UNIVERSITÄT DES SAARLANDES
Prof. Dr.-Ing. Philipp Slusallek
Computer Graphics Group
Stefan Lemme
Arsène Pérard-Gayot

Introduction to Computer Graphics
Final Exam Sample

First Name:

Surname:

Matriculation Number:

Please read the instructions on the next page carefully!

Duration of the exam: **120 minutes**

Please do **not** fill in the following table!

exercise	part	max. points	points gained
1	a	4	
1	b	4	
1	c	1	
1	d	7	
2		8	
3	a	3	
3	b	5	
3	c	4	
4	a	2	
4	b	7	
4	c	7	
5	a	4	
5	b	3	
6	a	10	
7		12	
8	a	10	
8	b	4	
8	c	2	
8	d	4	
Sum		**101**	

Instructions for the exam:

- Don't panic!
- Check whether you received all sheets.
- Fill in your name and matriculation number on the cover sheet!
- Write your name and matriculation number on *every* sheet of paper, this includes all sheets of the exam and the additional paper you get from us.
- No tools, books, and lecture notes are allowed for this exam.
- On your desk should be no more than: the exam, some pens, a ruler, and your student id. Nothing else!
- If you try to cheat you will fail the exam.
- Be careful to write in a readable way! For things we cannot read you get no points.
- Answer only the questions that are posed. Giving more information than asked for will not be rewarded by extra points, and you will lose valuable time for giving these answers.
- If you are asked to calculate something, on every step write down what you are currently calculating. Giving only the final result will not be enough to get full points.
- Always write short and precise answers and do not spend too much time on any specific question.
- The answers can be given either in English or in German.

Enough details, let's get started!

Exercise 1: Rendering Equation (4 + 4 + 1 + 7 points)

a) Write down the rendering equation expressed in terms of solid angles and explain all terms and factors.

b) Given the definition of the rendering equation:

1. Name two light-surface interaction effects that *can* be captured by it;
2. Name two light-surface interaction effects that *cannot* be captured by it and argue why.

c) Explain why the Helmholtz reciprocity principle is important to ray tracing.

d) Derive the BRDF function of a perfectly diffuse (Lambertian) material with no absorption (i.e. reflecting 100% of the incoming light).

Exercise 2: Camera (8 points)

A perspective camera is parametrized as follows:

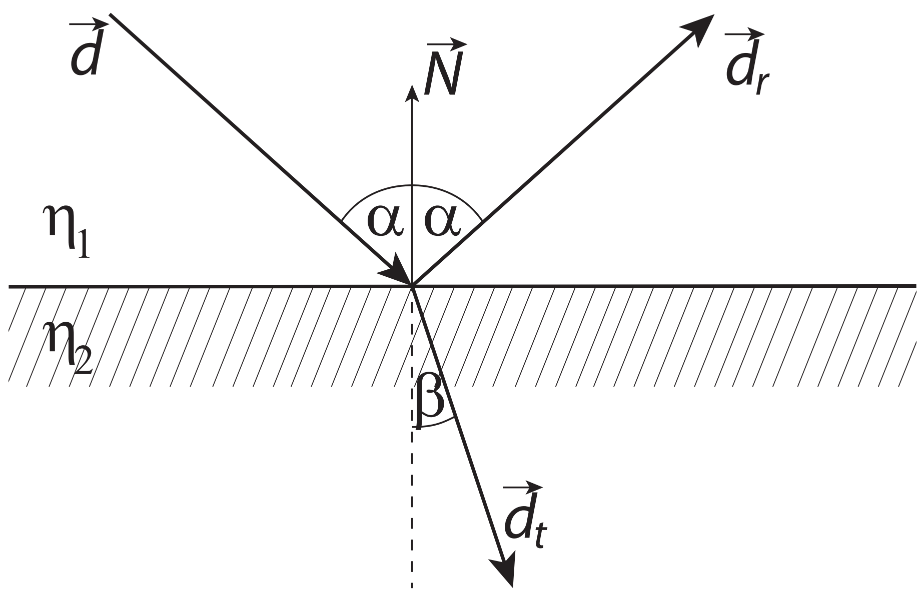
- origin $= (0, 0, -3)$
- direction $= (0, 0, 1)$
- full vertical opening angle $= 60°$
- aspect ratio $1 : 1$

For each primitive below, decide if it is going to be rendered fully, partially or not at all. Justify your answer.

1. Infinite plane defined by point $a = (0, -2, -4)$, and normal $n = (0, 1, 0)$
2. Infinite plane defined by point $a = (-1, 0, -4)$, and normal $n = (\frac{\sqrt{2}}{2}, 0, \frac{\sqrt{2}}{2})$
3. A sphere centered at $c = (0, \frac{1}{2}, \frac{\sqrt{3}}{2} - 3)$ and radius $r = 0.125$
4. A sphere centered at $c = (0, -3, -3)$ and radius $r = 1.6\sqrt{3}$

Exercise 3: Reflection and Refraction (3 + 5 + 4 points)

When a light ray hits a dielectric object, such as glass, it exhibits perfect specular reflection and/or transmission on the object's surface. The figure below illustrates this, where the incoming ray's (normalized) direction is d, the (normalized) surface normal is N, and the reflection and transmission rays are d_r and d_t, respectively. The refractive indices of the media of incidence and transmission are denoted by η_1 and η_2, respectively.



a) Depending on the scene description or whether the surface of the dielectric is hit from inside or outside, the surface normal may face forward or backward with respect to the incoming ray. How are the computations of d_r and d_t affected by the direction of the normal?

b) Derive the formula for computing the reflection ray direction d_r. Your result should not contain any angle other than $\cos\alpha$ which can be in turn derived using the incoming direction and the surface normal.

c) Prove that the formula:

$$d_t = \frac{\eta_1}{\eta_2}\, d - \left(\sqrt{1 - \left(\frac{\eta_1}{\eta_2}\right)^2 (1 - (d \cdot N)^2)} - \frac{\eta_1}{\eta_2}\, (d \cdot N) \right) N$$

correctly computes the transmission ray direction.

Exercise 4: Homogeneous Transformations (2 + 7 + 7 points)

a) How do you test whether two homogeneous vectors represent the same affine point?

b) Derive the transformation $T(a)$, where $a = (a_1\ a_2\ a_3\ a_4)^\top$, that performs the following operations in the order they are given:

1. Translation by 1 unit along the y-axis
2. Rotation by 45 degrees around the z-axis
3. Mirroring about the XY-plane through the origin
4. Uniform scaling by 2

Note: You do not have to compute the final transformation matrix.

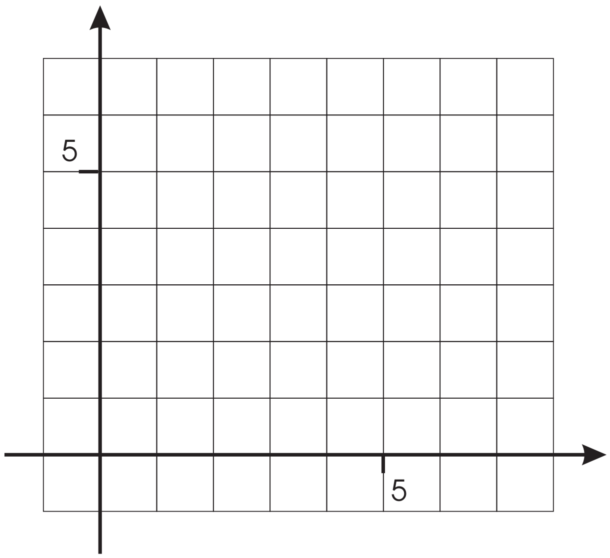
c) Let $T(a)$, where $a = (a_1\ a_2\ a_3\ a_4)^\top$, be the following transformation:

$$
\begin{aligned}
T(a) &= \begin{pmatrix} 2 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0.5 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \end{pmatrix} \\
&= \begin{pmatrix} 0 & 0 & 2 & 0 \\ 0 & 1 & 0 & 0 \\ -0.5 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \end{pmatrix}.
\end{aligned}
$$

1. Can T be directly used to correctly transform both the vertices and the normals of a 3D object? Why?
2. Does T preserve volumes? Why?

Exercise 5: Splines (4 + 3 points)

a) Given are four control points $P_0 = (6,5), P_1 = (1,1), P_2 = (7,1)$, and $P_3 = (3,4)$, which represent a Bezier-Spline $S(\lambda)$ in the order they are given. Draw the 4 control points on the coordinate system below and apply the DeCasteljau algorithm graphically to compute the point $S\left(\frac{1}{2}\right)$ on the spline.



b) Explain how the direction of the derivative at point $S\left(\frac{1}{2}\right)$ of the Bezier Spline can be constructed using the DeCastaljau algorithm. Draw the direction of the derivative into the coordinate system above.

Exercise 6: Rasterization (10 points)

a) You are given a line segment L, specified by its endpoints $L_0(x_0, y_0)$ and $L_1(x_1, y_1)$, where $x_0 < x_1$, $y_0 < y_1$, and $x_1 - x_0 < y_1 - y_0$. Develop a mid-point algorithm that draws L on a raster grid. You have to derive the formulas for computing x_{i+1}, y_{i+1}, and d_{i+1} out of x_i, y_i, d_i. The computation of d_{i+1} must depend only on the decision variable (d_i) from the previous step. You can use the piece of code below to write your answers.

```
drawLine(x0, y0, x1, y1)
    d =
    x =
    y =
    while x <=
        plot(x, y)
        x =
        d =
        if d >= 0.5
            y =
            d =
```

Exercise 7: Volume Rendering (12 points)

We consider a point x on an infinite plane embedded in a homogenous fog of optical density κ. The point x is illuminated by an isotropic point light source of total power Φ located at a distance d, along the normal at x. If the light source is now moved to a distance $2d$, how should the optical density of the fog change so that the illumination at the point x remains the same.

Exercise 8: Signal Theory (10 + 4 + 2 + 4 points)

a) The convolution of two continuous signals f and g is given by:

$$(f * g)(t) = \int_{-\infty}^{+\infty} f(\tau)g(t - \tau)\, d\tau.$$

Given the function

$$b(x) = \begin{cases} 1 & \text{if } -0.5 \leq x \leq 0.5, \\ 0 & \text{else} \end{cases}$$

1. Compute the convolution of b with itself;
2. Argue what the Fourier transform of the result is. You do not have to compute the Fourier transform.

b) For a one-dimensional signal, explain in the spatial and the Fourier domains:

1. What sampling of the signal means;
2. What aliasing in the sampled signal means.

c) You are given a one-dimensional continuous signal which is low-pass filtered to frequencies below 2 Hz. To be able to perform full reconstruction after sampling, what should be the minimum sampling frequency for this signal? Explain why.

d) Name two different techniques that help reduce aliasing. Explain their basic principles.