Computer Graphics

- Distribution Ray Tracing -

Arsène Pérard-Gayot

Problems

- **Anti-aliasing**
- **Depth of field**
- **Motion blur**
- **BRDF**
- **Area Lights**

Anti-aliasing

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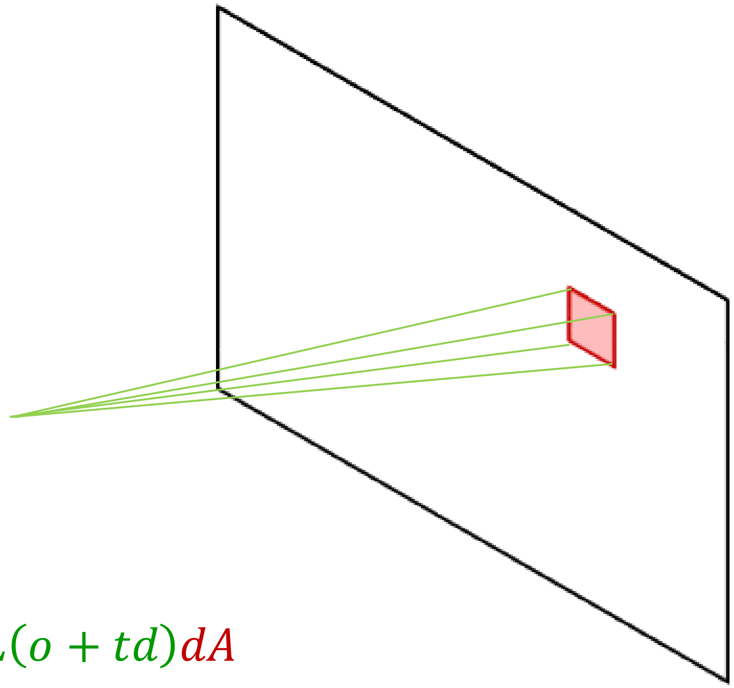

$$I \approx \int_{A} L(o + td)\, dA$$

Depth of field

- **Anti-aliasing**
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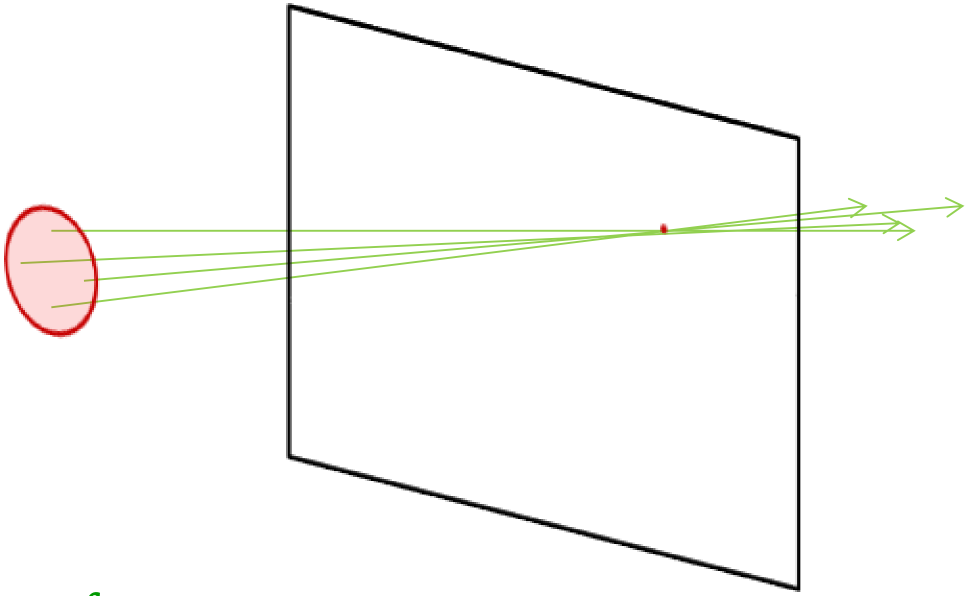
- **Motion blur**
- **BRDF**
- **Area Lights**



$$I \approx \int_{A} L(o + td)\,dA$$

Motion blur

- **Anti-aliasing**
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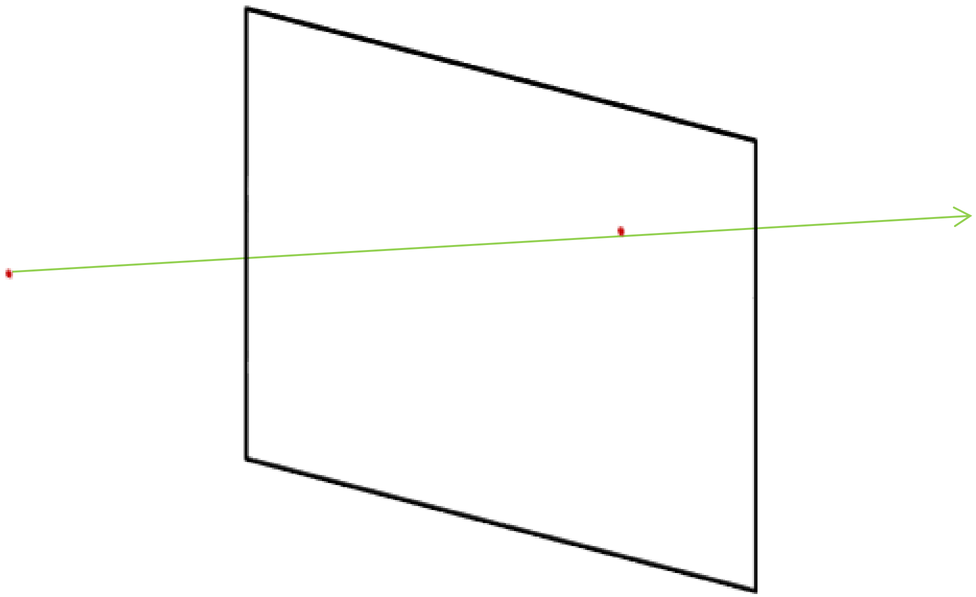

$$L \approx \int_{[t_o,t_1]} L_T(o + td)\,dT$$

BRDF

- **Anti-aliasing**
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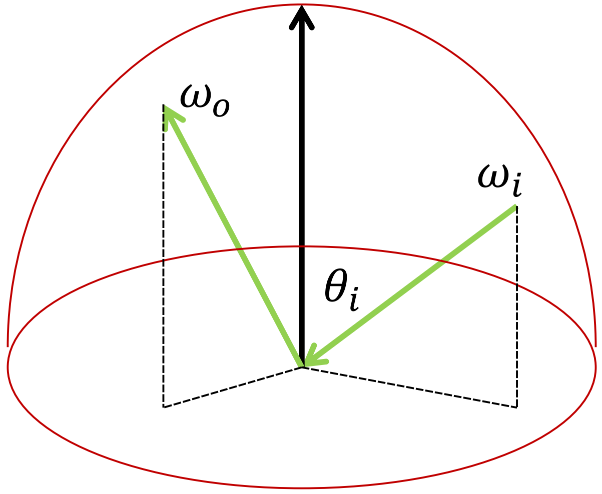

$$L_o = L_e + \int_{\Omega_+} f_r L_i \cos\theta_i \, d\omega_i$$

Area Lights

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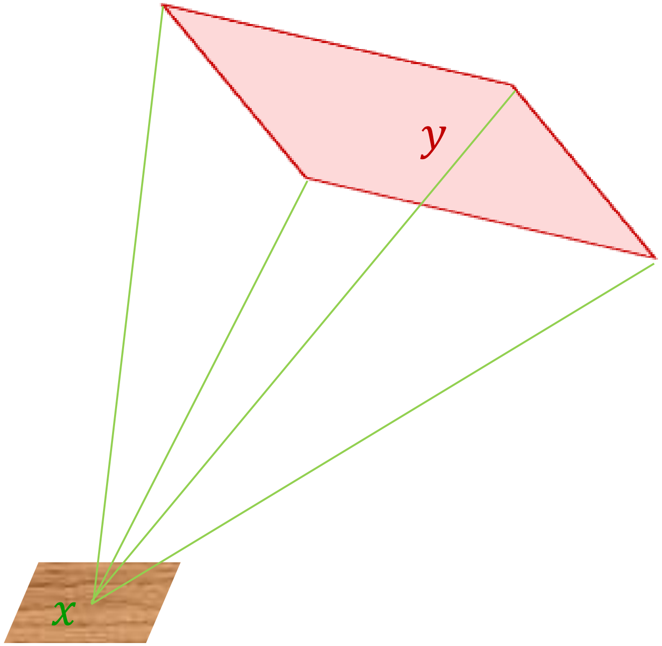

$$E_i = \int_A V(x, y) \frac{\cos \theta_A}{\|x - y\|^2} dA$$

Sampling

- **Anti-aliasing**
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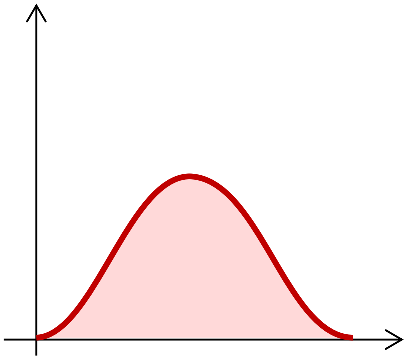
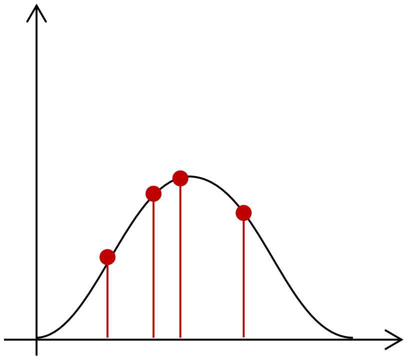
- **Area Lights**

$$R = \int_D f(x)dx$$



$$R \approx \frac{D}{n}\sum_{i=1}^{n} f(x_i)$$

$$x_i = \text{uniform}(D)$$



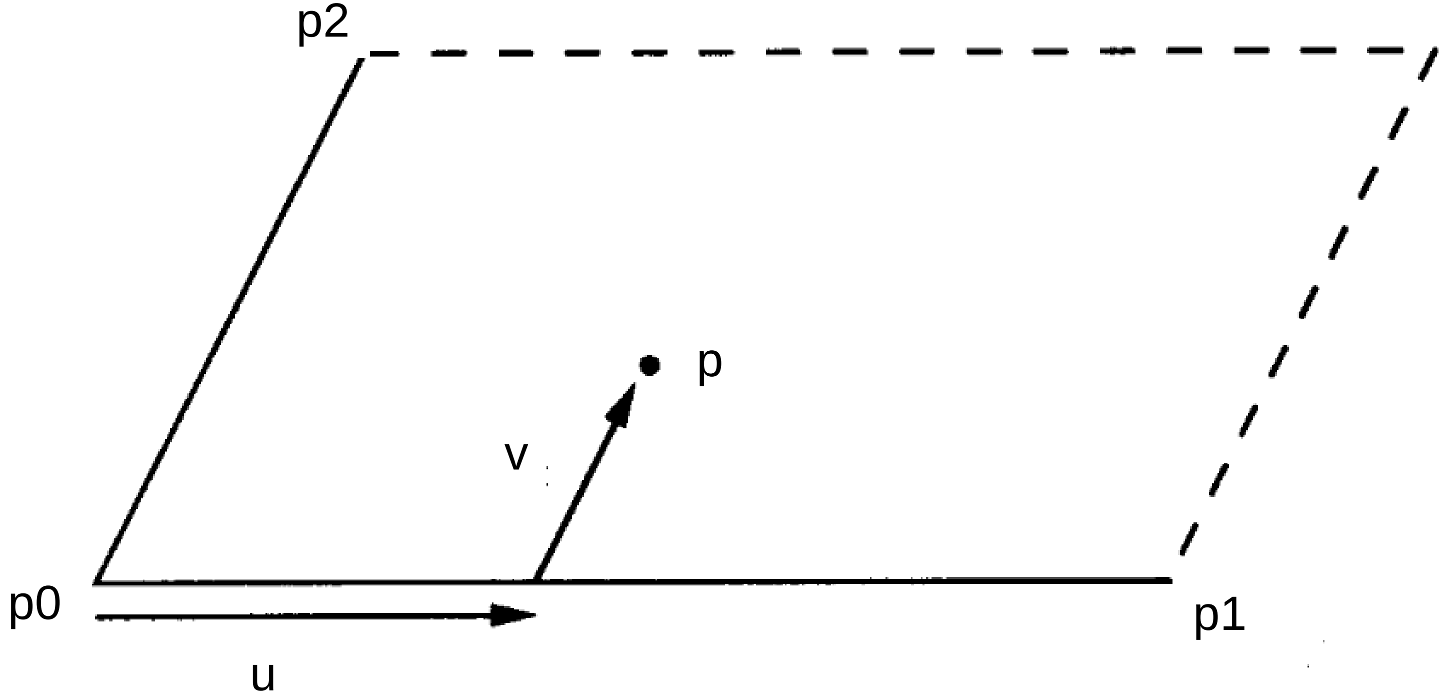
STOCHASTIC SAMPLING

Random Number

- **Random Number**
  - Uniformly distributed
  - ξ in [0, 1)
  - D=1
- **Pseudo-Random Number**
  - Linear congruential
  - Mersenne-Twister
  - …
  - Speed / evenness trade-off

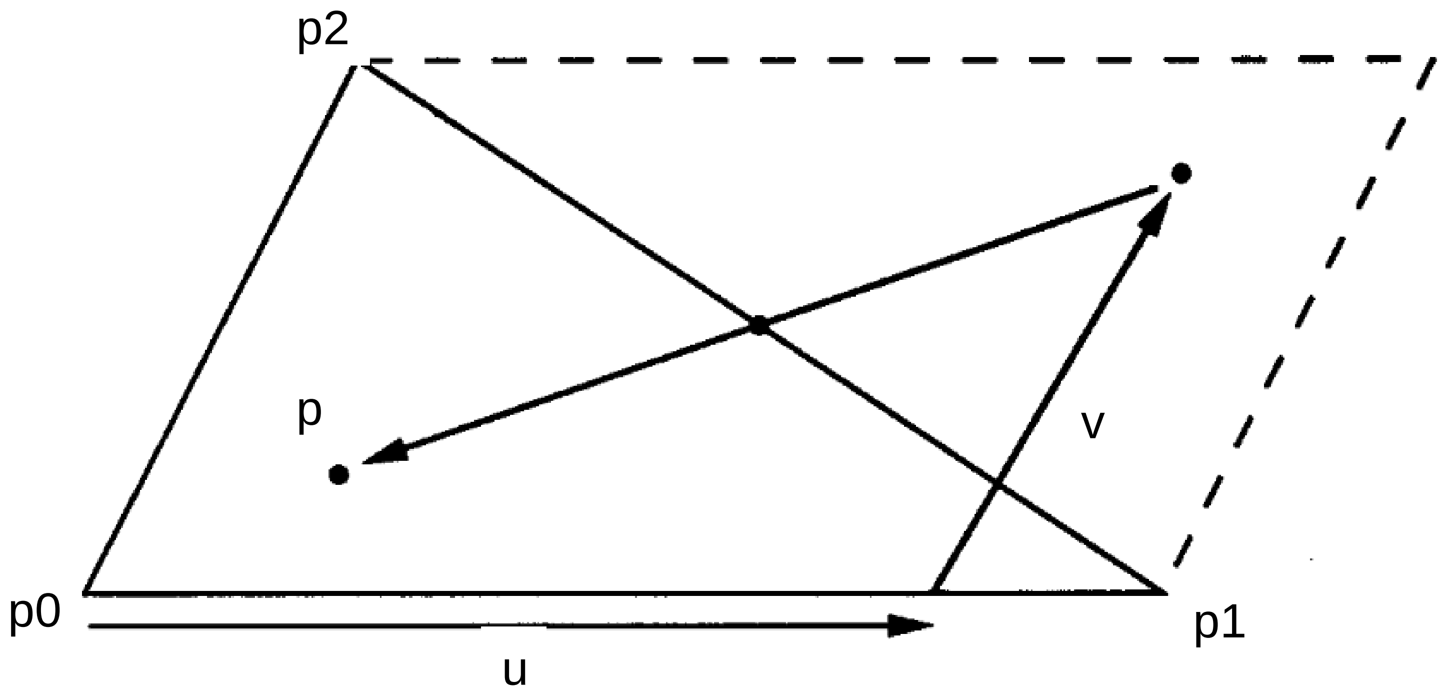
Parallelogram Sampling

- **Parametric Form**
  - $p(u, v) = (1 - u - v)p_0 + up_1 + v\,p_2$
- **Random Sampling**
  - $p(\xi_1, \xi_2)$



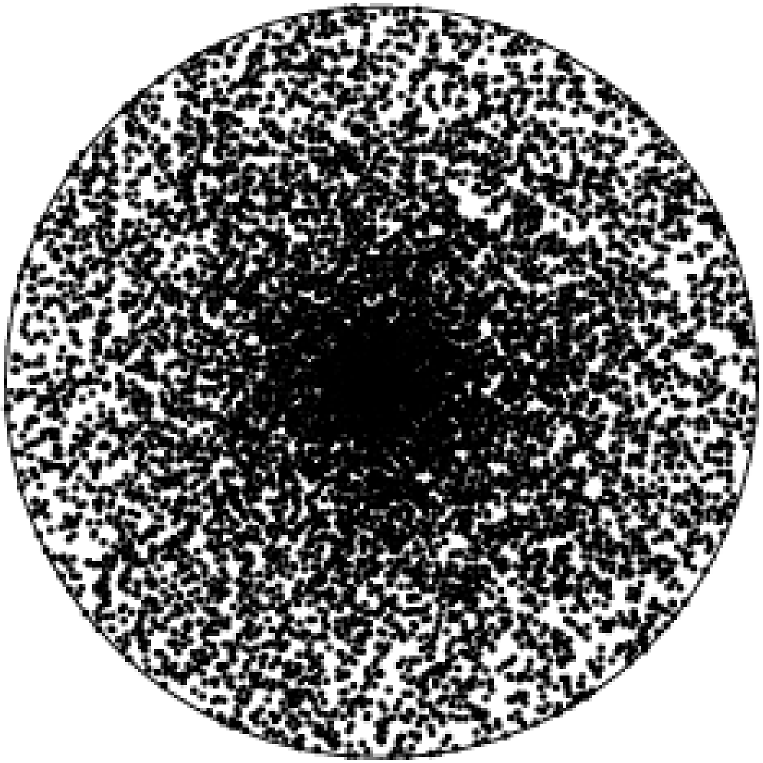
Triangle Sampling

- **Parametric Form**
  - $p(u, v) = (1 - u - v)p_0 + up_1 + v\, p_2$
- **Random Sampling**
  - if $\xi_1 + \xi_2 < 1$: $p(\xi_1, \xi_2)$
  - if $\xi_1 + \xi_2 > 1$: $p(1 - \xi_1, 1 - \xi_2)$



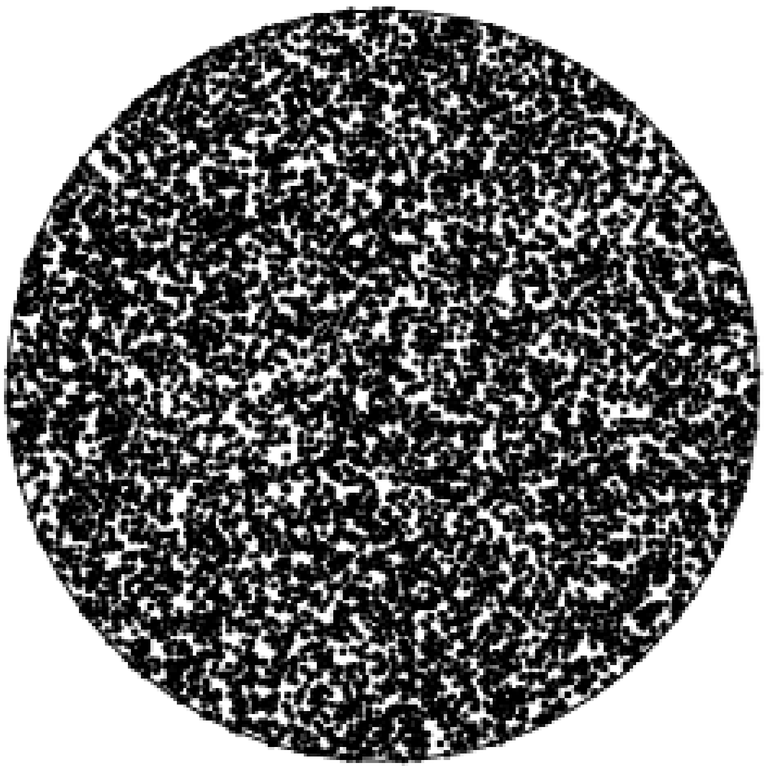
Disc Sampling

- **Parametric Form**
  - $\mathrm{p}_{\circ}(u, v) = \text{Polar\_to\_Cartesian}(Rv, 2\pi u)$
- **Naïve Sampling**
  - $\mathrm{p}_{\circ}(\xi_1, \xi_2)$



Disc Sampling

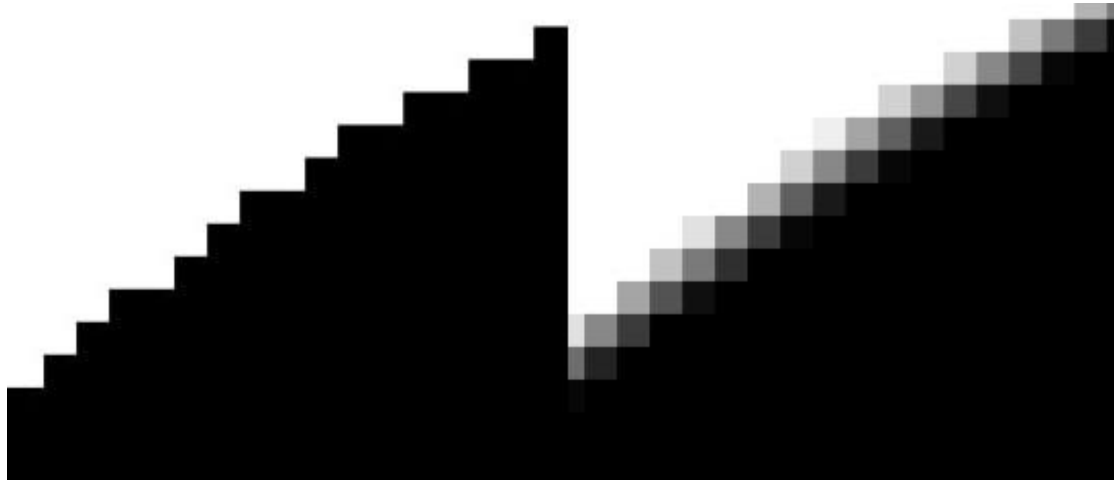
- **Parametric Form**
  - $\mathrm{p}_{\circ}(u, v) = \mathrm{Polar\_to\_Cartesian}(Rv, 2\pi u)$
- **Random Sampling**
  - $\mathrm{p}_{\circ}\left(\xi_1, \sqrt{\xi_2}\right)$



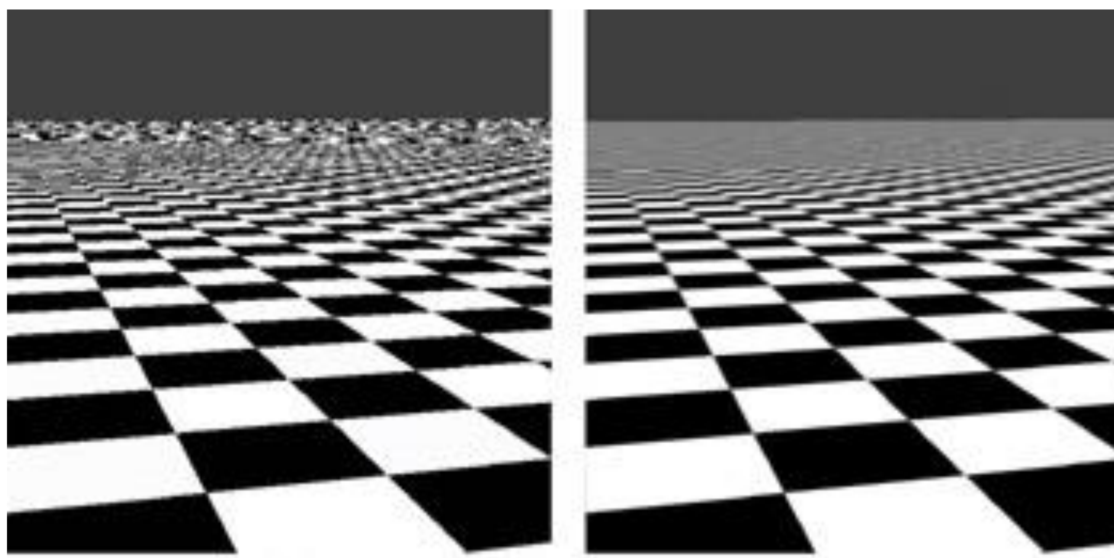
DISTRIBUTION RAY-TRACING

Anti-Aliasing

- **Artifacts**
  - Jagged edges



  - Aliased patterns

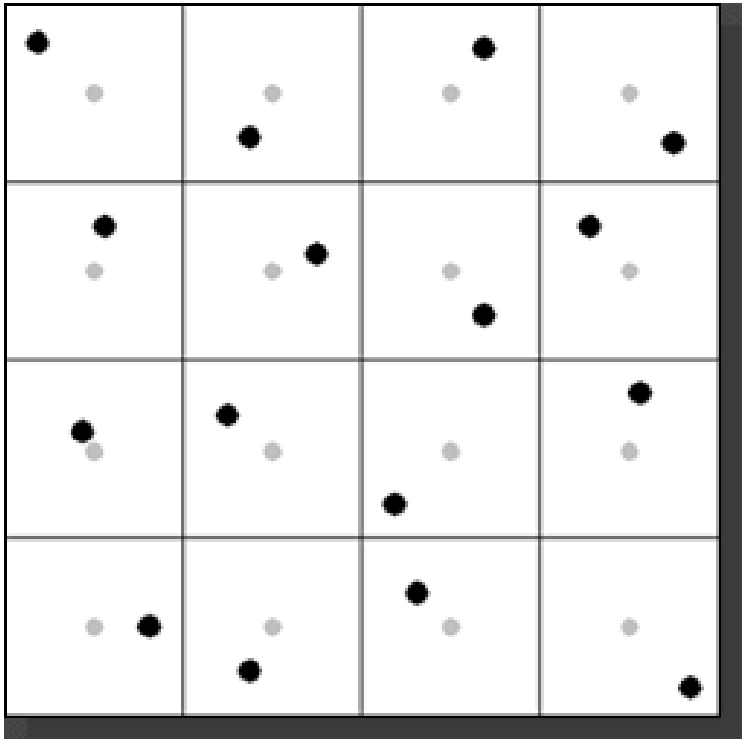


(a) (b)

Anti-Aliasing

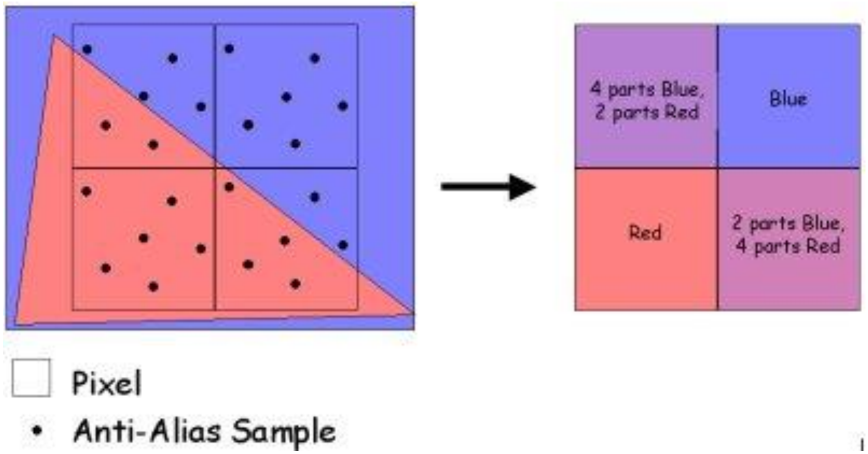
- **Approach**
  - Average samples over pixel area
  - Sample offset in the pixel area

$$(x + 0.5, y + 0.5) \rightarrow (x + \xi_1, y + \xi_2)$$

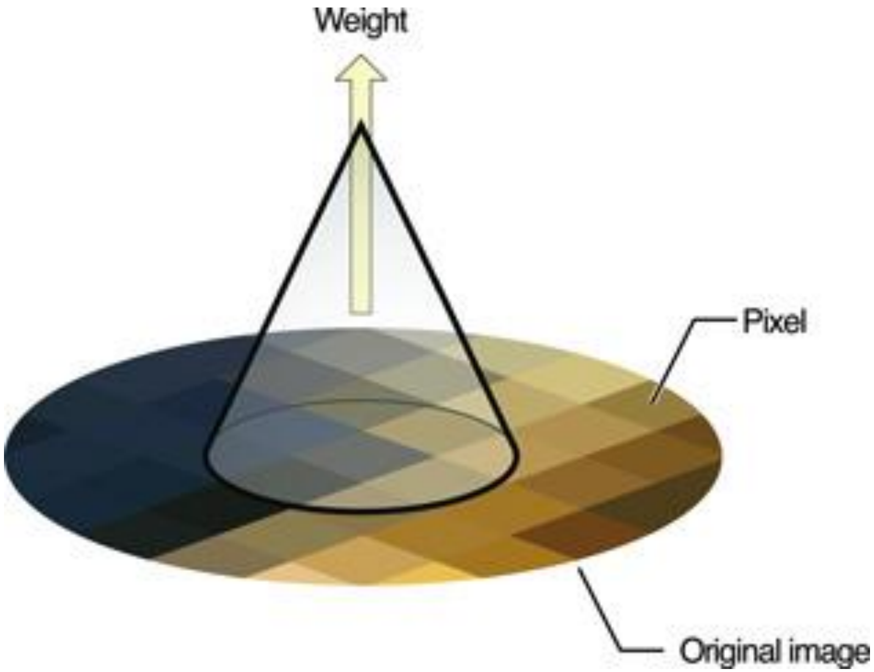


Anti-Aliasing

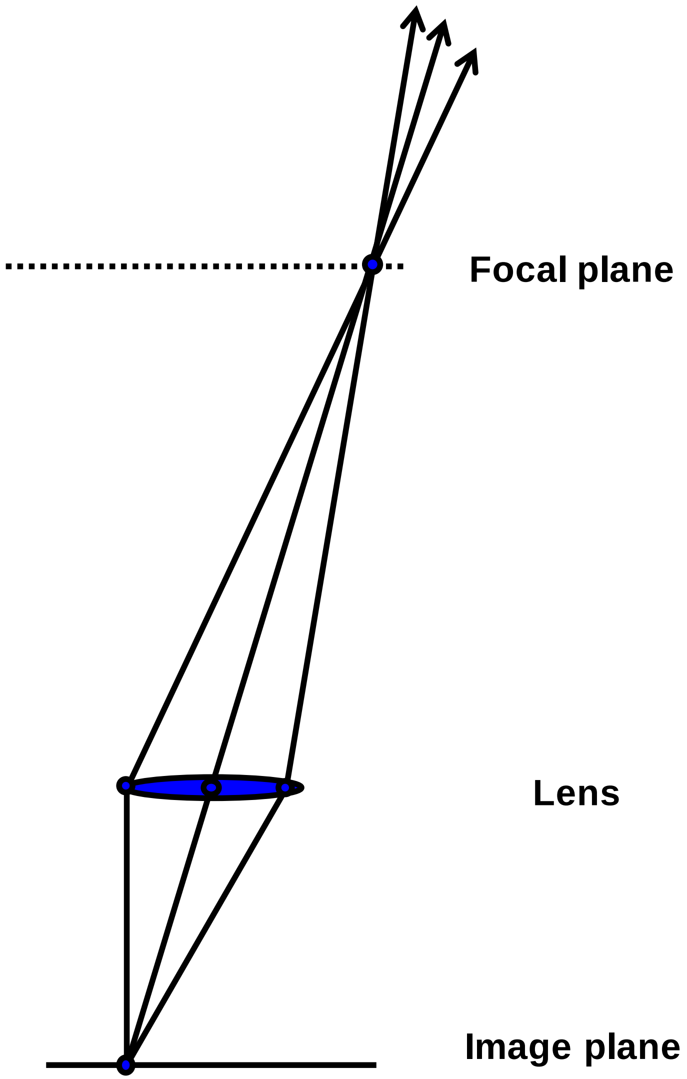
- **Basic Method**
  - Plain average
  - Box filter f(x, y) = 1
  - $L = \frac{\sum_{i=1}^{n} L(\xi_{i1},\xi_{i2})}{n}$

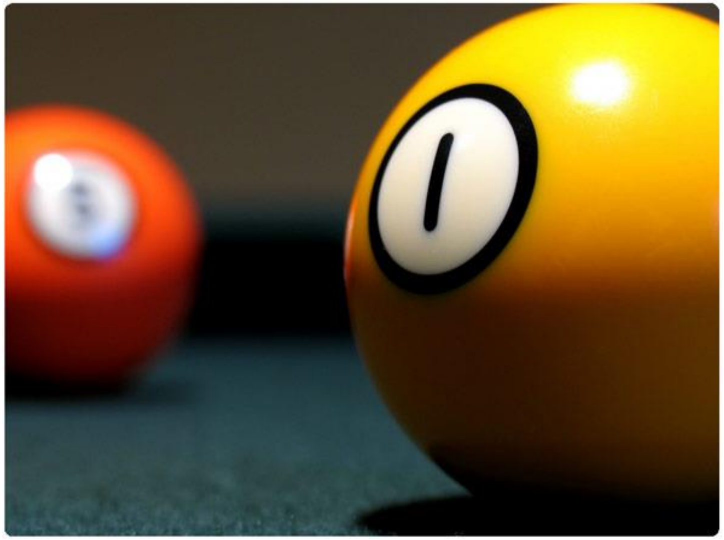


- **Filtering**
  - Weighted average
  - Filter f(x, y)
  - $L = \frac{\sum_{i=1}^{n} f(\xi_{i1},\xi_{i2}) L(\xi_{i1},\xi_{i2})}{\sum_{i=1}^{n} f(\xi_{i1},\xi_{i2})}$

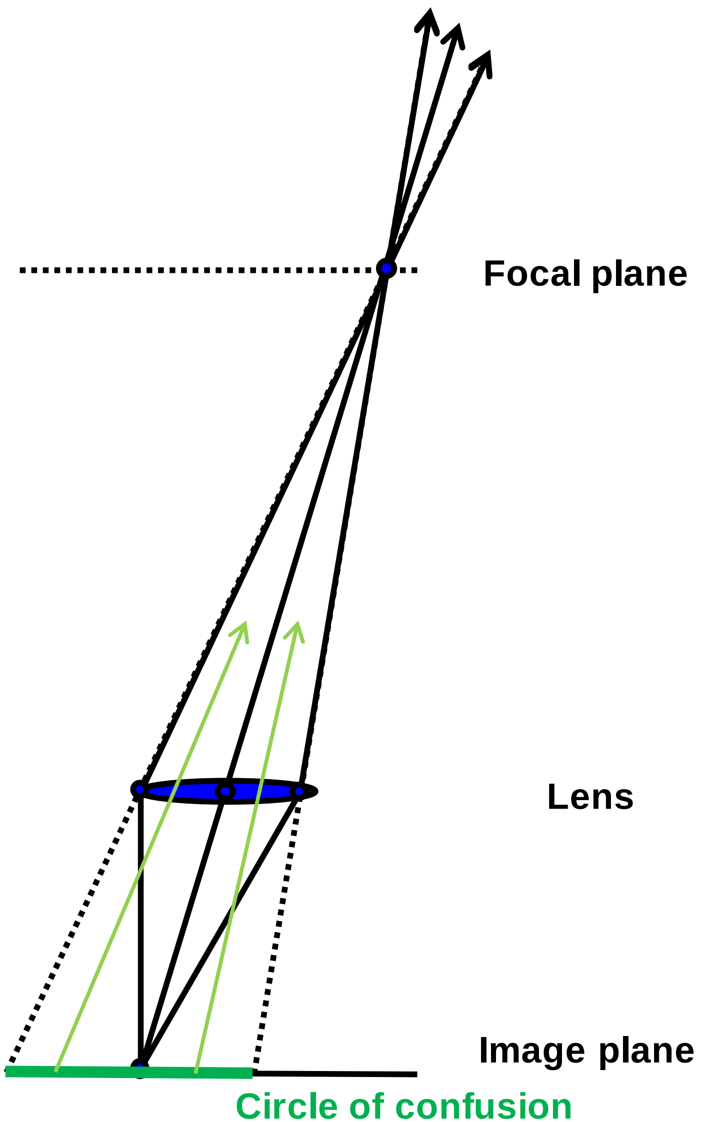


Depth of Field





Depth of Field



Depth of Field

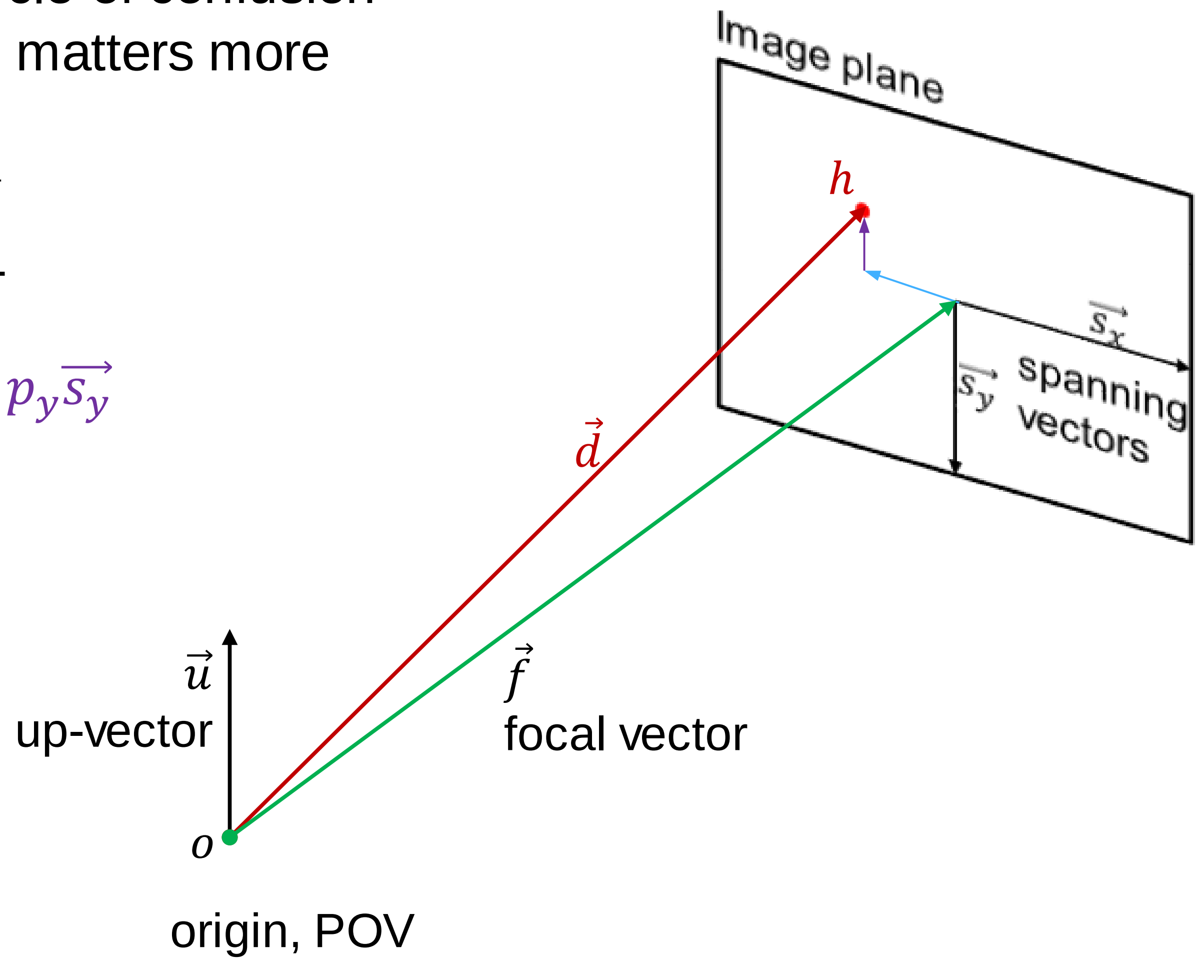
- **Specified by**
  - Pinhole camera parameters
  - Radius of circle of confusion
  - Focal length matters more

$$p_x = 2\,\frac{x+0.5}{X} - 1$$

$$p_y = 2\,\frac{y+0.5}{Y} - 1$$

$$\vec{d} = \vec{f} + p_x\overrightarrow{s_x} + p_y\overrightarrow{s_y}$$

$$h = o + \vec{d}$$



Depth of Field

- **Specified by**
  - Pinhole camera parameters
  - Radius of circle of confusion – r
  - Focal length matters more

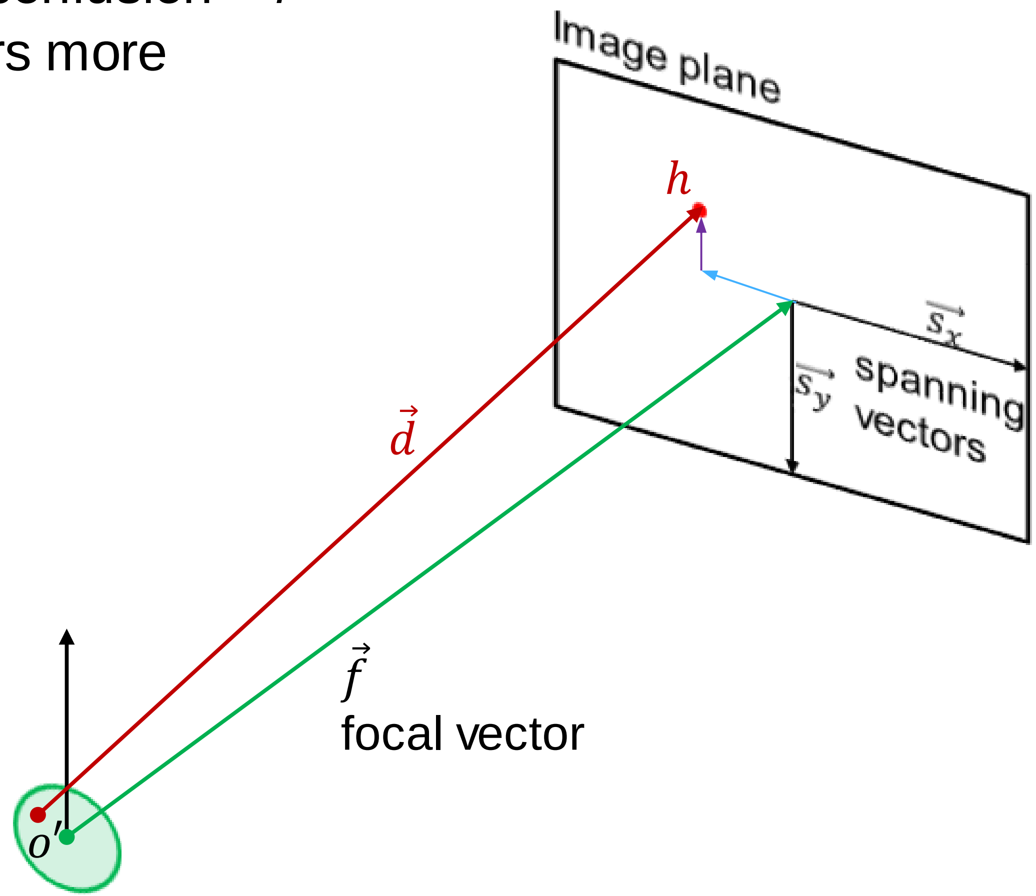
$$\vec{d} = \vec{f} + p_x \overrightarrow{s_x} + p_y \overrightarrow{s_y}$$

$$h = o + \vec{d}$$

$$(\zeta_1, \zeta_2) = \mathrm{p}_\circ\left(\xi_1, \sqrt{\xi_2}\right)$$

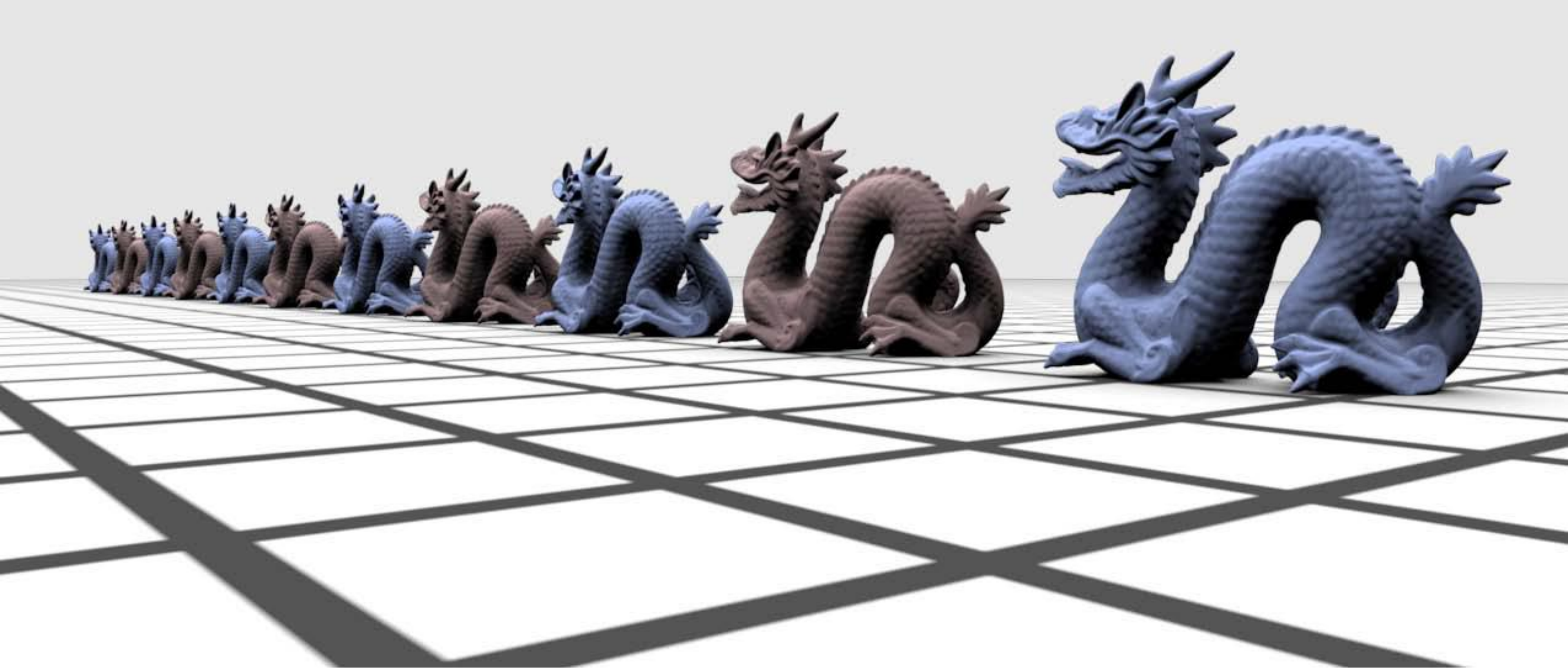
$$o' = o + \zeta_1 r \frac{\overrightarrow{s_x}}{|\overrightarrow{s_x}|} + \zeta_2 r \frac{\overrightarrow{s_y}}{|\overrightarrow{s_y}|}$$

$$\overrightarrow{d'} = h - o'$$



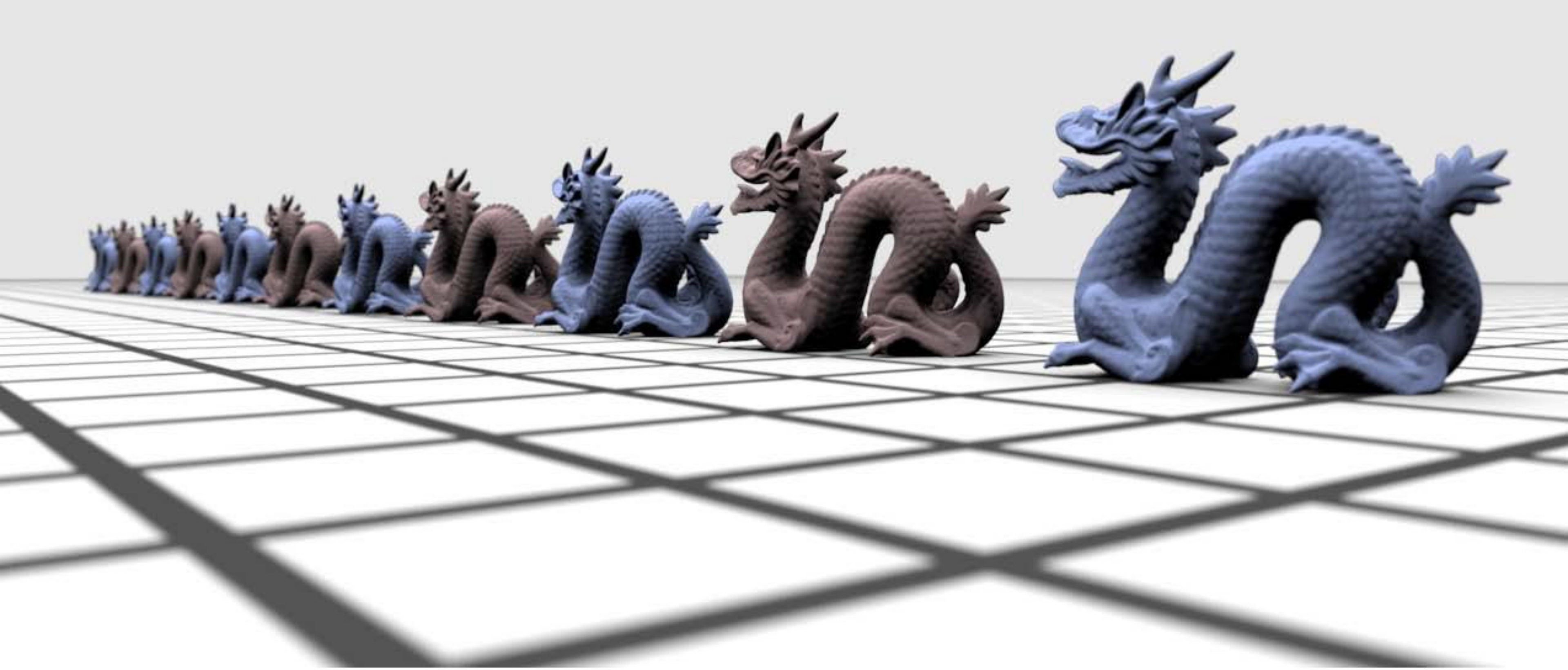
Depth of Field

- **Very Small Aperture**



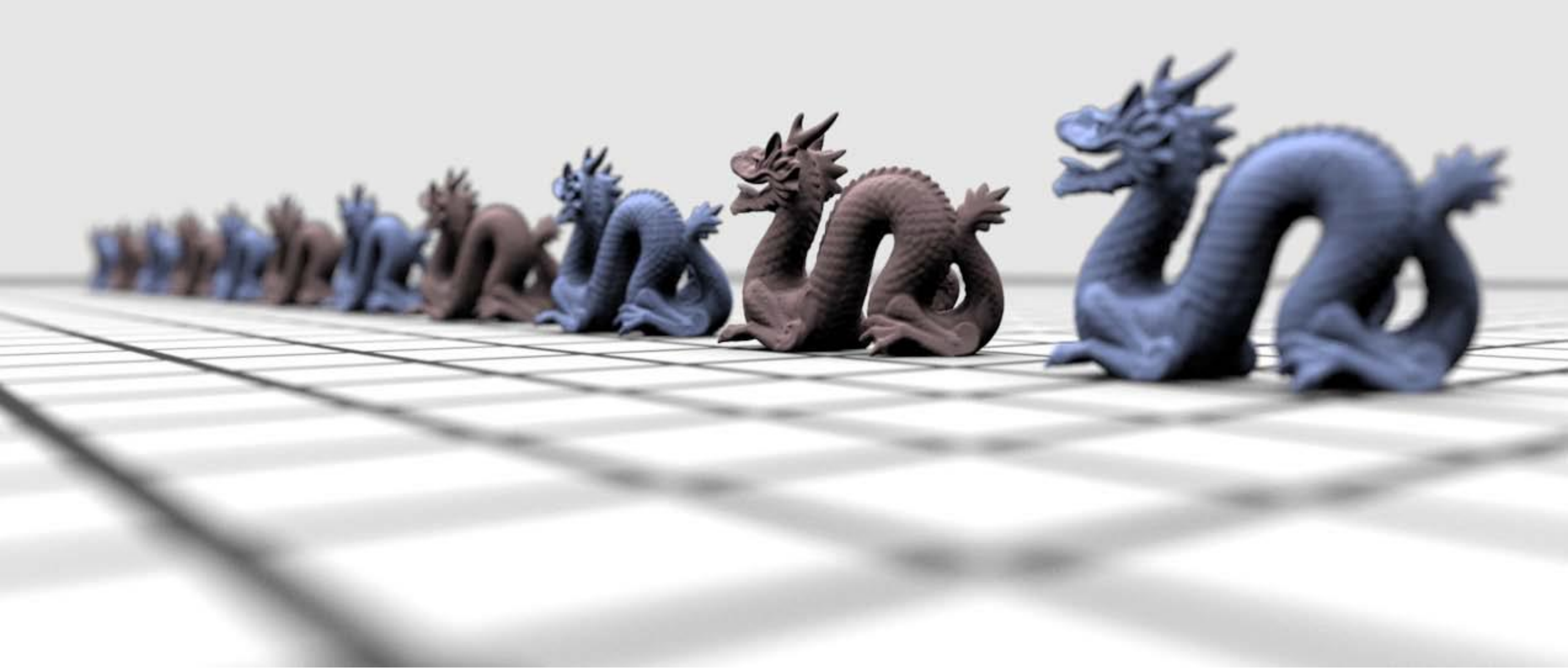
Depth of Field

- **Small Aperture**



Depth of Field

- **Large Aperture**

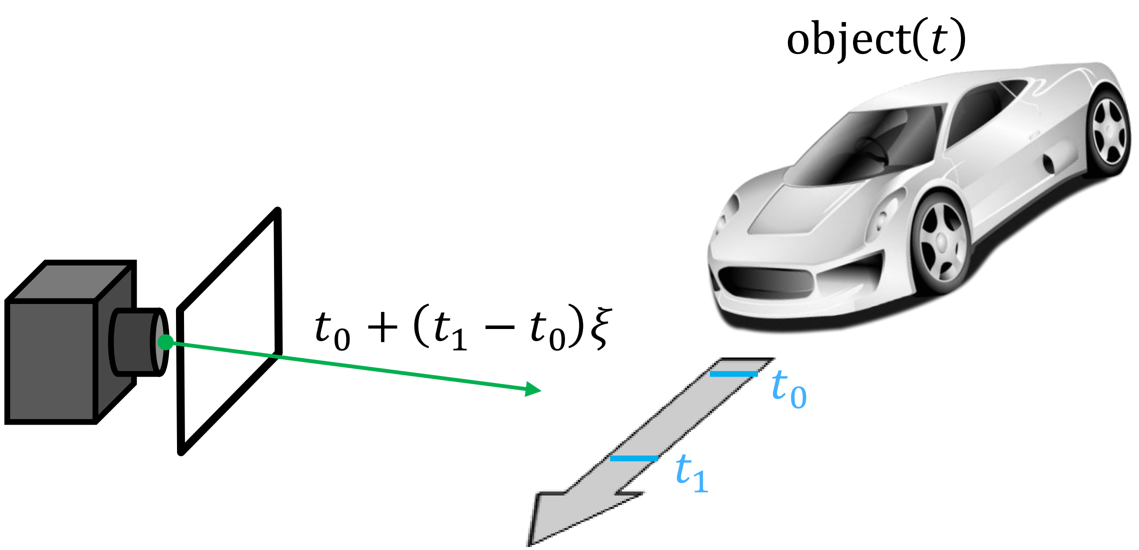


Depth of Field

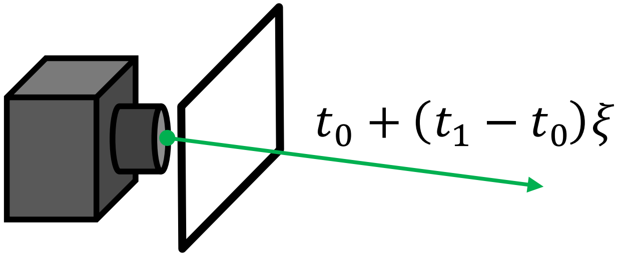
- **Very Large Aperture**

Motion Blur

- **Real Camera**
  - Finite exposure time
  - Shutter opening at t_0
  - Shutter closing at t_1



Motion Blur

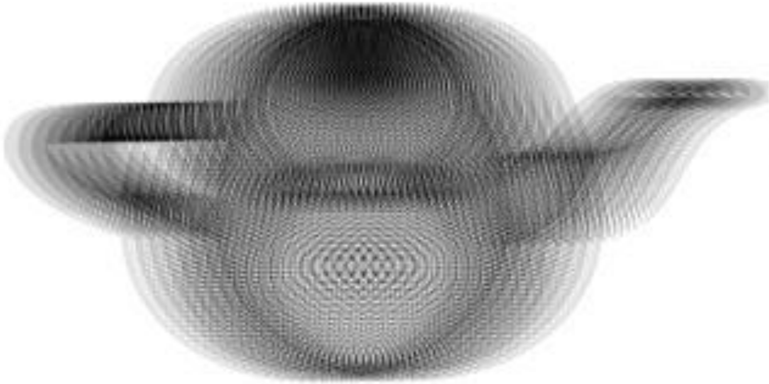


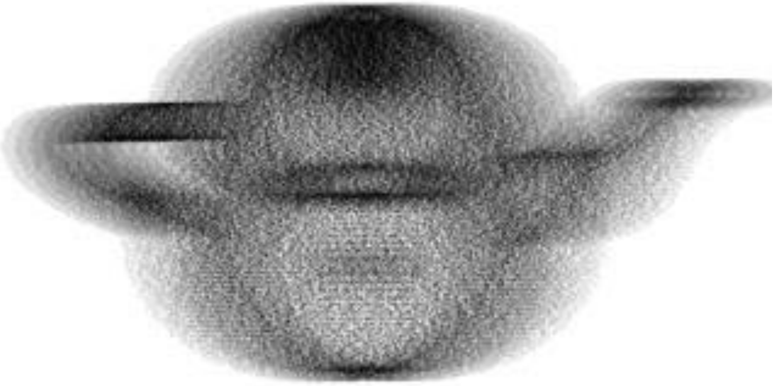
constant sample ξ per image
several images superimposed

segmented result
one index per frame

new sample ξ per ray
several rays per pixel

smooth results
Indexing structures?





Reflections/Refractions

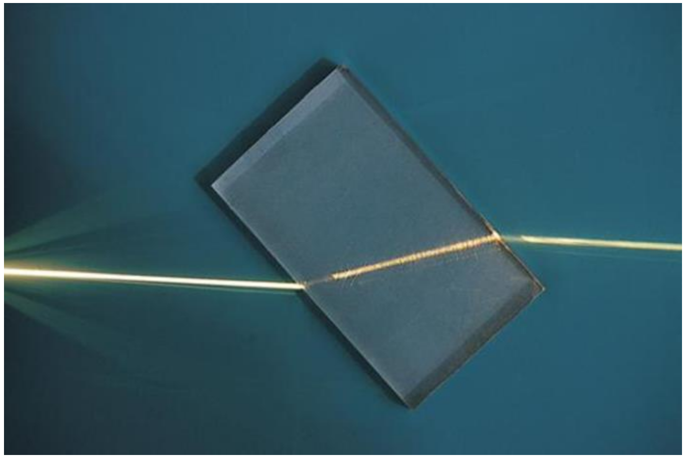
- **Dielectric Materials**
  - η_i – refractive index $\frac{c}{v}$
  - Light: fastest path
  - Snell's law:

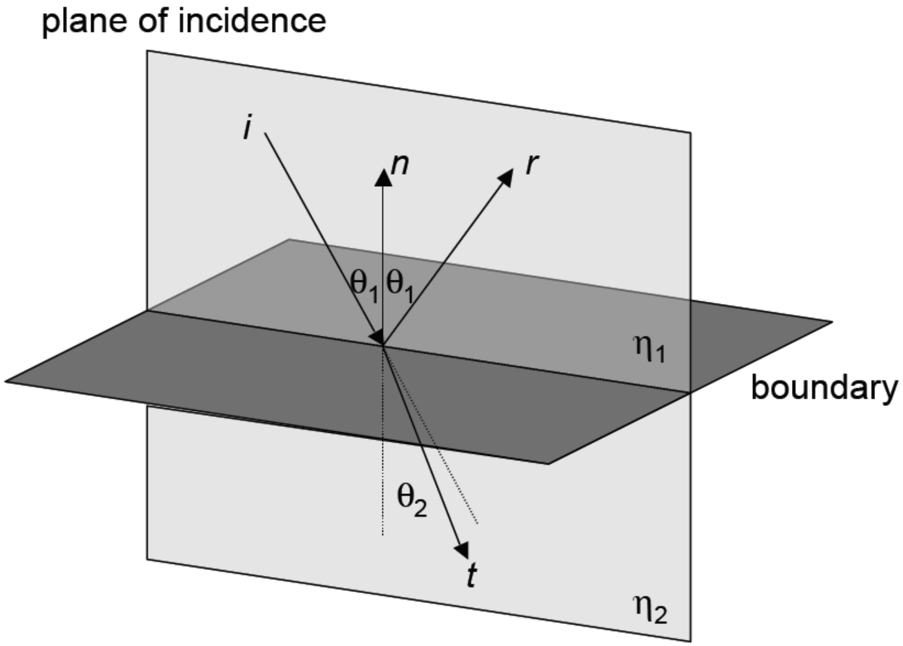
$$\frac{\sin\theta_1}{\sin\theta_2} = \frac{\eta_2}{\eta_1}$$

  - if

$$\sin\theta_2 = \frac{\eta_1}{\eta_2}\sin\theta_1 > 1$$

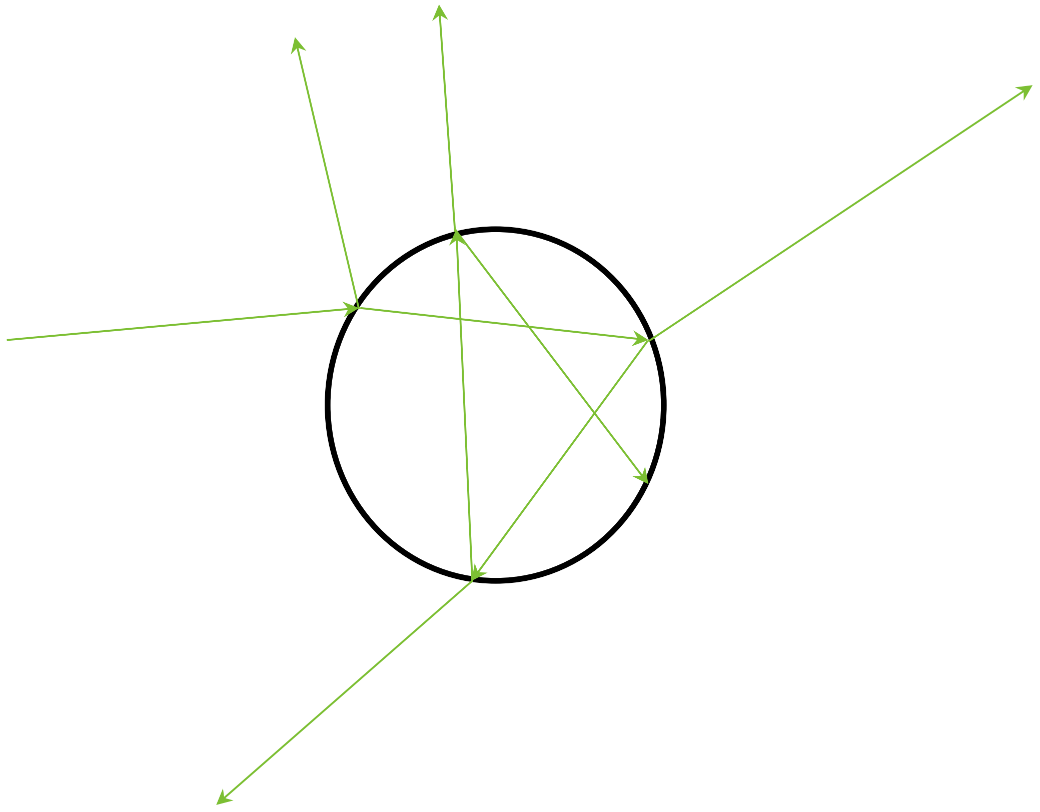
... then total inner reflection





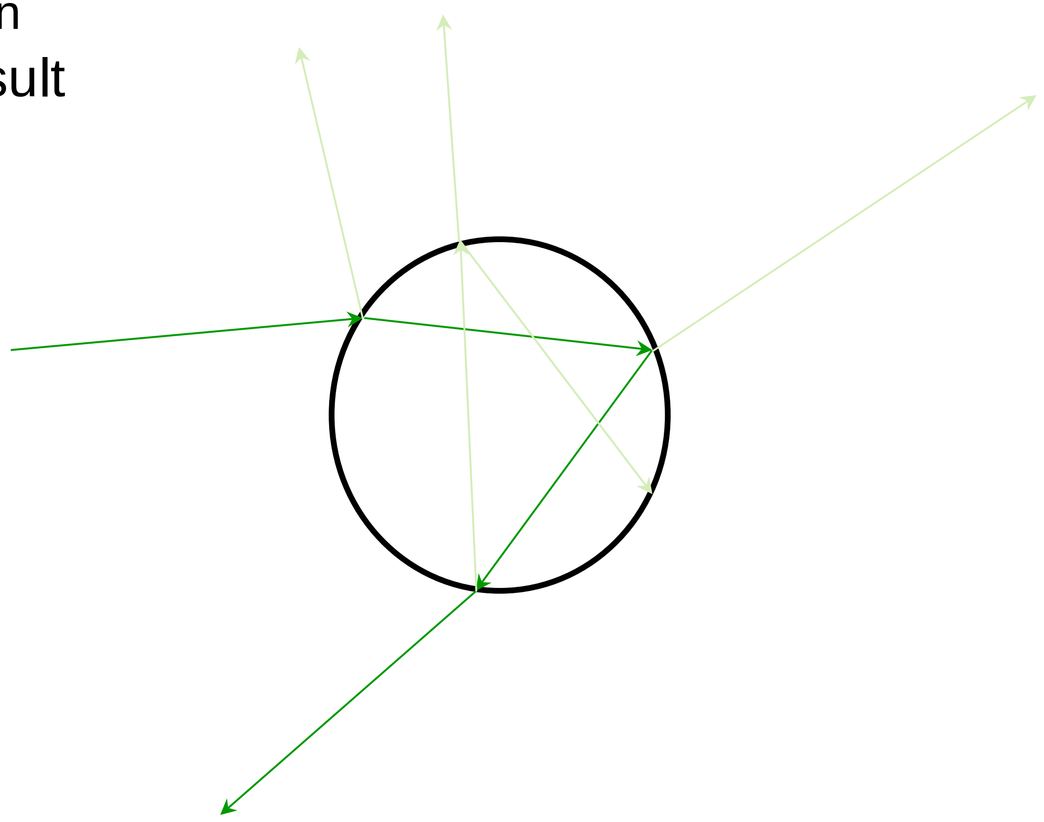
Reflections/Refractions

- **Which ray to trace?**
  - Both: may be exponential



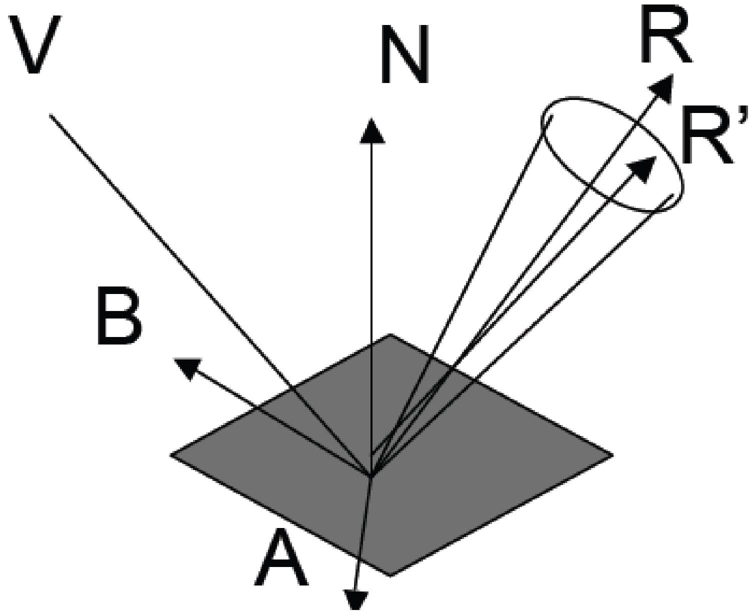
Reflections/Refractions

- **Which ray to trace?**
  - Pick one at random:
    - $\xi < 0.5$ – reflection
    - $\xi \geq 0.5$ – refraction
  - Compensate the result
    - $L_o = 2 \cdot L_i \cdot f_r$



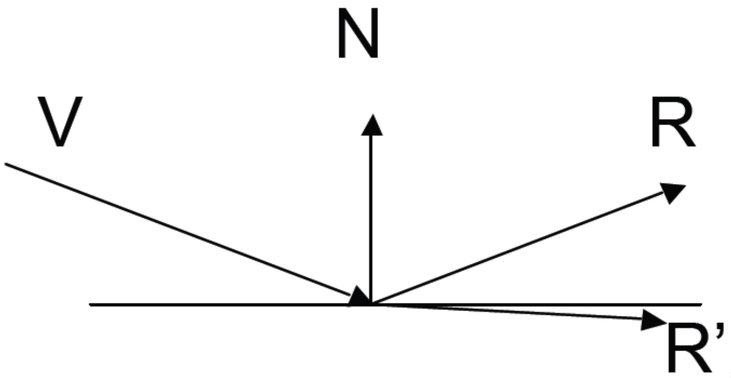
Fuzzy Reflections/Refractions

- **Real Materials**
  - Never perfectly smooth surfaces
- **Approach**
  - Empirical approximation of non-ideal mirrors and glass (see RIS)
  - Sample direction in blur disc

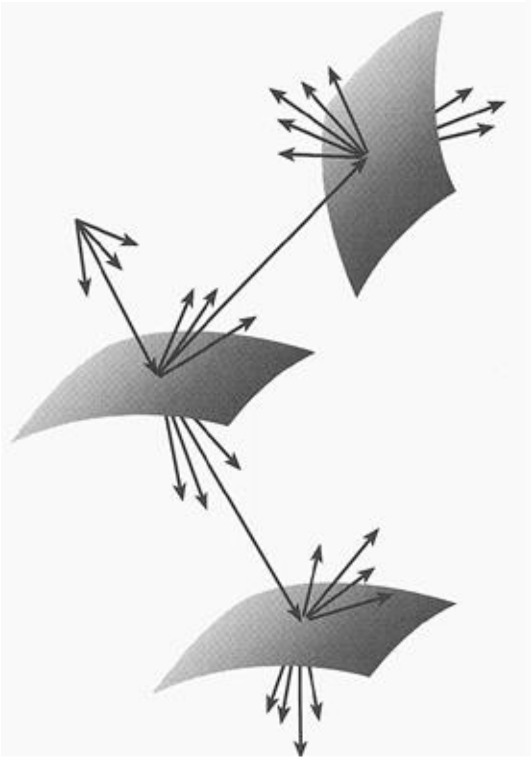


Fuzzy Reflections/Refractions

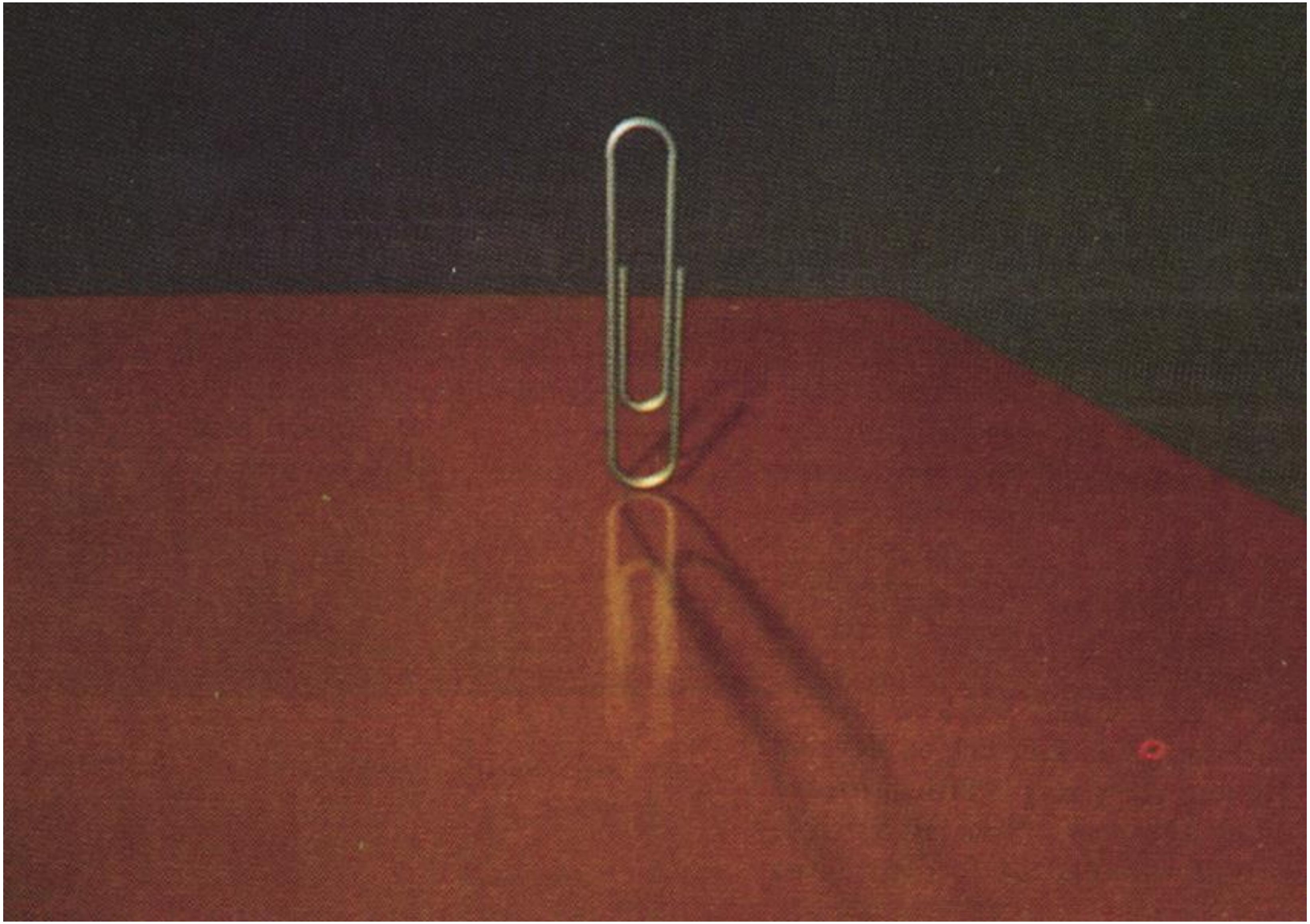
- **Gotchas**
  - Perturbed ray may flip side
  - Check sign of dot product with N
  - Ignore rays on wrong side



- **Inter-Reflections/Refractions**
  - Recursively repeat process
  - At surfaces with corresponding materials

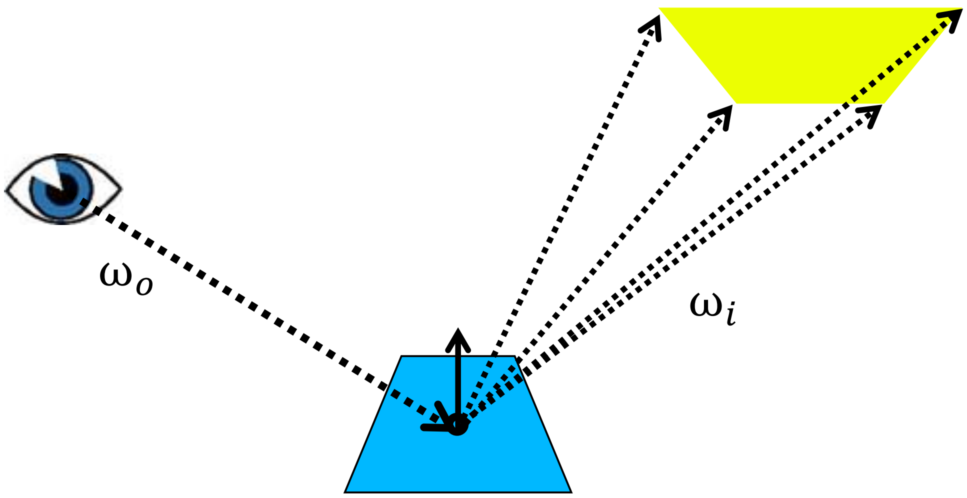


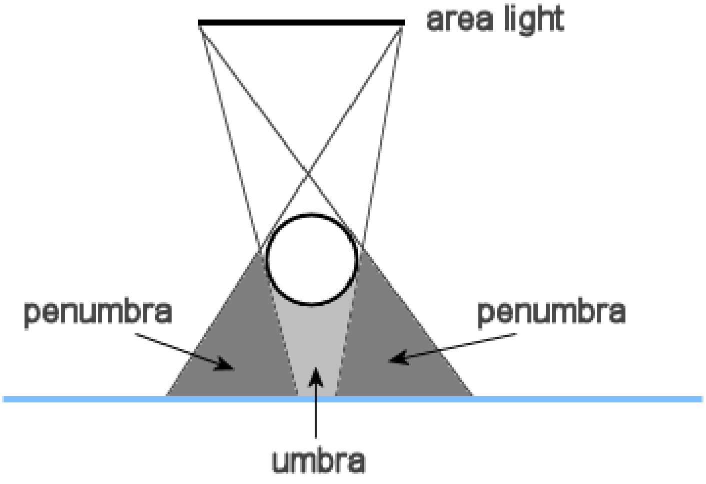
Fuzzy Reflections/Refractions



Area Light Sources

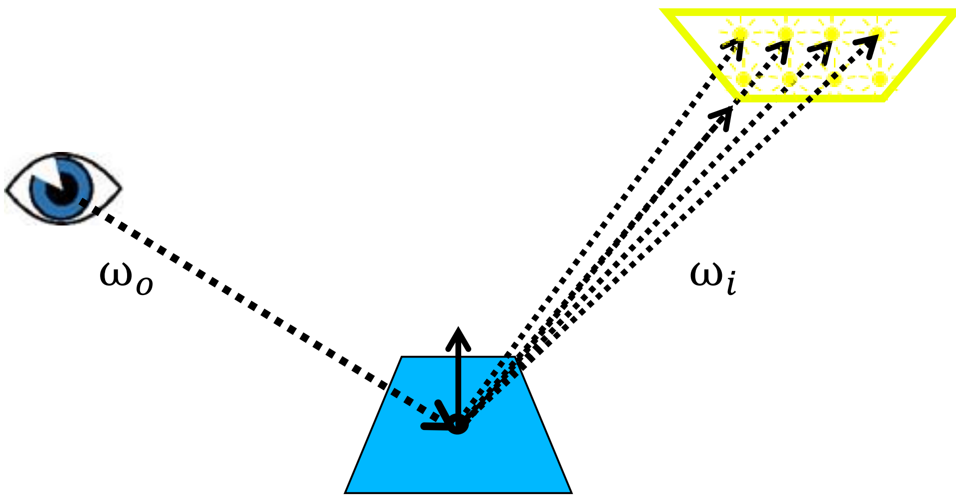
- **Real Light Sources**
  - Finite area

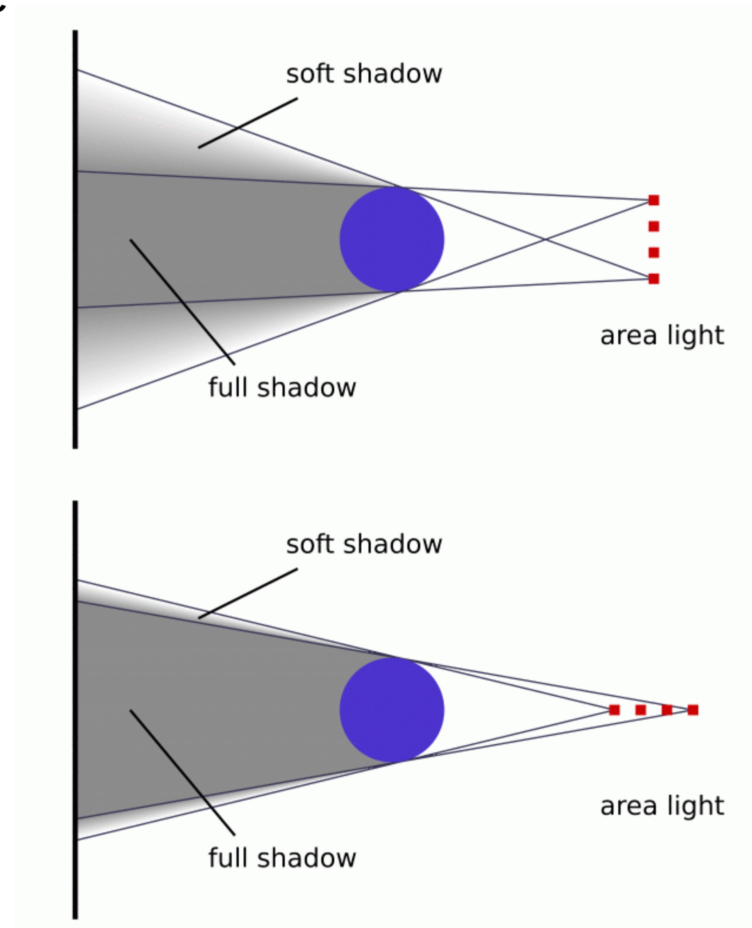




Area Light Sources

- **Approach**
  - Random sample point on surface of light source
  - Scale intensity by area and cosine





Soft Shadows

- **Small vs. Large Area Light**