec{d},$

$$\mathbf{t} \in \mathbb{R}; \vec{o}, \vec{d} \in \mathbb{R}^3$$

- Sphere: $\vec{c} \in \mathbb{R}^3$, $r \in \mathbb{R}$:
 - $\forall \vec{p} \in \mathbb{R}^3 : \vec{p} \in S \Leftrightarrow (\vec{p} \vec{c}) \cdot (\vec{p} \vec{c}) r^2 = 0$
- Find closest intersection point
 - Algebraic approach: substitute ray equation

•
$$(\vec{p} - \vec{c}) \cdot (\vec{p} - \vec{c}) - r^2 = 0$$
 with $\vec{p} = \vec{o} + t\vec{d}$

•
$$t^2 \vec{d} \cdot \vec{d} + 2t \vec{d} \cdot (\vec{o} - \vec{c}) + (\vec{o} - \vec{c}) \cdot (\vec{o} - \vec{c}) - r^2 = 0$$

• Solve for t

Ray-Sphere Intersection (2)

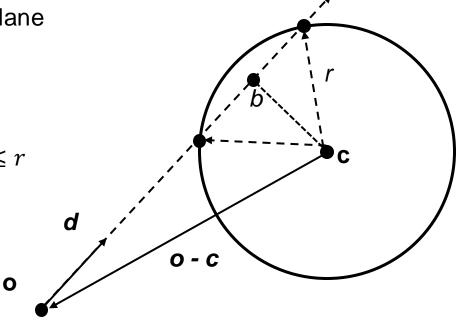
- Given
 - $\operatorname{Ray:} r(t) = \vec{o} + t\vec{d},$

 $\mathbf{t} \in \mathbb{R}; \vec{o}, \vec{d} \in \mathbb{R}^3$

- Sphere: $\vec{c} \in \mathbb{R}^3$, $r \in \mathbb{R}$:
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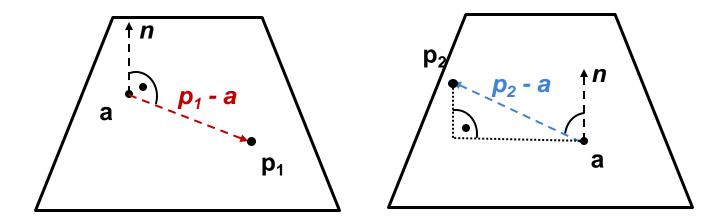
Find closest intersection point

- Geometric approach
 - Ray and center span a plane
 - Solve in 2D
 - Compute $|\vec{b} \vec{o}|$, $|\vec{b} \vec{c}|$
 - Such that $\angle OBC = 90^{\circ}$
 - Intersection(s) if $\left| \vec{b} \vec{c} \right| \le r$
- Be aware of floating point issues if o is far from sphere



Basic Math - Plane

- Plane P
 - $-\vec{n}, \vec{a} \in \mathbb{R}^3$: normal and point in *P* (Hesse normal form for plane)
 - $\ \forall \vec{p} \in \mathbb{R}^3 \colon \vec{p} \in P \Leftrightarrow (\vec{p} \vec{a}) \cdot \vec{n} = 0$
 - The difference vector between any two points on the plane is either 0 or orthogonal to the plane's normal



Ray-Plane Intersection

- Given
 - Ray: $r(t) = \vec{o} + t\vec{d}$, $t \in \mathbb{R}; \vec{o}, \vec{d} \in \mathbb{R}^3$
 - Plane: $\vec{n}, \vec{a} \in \mathbb{R}^3$: normal and point in *P*

Compute intersection point

- Plane equation: $\vec{p} \in P \Leftrightarrow (\vec{p} \vec{a}) \cdot \vec{n} = 0$ $\Leftrightarrow \vec{p} \cdot \vec{n} - D = 0$, with $D = \vec{a} \cdot \vec{n}$
- Substitute ray parameterization: $(\vec{o} + t\vec{d}) \cdot \vec{n} D = 0$
- Solve for t
 - What are possible solutions?

Ray-Plane Intersection

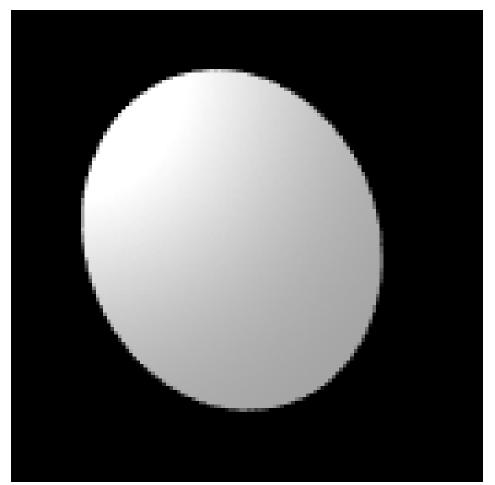
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- Substitute ray parameterization: $(\vec{o} + t\vec{d}) \cdot \vec{n} D = 0$
- Solve for t
 - 0,1, or infinitely many solutions
 - One intersection or parallel to/in plane

Ray-Disc Intersection

- Intersect ray with plane
- Discard intersection if ||p a|| > r



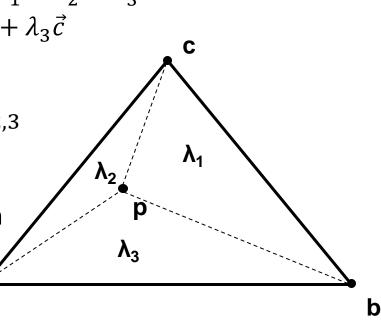
Basic Math - Triangle

- Triangle T
 - $-\vec{a},\vec{b},\vec{c}\in\mathbb{R}^3$: vertices
 - Affine combinations of $\vec{a}, \vec{b}, \vec{c} \rightarrow$ points in the plane
 - Non-negative coefficients that sum up to $1 \rightarrow \text{points}$ in the triangle

$$- \forall \vec{p} \in \mathbb{R}^3 : \vec{p} \in T \Leftrightarrow \exists \lambda_{1,2,3} \in \mathbb{R}_{0+}, \ \lambda_1 + \lambda_2 + \lambda_3 = 1 \text{ and} \\ \vec{p} = \lambda_1 \vec{a} + \lambda_2 \vec{b} + \lambda_3 \vec{c}$$

a

- Barycentric coordinates $\lambda_{1,2,3}$
 - $-\lambda_1 = S_{pbc}/S_{abc}$, etc.
 - S: signed area of triangles, based on CLW/CCW orientation

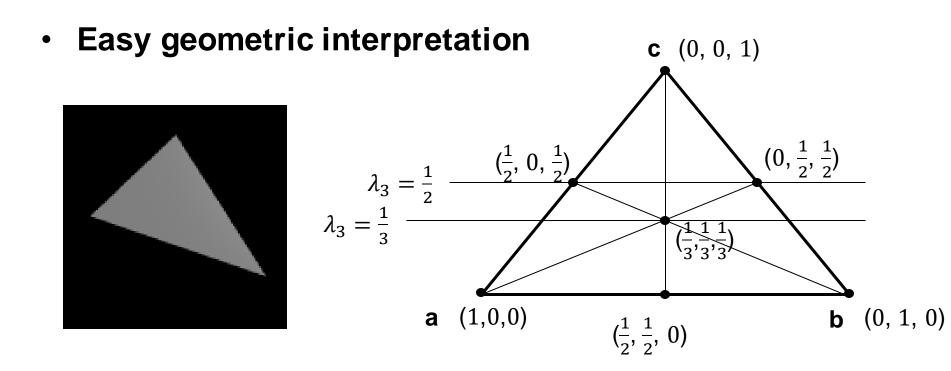


Barycentric Coordinates

- Triangle T
 - $-\vec{a},\vec{b},\vec{c}\in\mathbb{R}^3$: vertices
 - $-\lambda_{1,2,3}$: Barycentric coordinates

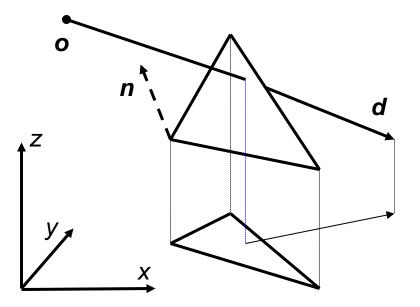
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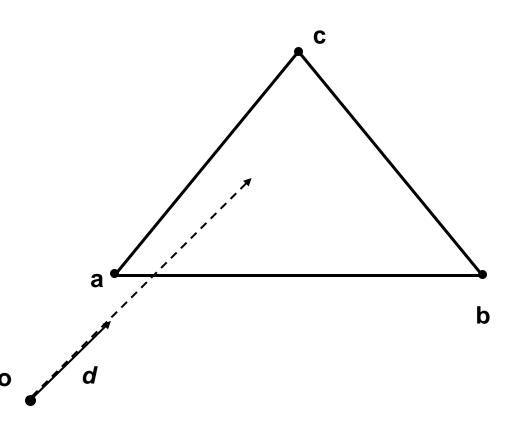


Triangle Intersection: Plane-Based

- Compute intersection with triangle plane
- Compute barycentric coordinates
 - Signed areas of subtriangles
 - Can be done in 2D, after
 "projection" onto major plane, depending on largest component of normal vector
- Test for positive BCs
- Issues:
 - Edges of neighboring triangles might not be identical
 - Due to inaccuracies of floats
 - Need a better method!



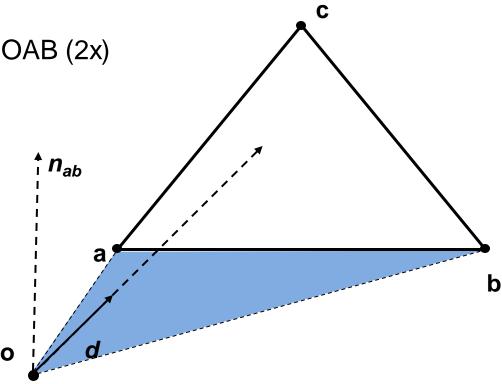
- 3D linear function across triangle (3D edge functions)
 - $\text{ Ray: } \vec{o} + t\vec{d} , \qquad \qquad t \in \mathbb{R}; \vec{o}, \vec{d} \in \mathbb{R}^3$
 - Triangle: $\vec{a}, \vec{b}, \vec{c} \in \mathbb{R}^3$



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$$- \overrightarrow{n_{ab}} = \left(\vec{b} - \vec{o}\right) \times \left(\vec{a} - \vec{o}\right)$$

 $- |\overrightarrow{n_{ab}}|$ is the signed area of OAB (2x)



• 3D linear function across triangle (3D edge functions)

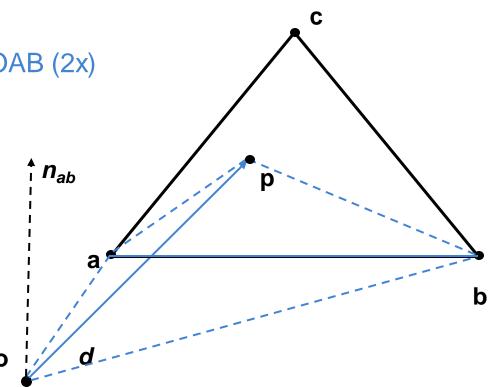
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$$-\lambda_3^*(t) = \overrightarrow{n_{ab}} \cdot t \vec{d}$$

- Volume of OBAP (6x)
- For $t = t_{hit}$



• 3D linear function across triangle (3D edge functions)

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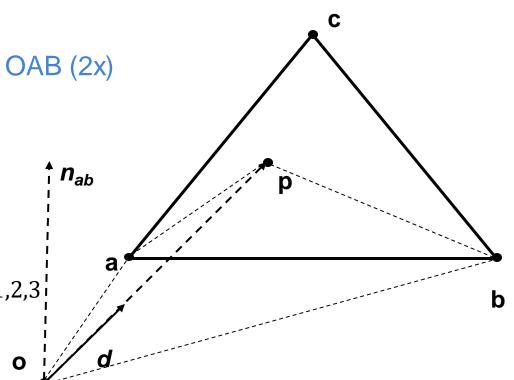
- Volume of OBAP (6x)
- For $t = t_{hit}$

$$- \lambda_{1,2}^*(t) = \overrightarrow{n_{bc,ac}} \cdot t \vec{d}$$

Normalize

•
$$\lambda_i = \frac{\lambda_i^*(t)}{\lambda_1^*(t) + \lambda_2^*(t) + \lambda_3^*(t)}$$
, $i = 1, 2, 3$

• Length of $t\vec{d}$ cancels out



• 3D linear function across triangle (3D edge functions)

- $\text{ Ray: } \vec{o} + t\vec{d} , \qquad \qquad t \in \mathbb{R}; \vec{o}, \vec{d} \in \mathbb{R}^3$
- Triangle: $\vec{a}, \vec{b}, \vec{c} \in \mathbb{R}^3$

$$-\overrightarrow{n_{ab}} = \left(\vec{b} - \vec{o}\right) \times \left(\vec{a} - \vec{o}\right)$$

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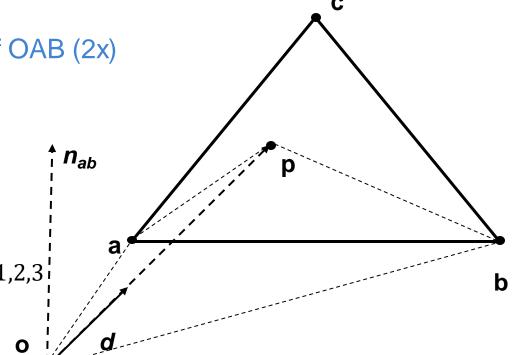
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- For $t = t_{hit}$

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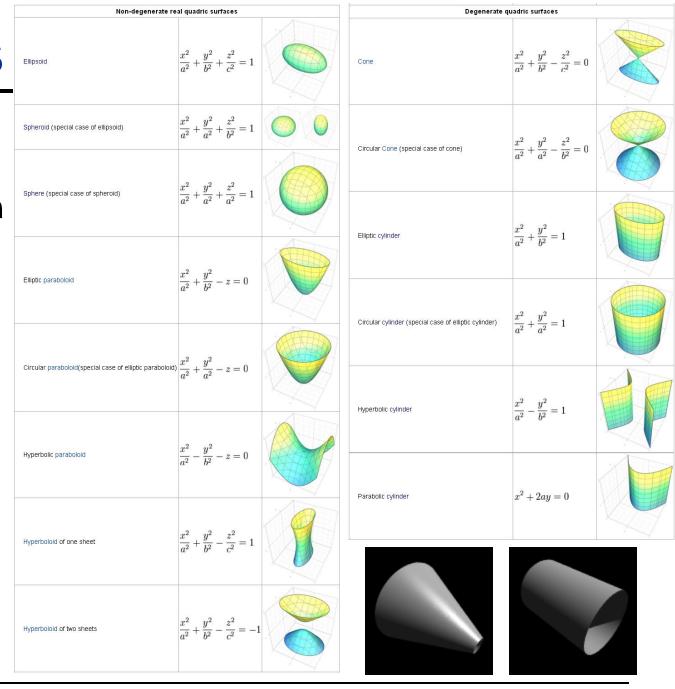
- For all-positive BCs:
 - Compute $\vec{p} = \lambda_1 \vec{a} + \lambda_2 \vec{b} + \lambda_3 \vec{c}$



Quadrics

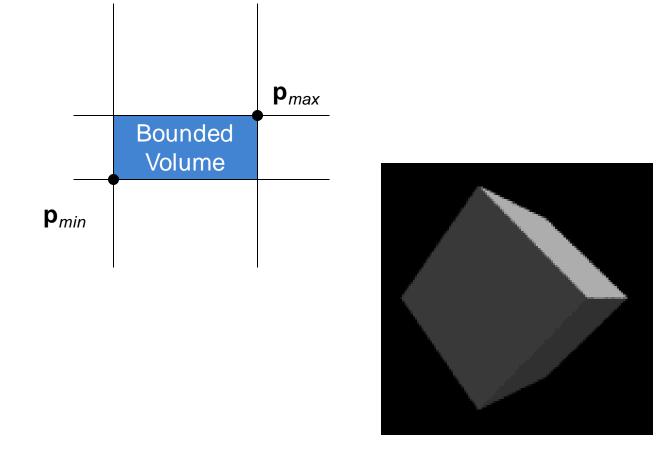
- Implicit

 f(x, y, z) = v
- Ray equation
 - x = xo + t xd
 - y = yo + t yd
 - -z = zo + t zd
- Solve for t
- Assignment



Axis Aligned Bounding Box

- Given
 - Ray: $\vec{o} + t\vec{d}$, $t \in \mathbb{R}; \vec{o}, \vec{d} \in \mathbb{R}^3$
 - Axis aligned bounding box (AABB): $\overrightarrow{p_{min}}, \overrightarrow{p_{max}} \in \mathbb{R}^3$

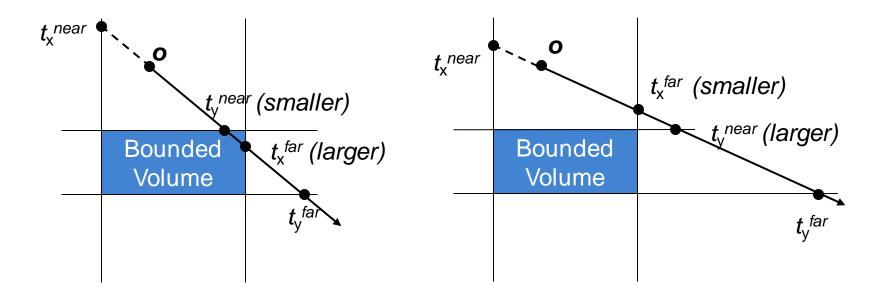


Ray-Box Intersection

- Given
 - $\text{ Ray: } \vec{o} + t\vec{d} , \qquad \qquad t \in \mathbb{R}; \vec{o}, \vec{d} \in \mathbb{R}^3$
 - Axis aligned bounding box (AABB): $\overrightarrow{p_{min}}, \overrightarrow{p_{max}} \in \mathbb{R}^3$

"Slabs test" for ray-box intersection

- Ray enters the box in all dimensions before exiting in any
- $\max(\{t_i^{near} | i = x, y, z\}) < \min(\{t_i^{far} | i = x, y, z\})$



History of Intersection Algorithms

Ray-geometry intersection algorithms

- Polygons:
- Recursive Ray Tracing: [Whitted '79]
- Tori:
- Bicubic patches:
- Algebraic surfaces:
- Swept surfaces:
- Fractals:
- Deformations:
- NURBS:
- Subdivision surfaces:

[Appel '68] Quadrics, CSG: [Goldstein & Nagel '71] [Roth '82] [Whitted '80, Kajiya '82] [Hanrahan '82] [Kajiya '83, van Wijk '84] [Kajiya '83] [Barr '86] [Stürzlinger '98] [Kobbelt et al '98]

Precision Problems

• E.g., Cause of "surface acne"

