

# Computer Graphics

- Light Transport -

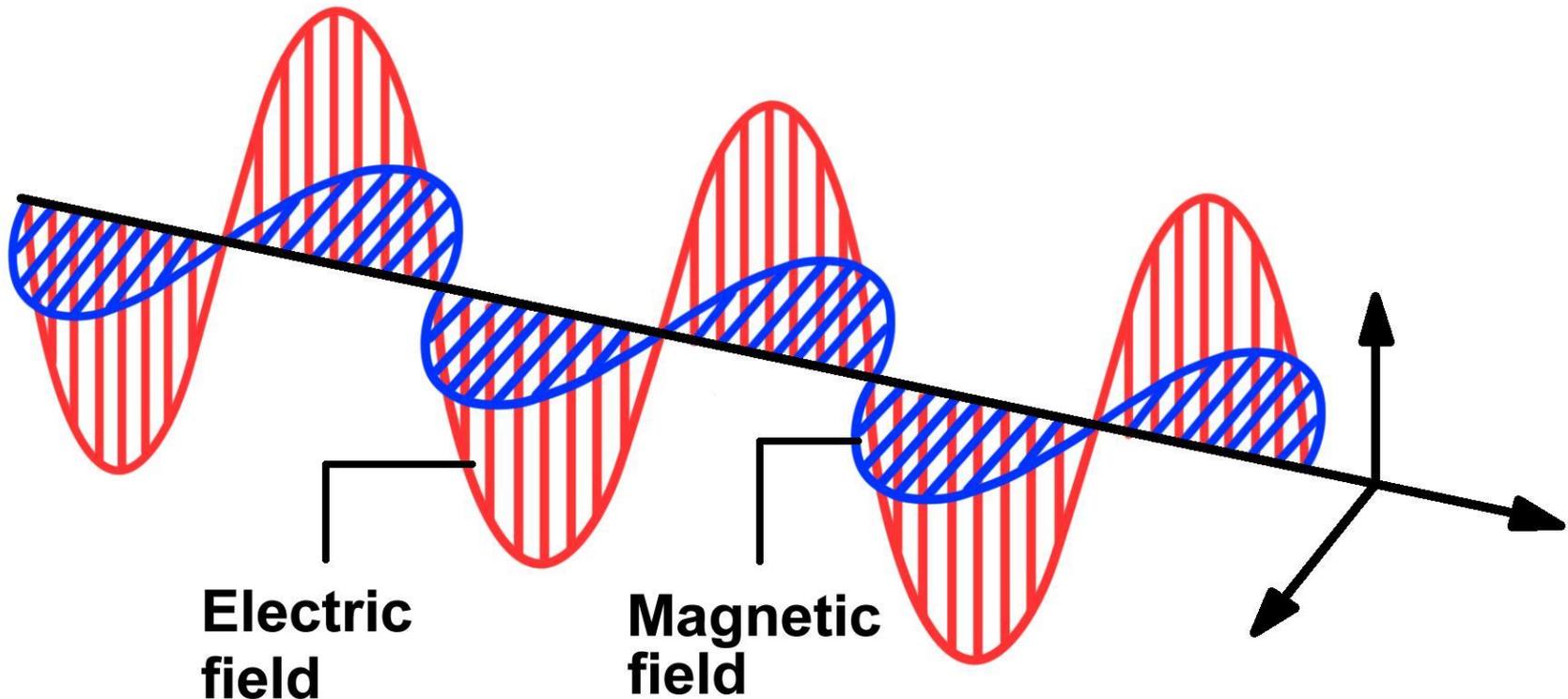
**Philipp Slusallek**

**LIGHT**

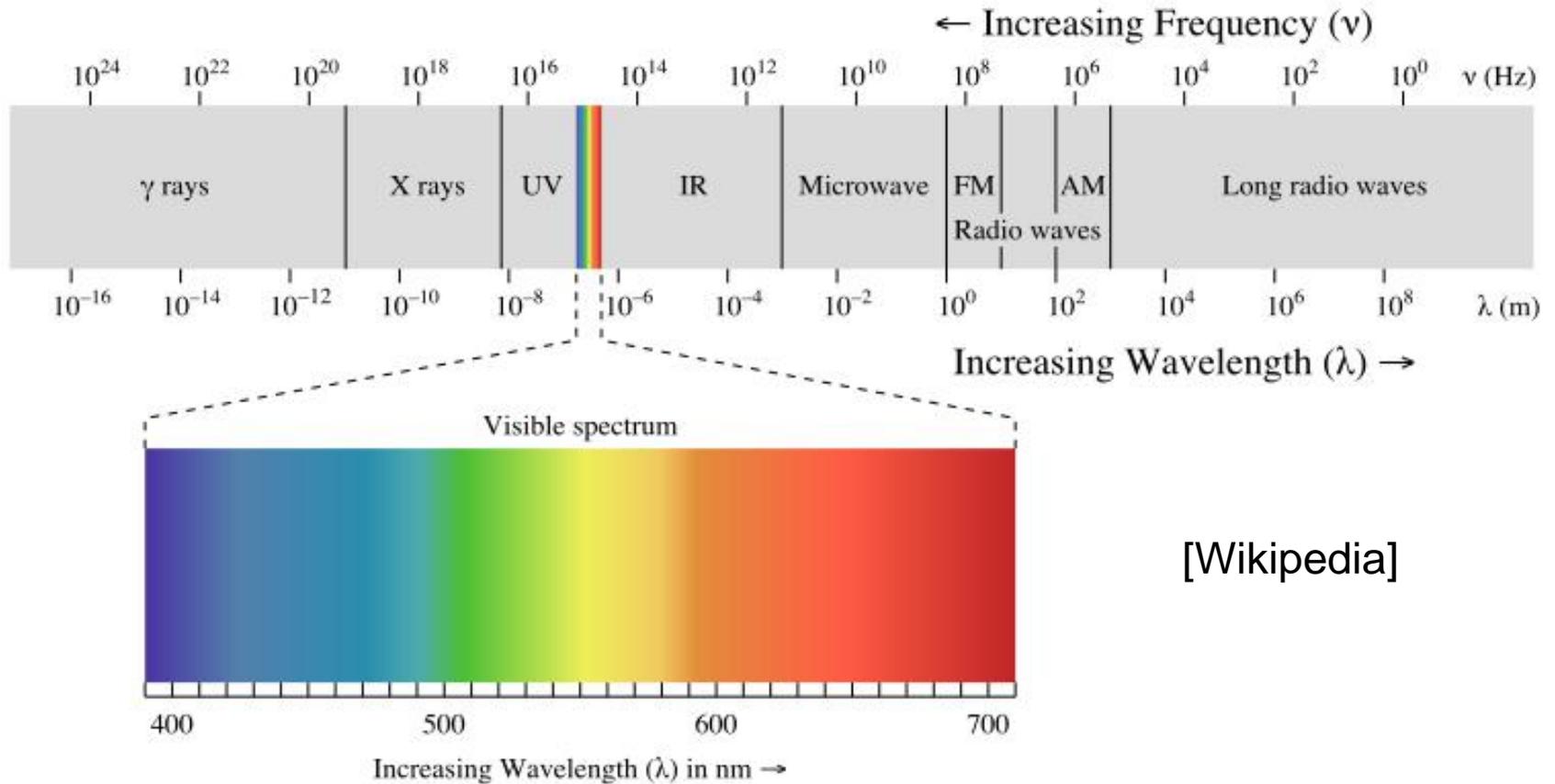
# What is Light ?

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- **Electro-magnetic wave propagating at speed of light**



# What is Light ?



[Wikipedia]

# What is Light ?

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- **Ray**
  - Linear propagation
  - Geometrical optics
- **Vector**
  - Polarization
  - **Jones Calculus**: matrix representation
- **Wave**
  - Diffraction, interference
  - **Maxwell equations**: propagation of light
- **Particle**
  - Light comes in discrete energy quanta: photons
  - **Quantum theory**: interaction of light with matter
- **Field**
  - Electromagnetic force: exchange of virtual photons
  - **Quantum Electrodynamics (QED)**: interaction between particles

# What is Light ?

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# Light in Computer Graphics

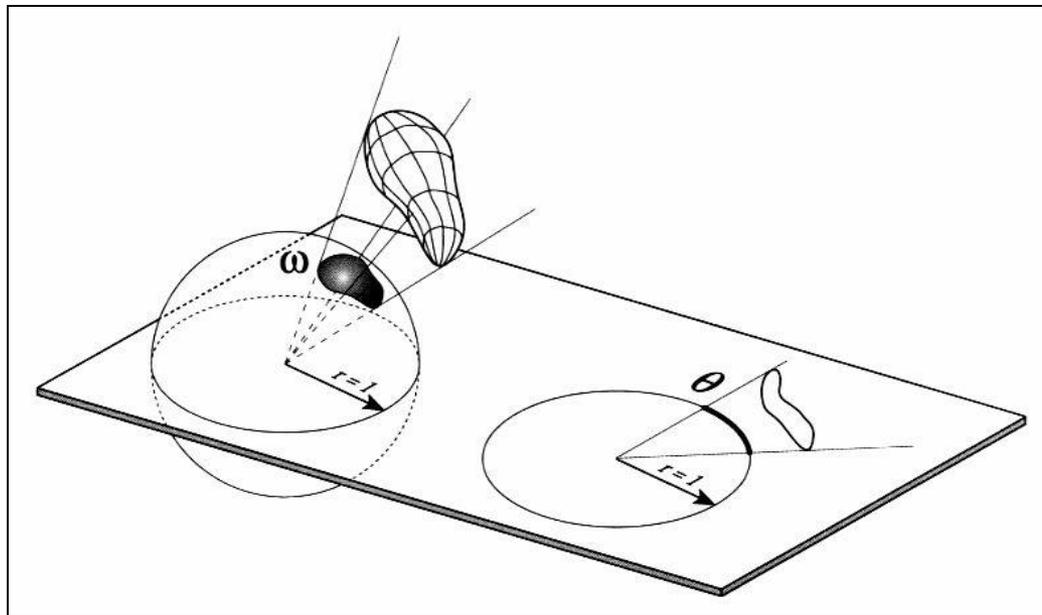
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- **Based on human visual perception**
  - Macroscopic geometry (→ Reflection Models)
  - Tristimulus color model (→ Human Visual System)
  - Psycho-physics: tone mapping, compression, ... (→ RIS course)
- **Ray optic assumptions**
  - Macroscopic objects
  - Incoherent light
  - Light: scalar, real-valued quantity
  - Linear propagation
  - Superposition principle: light contributions add, do not interact
  - No attenuation in free space
- **Limitations**
  - No microscopic structures ( $\approx \lambda$ ): diffraction, interference
  - No polarization
  - No dispersion, ...

# Angle and Solid Angle

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- The **angle**  $\theta$  (in radians) subtended by a curve in the plane is the length of the corresponding arc on the unit circle:  $l = \theta r = \theta$
- The **solid angle**  $\Omega$ ,  $d\omega$  subtended by an object is the surface area of its projection onto the unit sphere
  - Units for measuring solid angle: steradian [sr] (dimensionless)



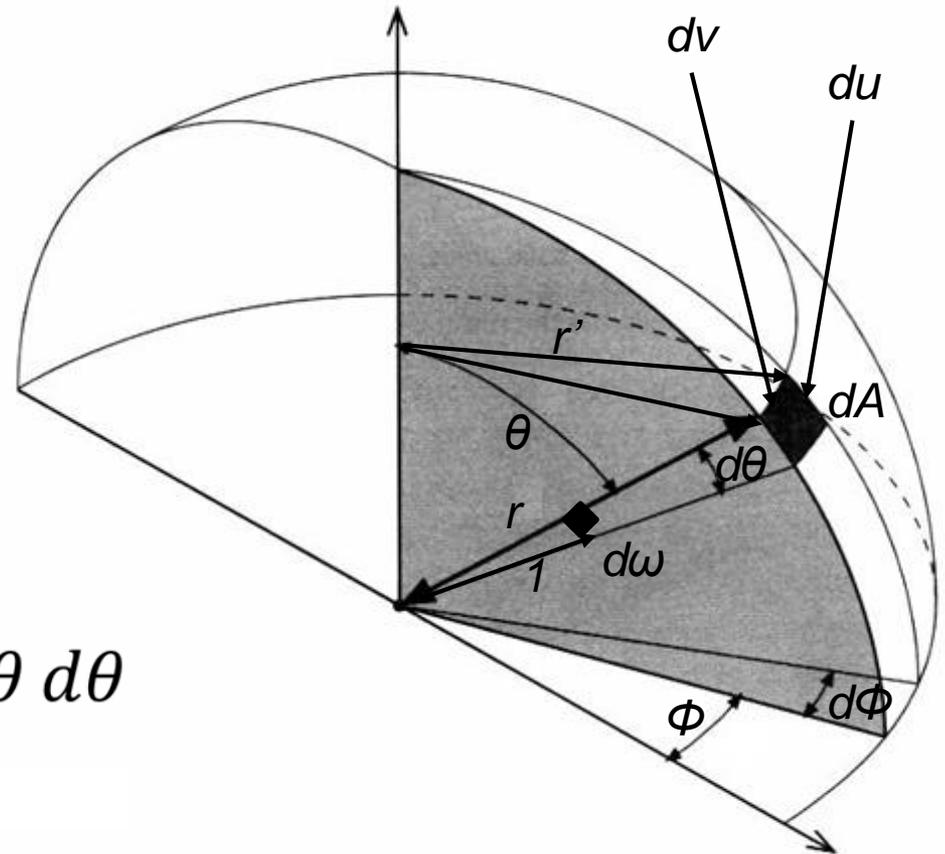
# Solid Angle in Spherical Coords

- **Infinitesimally small solid angle  $d\omega$**

- $du = r d\theta$
- $dv = r' d\Phi = r \sin \theta d\Phi$
- $dA = du dv = r^2 \sin \theta d\theta d\Phi$
- $d\omega = dA/r^2 = \sin \theta d\theta d\Phi$

- **Finite solid angle**

$$\Omega = \int_{\phi_0}^{\phi_1} d\phi \int_{\theta_0(\phi)}^{\theta_1(\phi)} \sin \theta d\theta$$



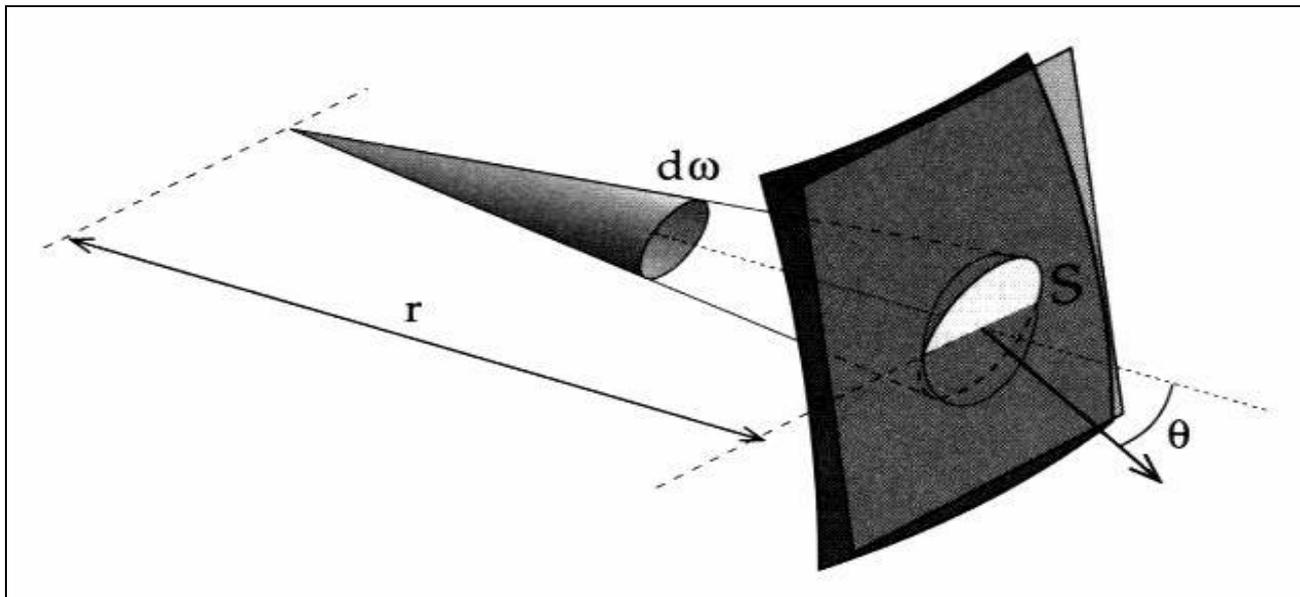
# Solid Angle for a Surface

- The solid angle subtended by a small surface patch  $S$  with area  $dA$  is obtained (i) by projecting it orthogonal to the vector  $r$  from the origin:

$$dA \cos \theta$$

and (ii) dividing by the squared distance to the origin:  $d\omega = \frac{dA \cos \theta}{r^2}$

$$\Omega = \iint_S \frac{\vec{r} \cdot \vec{n}}{r^3} dA$$



# Radiometry

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- **Definition:**

- Radiometry is the science of measuring radiant energy transfers. Radiometric quantities have physical meaning and can be directly measured using proper equipment such as spectral photometers.

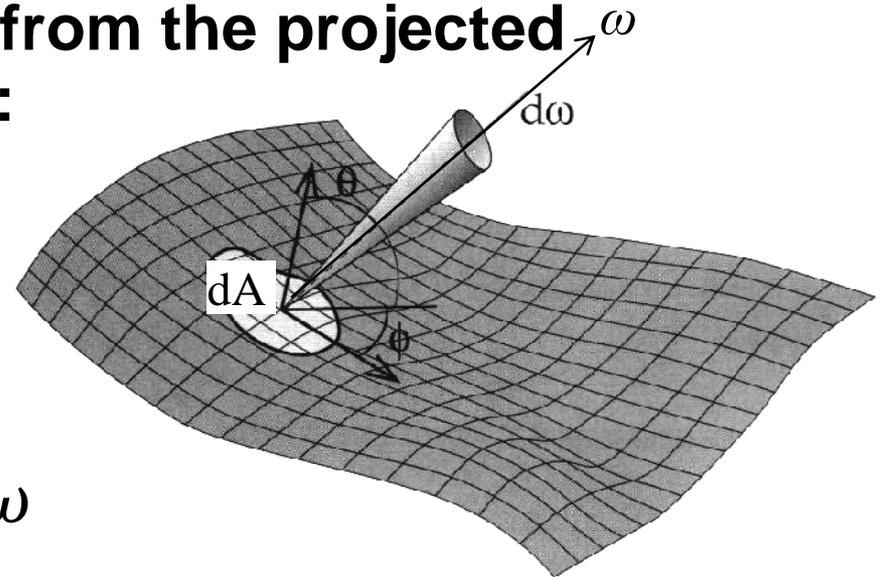
- **Radiometric Quantities**

- |                 |                            |        |                                       |
|-----------------|----------------------------|--------|---------------------------------------|
| – Energy        | [J]                        | $Q$    | (#Photons x Energy = $n \cdot h\nu$ ) |
| – Radiant power | [watt = J/s]               | $\Phi$ | (Total Flux)                          |
| – Intensity     | [watt/sr]                  | $I$    | (Flux from a point per s.angle)       |
| – Irradiance    | [watt/m <sup>2</sup> ]     | $E$    | (Incoming flux per area)              |
| – Radiosity     | [watt/m <sup>2</sup> ]     | $B$    | (Outgoing flux per area)              |
| – Radiance      | [watt/(m <sup>2</sup> sr)] | $L$    | (Flux per area & proj. s. angle)      |

# Radiometric Quantities: Radiance

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- Radiance is used to describe radiant energy transfer
- Radiance  $L$  is defined as
  - The power (flux) traveling through some point  $x$
  - In a specified direction  $\omega = (\theta, \varphi)$
  - Per unit area **perpendicular** to the direction of travel
  - Per unit solid angle
- Thus, the differential power  $d^2\Phi$  radiated through the **differential solid angle  $d\omega$** , from the projected differential area  $dA \cos \theta$  is:



$$d^2\Phi = L(x, \omega) dA \cos \theta d\omega$$

# Radiometric Quantities: Irradiance

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- Irradiance  $E$  is defined as the **total power per unit area** (flux density) incident onto a surface. To obtain the total flux incident to  $dA$ , the **incoming** radiance  $L_i$  is integrated over the upper hemisphere  $\Omega_+$  above the surface:

$$E \equiv \frac{d\Phi}{dA}$$

$$d\Phi = \left[ \int_{\Omega_+} L_i(x, \omega) \cos \theta d\omega \right] dA$$

$$E(x) = \int_{\Omega_+} L_i(x, \omega) \cos \theta d\omega = \int_0^{\frac{\pi}{2}} \int_0^{2\pi} L_i(x, \omega) \cos \theta \sin \theta d\theta d\phi$$

# Radiometric Quantities: Radiosity

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- **Radiosity  $B$**  is defined as the **total power per unit area** (flux density) **exitant from** a surface. To obtain the total flux incident to  $dA$ , the **outgoing** radiance  $L_o$  is integrated over the upper hemisphere  $\Omega_+$  above the surface:

$$B \equiv \frac{d\Phi}{dA}$$

$$d\Phi = \left[ \int_{\Omega_+} L_o(x, \omega) \cos \theta d\omega \right] dA$$

$$B(x) = \int_{\Omega_+} L_o(x, \omega) \cos \theta d\omega = \int_0^{\frac{\pi}{2}} \int_0^{2\pi} L_o(x, \omega) \cos \theta \sin \theta d\theta d\phi$$

# Spectral Properties

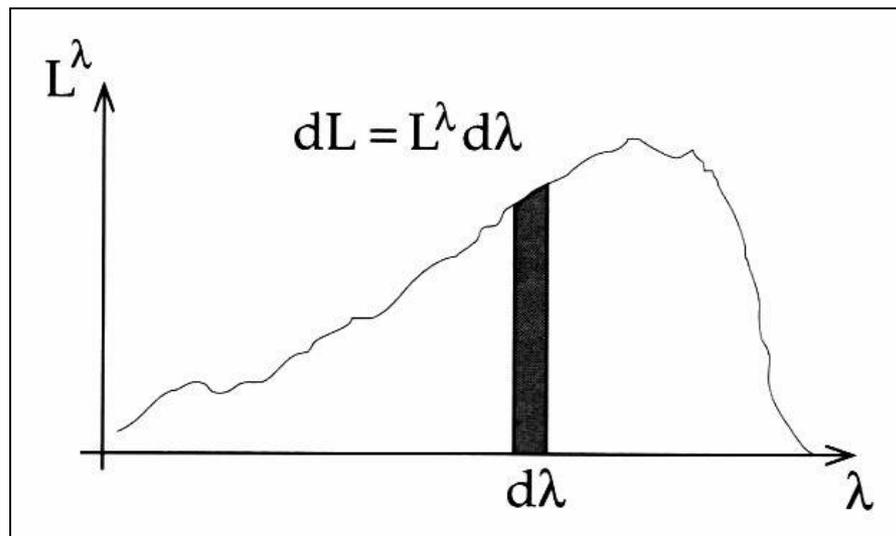
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- **Wavelength**

- Light is composed of electromagnetic waves
- These waves have different frequencies and wavelengths
- Most transfer quantities are continuous functions of wavelength

- **In graphics**

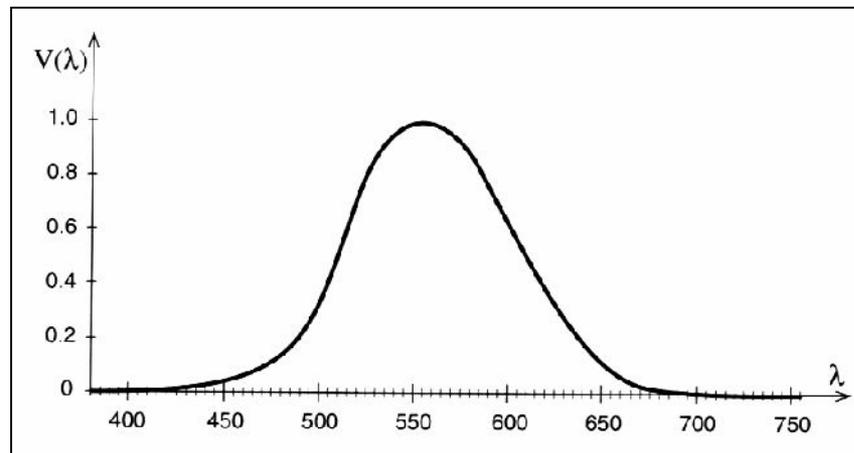
- Each measurement  $L(x, \omega)$  is for a discrete band of wavelength only
  - Often **R**(ed, long), **G**(reen, medium), **B**(lue, short) (but see later)



# Photometry

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- The human eye is sensitive to a limited range of wavelengths
  - Roughly from 380 nm to 780 nm
- Our visual system responds differently to different wavelengths
  - Can be characterized by the *Luminous Efficiency Function*  $V(\lambda)$
  - Represents the average human spectral response
  - Separate curves exist for light and dark adaptation of the eye
- Photometric quantities are derived from radiometric quantities by *integrating* them against this function



# Radiometry vs. Photometry

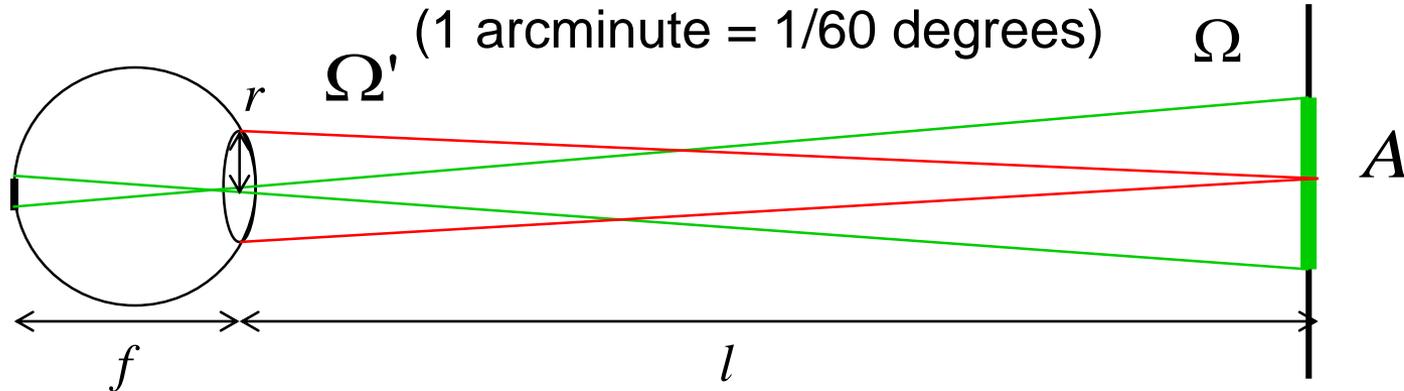
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**Physics-based quantities**

**Perception-based quantities**

Radiometry		→	Photometry	
W	Radiant power	→	Luminous power	Lumens (lm)
W/m <sup>2</sup>	Radiosity	→	Luminosity	Lux (lm/m <sup>2</sup> )
	Irradiance		Illuminance	
W/m <sup>2</sup> /sr	Radiance	→	Luminance	cd/m <sup>2</sup> (lm/m <sup>2</sup> /sr)

# Perception of Light



photons / second = **flux** = **energy / time** = **power** ( $\Phi$ )

rod sensitive to flux

angular extent of rod = **resolution** ( $\approx 1$  arcminute<sup>2</sup>)

$\Omega$

projected rod size = **area**

$$A \approx l^2 \cdot \Omega$$

angular extent of pupil aperture ( $r \leq 4$  mm) = **solid angle**

$$\Omega' \approx \pi \cdot r^2 / l^2$$

flux proportional to area and solid angle

$$\Phi = L A \Omega'$$

**radiance** = flux per unit area per unit solid angle

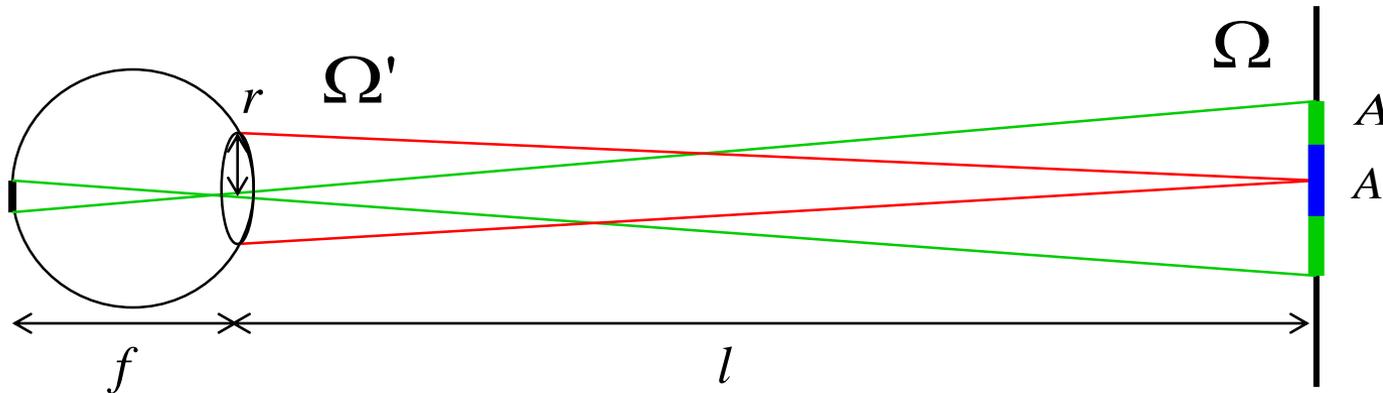
$$L = \frac{\Phi}{\Omega' \cdot A}$$

The eye detects radiance

As  $l$  increases:

$$\Phi_0 = L \cdot l^2 \cdot \Omega \cdot \pi \frac{r^2}{l^2} = L \cdot \text{const}$$

# Brightness Perception

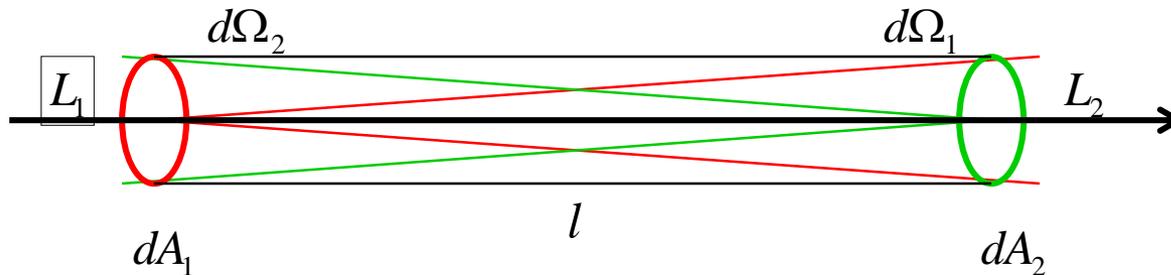


- $A' > A$  : photon flux per rod stays constant
- $A' < A$  : photon flux per rod decreases

## Where does the Sun turn into a star ?

- Depends on apparent Sun disc size on retina
- Photon flux per rod stays the same on Mercury, Earth or Neptune
- Photon flux per rod decreases when  $\Omega' < 1$  arcminute<sup>2</sup> (beyond Neptune)

# Radiance in Space



Flux leaving surface 1 must be equal to flux arriving on surface 2

$$L_1 d\Omega_1 dA_1 = L_2 d\Omega_2 dA_2$$

From geometry follows  $d\Omega_1 = \frac{dA_2}{l^2}$      $d\Omega_2 = \frac{dA_1}{l^2}$

Ray throughput  $T$ :  $T = d\Omega_1 \cdot dA_1 = d\Omega_2 \cdot dA_2 = \frac{dA_1 \cdot dA_2}{l^2}$

$$L_1 = L_2$$

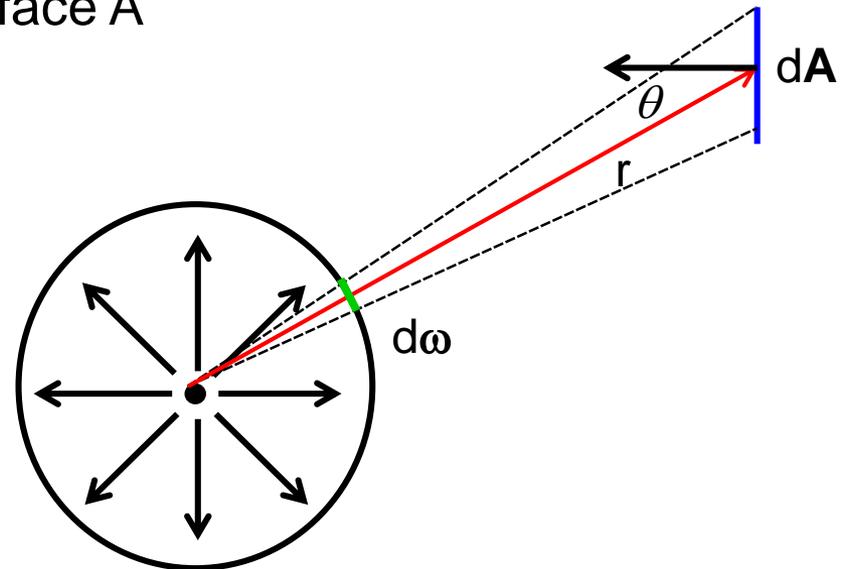
The **radiance** in the direction of a light ray  
**remains constant** as it propagates along the ray

# Point Light Source

- **Point light with *isotropic* radiance**

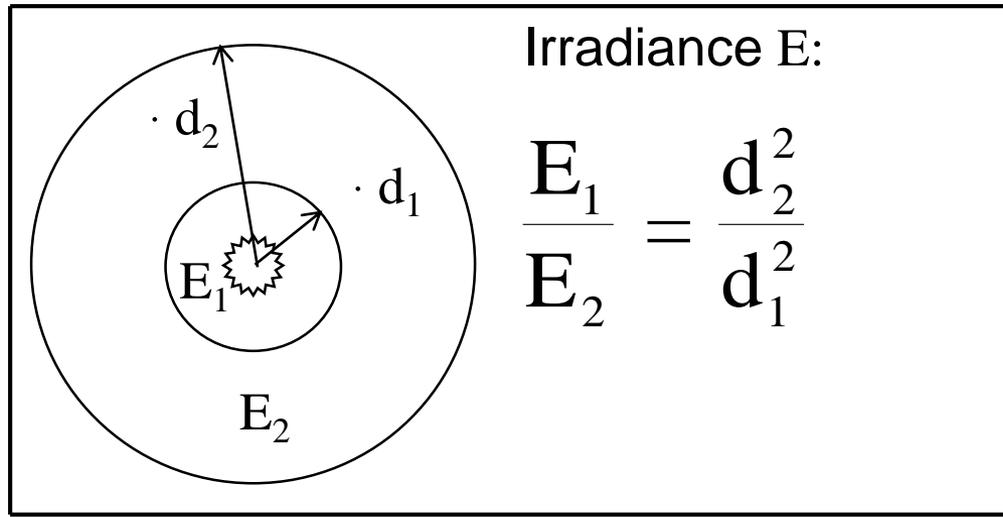
- Power (total flux) of a point light source
  - $\Phi_g =$  Power of the light source [watt]
- Intensity of a light source (radiance cannot be defined, no area)
  - $I = \Phi_g / 4\pi$  [watt/sr]
- Irradiance on a sphere with radius  $r$  around light source:
  - $E_r = \Phi_g / (4 \pi r^2)$  [watt/m<sup>2</sup>]
- Irradiance on some other surface A

$$\begin{aligned} E(x) &= \frac{d\Phi_g}{dA} = \frac{d\Phi_g}{d\omega} \frac{d\omega}{dA} = I \frac{d\omega}{dA} \\ &= \frac{\Phi_g}{4\pi} \cdot \frac{dA \cos \theta}{r^2 dA} \\ &= \frac{\Phi_g}{4\pi} \cdot \frac{\cos \theta}{r^2} \end{aligned}$$



# Inverse Square Law

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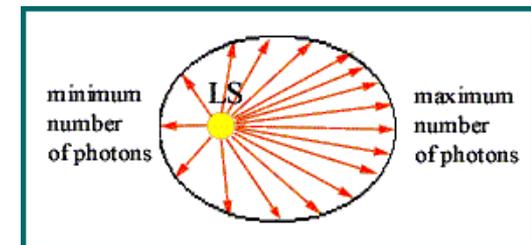
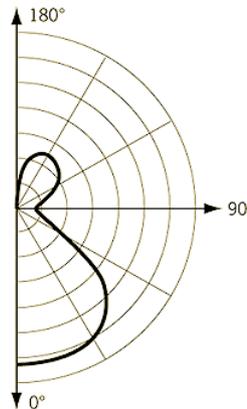
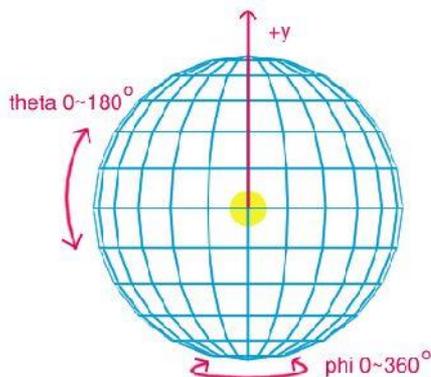
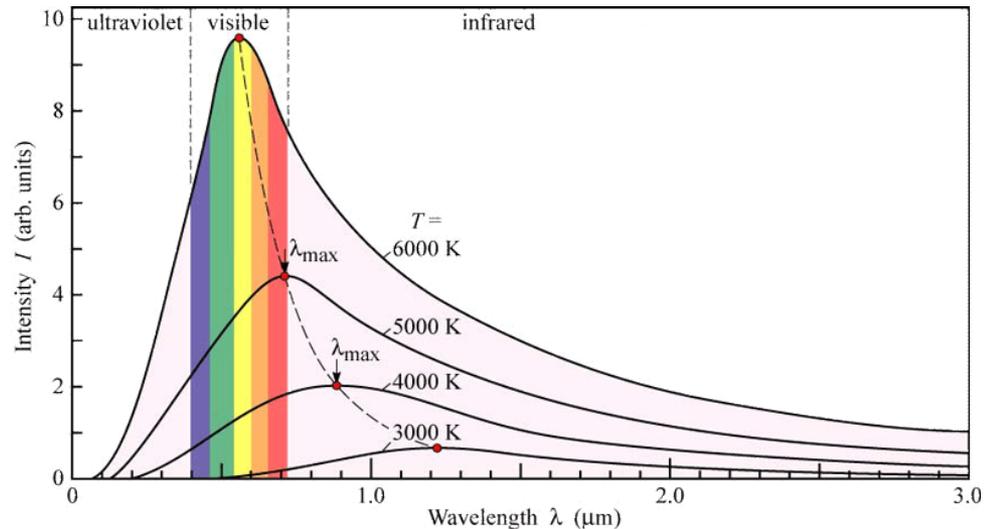


- **Irradiance  $E$ : power per  $m^2$** 
  - Illuminating quantity
- **Distance-dependent**
  - Double distance from emitter: area of sphere is four times bigger
- **Irradiance falls off with inverse of squared distance**
  - Only for point light sources (!)

# Light Source Specifications

- **Power (total flux)**
  - Emitted energy / time
- **Active emission size**
  - Point, line, area, volume
- **Spectral distribution**
  - Thermal, line spectrum
- **Directional distribution**
  - Goniometric diagram

Black body radiation (see later)



# Light Source Classification

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## Radiation characteristics

- **Directional light**
  - Spot-lights
  - Projectors
  - Distant sources
- **Diffuse emitters**
  - Torchieres
  - Frosted glass lamps
- **Ambient light**
  - “Photons everywhere”

## Emitting area

- **Volume**
    - Neon advertisements
    - Sodium vapor lamps
  - **Area**
    - CRT, LCD display
    - (Overcast) sky
  - **Line**
    - Clear light bulb, filament
  - **“Point”**
    - Xenon lamp
    - Arc lamp
    - Laser diode
-

# Sky Light

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- **Sun**
  - Point source (approx.)
  - White light (by def.)
- **Sky**
  - Area source
  - Scattering: blue
- **Horizon**
  - Brighter
  - Haze: whitish
- **Overcast sky**
  - Multiple scattering in clouds
  - Uniform grey
- **Several sky models are available**



Courtesy Lynch & Livingston

# LIGHT TRANSPORT

# Light Transport in a Scene

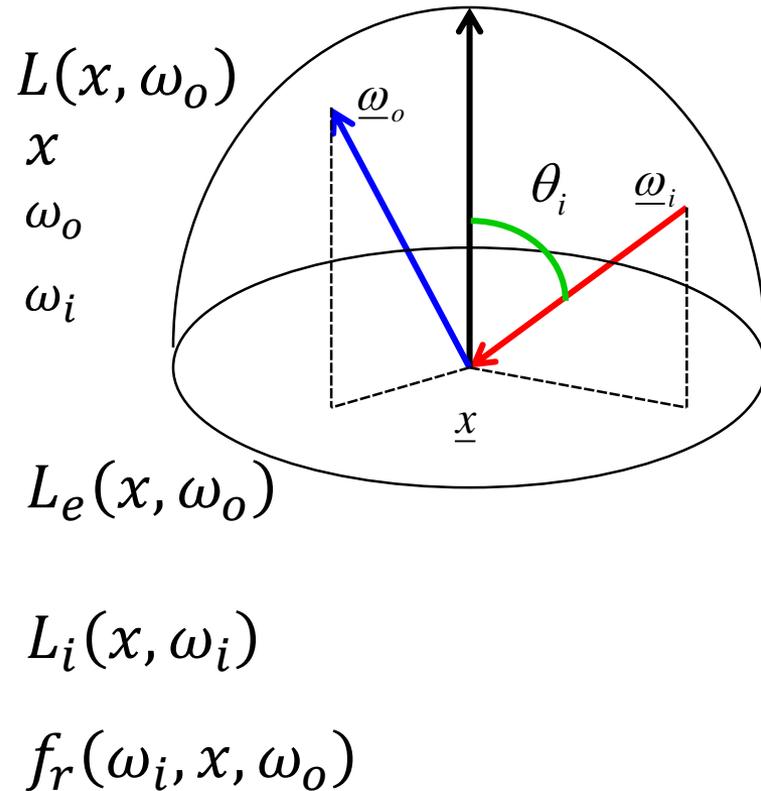
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- **Scene**
  - Lights (emitters)
  - Object surfaces (partially absorbing)
- **Illuminated object surfaces become emitters, too!**
  - Radiosity = Irradiance minus absorbed photons flux density
    - Radiosity: photons per second per  $\text{m}^2$  leaving surface
    - Irradiance: photons per second per  $\text{m}^2$  incident on surface
- **Light bounces between all mutually visible surfaces**
- **Invariance of radiance in free space**
  - No absorption in-between objects
- **Dynamic energy equilibrium**
  - Emitted photons = absorbed photons (+ escaping photons)
    - **Global Illumination, discussed in RIS lecture**

# Surface Radiance

$$L(x, \omega_o) = L_e(x, \omega_o) + \int_{\Omega_+} f_r(\omega_i, x, \omega_o) L_i(x, \omega_i) \cos \theta_i d\omega_i$$

- **Visible surface radiance**
  - Surface position
  - Outgoing direction
- **Incoming illumination direction**
- **Self-emission**
- **Reflected light**
  - Incoming radiance from all directions
  - Direction-dependent reflectance  
(**BRDF: bidirectional reflectance distribution function**)



# Rendering Equation

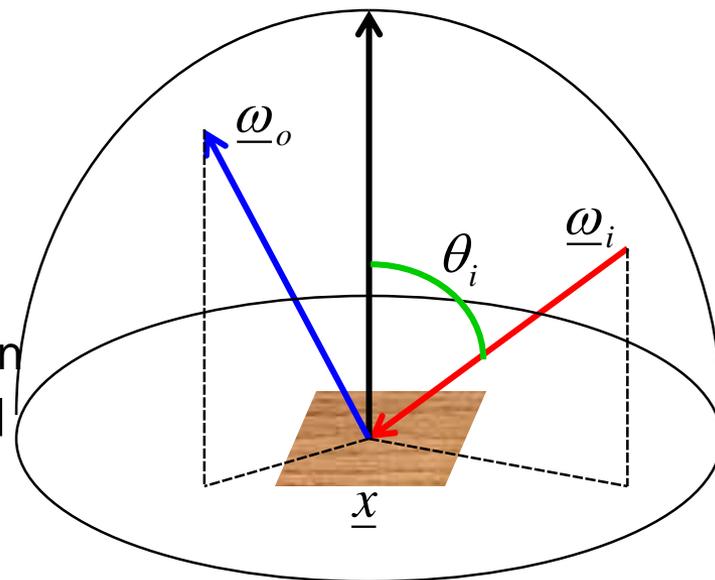
Typ. Exam  
Question!

- **Most important equation for graphics**
  - Expresses energy equilibrium in scene

$$L(x, \omega_o) = L_e(x, \omega_o) + \int_{\Omega_+} f_r(\omega_i, x, \omega_o) L_i(x, \omega_i) \cos \theta_i d\omega_i$$

total radiance = emitted + reflected radiance

- **First term: emissivity of the surface**
  - Non-zero only for light sources
- **Second term: reflected radiance**
  - Integral over all possible incoming directions of radiance times angle-dependent surface reflection function
- **Fredholm integral equation of 2nd kind**
  - Unknown radiance appears both on the left-hand side and inside the integral
  - Numerical methods necessary to compute approximate solution



# Rendering Equation: Approximations

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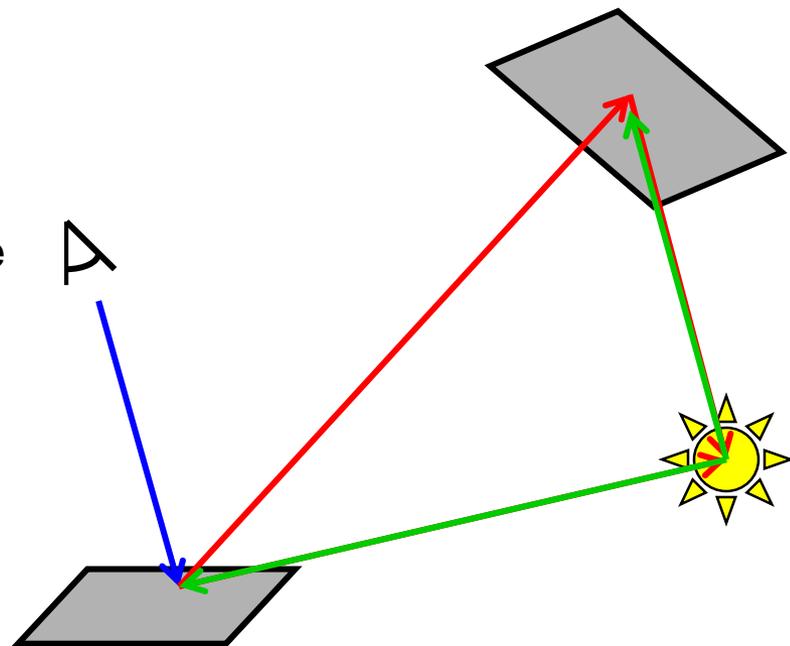
- **Approximations based only on empirical foundations**
  - An example: polygon rendering in OpenGL
- **Using RGB instead of full spectrum**
  - Follows roughly the eye's sensitivity
- **Sampling hemisphere along finite, discrete directions**
  - Simplifies integration to summation
- **Reflection function model (BRDF)**
  - Parameterized function
    - **Ambient**: constant, non-directional, background light
    - **Diffuse**: light reflected uniformly in all directions
    - **Specular**: light from mirror-reflection direction

# Ray Tracing

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$$L(x, \omega_o) = L_e(x, \omega_o) + \int_{\Omega_+} f_r(\omega_i, x, \omega_o) L_i(x, \omega_i) \cos \theta_i d\omega_i$$

- **Simple ray tracing**
  - Illumination from discrete point light sources only – **direct illumination only**
    - Integral → sum of contributions from each light
    - **No global illumination**
  - Evaluates angle-dependent reflectance function (BRDF) – **shading process**
- **Advanced ray tracing techniques**
  - Recursive ray tracing
    - Multiple reflections/refractions (for specular surfaces)
  - Ray tracing for global illumination
    - Stochastic sampling (Monte Carlo methods)



# RE: Integrating over Surfaces

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- **Outgoing illumination at a point**

$$L(x, \omega_o) = L_e(x, \omega_o) + L_r(x, \omega_o)$$

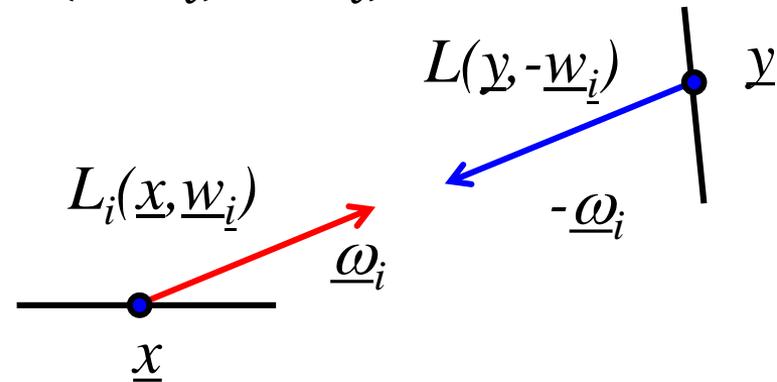
$$L(x, \omega_o) = L_e(x, \omega_o) + \int_{\Omega_+} f_r(\omega_i, x, \omega_o) L_i(x, \omega_i) \cos \theta_i d\omega_i$$

- **Linking with other surface points**

- Incoming radiance at x is outgoing radiance at y

$$L_i(x, \omega_i) = L(y, -\omega_i) = L(RT(x, \omega_i), -\omega_i)$$

- Ray-Tracing operator:  $y = RT(x, \omega_i)$



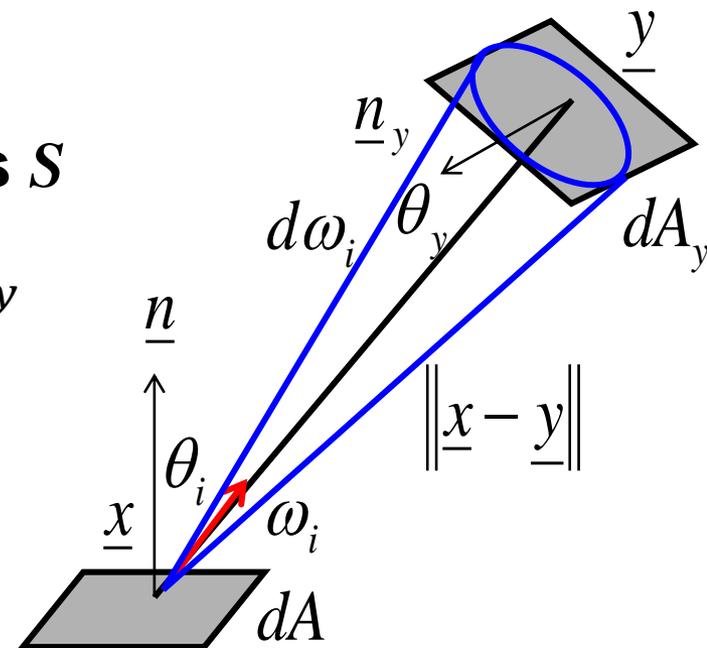
# Integrating over Surfaces

- **Outgoing illumination at a point**

$$L(x, \omega_o) = L_e(x, \omega_o) + \int_{\Omega_+} f_r(\omega_i, x, \omega_o) L_i(x, \omega_i) \cos \theta_i d\omega_i$$

- **Re-parameterization over surfaces  $S$**

$$d\omega_i = \frac{\cos \theta_y}{\|x - y\|^2} dA_y$$



$$L(x, \omega_o) = L_e(x, \omega_o)$$

$$+ \int_{y \in S} f_r(\omega(x, y), x, \omega_o) L_i(x, \omega(x, y)) V(x, y) \frac{\cos \theta_i \cos \theta_y}{\|x - y\|^2} dA_y$$

# Integrating over Surfaces

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$$\begin{aligned} L(x, \omega_o) &= L_e(x, \omega_o) \\ &+ \int_{y \in S} f_r(\omega(x, y), x, \omega_o) L_i(x, \omega(x, y)) V(x, y) \frac{\cos \theta_i \cos \theta_y}{\|x - y\|^2} dA_y \end{aligned}$$

- **Geometry term:**  $G(x, y) = V(x, y) \frac{\cos \theta_i \cos \theta_y}{\|x - y\|^2}$
  - **Visibility term:**  $V(x, y) = \begin{cases} 1, & \text{if visible} \\ 0, & \text{otherwise} \end{cases}$
  - **Integration over all surfaces:**  $\int_{y \in S} \cdots dA_y$
- $$L(x, \omega_o) = L_e(x, \omega_o) + \int_{y \in S} f_r(\omega(x, y), x, \omega_o) L_i(x, \omega(x, y)) G(x, y) dA_y$$

# Lighting Simulation

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# Lighting Simulation

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# Lighting Simulation

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# Wrap Up

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- **Physical Quantities in Rendering**
  - Radiance
  - Radiosity
  - Irradiance
  - Intensity
- **Light Perception**
- **Light Source Definition**
- **Rendering Equation**
  - Key equation in graphics (!)
  - Integral equation
  - Describes global balance of radiance