Computer Graphics

- Material Models -

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Overview

- **Last time**
  - Light: radiance & light sources
  - Light transport: rendering equation & formal solutions
- **Today**
  - Reflectance properties:
    - Material models
    - Bidirectional Reflectance Distribution Function (BRDF)
    - Reflection models
  - Shading
- **Next lecture**
  - Varying (reflection) properties over object surfaces: texturing

REFLECTANCE PROPERTIES

Appearance Samples

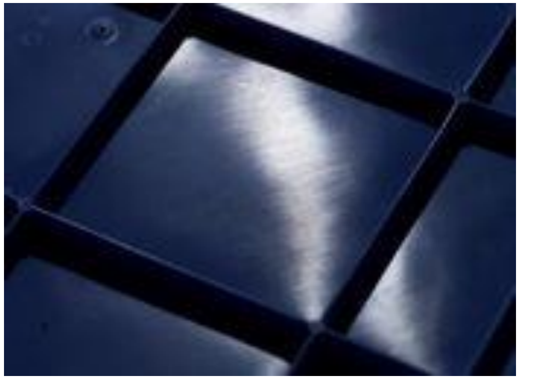
- **How do materials reflect light?**

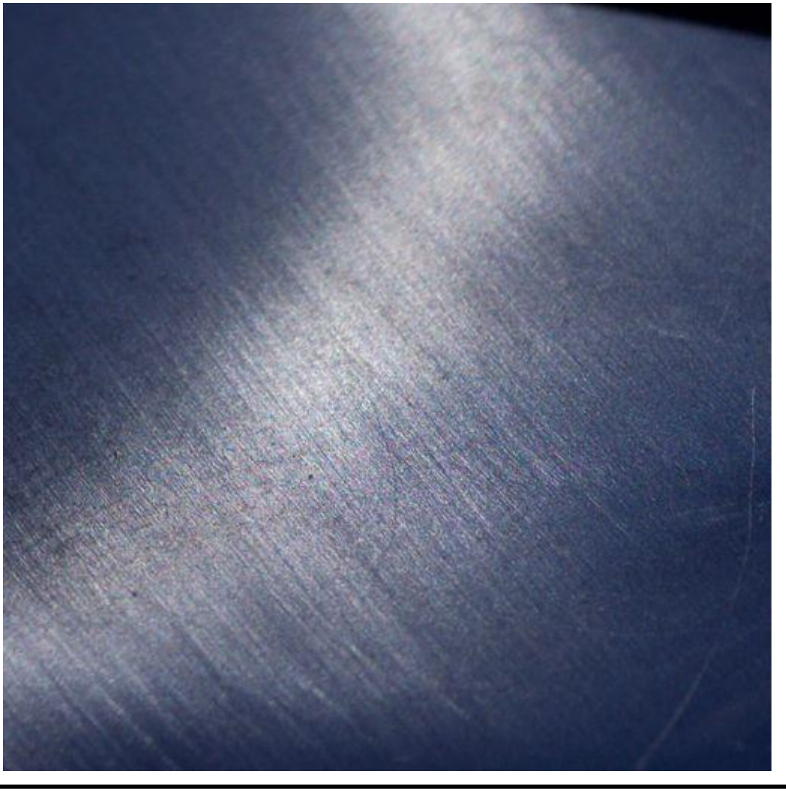
Opaque

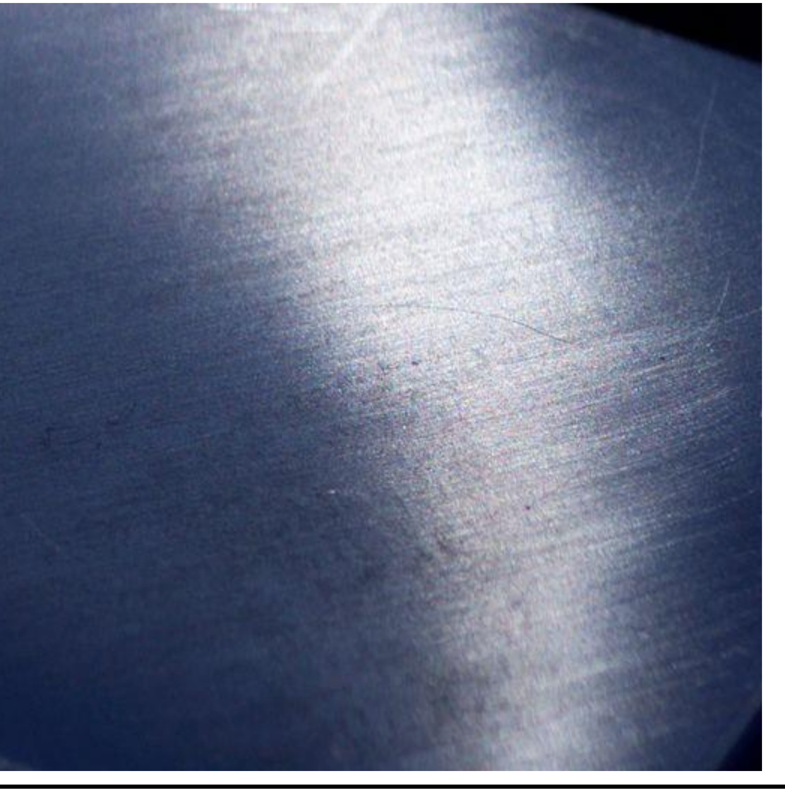
Translucency - subsurface scattering

Material Samples

- **Anisotropic**

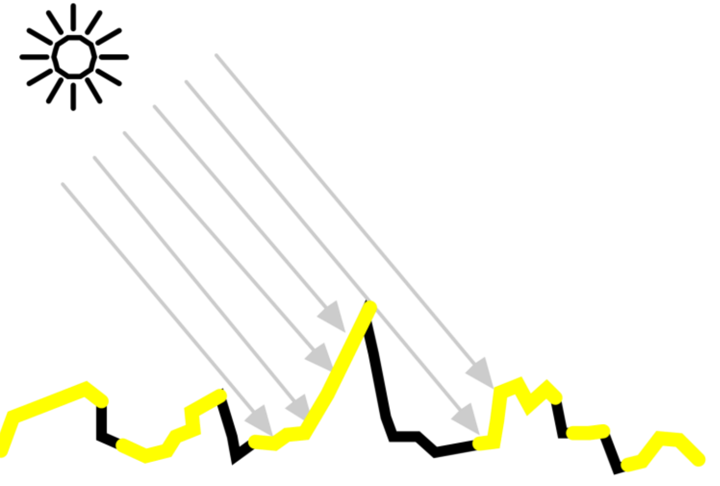






Material Samples

- **Complex surface meso-structure**



Material Samples

- **Fibers**

Material Samples

- **Photos of samples with light source at exactly the same position**

diffuse glossy mirror

How to describe materials?

- **Reflection properties**
- **Mechanical, chemical, electrical properties**
- **Surface roughness**
- **Geometry/meso-structure**

- **Goal: relightable representation of appearance**

Reflection Equation - Reflectance

- **Reflection equation**

$$L_o(x, \omega_o) = \int_{\Omega_+} f_r(\omega_i, x, \omega_o) L_i(x, \omega_i) cos\theta_i d\omega_i$$

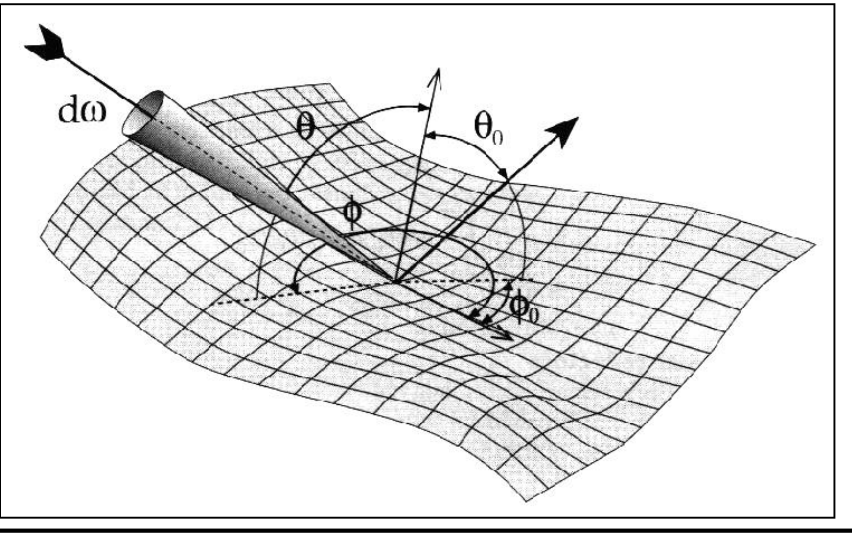
- **BRDF**
  - Ratio of reflected radiance to incident irradiance

$$f_r(\omega_i, x, \omega_o) = \frac{dL_o(x, \omega_o)}{dE_i(x, \omega_i)}$$

BRDF

- **BRDF describes surface reflection**
  - for light incident from direction $\boldsymbol{\omega_i} = (\boldsymbol{\theta_i}, \boldsymbol{\varphi_i})$
  - observed from direction $\boldsymbol{\omega_o} = (\boldsymbol{\theta_o}, \boldsymbol{\varphi_o})$
- **Bidirectional**
  - Depends on 2 directions $\omega_i,\ \omega_o$ and position x (6-D function)

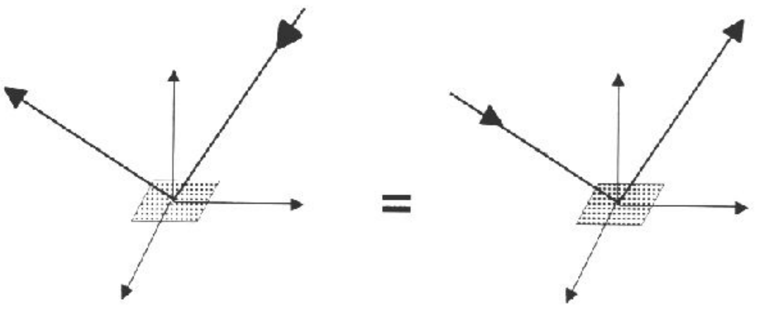
$$f_r(\omega_i, x, \omega_o) = \frac{dL_o(x, \omega_o)}{dE_i(x, \omega_i)} = \frac{dL_o(x, \omega_o)}{L_i(x, \omega_i) cos\theta_i d\omega_i}$$



BRDF Properties

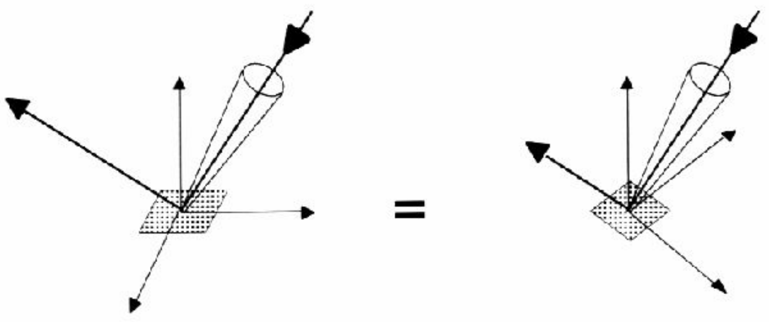
- **Helmholtz reciprocity principle**
  - BRDF remains unchanged if incident and reflected directions are interchanged
  - Due to physical law of time reversal

$$f_r(\omega_i, \omega_o) = f_r(\omega_o, \omega_i)$$



- **Smooth surface: isotropic BRDF**
  - Reflectivity independent of rotation around surface normal
  - BRDF has only 3 instead of 4 directional degrees of freedom

$$f_r(\theta_i, x, \theta_o, \varphi_o - \varphi_i)$$



BRDF Properties

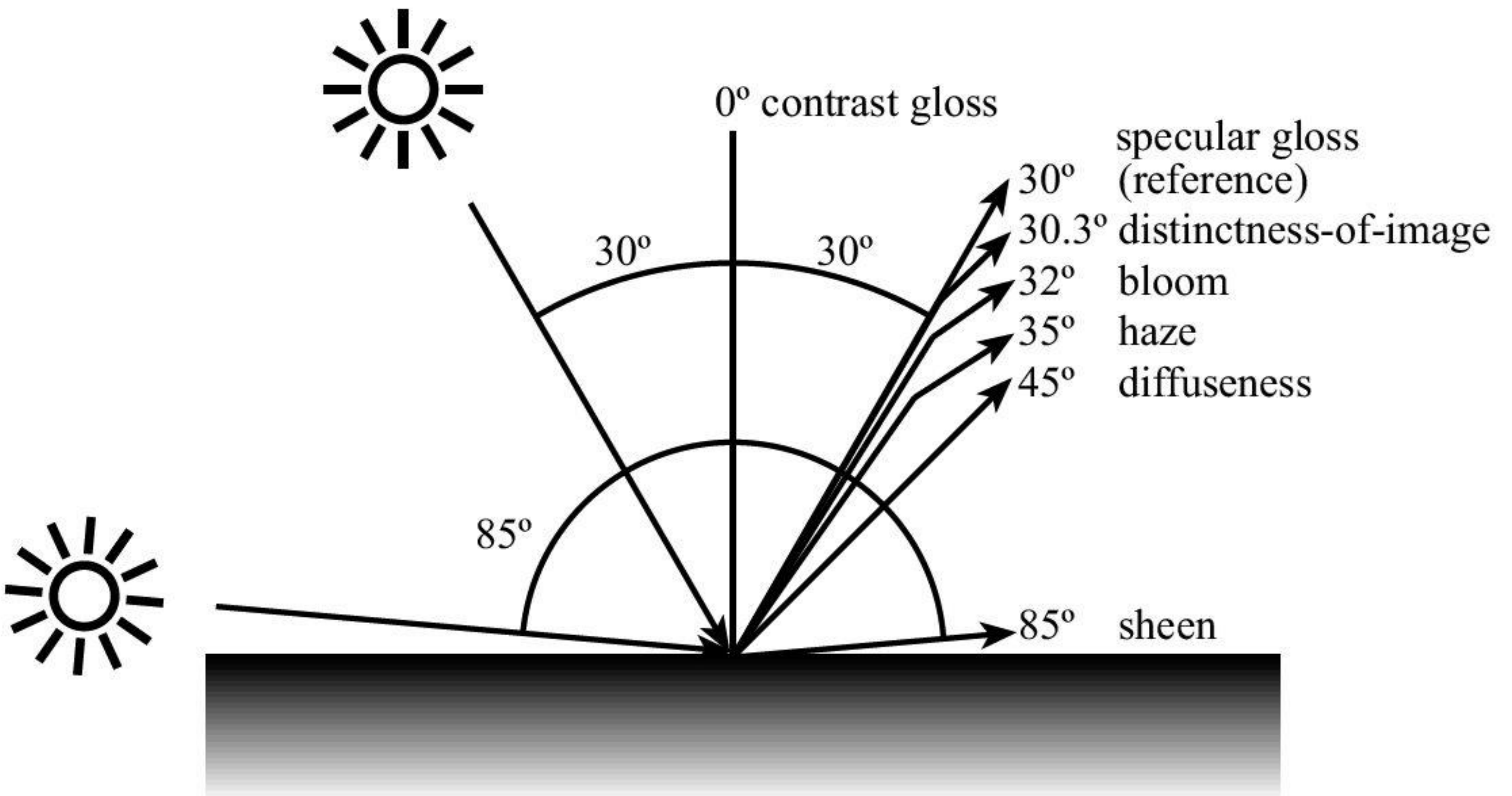
- **Characteristics**
  - BRDF units
    - Inverse steradian: sr^{-1} (not intuitive)
  - Range of values: distribution function is positive, can be infinite
    - From 0 (total absorption) to ∞ (reflection, δ -function)
  - Energy conservation law
    - No self-emission
    - Possible absorption

$$\int_{\Omega_+} f_r(\omega_i, x, \omega_o) cos\theta_o d\omega_o \leq 1, \qquad \forall \omega_i$$

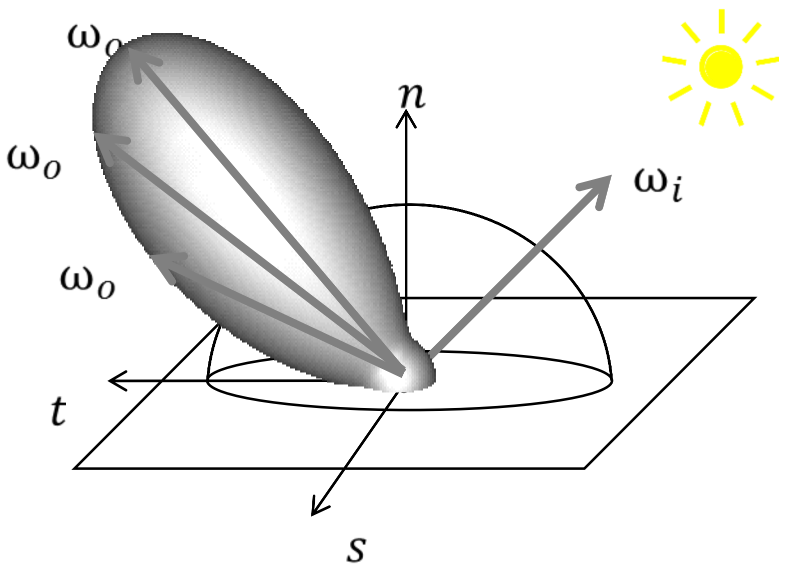
- **Reflection only at the point of entry ($\boldsymbol{x_i = x_o}$)**
  - No subsurface scattering

Standardized Gloss Model

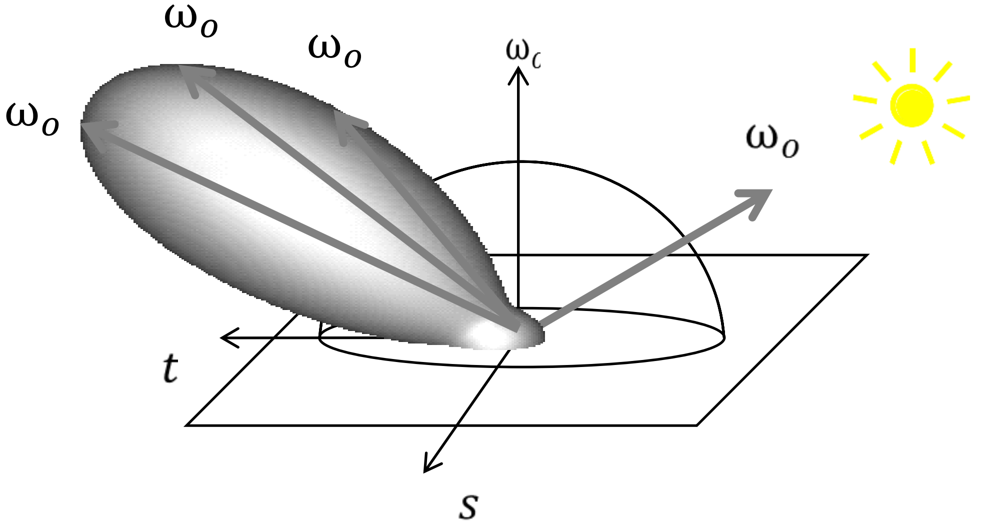
- **Industry often uses only a subset of BRDF values**
  - Reflection only measured at discrete set of angles



Reflection of an Opaque Surface



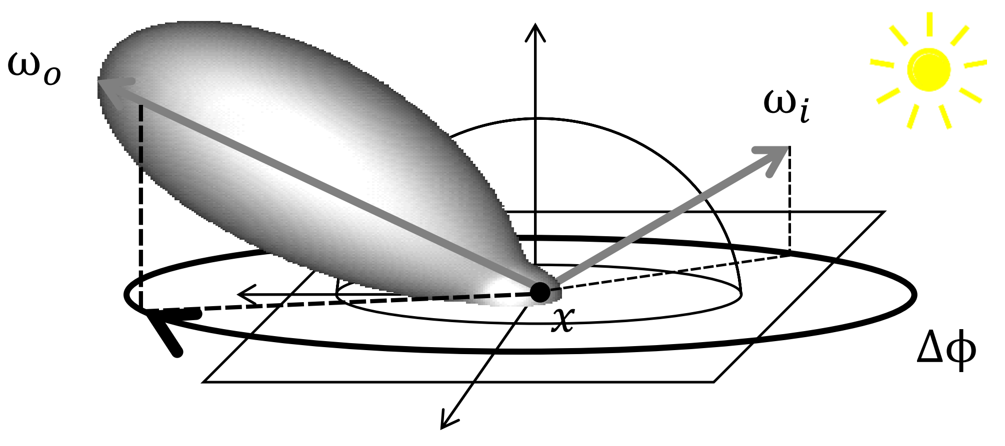
Reflection of an Opaque Surface



Isotropic BRDF – 3D

- **Invariant with respect to rotation about the normal**
  - Only depends on azimuth difference to incoming angle

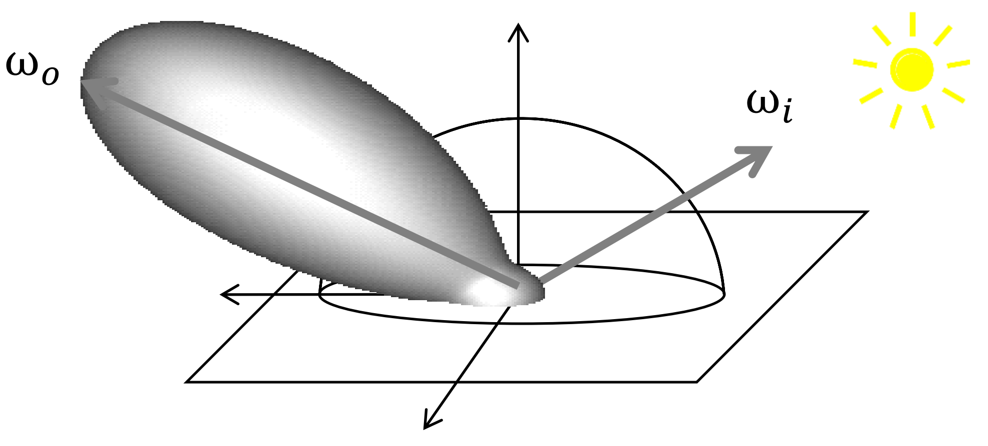
$$f_r\big((\theta_i,\varphi_i)\rightarrow(\theta_o,\varphi_o)\big)\Longrightarrow$$
$$f_r(\theta_i\rightarrow\theta_o,(\varphi_i-\varphi_o)) = f_r(\theta_i\rightarrow\theta_o,\Delta\varphi)$$



Homogeneous BRDF – 4D

- **Homogeneous bidirectional reflectance distribution function**
  - Ratio of reflected radiance to incident irradiance

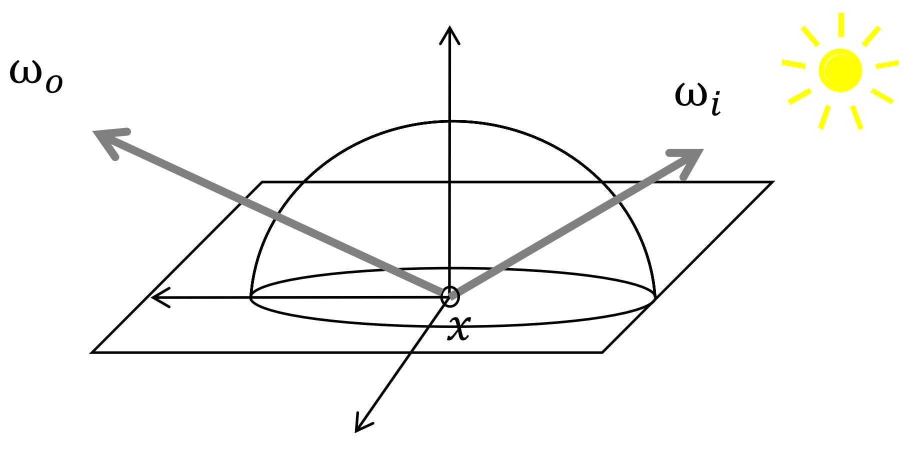
$$f_r(\omega_i \rightarrow \omega_o) = \frac{dL_o(\omega_o)}{dE_i(\omega_i)}$$



Spatially Varying BRDF – 6D

- **Heterogeneous materials (standard model for BRDF)**
  - Dependent on position, and two directions
  - Reflection at the point of incidence

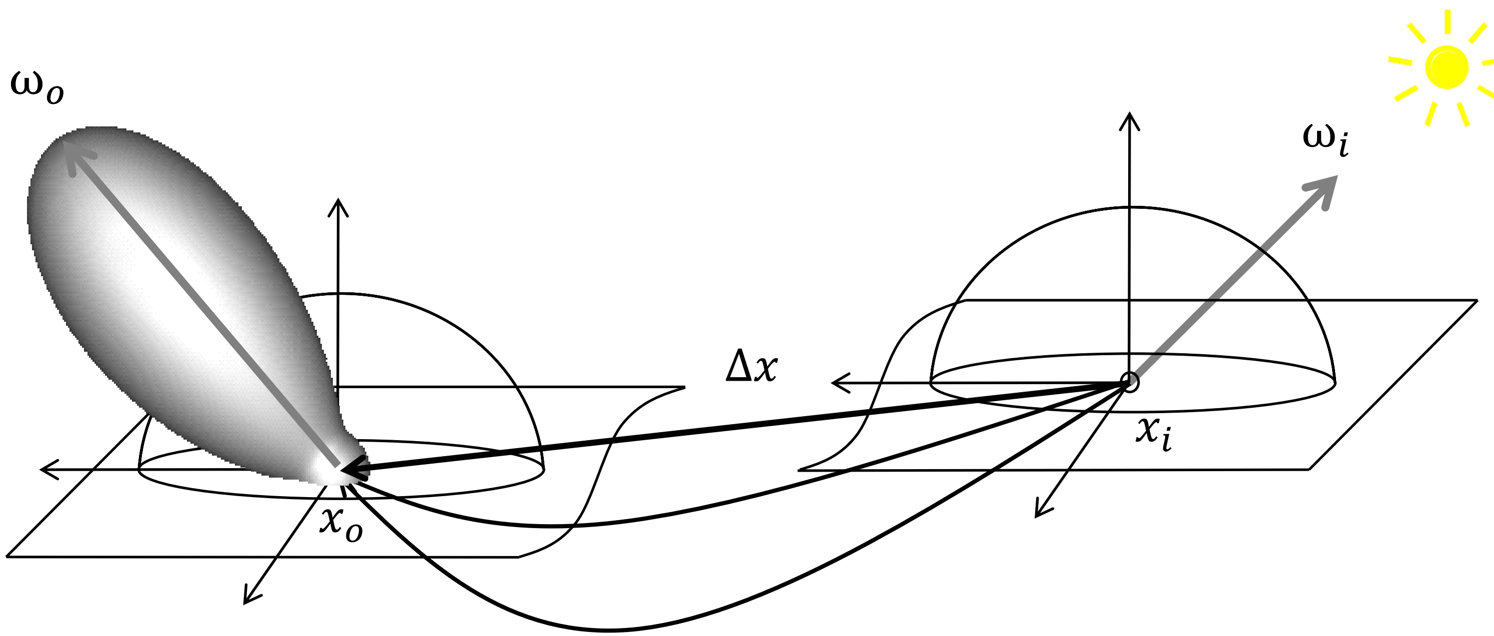
$$f_r(x, \omega_i \rightarrow \omega_o)$$



Homogeneous BSSRDF – 6D

- **Homogeneous bidirectional scattering surface reflectance distribution function**
  - Assumes a homogeneous and flat surface
  - Only depends on the difference vector to the outgoing point

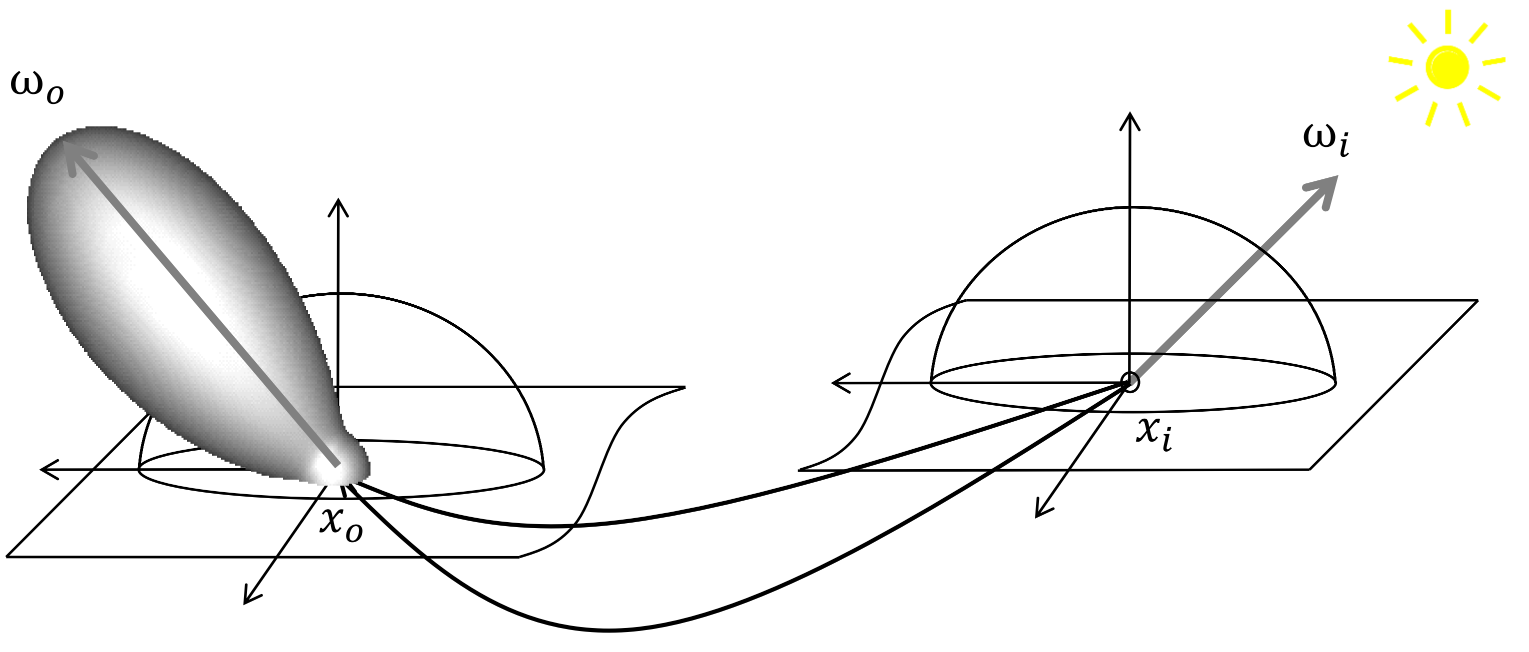
$$f_r(\Delta x, \omega_i \to \omega_o)$$



BSSRDF – 8D

- **Bidirectional scattering surface reflectance distribution function**

$$f_r((x_i, \omega_i) \rightarrow (x_o, \omega_o))$$



Generalization – 9D

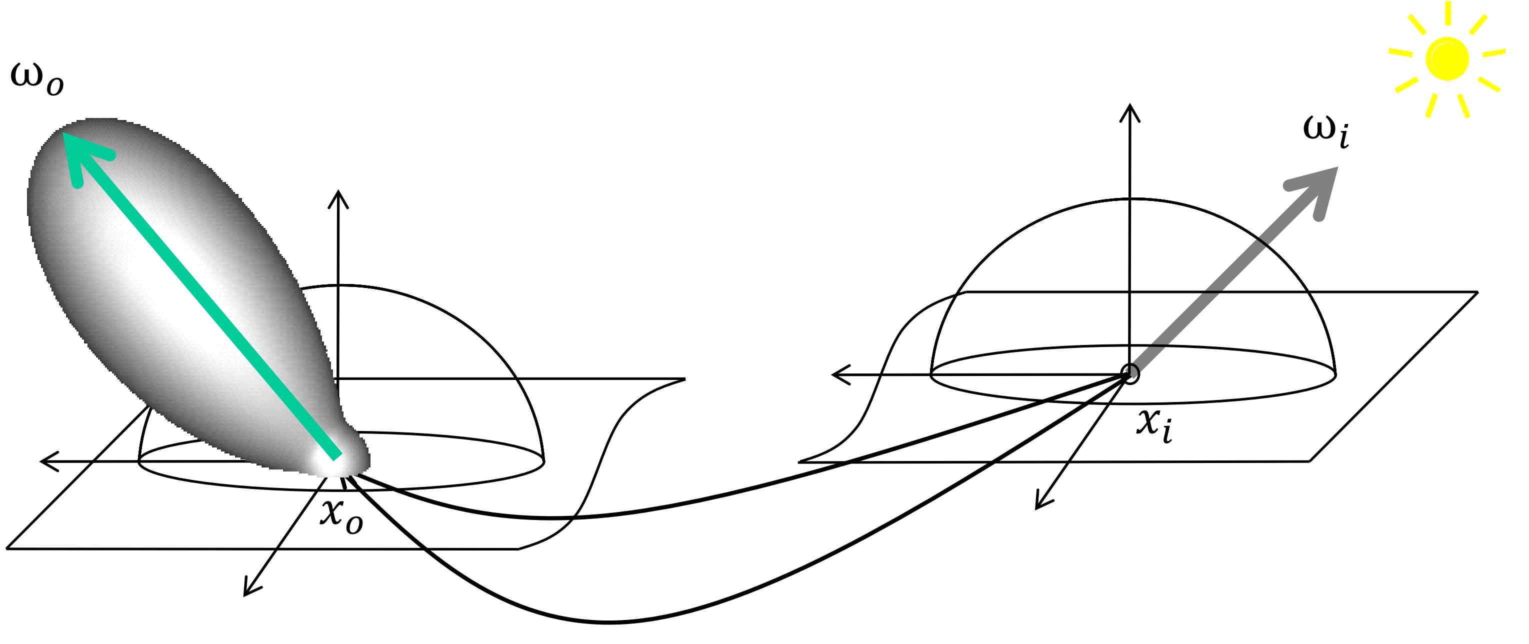
- **Generalizations**
  - Add wavelength dependence

$$f_r(\lambda, (x_i, \omega_i) \rightarrow (x_o, \omega_o))$$

Generalization – 10D

- **Generalizations**
  - Add wavelength dependence
  - Add **fluorescence**
    - Change to longer wavelength during scattering

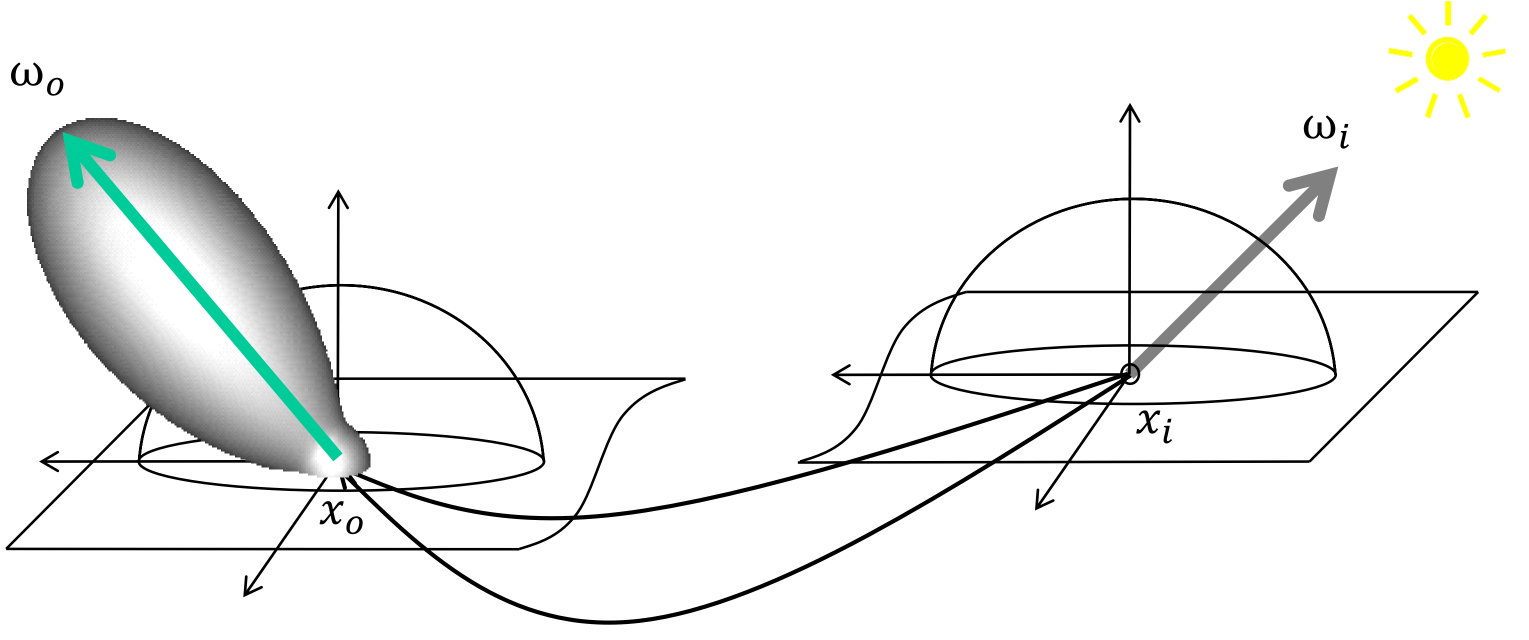
$$f_r\big((x_i, \omega_i, \lambda_i) \to (x_o, \omega_o, \lambda_o)\big)$$



Generalization – 11D

- **Generalizations**
  - Add wavelength dependence
  - Add fluorescence (change to longer wavelength for reflection)
  - Time varying surface characteristics

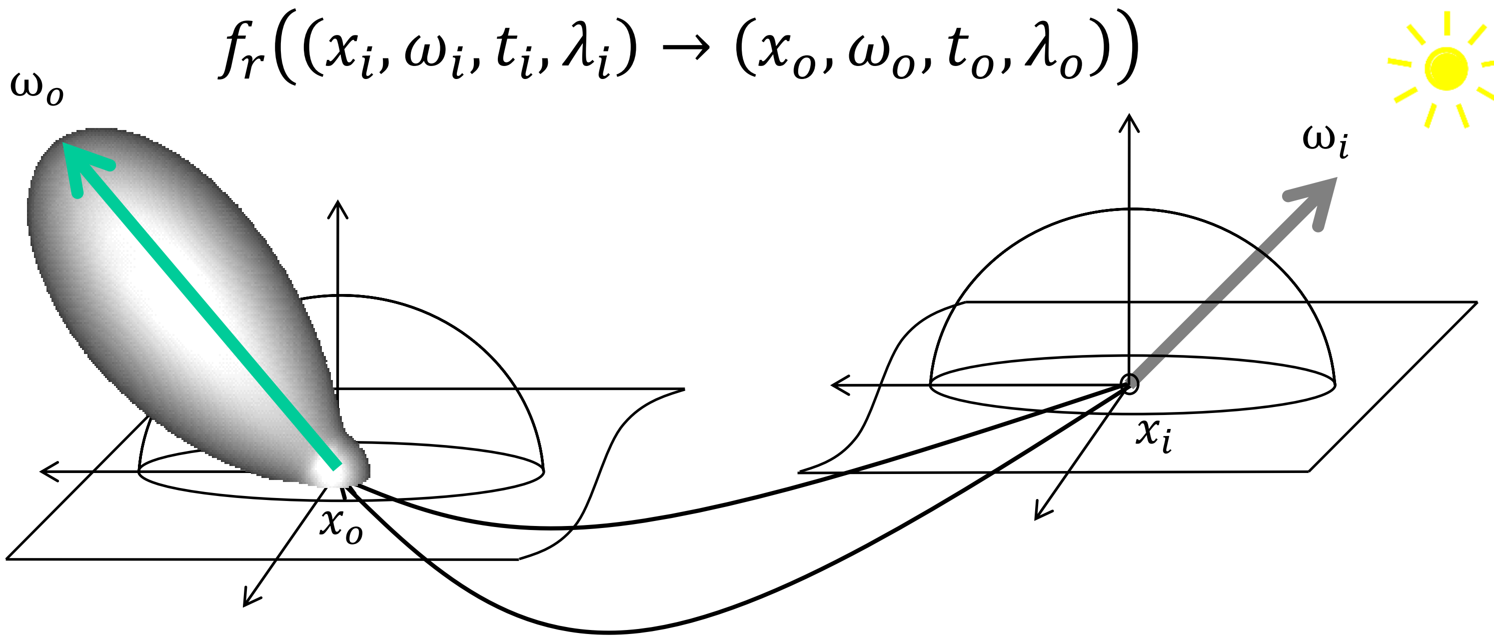
$$f_r\left(t, (x_i, \omega_i, \lambda_i) \rightarrow (x_o, \omega_o, \lambda_o)\right)$$



Generalization – 12D

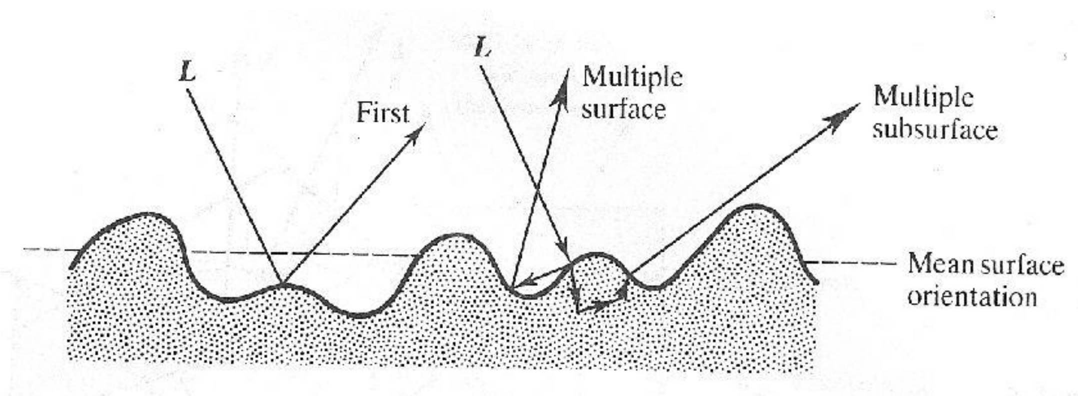
- **Generalizations**
  - Add wavelength dependence
  - Add fluorescence (change to longer wavelength for reflection)
  - Time varying surface characteristics
  - **Phosphorescence**
    - Temporal storage of light

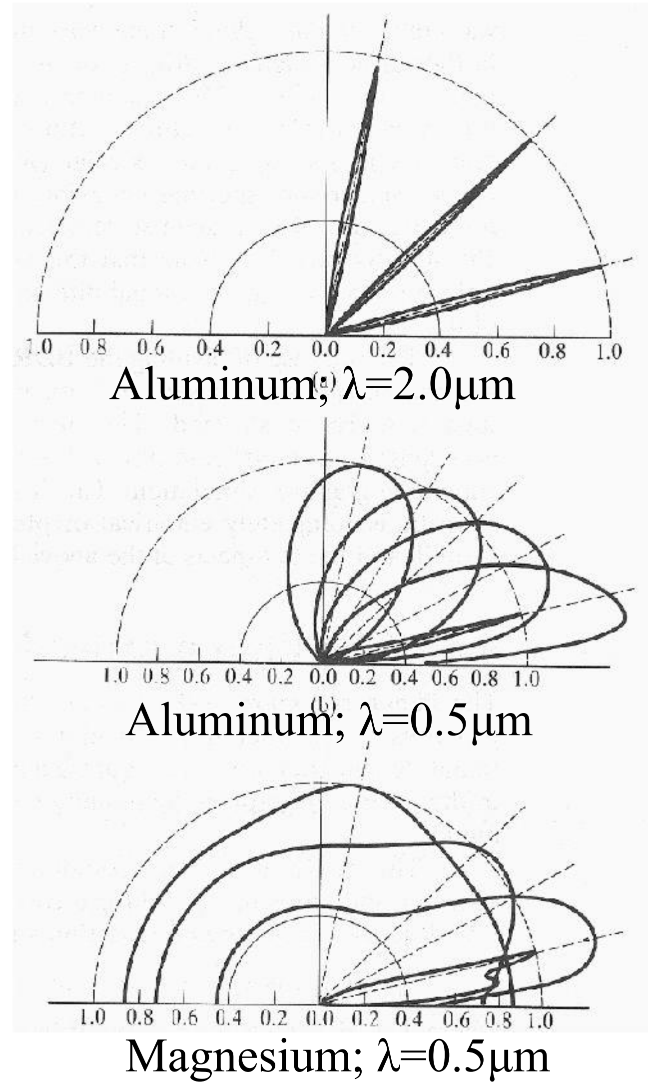
$$f_r\big((x_i, \omega_i, t_i, \lambda_i) \rightarrow (x_o, \omega_o, t_o, \lambda_o)\big)$$



Reflectance

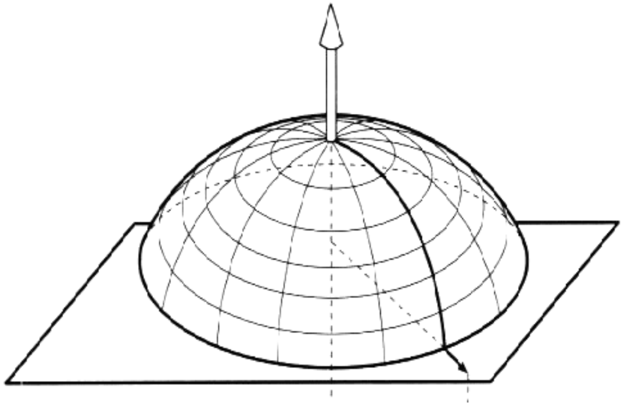
- **Reflectance may vary with**
  - Illumination angle
  - Viewing angle
  - Wavelength
  - (Polarization, ...)
- **Variations due to**
  - Absorption
  - Surface micro-geometry
  - Index of refraction / dielectric constant
  - Scattering





BRDF Measurement

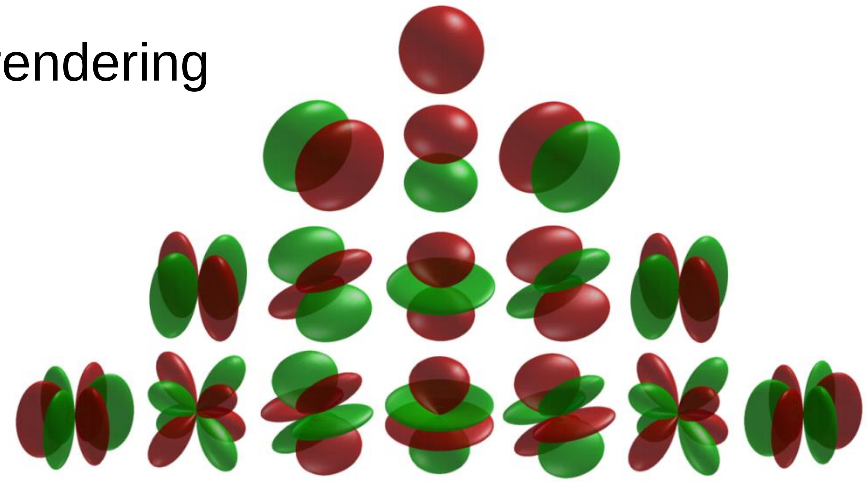
- **Gonio-Reflectometer**
- **BRDF measurement**
  - Point light source position (θ_i, φ_i)
  - Light detector position (θ_o, φ_o)
- **4 directional degrees of freedom**
- **BRDF representation**
  - *m* incident direction samples
  - *n* outgoing direction samples
  - *m*n* reflectance values (large!!!)



Stanford light gantry

Rendering from Measured BRDF

- **Linearity, superposition principle**
  - Complex illumination: integrating light distribution against BRDF
  - Sampled computation: superimposing many point light sources
- **Interpolation**
  - Look-up of BRDF values during rendering
  - Sampled BRDF must be filtered
- **BRDF Modeling**
  - Fitting of parameterized BRDF models to measured data
    - Continuous, analytic function
    - No interpolation
    - Typically fast evaluation

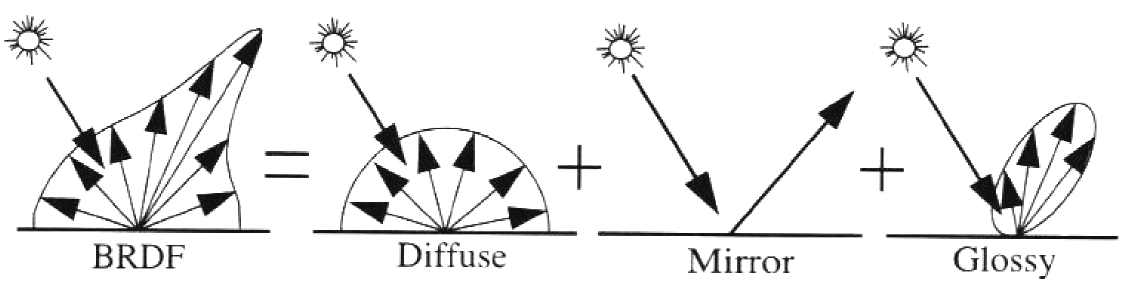


Spherical Harmonics
Red is positive, green negative [Wikipedia]

- **Representation in a basis**
  - Most appropriate: Spherical harmonics
    - Ortho-normal function basis on the sphere
  - Mathematically elegant filtering, illumination-BRDF integration

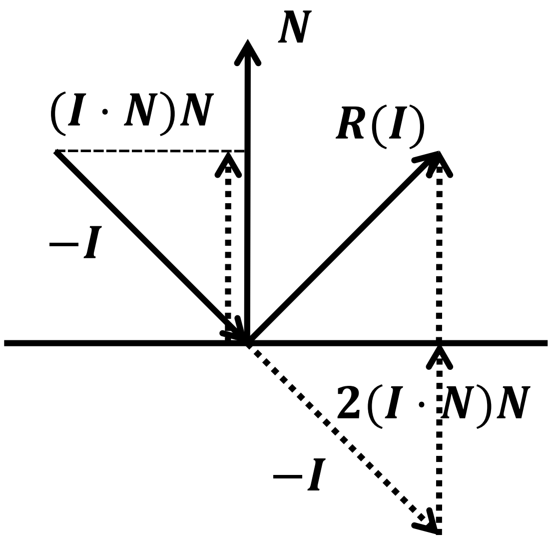
BRDF Modeling

- **Phenomenological approach**
  - Description of visual surface appearance
  - Composition of different terms:
- **Ideal diffuse reflection**
  - Lambert’s law, interactions within material
  - Matte surfaces
- **Ideal specular reflection**
  - Reflection law, reflection on a planar surface
  - Mirror
- **Glossy reflection**
  - Directional diffuse, reflection on surface that is somewhat rough
  - Shiny surface
  - Glossy highlights

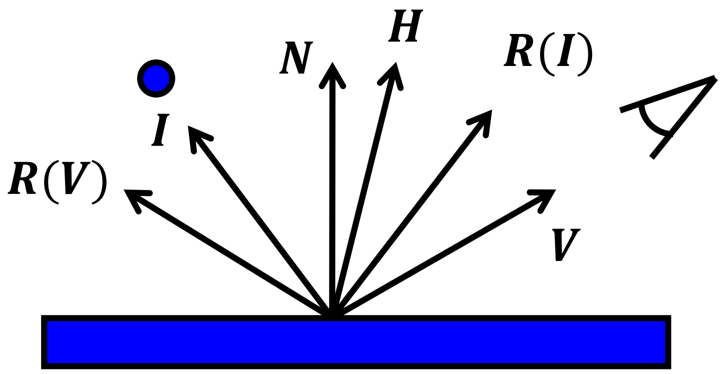


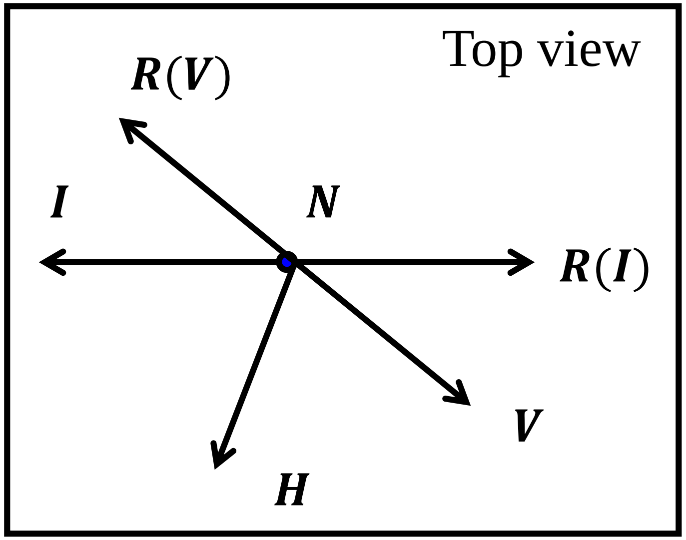
Reflection Geometry

- **Direction vectors (normalize):**
  - N: Surface normal
  - I: Light source direction vector
  - V: Viewpoint direction vector
  - $R(I)$: Reflection vector
    - $R(I) = -I + 2(I \cdot N)N$
  - H: Halfway vector
    - $H = (I + V) / |I + V|$



- **Tangential surface: local plane**

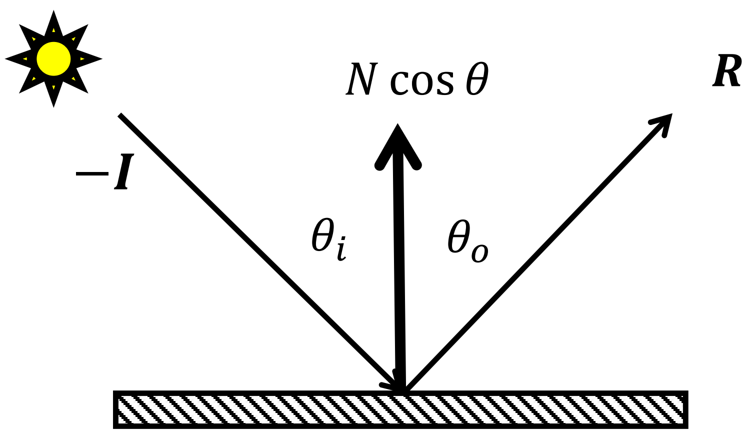




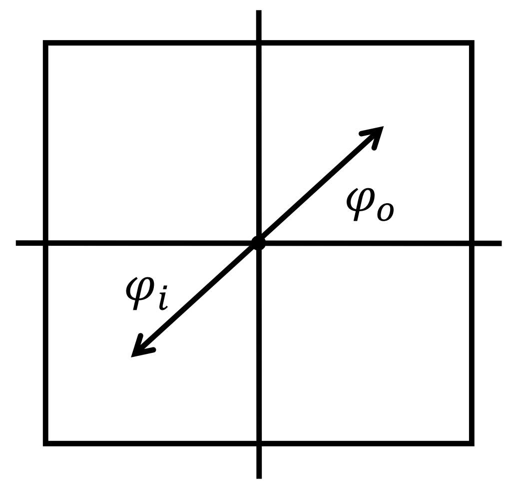
Ideal Specular Reflection

- **Angle of reflectance equal to angle of incidence**
- **Reflected vector in a plane with incident ray and surface normal vector**

$$R + I = 2\cos\boldsymbol{\theta}\, N = 2(I \cdot N)N \Longrightarrow$$
$$R(I) = -I + 2(I \cdot N)\, N$$



$\theta_i = \theta_o$



$\varphi_o = \varphi_i + 180°$

Mirror BRDF

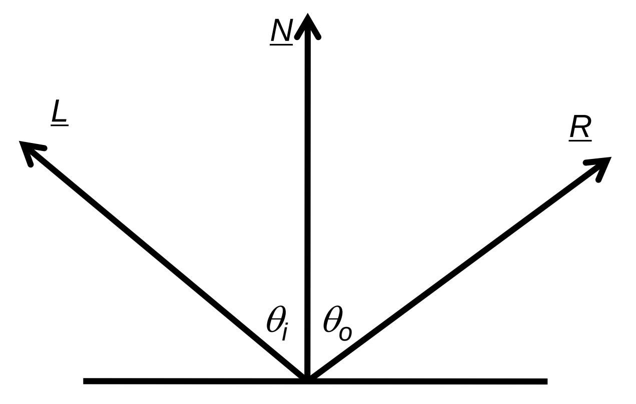
- **Dirac Delta function $\boldsymbol{\delta(x)}$**
  - $\boldsymbol{\delta(x)}$: zero everywhere except at $x = 0$
  - Unit integral iff domain contains $x = 0$ (else zero)

$$f_{r,m}(\omega_i, x, \omega_o) = \rho_s(\theta_i) \frac{\delta(cos\theta_i - cos\theta_o)}{\cos\theta_i} \delta(\varphi_i - \varphi_o \pm \pi)$$

$$L_o(x, \omega_o) = \int_{\Omega_+} f_{r,m}(\omega_i, x, \omega_o) L_i(x, \omega_i) cos\theta_i d\omega_i = \rho_s(\theta_o) L_i(x, \theta_o, \varphi_o \pm \pi)$$

- **Specular reflectance ρ_s**
  - Ratio of reflected radiance in specular direction and incoming radiance
  - Dimensionless quantity between 0 and 1

$$\rho_s(x, \theta_i) = \frac{L_o(x, \theta_o)}{L_i(x, \theta_o)}$$



Refraction in Dielectrics

- **Time required for light to travel from *A* to *B* through *C***

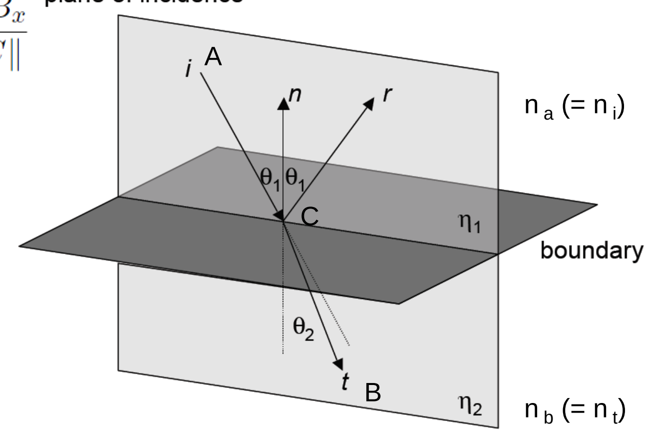
$$t_{AB} = t_{AC} + t_{CB} = \frac{\|AC\|}{c_a} + \frac{\|BC\|}{c_b} = \frac{\sqrt{(C_x - A_x)^2 + A_y^2}}{c_a} + \frac{\sqrt{(C_x - B_x)^2 + B_y^2}}{c_b}$$

(assuming $C_y = 0$)

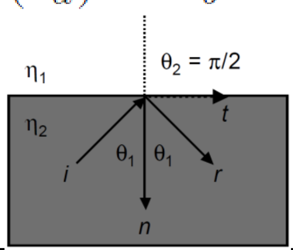
- **Fermat's principle: light path of least traversal time**

$$\frac{dt_{AB}}{dC_x} = \frac{C_x - A_x}{c_a\sqrt{(C_x - A_x)^2 + A_y^2}} + \frac{C_x - B_x}{c_b\sqrt{(C_x - B_x)^2 + B_y^2}} = \frac{C_x - A_x}{c_a\|AC\|} + \frac{C_x - B_x}{c_b\|BC\|}$$

$$= \frac{sin(\theta_a)}{c_a} + \frac{-sin(\theta_b)}{c_b} = \frac{n_a sin(\theta_a)}{c_0} - \frac{n_b sin(\theta_b)}{c_0} = 0$$



- **Snell's law:** $n_a sin(\theta_a) = n_b sin(\theta_b)$
- **Special case possible when $\eta_b < \eta_a$**
  - If $sin(\theta_a) > n_b/n_a$ then $n_b = n_a \frac{n_b}{n_a} < n_a sin(\theta_a) = n_b sin(\theta_b)$
  - Which is impossible since $sin(\theta_b) \leq 1$
    $\Rightarrow$ **total internal reflection**



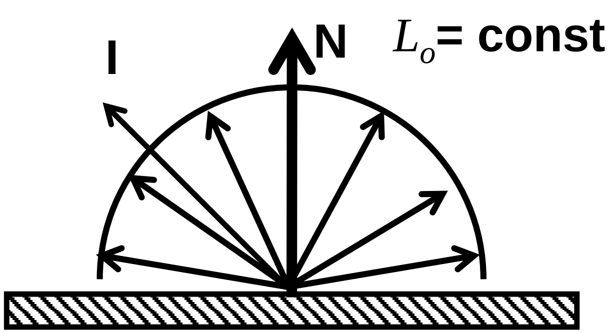
Lambertian Diffuse Reflection

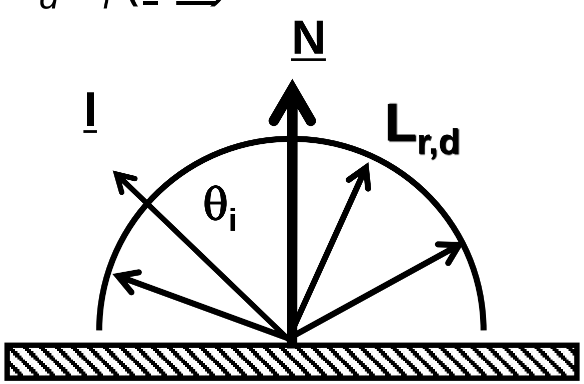
- **Light equally likely to be reflected in any output direction (independent of input direction)**
- **Constant BRDF**

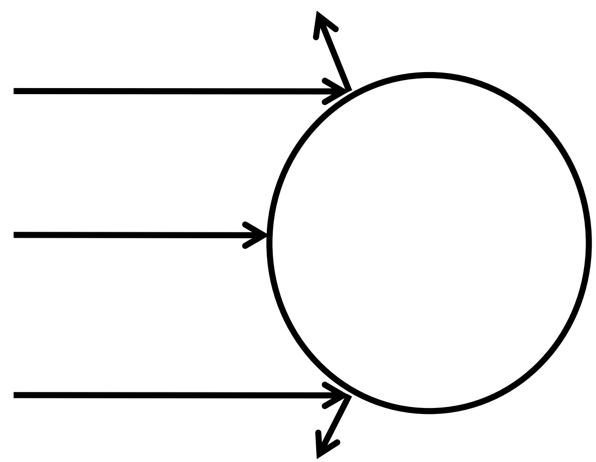
$$f_{r,d}(\omega_i, x, \omega_o) = k_d = const$$

$$L_o(x, \omega_o) = k_d \int_{\Omega_+} L_i(x, \omega_i) \cos\theta_i \, d\omega_i = k_d E$$

  - k_d: diffuse coefficient, material property [1/sr]
- **For each point light source**
  - $L_{r,d} = k_d L_i \cos\theta_i = k_d L_i (\underline{I} \bullet \underline{N})$



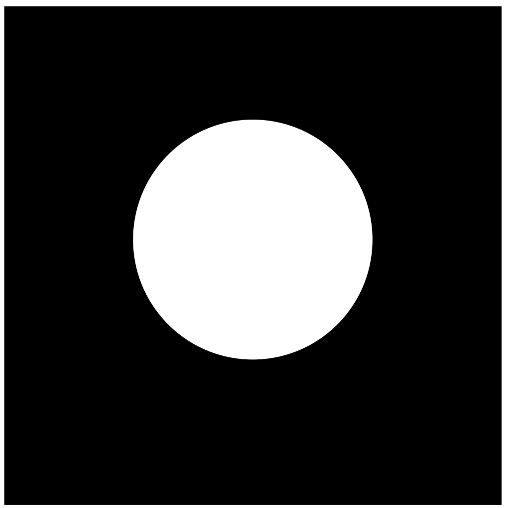




Lambertian Objects

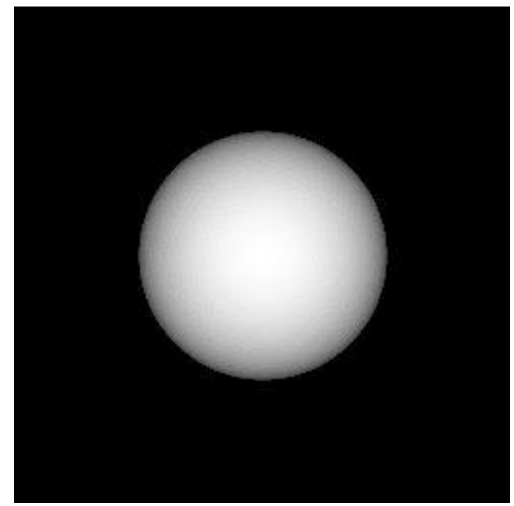
Self-luminous
spherical Lambertian light source

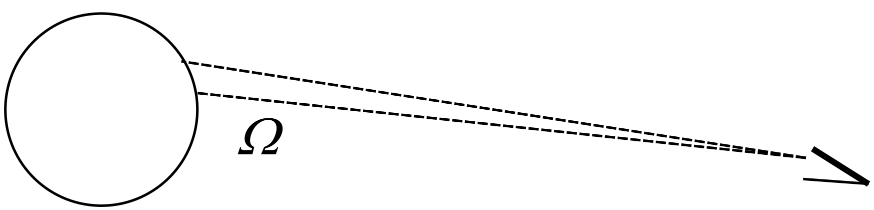
$\Phi_0 \propto L_0 \cdot \Omega$

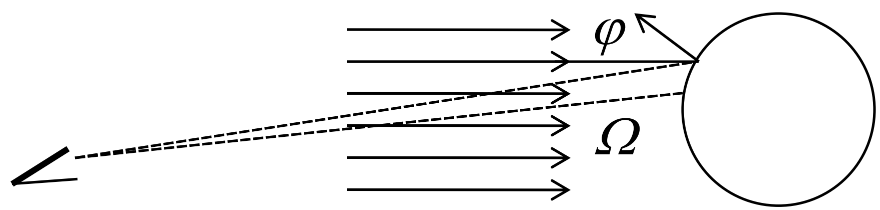


Eye-light illuminated
spherical Lambertian reflector

$\Phi_1 \propto L_i \cdot cos\,\theta \cdot \Omega$

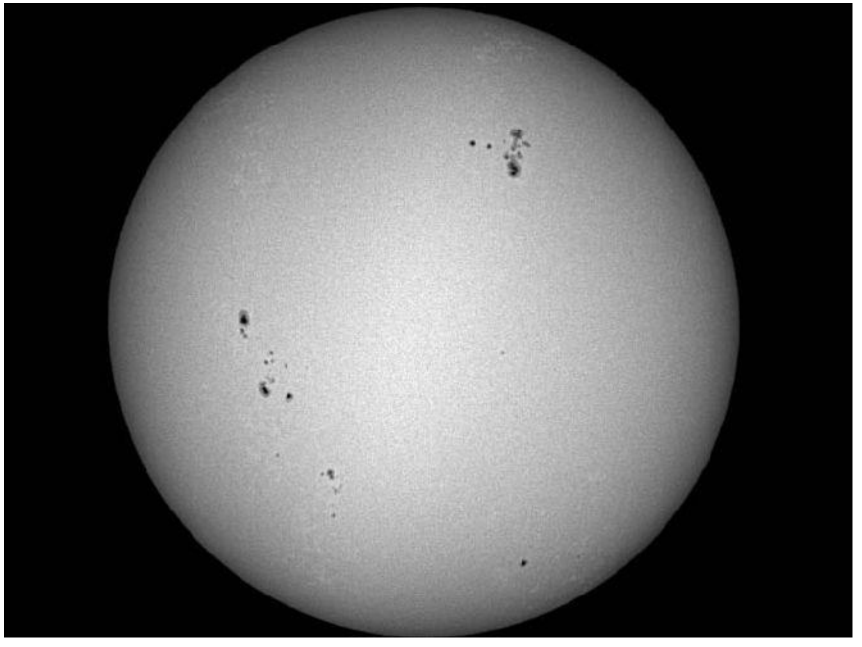






Lambertian Objects (?)

The Sun



The Moon

- Some absorption in photosphere
- Path length through photosphere longer from the Sun's rim

- Surface covered with fine dust
- Dust visible best from slanted viewing angle

⇒ Neither the Sun nor the Moon are Lambertian

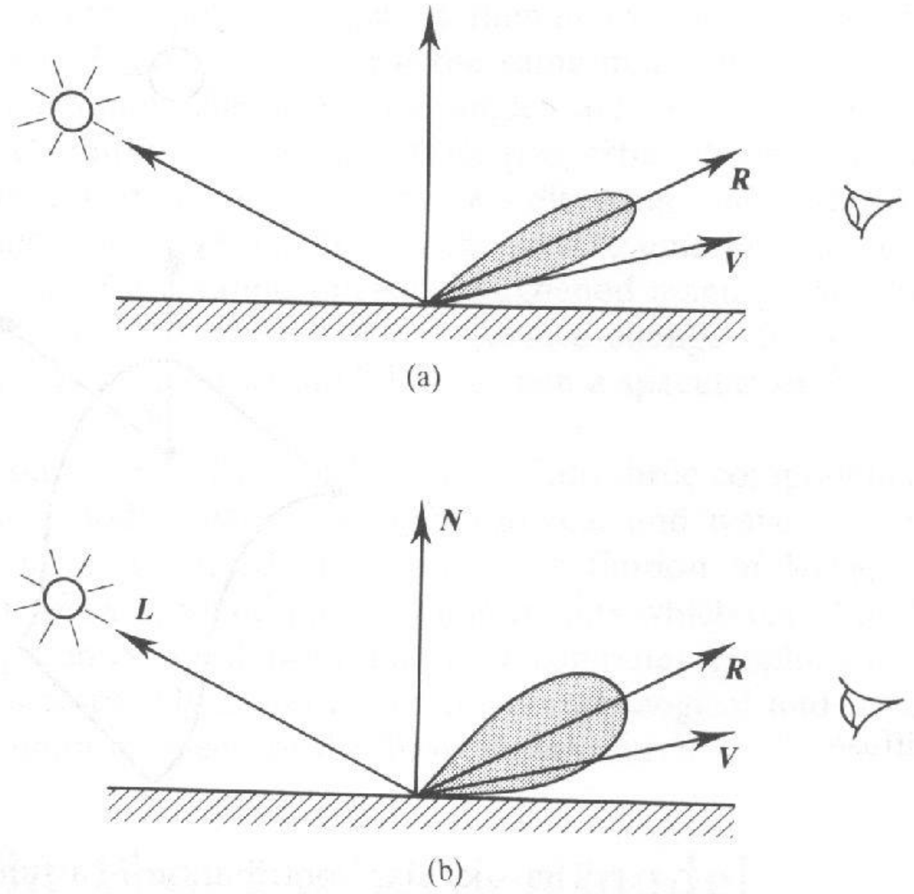
“Diffuse” Reflection

- **Theoretical explanation**
  - Multiple scattering with in the material (at very short range)
- **Experimental realization**
  - Pressed magnesium oxide powder
    - Random mixture of tiny, highly reflective particles
  - Almost never valid at grazing angles of incidence
  - Paint manufacturers attempt to create ideal diffuse paints

Glossy Reflection

Glossy Reflection

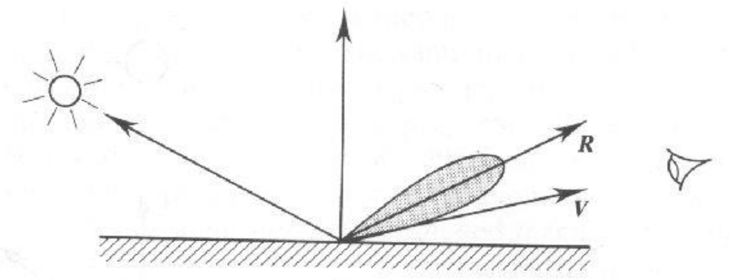
- **Due to surface roughness**
- **Empirical models (phenomenological)**
  - Phong
  - Blinn-Phong
- **Physically-based models**
  - Blinn
  - Cook & Torrance



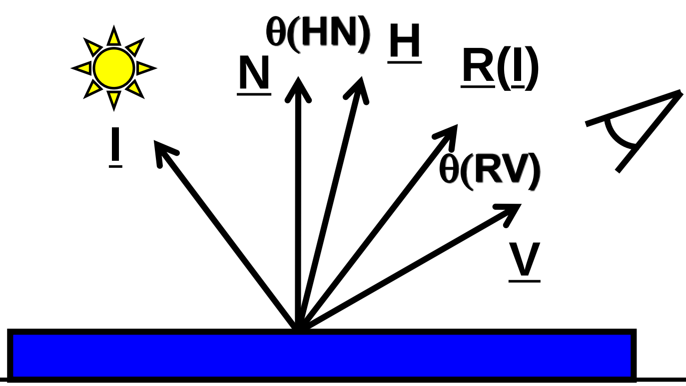
Phong Glossy Reflection Model

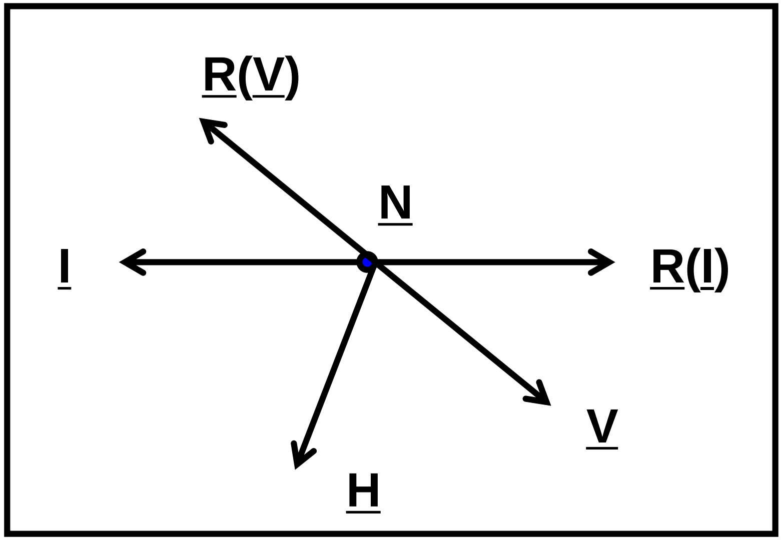
- **Simple description: Cosine power lobe**

$$f_r(\omega_i, x, \omega_o) = k_s (R(I) \cdot V)^{k_e} / I \cdot N$$



- Take angle to reflection direction to some
  - $L_{r,s} = L_i\, k_s \cos^{ke} \theta_{RV}$
- **Issues**
  - Not energy conserving/reciprocal
  - Plastic-like appearance
- **Dot product & power**
  - Still widely used in CG

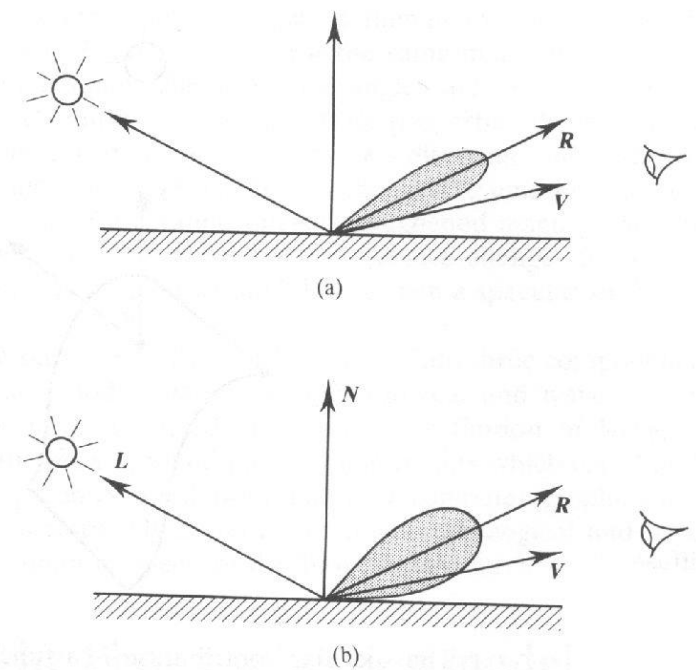


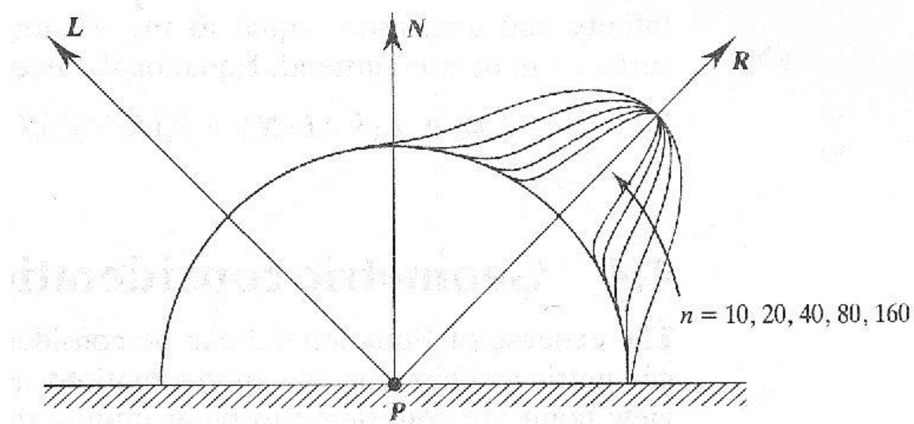


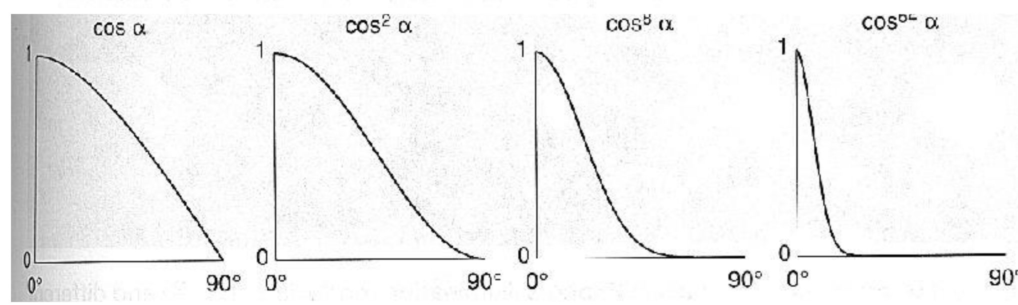
Phong Exponent k_e

$$f_r(\omega_i, x, \omega_o) = k_s (R(I) \cdot V)^{k_e} \,/\, I \cdot N$$

- **Determines size of highlight**







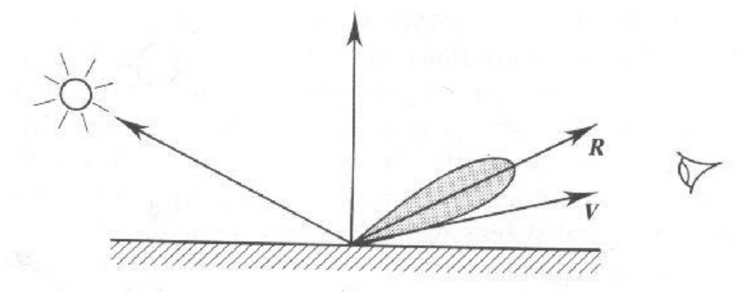
- **Beware: Non-zero contribution into the material !!!**

Blinn-Phong Glossy Reflection

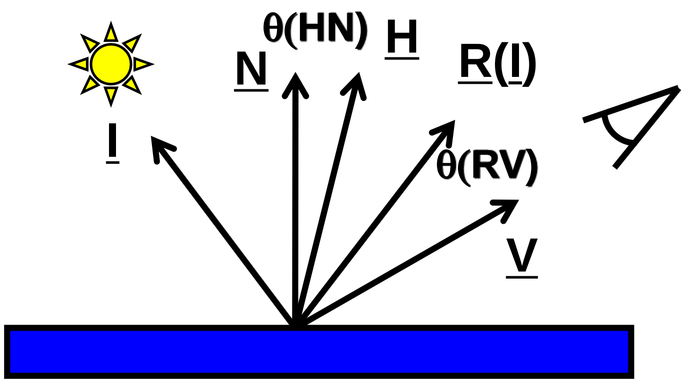
- **Same idea: Cosine power lobe**

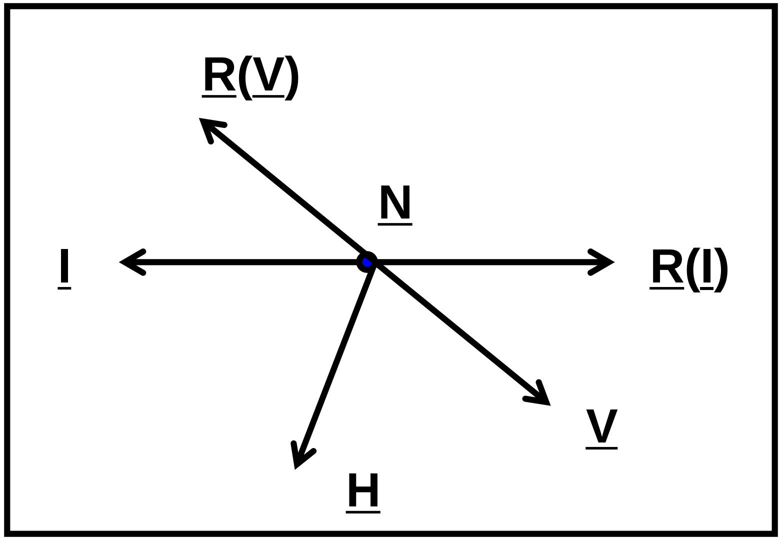
$$f_r(\omega_i, x, \omega_o) = k_s(H \cdot N)^{k_e} / I \cdot N$$

  - $L_{r,s} = L_i\, k_s \cos^{ke} \theta_{HN}$



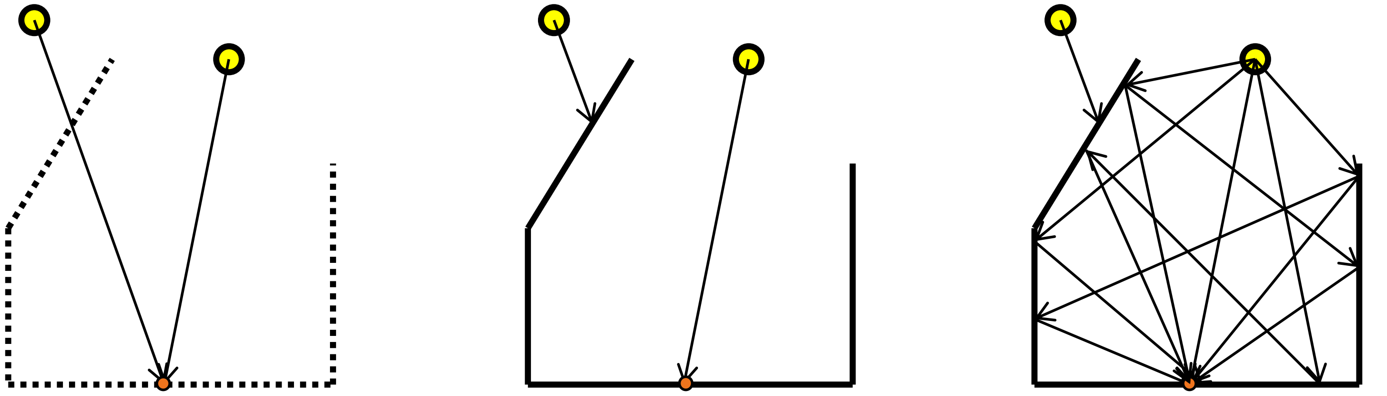
- **Dot product & power**
  - $\theta_{RV} \rightarrow \theta_{HN}$
  - Special case: Light source, viewer far away
    - *I*, *R* constant: *H* constant
    - θ_{HN} less expensive to compute





Different Types of Illumination

- **Three types of illumination**



Local
(without shadows)

Direct
(with shadows)

Global
(with all interreflecions)

- **Ambient Illumination**
  - Global illumination is costly to compute
  - Indirect illumination (through interreflections) is typically smooth
  - ➔ Approximate via a constant term $L_{i,a}$ (incoming ambient illum)
  - Has no incoming direction, provide ambient reflection term k_a

$$L_o(x, \omega_o) = k_a L_{i,a}$$

Full Phong Illumination Model

- **Phong illumination model for *multiple* point light sources**

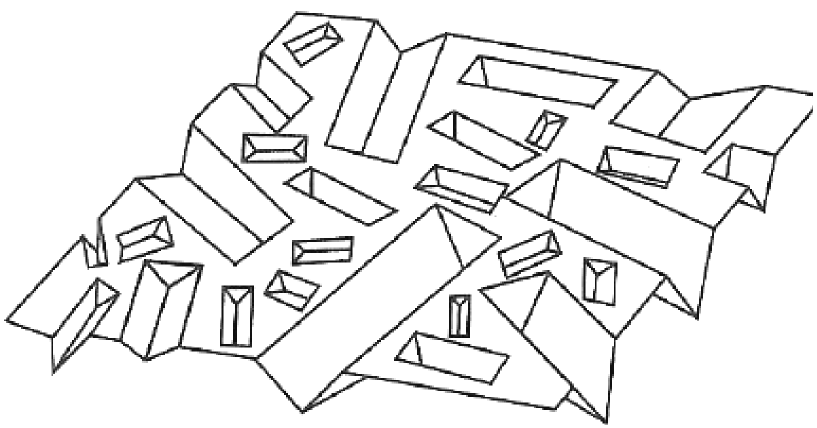
$$L_r = k_a L_{i,a} + k_d \sum_l L_l (I_l \cdot N) + k_s \sum_l L_l (R(I_l) \cdot V)^{k_e} \ (Phong)$$

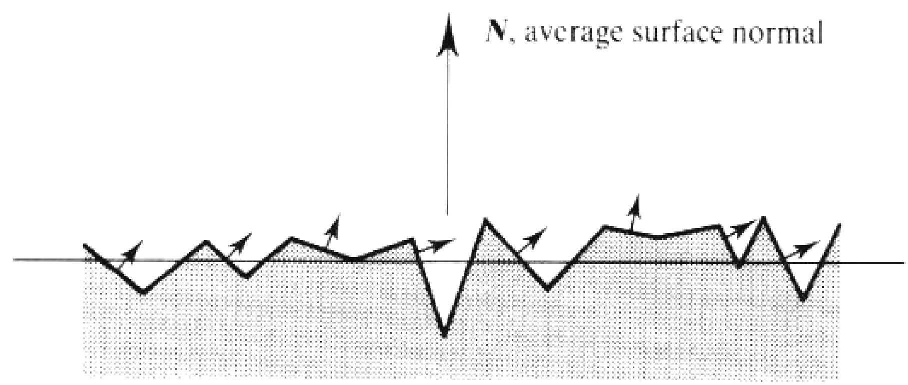
$$L_r = k_a L_{i,a} + k_d \sum_l L_l (I_l \cdot N) + k_s \sum_l L_l (H_l \cdot N)^{k_e} \ (Blinn)$$

  - Diffuse reflection (contribution only depends on incoming cosine)
  - Ambient and Glossy reflection (Phong or Blinn-Phong)
- **Typically: Color of specular reflection k_s is white**
  - Often separate specular and diffuse color (common extension, OGL)
- **Empirical model!**
  - Contradicts physics
  - Purely local illumination
    - Only direct light from the light sources, constant ambient term
- **Optimization: Lights & viewer assumed to be far away**

Microfacet BRDF Model

- **Physically-Inspired Models**
  - Isotropic microfacet collection
  - Microfacets assumed as perfectly smooth reflectors
- **BRDF**
  - Distribution of microfacets
    - Often probabilistic distribution of orientation or V-groove assumption
  - Planar reflection properties
  - Self-masking, shadowing



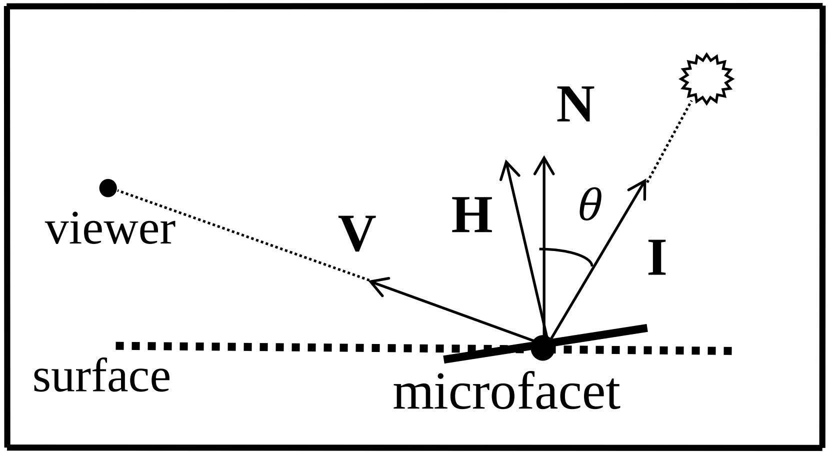


Ward Reflection Model

- **BRDF**

$$f_r = \frac{\rho_d}{\pi} + \frac{\rho_s}{\sqrt{(I \cdot N)(V \cdot N)}} \frac{\exp\left(-\frac{tan^2 \angle H, N}{\sigma^2}\right)}{4\pi\sigma^2}$$

  - σ standard deviation (RMS) of surface slope
  - Simple expansion to anisotropic model (σ_x, σ_y)
  - Empirical, not physics-based
- **Inspired by notion of reflecting microfacets**
  - Convincing results
  - Good match to measured data



Cook-Torrance Reflection Model

- **Cook-Torrance reflectance model**
  - Is based on the *microfacet* model
  - BRDF is defined as the sum of a diffuse and a specular component:

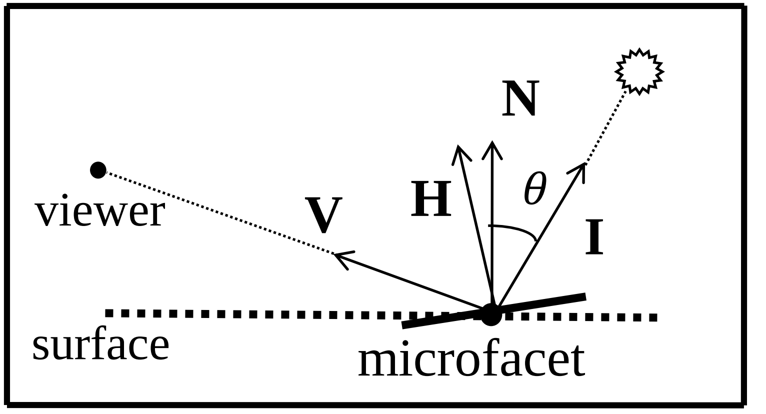
$$f_r = \kappa_d \rho_d + \kappa_s \rho_s; \qquad \rho_d + \rho_s \leq 1$$

where ρ_s and ρ_d are the specular and diffuse coefficients.

  - Derivation of the specular component κ_s is based on a physically derived theoretical reflectance model

Cook-Torrance Specular Term

$$\kappa_s = \frac{F_\lambda D G}{\pi (N \cdot V)(N \cdot I)}$$



- **D : Distribution function of microfacet orientations**
- **G : Geometrical attenuation factor**
  - represents self-masking and shadowing effects of microfacets
- **F_λ : Fresnel term**
  - computed by Fresnel equation
  - Fraction of specularly reflected light for each planar microfacet
- **N·V : Proportional to visible surface area**
- **N·I : Proportional to illuminated surface area**

Electric Conductors (e.g. Metals)

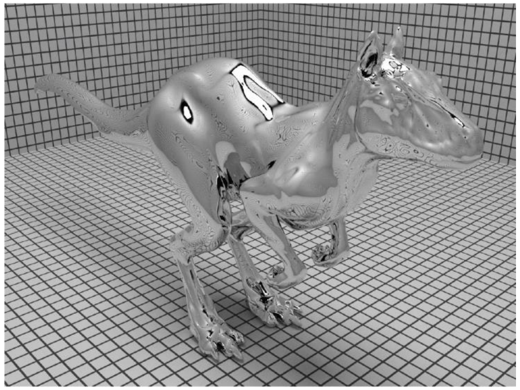
- **Assume ideally smooth surface**
- **Perfect specular reflection of light, rest is absorbed**
- **Reflectance is defined by Fresnel formula based on:**
  - Index of refraction η
  - Absorption coefficient κ
  - Both wavelength dependent

Object	η	k
Gold	0.370	2.820
Silver	0.177	3.638
Copper	0.617	2.63
Steel	2.485	3.433

- **Given for parallel and perpendicular polarized light**

$$r_{\parallel}^2 = \frac{(\eta^2 + k^2)\cos\theta_i^2 - 2\eta\cos\theta_i + 1}{(\eta^2 + k^2)\cos\theta_i^2 + 2\eta\cos\theta_i + 1}$$

$$r_{\perp}^2 = \frac{(\eta^2 + k^2) - 2\eta\cos\theta_i + \cos\theta_i^2}{(\eta^2 + k^2) + 2\eta\cos\theta_i + \cos\theta_i^2}.$$

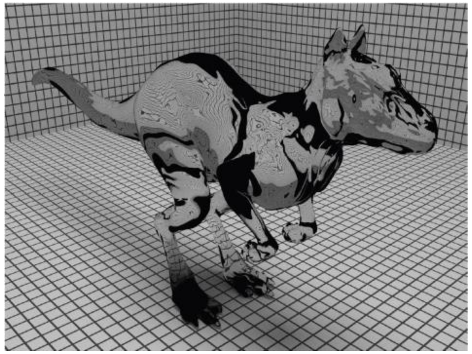


  - θ_i, θ_t: Angle between ray & plane, incident & transmitted
- **For unpolarized light:**

$$F_r = \frac{1}{2}(r_{\parallel}^2 + r_{\perp}^2)$$

Dielectrics (e.g. Glass)

- **Assume ideally smooth surface**
- **Non-reflected light is perfectly transmitted: *1 - F_r***
  - They do not conduct electricity
- **Fresnel formula depends on:**
  - Refr. index: speed of light in vacuum vs. medium
  - Refractive index in incident medium $\eta_i = c_0 / c_i$
  - Refractive index in transmitted medium $\eta_t = c_0 / c_t$



- **Given for parallel and perpendicular polarized light**

$$r_{\|} = \frac{\eta_t \cos\theta_i - \eta_i \cos\theta_t}{\eta_t \cos\theta_i + \eta_i \cos\theta_t}$$

$$r_{\perp} = \frac{\eta_i \cos\theta_i - \eta_t \cos\theta_t}{\eta_i \cos\theta_i + \eta_t \cos\theta_t},$$

Medium	Index of refraction η
Vacuum	1.0
Air at sea level	1.00029
Ice	1.31
Water (20° C)	1.333
Fused quartz	1.46
Glass	1.5–1.6
Sapphire	1.77
Diamond	2.42

- **For unpolarized light:**

$$F_r = \frac{1}{2}(r_{\|}^2 + r_{\perp}^2)$$

Microfacet Distribution Functions

- **Isotropic Distributions** $D(\omega) \Rightarrow D(\alpha) \quad \alpha = \angle N, H$
  - α : angle to average normal of surface
  - m : average slope of the microfacets

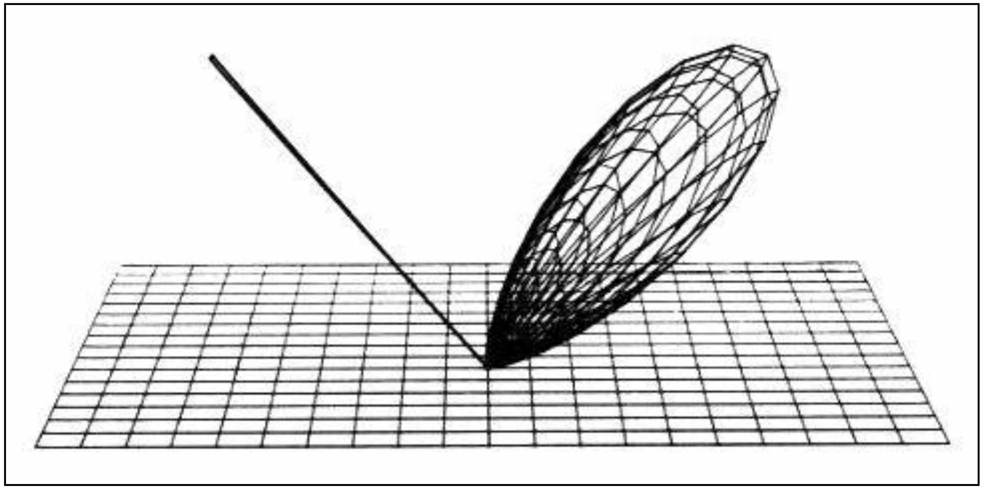
- **Blinn:** $D(\alpha) = \cos^{-\frac{\ln 2}{\ln \cos m}}(\alpha)$

- **Torrance-Sparrow** $D(\alpha) = e^{-\ln 2\left(\frac{\alpha}{m}\right)^2}$
  - Gaussian

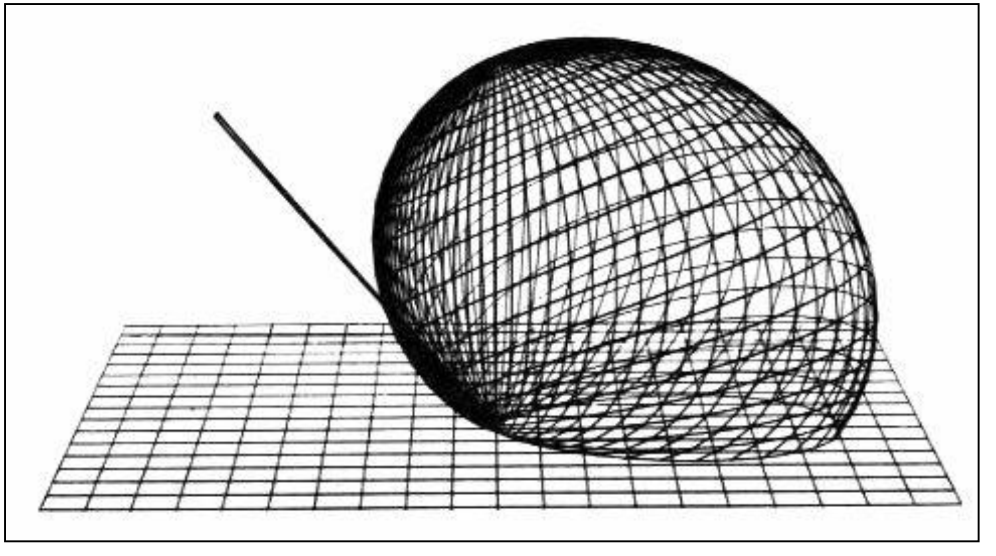
- **Beckmann** $D(\alpha) = \frac{1}{\pi m^2 \cos^4 \alpha} e^{-(\frac{\tan \alpha}{m})^2}$
  - Used by Cook-Torrance

Beckman Microfacet Distribution

m=0.2



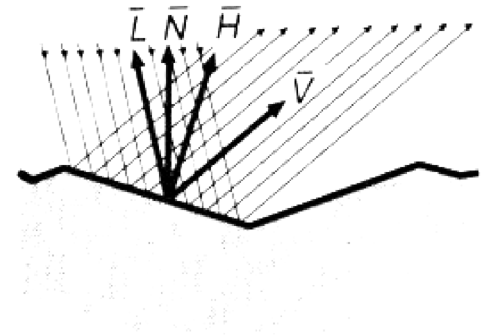
m=0.6



Geometric Attenuation Factor

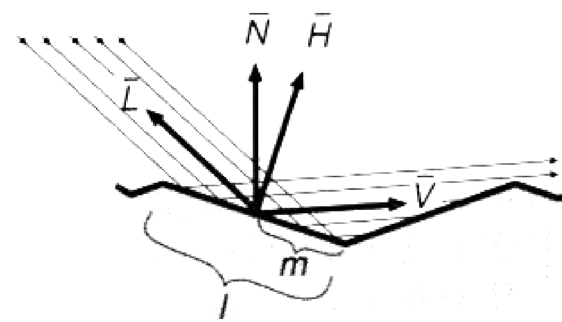
- **V-shaped grooves**
- **Fully illuminated and visible**

$$G = 1$$



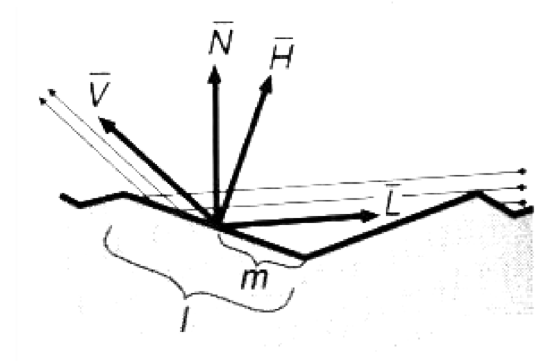
- **Partial masking of reflected light**

$$G = \frac{2(\underline{N} \cdot \underline{H})(\underline{N} \cdot \underline{V})}{(\underline{V} \cdot \underline{H})}$$



- **Partial shadowing of incident light**

$$G = \frac{2(\underline{N} \cdot \underline{H})(\underline{N} \cdot \underline{I})}{(\underline{V} \cdot \underline{H})}$$

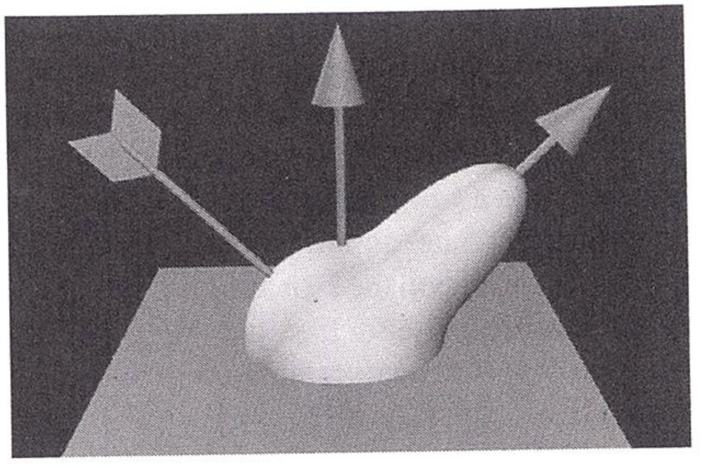


- **Final**

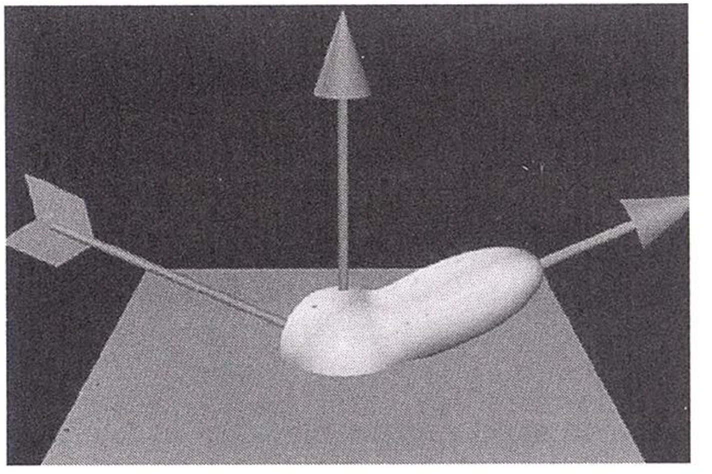
$$G = min\left\{1, \frac{2(\underline{N} \cdot \underline{H})(\underline{N} \cdot \underline{V})}{(\underline{V} \cdot \underline{H})}, \frac{2(\underline{N} \cdot \underline{H})(\underline{N} \cdot \underline{I})}{(\underline{V} \cdot \underline{H})}\right\}$$

Comparison Phong vs. Torrance

Phong:

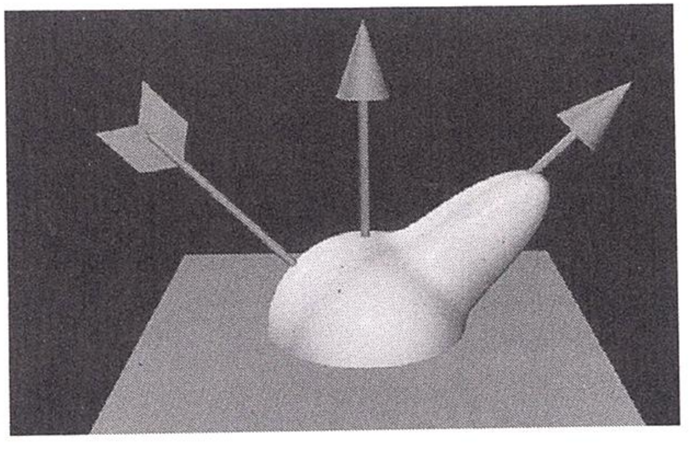


(a)

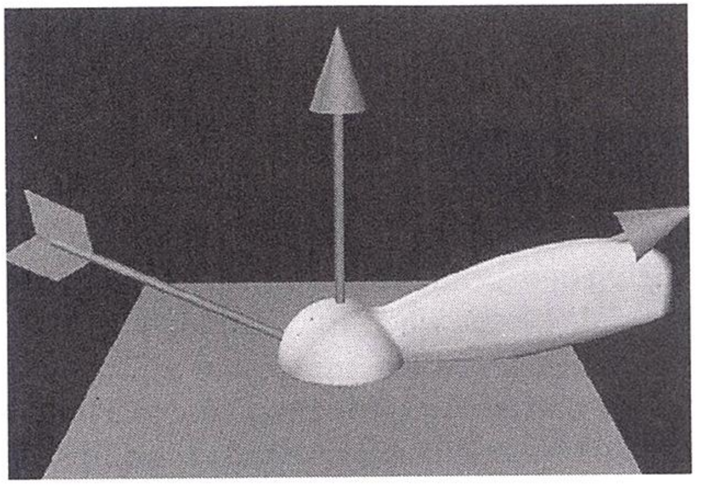


(b)

Torrance:



(c)



(d)

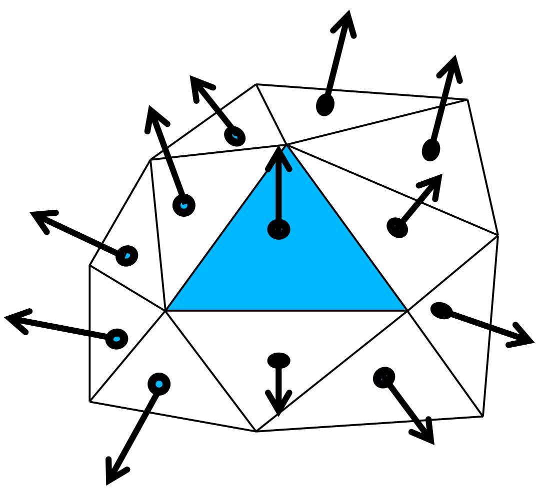
SHADING

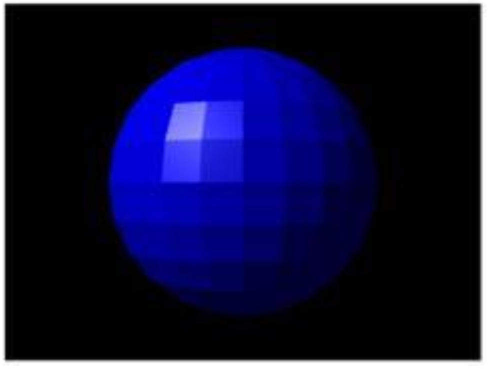
What is necessary?

- **View point position**
- **Light source description**
- **Reflectance model**
- **Surface normal / local coordinate frame**

Surface Normals – Triangle Mesh

- **Most simple: Constant Shading**
  - Fixed color per polygon/triangle
- **Shading Model: Flat Shading**
  - Single per-surface normal
  - Single color per polygon
  - Evaluated at one of the vertices (➔ OGL) or at center





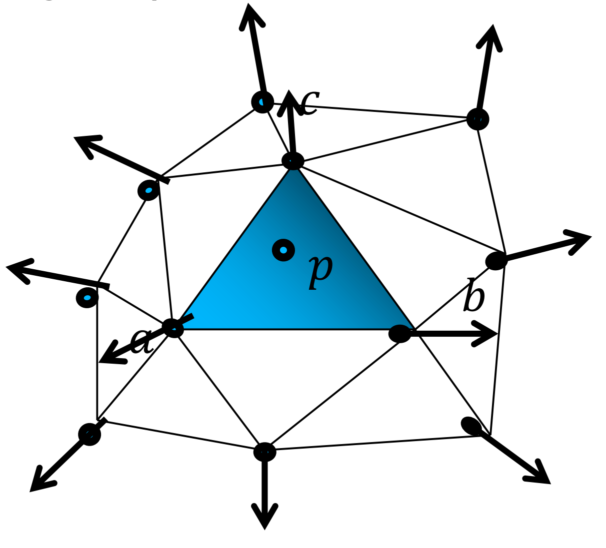
[wikipedia]

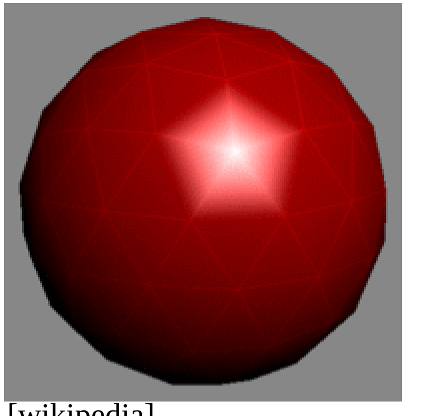
Surface Normals – Triangle Mesh

- **Shading Model: Gouraud shading**
  - Per-vertex normal
    - Can be computed from adjacent triangle normals (e.g. by averaging)
  - Linear interpolation of the shaded colors
    - Computed at all vertices and interpolated
  - Often results in shading artifacts along edges
    - **Mach Banding** (i.e. discontinuous 1st derivative)
    - Flickering of highlights (when one of the normal generates strong reflection)

$$L_x \sim f_r(\omega_o, n_x, \omega_i) L_i \cos\theta_i$$
$$L_p = \lambda_1 L_a + \lambda_2 L_b + \lambda_3 L_c$$

  - **Barycentric interpolation** within triangle





[wikipedia]

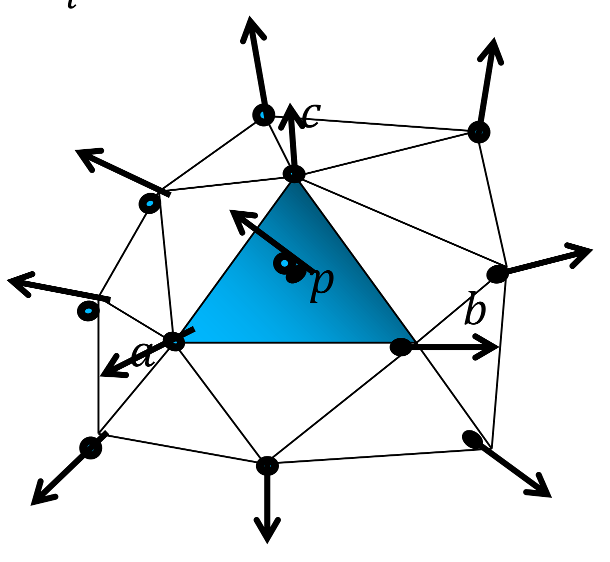
Surface Normals – Triangle Mesh

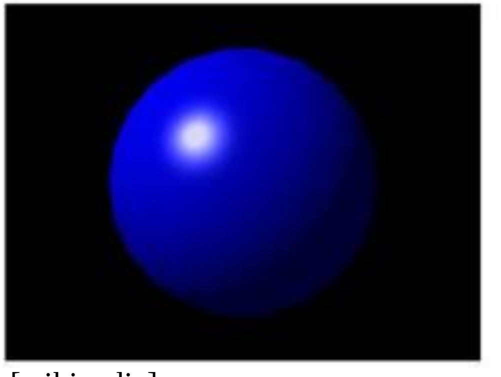
- **Shading Model: Phong shading**
  - Linear interpolation of the surface normal
  - Shading is evaluated at every point separately
  - Smoother but still off due to hit point offset from apparent surface

$$n_p = \frac{\lambda_1 n_1 + \lambda_2 n_2 + \lambda_3 n_3}{\| \lambda_1 n_1 + \lambda_2 n_2 + \lambda_3 n_3 \|}$$

$$L_p \sim f_r(\omega_o, n_p, \omega_i) L_i \cos\theta_i$$

  - Barycentric interpolation within triangle

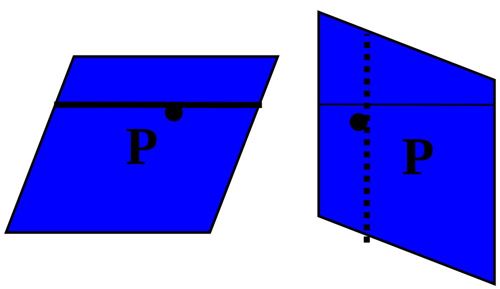




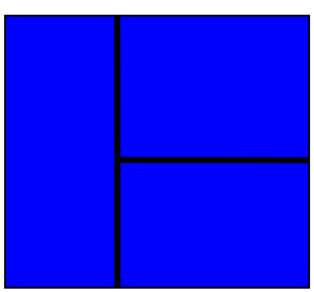
[wikipedia]

Problems in Interpolated Shading

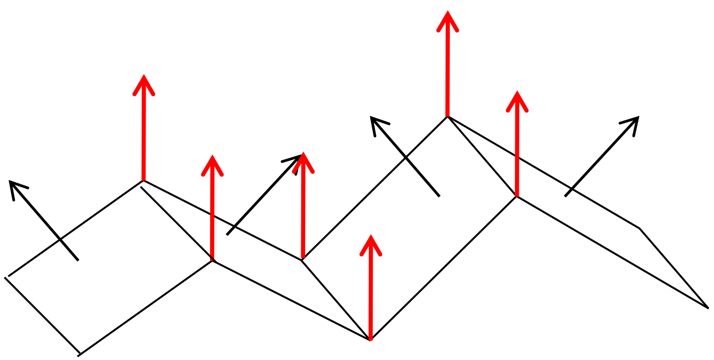
- **Issues**
  - Polygonal silhouette may not match the smooth shading
  - Perspective distortion
    - Interpolation in 2-D screen space rather than world space (==> later)
  - Orientation dependence
    - Only for polygons
    - Not with triangles (here linear interpolation is rotation-invariant)
  - Shading discontinuities at shared vertices (T-edges)
  - Unrepresentative normal vectors



Shading at **P** is interpolated along different scan-lines when polygon rotates.



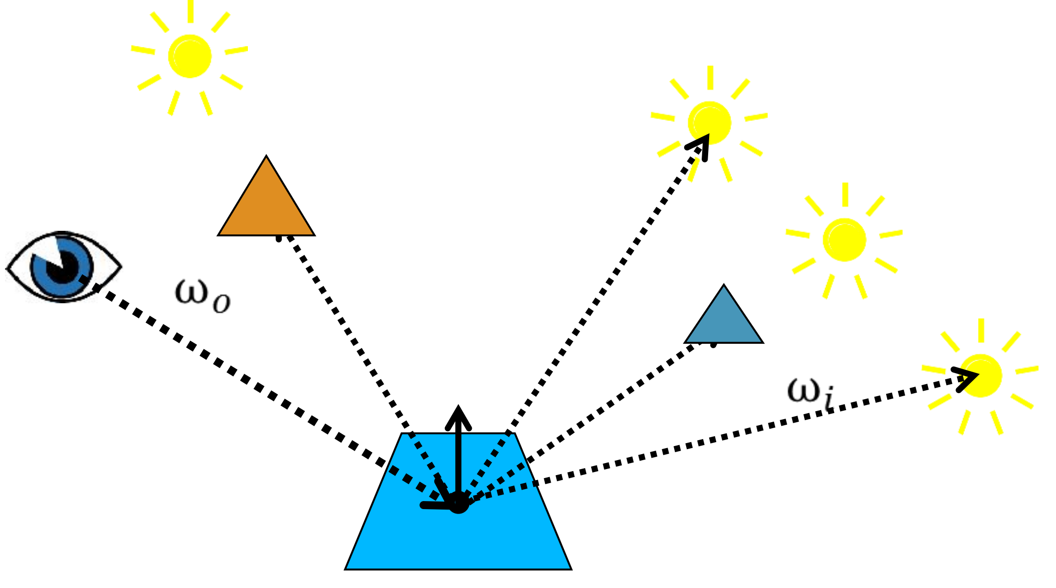
T-edges



Vertex normals are all parallel

Occlusions

- **The point on the surface might be in shadow**
  - Rasterization (OpenGL):
    - Not easily done
    - Can use shadow map or shadow volumes (➔ later)
  - Ray tracing
    - Simply trace ray to light source and test for occlusion



Area Light sources

- **Typically approximated by sampling**
  - Replacing it with some point light sources
    - Often randomly sampled
    - Cosine distribution of power over angular directions