UNIVERSITÄT DES SAARLANDES
Prof. Dr.-Ing. Philipp Slusallek
Computer Graphics Group
Stefan Lemme
Arsène Pérard-Gayot

Introduction to Computer Graphics

Sample Mid-term Exam

First Name:

Surname:

Matriculation Number:

Please read the instructions on the next page carefully!

Duration of the exam: **60 minutes**

Please do **not** fill in the following table!

exercise	part	max. points	points gained
1		8	
2	a	8	
2	b	5	
3	a	4	
3	b	2	
4	a	3	
4	b	3	
Sum		**33**	

Instructions for the exam:

- Don't panic!
- Check whether you received all sheets.
- Fill in your name and matriculation number on the cover sheet!
- Write your name and matriculation number on *every* sheet of paper, this includes all sheets of the exam and the additional paper you get from us.
- No tools, books, and lecture notes are allowed for this exam.
- On your desk should be no more than: the exam, some pens, a ruler, and your student id. Nothing else!
- If you try to cheat you will fail the exam.
- Be careful to write in a readable way! For things we cannot read you get no points.
- Answer only the questions that are posed. Giving more information than asked for will not be rewarded by extra points, and you will lose valuable time for giving these answers.
- If you are asked to calculate something, on every step write down what you are currently calculating. Giving only the final result will not be enough to get full points.
- Always write short and precise answers and do not spend too much time on any specific question.
- The answers can be given either in English or in German.

Enough details, let's get started!

Exercise 1: Camera (8 points)

A perspective camera is parametrized as follows:

- origin = $(0, 0, -3)$
- direction = $(0, 0, 1)$
- full vertical opening angle = $60°$
- aspect ratio 1 : 1

For each primitive below, decide if it is going to be rendered fully, partially or not at all. Justify your answer.

1. Infinite plane defined by point $a = (0, -2, -4)$, and normal $n = (0, 1, 0)$
2. Infinite plane defined by point $a = (-1, 0, -4)$, and normal $n = (\frac{\sqrt{2}}{2}, 0, \frac{\sqrt{2}}{2})$
3. A sphere centered at $c = (0, \frac{1}{2}, \frac{\sqrt{3}}{2} - 3)$ and radius $r = 0.125$
4. A sphere centered at $c = (0, -3, -3)$ and radius $r = 1.6\sqrt{3}$

Exercise 2: Ray-Primitive Intersection (8 + 5 points)

You are given a ray R defined by its origin o and direction $\vec{d}$ parameterized by the distance t.

a) An infinite cylinder can be defined by its axis vector $\vec{a}$, a point on the axis $\vec{p}$ and a radius r.

Derive the expression of the distance at which the ray R intersects the cylinder.

You may assume that $\vec{a}$ is normalized.

b) Provide an implicit equation of a sphere of radius r centered at the origin.

Derive the expression of the distance at which the ray R intersects the sphere.

Exercise 3: Homogeneous Transformations (4 + 2 points)

a) Derive the transformation $T(a)$, where $a = (a_1\ a_2\ a_3\ a_4)^\top$, that performs the following operations in the order they are given:

1. Translation by 1 unit along the y-axis
2. Rotation by 45 degrees around the z-axis
3. Mirroring about the XY-plane through the origin
4. Uniform scaling by 2

Note: You do not have to compute the final transformation matrix.

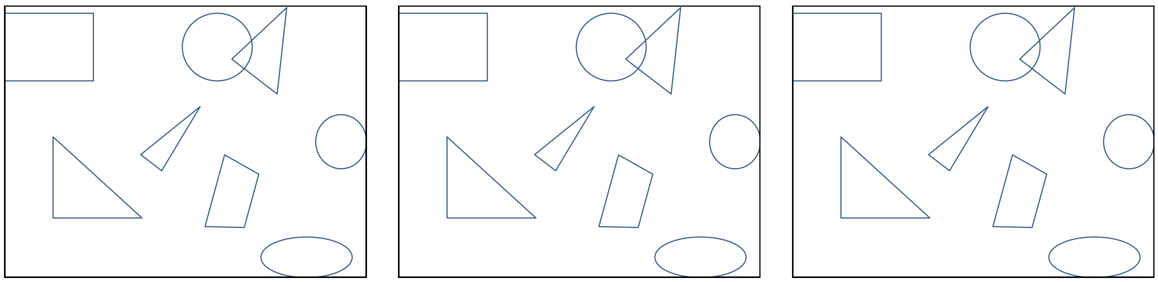
b) Let $T(a)$, where $a = (a_1\ a_2\ a_3\ a_4)^\top$, be the following transformation:

$$T(a) = \begin{pmatrix} 2 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0.5 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \end{pmatrix}$$
$$= \begin{pmatrix} 0 & 0 & 2 & 0 \\ 0 & 1 & 0 & 0 \\ -0.5 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \end{pmatrix}.$$

1. Can T be directly used to correctly transform both the vertices and the normals of a 3D object? Why?
2. Does T preserve volumes? Why?

Exercise 4: Spatial Index Structures (3 + 3 points)

a) Given a scene composed of the geometric entities sketched below, draw good partitions that would result from using a grid, BVH and k-D tree as an acceleration structure. Add *one* short sentence to justify your choice of partition.



b) Considering a k-D tree, a single partitioning may be devised based on a middle-split or median-split strategy. Draw such a split for each of the 2 strategies on the first 2 following diagrams and explain what the limitations of each approach are. As an alternative, the split may be devised using the surface area heuristic (SAH). Draw such a split on the last diagram and explain why this approach is more optimal.